Government of Karnataka

MATHEMATICS

9

Ninth Standard

Part-II

राष्ट्रीय शैक्षिक अनुसंधान और प्रशिक्षण परिषद्
NATIONAL COUNCIL OF EDUCATIONAL RESEARCH AND TRAINING

Karnataka Textbook Society (R.)
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FOREWORD

The National Curriculum Framework (NCF), 2005, recommends that children's life at school must be linked to their life outside the school. This principle marks a departure from the legacy of bookish learning which continues to shape our system and causes a gap between the school, home and community. The syllabi and textbooks developed on the basis of NCF signify an attempt to implement this basic idea. They also attempt to discourage rote learning and the maintenance of sharp boundaries between different subject areas. We hope these measures will take us significantly further in the direction of a child-centred system of education outlined in the national Policy on Education (1986).

The success of this effort depends on the steps that school principals and teachers will take to encourage children to reflect on their own learning and to pursue imaginative activities and questions. We must recognize that, given space, time and freedom, children generate new knowledge by engaging with the information passed on to them by adults. Treating the prescribed textbook as the sole basis of examination is one of the key reasons why other resources and sites of learning are ignored. Inculcating creativity and initiative is possible if we perceive and treat children as participants in learning, not as receivers of a fixed body of knowledge.

This aims imply considerable change is school routines and mode of functioning. Flexibility in the daily time-table is as necessary as rigour in implementing the annual calendar so that the required number of teaching days are actually devoted to teaching. The methods used for teaching and evaluation will also determine how effective this textbook proves for making children's life at school a happy experience, rather then a source of stress or boredom. Syllabus designers have tried to address the problem of curricular burden by restructuring and reorienting knowledge at different stages with greater consideration for child psychology and the time available for teaching. The textbook attempts to enhance this endeavour by giving higher priority and space to opportunities for contemplation and wondering, discussion in small groups, and activities requiring hands-on experience.

The National Council of Educational Research and Training (NCERT) appreciates the hard work done by the textbook development committee responsible for this book. We wish to thank the Chairperson of the advisory group in science and mathematics, Professor J.V. Narlikar and the Chief Advisor for this book, Professor P. Sinclair of IGNOU, New Delhi for guiding the work of this committee. Several teachers contributed to the development of this textbook; we are grateful to their principals for making this possible. We are indebted to the institutions and organizations which have generously permitted us to draw upon their resources, material and personnel. We are especially grateful to the members of the National Monitoring Committee, appointed by the Department of Secondary and Higher Education, Ministry of Human Resource Development under the Chairpersonship of Professor Mrinal Miri and Professor G.P. Deshpande, for their valuable time and contribution. As an organisation committed to systemic reform and continuous improvement in the quality of its products, NCERT welcomes comments and suggestions which will enable us to undertake further revision and refinement.

Director
National Council of Educational
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New Delhi
20 December 2005

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CHAPTER 8

HERON'S FORMULA

8.1 Introduction

You have studied in earlier classes about figures of different shapes such as squares, rectangles, triangles and quadrilaterals. You have also calculated perimeters and the areas of some of these figures like rectangle, square etc. For instance, you can find the area and the perimeter of the floor of your classroom.

Let us take a walk around the floor along its sides once; the distance we walk is its perimeter. The size of the floor of the room is its area.

So, if your classroom is rectangular with length 10 m and width 8 m, its perimeter would be 2(10 m + 8 m) = 36 m and its area would be 10 m × 8 m, i.e., 80 m^2.

Unit of measurement for length or breadth is taken as metre (m) or centimetre (cm) etc.

Unit of measurement for area of any plane figure is taken as square metre (m^2) or square centimetre (cm^2) etc.

Suppose that you are sitting in a triangular garden. How would you find its area? From Chapter 9 and from your earlier classes, you know that:

$$\textbf{Area of a triangle} = \frac{1}{2} \times \textbf{base} \times \textbf{height} \qquad \text{(I)}$$

We see that when the triangle is **right angled**, we can directly apply the formula by using two sides containing the right angle as base and height. For example, suppose that the sides of a right triangle ABC are 5 cm, 12 cm and 13 cm; we take base as 12 cm and height as 5 cm (see Fig. 8.1). Then the

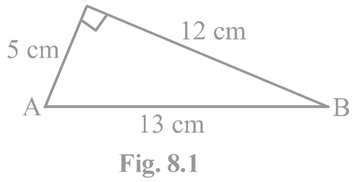


Fig. 8.1

area of Δ ABC is given by

$$\frac{1}{2} \times \text{base} \times \text{height} = \frac{1}{2} \times 12 \times 5 \text{ cm}^2\text{, i.e., } 30 \text{ cm}^2$$

Note that we could also take 5 cm as the base and 12 cm as height.

Now suppose we want to find the area of an **equilateral triangle** PQR with side 10cm (see Fig. 8.2). To find its area we need its height. Can you find the height of this triangle?

Let us recall how we find its height when we know its sides. This is possible in an equilateral triangle. Take the mid-point of QR as M and join it to P. We know that PMQ is a right triangle. Therefore, by using Pythagoras Theorem, we can find the length PM as shown below:

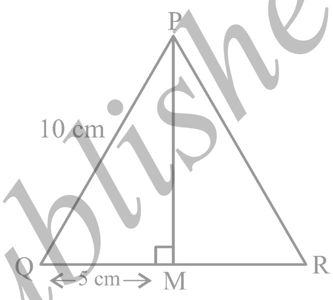


Fig. 8.2

$$PQ^2 = PM^2 + QM^2$$

i.e., $(10)^2 = PM^2 + (5)^2$, since QM = MR.

Therefore, we have $PM^2 = 75$

i.e., $PM = \sqrt{75}$ cm $= 5\sqrt{3}$ cm.

Then area of Δ PQR $= \frac{1}{2} \times \text{base} \times \text{height} = \frac{1}{2} \times 10 \times 5\sqrt{3} \text{ cm}^2 = 25\sqrt{3}$ cm².

Let us see now whether we can calculate the area of an **isosceles triangle** also with the help of this formula. For example, we take a triangle XYZ with two equal sides XY and XZ as 5 cm each and unequal side YZ as 8 cm (see Fig. 8.3).

In this case also, we want to know the height of the triangle. So, from X we draw a perpendicular XP to side YZ. You can see that this perpendicular XP divides the base YZ of the triangle in two equal parts.

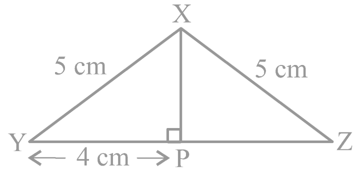


Fig. 8.3

Therefore, $YP = PZ = \frac{1}{2} YZ = 4$ cm

Then, by using Pythagoras theorem, we get

$$XP^2 = XY^2 - YP^2$$
$$= 5^2 - 4^2 = 25 - 16 = 9$$

So, XP = 3 cm

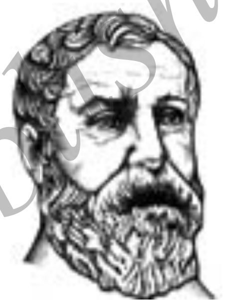
Now, area of Δ XYZ $= \frac{1}{2} \times \text{base YZ} \times \text{height XP}$

$$= \frac{1}{2} \times 8 \times 3 \text{ cm}^2 = 12 \text{ cm}^2.$$

Now suppose that we know the lengths of the sides of a scalene triangle and not the height. Can you still find its area? For instance, you have a triangular park whose sides are 40 m, 32 m, and 24 m. How will you calculate its area? Definitely if you want to apply the formula, you will have to calculate its height. But we do not have a clue to calculate the height. Try doing so. If you are not able to get it, then go to the next section.

8.2 Area of a Triangle — by Heron's Formula

Heron was born in about 10AD possibly in Alexandria in Egypt. He worked in applied mathematics. His works on mathematical and physical subjects are so numerous and varied that he is considered to be an encyclopedic writer in these fields. His geometrical works deal largely with problems on mensuration written in three books. Book I deals with the area of squares, rectangles, triangles, trapezoids (trapezia), various other specialised quadrilaterals, the regular polygons, circles, surfaces of cylinders, cones, spheres etc. In this book, Heron has derived the famous formula for the area of a triangle in terms of its three sides.



Heron (10 A.D. – 75 A.D.)

Fig. 8.4

The formula given by Heron about the area of a triangle, is also known as *Hero's formula*. It is stated as:

$$\textbf{Area of a triangle} = \sqrt{s(s-a)(s-b)(s-c)} \qquad \text{(II)}$$

where a, b and c are the sides of the triangle, and *s = semi-perimeter, i.e., half the perimeter of the triangle* = $\frac{a+b+c}{2}$,

This formula is helpful where it is not possible to find the height of the triangle easily. Let us apply it to calculate the area of the triangular park ABC, mentioned above (see Fig. 8.5).

Let us take a = 40 m, b = 24 m, c = 32 m,

so that we have $s = \frac{40+24+32}{2}$ m = 48 m.

$s - a = (48 - 40)$ m = 8 m,

$s - b = (48 - 24)$ m = 24 m,

$s - c = (48 - 32)$ m = 16 m.

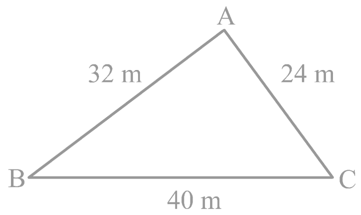


Fig. 8.5

Therefore, area of the park ABC

$$= \sqrt{s(s-a)(s-b)(s-c)}$$

$$= \sqrt{48 \times 8 \times 24 \times 16}\ \text{m}^2 = 384\ \text{m}^2$$

We see that $32^2 + 24^2 = 1024 + 576 = 1600 = 40^2$. Therefore, the sides of the park make a right triangle. The largest side, i.e., BC which is 40 m will be the hypotenuse and the angle between the sides AB and AC will be 90°.

By using Formula I, we can check that the area of the park is $\frac{1}{2} \times 32 \times 24\ \text{m}^2$ = 384 m^2.

We find that the area we have got is the same as we found by using Heron's formula.

Now using Heron's formula, you verify this fact by finding the areas of other triangles discussed earlier *viz*.,

(i) equilateral triangle with side 10 cm.

(ii) isosceles triangle with unequal side as 8 cm and each equal side as 5 cm.

You will see that

For (i), we have $s = \frac{10 + 10 + 10}{2}$ cm = 15 cm.

Area of triangle $= \sqrt{15(15-10)(15-10)(15-10)}\ \text{cm}^2$

$$= \sqrt{15 \times 5 \times 5 \times 5}\ \text{cm}^2 = 25\sqrt{3}\ \text{cm}^2$$

For (ii), we have $s = \frac{8 + 5 + 5}{2}$ cm = 9 cm.

Area of triangle = $\sqrt{9(9-8)(9-5)(9-5)}\ \text{cm}^2 = \sqrt{9 \times 1 \times 4 \times 4}\ \text{cm}^2 = 12\ \text{cm}^2$.

Let us now solve some more examples:

Example 1 : Find the area of a triangle, two sides of which are 8 cm and 11 cm and the perimeter is 32 cm (see Fig. 8.6).

Solution : Here we have perimeter of the triangle = 32 cm, a = 8 cm and b = 11 cm.

Third side c = 32 cm – (8 + 11) cm = 13 cm

So, $2s$ = 32, i.e., s = 16 cm,

$s - a$ = (16 – 8) cm = 8 cm,

$s - b$ = (16 – 11) cm = 5 cm,

$s - c$ = (16 – 13) cm = 3 cm.

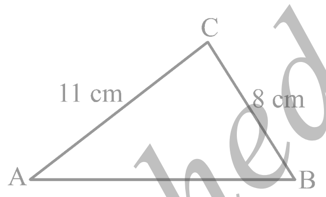


Fig. 8.6

Therefore, area of the triangle = $\sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{16 \times 8 \times 5 \times 3}\ \text{cm}^2 = 8\sqrt{30}\ \text{cm}^2$

Example 2 : A triangular park ABC has sides 120m, 80m and 50m (see Fig. 8.7). A gardener *Dhania* has to put a fence all around it and also plant grass inside. How much area does she need to plant? Find the cost of fencing it with barbed wire at the rate of Rs 20 per metre leaving a space 3m wide for a gate on one side.

Solution : For finding area of the park, we have

$2s$ = 50 m + 80 m + 120 m = 250 m.

i.e., s = 125 m

Now, $s - a$ = (125 – 120) m = 5 m,

$s - b$ = (125 – 80) m = 45 m,

$s - c$ = (125 – 50) m = 75 m.

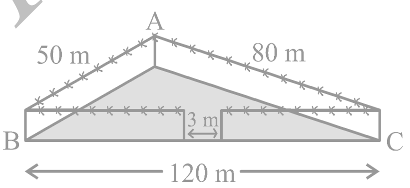


Fig. 8.7

Therefore, area of the park = $\sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{125 \times 5 \times 45 \times 75}\ \text{m}^2$

$= 375\sqrt{15}\ \text{m}^2$

Also, perimeter of the park = AB + BC + CA = 250 m

Therefore, length of the wire needed for fencing = 250 m – 3 m (to be left for gate)

= 247 m

And so the cost of fencing = Rs 20 × 247 = Rs 4940

Example 3 : The sides of a triangular plot are in the ratio of 3 : 5 : 7 and its perimeter is 300 m. Find its area.

Solution : Suppose that the sides, in metres, are $3x$, $5x$ and $7x$ (see Fig. 8.8).

Then, we know that $3x + 5x + 7x = 300$ (perimeter of the triangle)

Therefore, $15x = 300$, which gives $x = 20$.

So the sides of the triangle are 3×20 m, 5×20 m and 7×20 m

i.e., 60 m, 100 m and 140 m.

Can you now find the area [Using Heron's formula]?

Fig. 8.8

We have $s = \frac{60 + 100 + 140}{2}$ m = 150 m,

and area will be $\sqrt{150(150 - 60)(150 - 100)(150 - 140)}$ m^2

$= \sqrt{150 \times 90 \times 50 \times 10}$ m^2

$= 1500\sqrt{3}$ m^2

EXERCISE 8.1

1. A traffic signal board, indicating 'SCHOOL AHEAD', is an equilateral triangle with side 'a'. Find the area of the signal board, using Heron's formula. If its perimeter is 180 cm, what will be the area of the signal board?
2. The triangular side walls of a flyover have been used for advertisements. The sides of the walls are 122 m, 22 m and 120 m (see Fig. 8.9). The advertisements yield an earning of ₹ 5000 per m^2 per year. A company hired one of its walls for 3 months. How much rent did it pay?

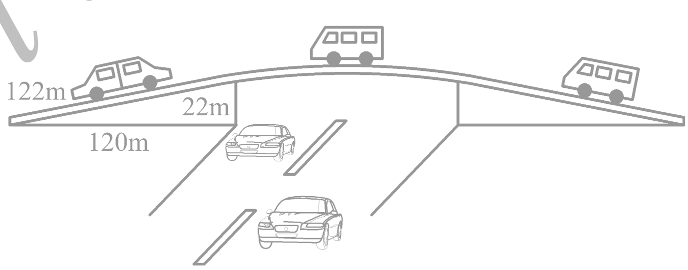


Fig. 8.9

3. There is a slide in a park. One of its side walls has been painted in some colour with a message "KEEP THE PARK GREEN AND CLEAN" (see Fig. 8.10). If the sides of the wall are 15 m, 11 m and 6 m, find the area painted in colour.

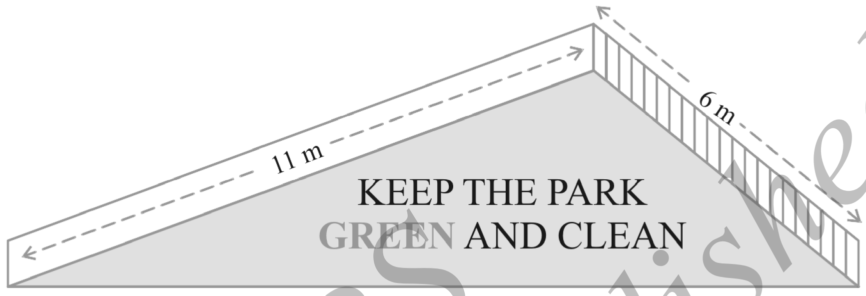


Fig. 8.10

4. Find the area of a triangle two sides of which are 18cm and 10cm and the perimeter is 42cm.
5. Sides of a triangle are in the ratio of 12 : 17 : 25 and its perimeter is 540cm. Find its area.
6. An isosceles triangle has perimeter 30 cm and each of the equal sides is 12 cm. Find the area of the triangle.

8.3 Application of Heron's Formula in Finding Areas of Quadrilaterals

Suppose that a farmer has a land to be cultivated and she employs some labourers for this purpose on the terms of wages calculated by area cultivated per square metre. How will she do this? Many a time, the fields are in the shape of quadrilaterals. We need to divide the quadrilateral in triangular parts and then use the formula for area of the triangle. Let us look at this problem:

Example 4 : Kamla has a triangular field with sides 240 m, 200 m, 360 m, where she grew wheat. In another triangular field with sides 240 m, 320 m, 400 m adjacent to the previous field, she wanted to grow potatoes and onions (see Fig. 8.11). She divided the field in two parts by joining the mid-point of the longest side to the opposite vertex and grew patatoes in one part and onions in the other part. How much area (in hectares) has been used for wheat, potatoes and onions? (1 hectare = 10000 m^2)

Solution : Let ABC be the field where wheat is grown. Also let ACD be the field which has been divided in two parts by joining C to the mid-point E of AD. For the area of triangle ABC, we have

$$a = 200 \text{ m},\ b = 240 \text{ m},\ c = 360 \text{ m}$$

Therefore, $s = \dfrac{200 + 240 + 360}{2}$ m = 400 m.

So, area for growing wheat

$$= \sqrt{400(400-200)(400-240)(400-360)}\ \text{m}^2$$

$$= \sqrt{400 \times 200 \times 160 \times 40}\ \text{m}^2$$

$$= 16000\sqrt{2}\ \text{m}^2 = 1.6 \times \sqrt{2} \text{ hectares}$$

$$= 2.26 \text{ hectares (nearly)}$$

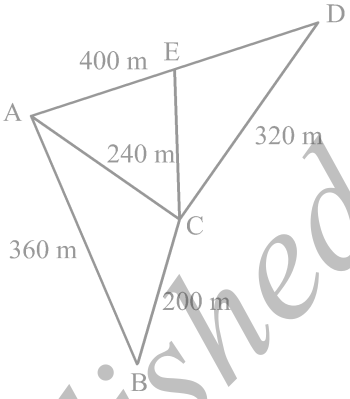


Fig. 8.11

Let us now calculate the area of triangle ACD.

Here, we have $s = \dfrac{240 + 320 + 400}{2}$ m = 480 m.

So, area of Δ ACD = $\sqrt{480(480-240)(480-320)(480-400)}\ \text{m}^2$

$$= \sqrt{480 \times 240 \times 160 \times 80}\ \text{m}^2 = 38400\ \text{m}^2 = 3.84 \text{ hectares}$$

We notice that the line segment joining the mid-point E of AD to C divides the triangle ACD in two parts equal in area. Can you give the reason for this? In fact, they have the bases AE and ED equal and, of course, they have the same height.

Therefore, area for growing potatoes = area for growing onions

$$= (3.84 \div 2) \text{ hectares} = 1.92 \text{ hectares.}$$

Example 5 : Students of a school staged a rally for cleanliness campaign. They walked through the lanes in two groups. One group walked through the lanes AB, BC and CA; while the other through AC, CD and DA (see Fig. 8.12). Then they cleaned the area enclosed within their lanes. If AB = 9 m, BC = 40 m, CD = 15 m, DA = 28 m and $\angle$ B = 90º, which group cleaned more area and by how much? Find the total area cleaned by the students (neglecting the width of the lanes).

Solution : Since AB = 9 m and BC = 40 m, $\angle$ B = 90°, we have:

$$\text{AC} = \sqrt{9^2 + 40^2}\ \text{m}$$

$$= \sqrt{81 + 1600}\ \text{m}$$

$$= \sqrt{1681}\ \text{m} = 41\text{m}$$

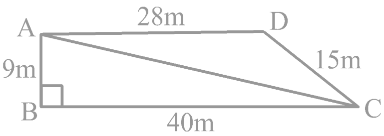


Fig. 8.12

Therefore, the first group has to clean the area of triangle ABC, which is right angled.

Area of Δ ABC = $\dfrac{1}{2}$ × base × height

$$= \frac{1}{2} \times 40 \times 9\ \text{m}^2 = 180\ \text{m}^2$$

The second group has to clean the area of triangle ACD, which is scalene having sides 41 m, 15 m and 28 m.

Here, $s = \dfrac{41 + 15 + 28}{2}$ m = 42 m

Therefore, area of Δ ACD = $\sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{42(42-41)(42-15)(42-28)}$ m^2

$= \sqrt{42 \times 1 \times 27 \times 14}$ m^2 = 126 m^2

So first group cleaned 180 m^2 which is (180 – 126) m^2, i.e., 54 m^2 more than the area cleaned by the second group.

Total area cleaned by all the students = (180 + 126) m^2 = 306 m^2.

Example 6 : Sanya has a piece of land which is in the shape of a rhombus (see Fig. 8.13). She wants her one daughter and one son to work on the land and produce different crops. She divided the land in two equal parts. If the perimeter of the land is 400 m and one of the diagonals is 160 m, how much area each of them will get for their crops?

Solution : Let ABCD be the field.

Perimeter = 400 m

So, each side = 400 m ÷ 4 = 100 m.

i.e. AB = AD = 100 m.

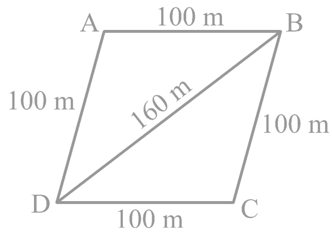


Fig. 8.13

Let diagonal BD = 160 m.

Then semi-perimeter s of Δ ABD is given by

$$s = \frac{100 + 100 + 160}{2} \text{ m} = 180 \text{ m}$$

Therefore, area of Δ ABD = $\sqrt{180(180-100)(180-100)(180-160)}$

$= \sqrt{180 \times 80 \times 80 \times 20}$ m^2 = 4800 m^2

Therefore, each of them will get an area of 4800 m^2.

Alternative method : Draw CE ⊥ BD (see Fig. 8.14).

As BD = 160 m, we have

DE = 160 m ÷ 2 = 80 m

And, $DE^2 + CE^2 = DC^2$, which gives

$$CE = \sqrt{DC^2 - DE^2}$$

or, $$CE = \sqrt{100^2 - 80^2}\ m = 60\ m$$

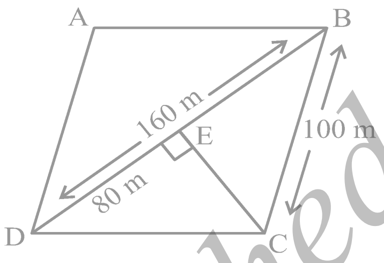


Fig. 8.14

Therefore, area of Δ BCD = $\frac{1}{2} \times 160 \times 60\ m^2 = 4800\ m^2$

EXERCISE 8.2

1. A park, in the shape of a quadrilateral ABCD, has ∠ C = 90°, AB = 9 m, BC = 12 m, CD = 5 m and AD = 8 m. How much area does it occupy?
2. Find the area of a quadrilateral ABCD in which AB = 3 cm, BC = 4 cm, CD = 4 cm, DA = 5 cm and AC = 5 cm.
3. Radha made a picture of an aeroplane with coloured paper as shown in Fig 8.15. Find the total area of the paper used.

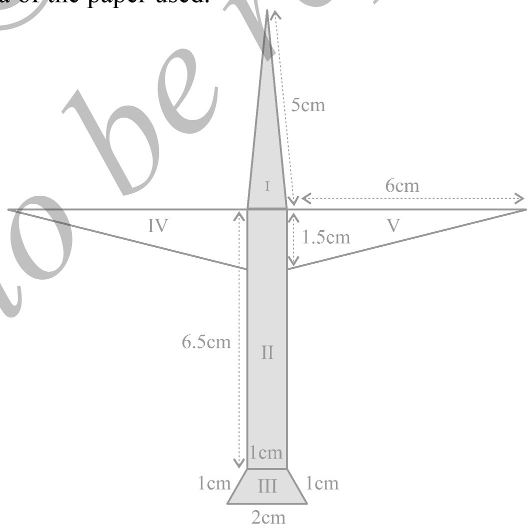


Fig. 8.15

4. A triangle and a parallelogram have the same base and the same area. If the sides of the triangle are 26 cm, 28 cm and 30 cm, and the parallelogram stands on the base 28 cm, find the height of the parallelogram.

5. A rhombus shaped field has green grass for 18 cows to graze. If each side of the rhombus is 30 m and its longer diagonal is 48 m, how much area of grass field will each cow be getting?

6. An umbrella is made by stitching 10 triangular pieces of cloth of two different colours (see Fig. 8.16), each piece measuring 20 cm, 50 cm and 50 cm. How much cloth of each colour is required for the umbrella?

7. A kite in the shape of a square with a diagonal 32 cm and an isosceles triangle of base 8 cm and sides 6 cm each is to be made of three different shades as shown in Fig. 8.17. How much paper of each shade has been used in it?

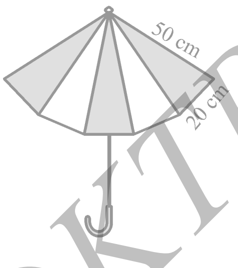


Fig. 8.16

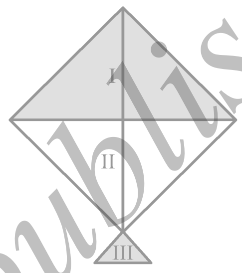


Fig. 8.17

8. A floral design on a floor is made up of 16 tiles which are triangular, the sides of the triangle being 9 cm, 28 cm and 35 cm (see Fig. 8.18). Find the cost of polishing the tiles at the rate of 50p per cm^2.

9. A field is in the shape of a trapezium whose parallel sides are 25 m and 10 m. The non-parallel sides are 14 m and 13 m. Find the area of the field.

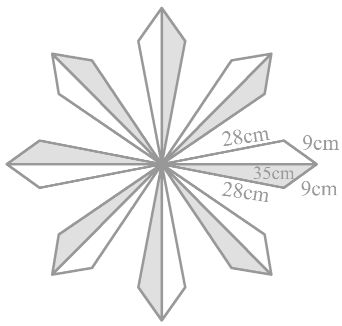


Fig. 8.18

8.4 Summary

In this chapter, you have studied the following points :

1. Area of a triangle with its sides as a, b and c is calculated by using Heron's formula, stated as

$$\text{Area of triangle} = \sqrt{s(s-a)(s-b)(s-c)}$$

where
$$s = \frac{a+b+c}{2}$$

2. Area of a quadrilateral whose sides and one diagonal are given, can be calculated by dividing the quadrilateral into two triangles and using the Heron's formula.

CHAPTER 9

COORDINATE GEOMETRY

> What's the good of Mercator's North Poles and Equators, Tropics, Zones and Meridian Lines?' So the Bellman would cry; and crew would reply ' They are merely conventional signs!'
>
> LEWIS CARROLL, *The Hunting of the Snark*

9.1 Introduction

You have already studied how to locate a point on a number line. You also know how to describe the position of a point on the line. There are many other situations, in which to find a point we are required to describe its position with reference to more than one line. For example, consider the following situations:

I. In Fig. 9.1, there is a main road running in the East-West direction and streets with numbering from West to East. Also, on each street, house numbers are marked. To look for a friend's house here, is it enough to know only one reference point? For instance, if we only know that she lives on Street 2, will we be able to find her house easily? Not as easily as when we know two pieces of information about it, namely, the number of the street on which it is situated, and the house number. If we want to reach the house which is situated in the 2nd street and has the number 5, first of all we would identify the 2nd street and then the house numbered 5 on it. In Fig. 9.1, H shows the location of the house. Similarly, P shows the location of the house corresponding to Street number 7 and House number 4.

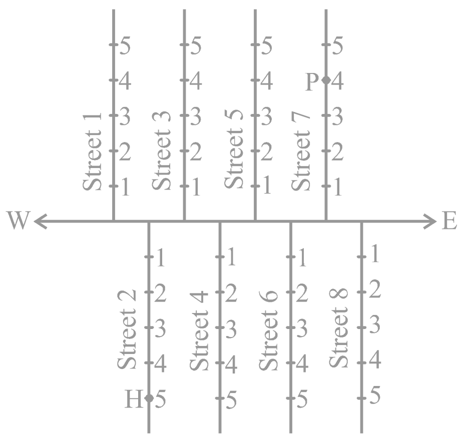


Fig. 9.1

II. Suppose you put a dot on a sheet of paper [Fig.9.2 (a)]. If we ask you to tell us the position of the dot on the paper, how will you do this? Perhaps you will try in some such manner: "The dot is in the upper half of the paper", or "It is near the left edge of the paper", or "It is very near the left hand upper corner of the sheet". Do any of these statements fix the position of the dot precisely? No! But, if you say " The dot is nearly 5 cm away from the left edge of the paper", it helps to give some idea but still does not fix the position of the dot. A little thought might enable you to say that the dot is also at a distance of 9 cm above the bottom line. We now know exactly where the dot is!

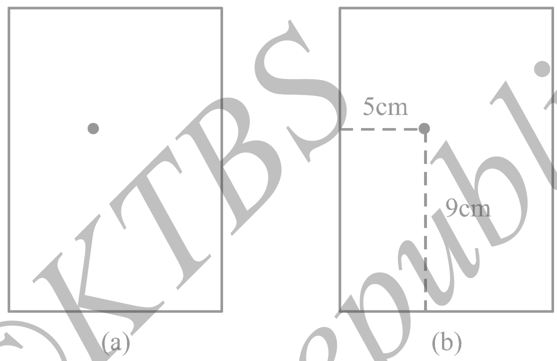


Fig. 9.2

For this purpose, we fixed the position of the dot by specifying its distances from two fixed lines, the left edge of the paper and the bottom line of the paper [Fig.9.2 (b)]. In other words, we need **two** independent informations for finding the position of the dot.

Now, perform the following classroom activity known as 'Seating Plan'.

Activity 1 (Seating Plan) : Draw a plan of the seating in your classroom, pushing all the desks together. Represent each desk by a square. In each square, write the name of the student occupying the desk, which the square represents. Position of each student in the classroom is described precisely by using two independent informations:

(i) the column in which she or he sits,

(ii) the row in which she or he sits.

If you are sitting on the desk lying in the 5^{th} column and 3^{rd} row (represented by the shaded square in Fig. 9.3), your position could be written as (5, 3), first writing the column number, and then the row number. Is this the same as (3, 5)? Write down the names and positions of other students in your class. For example, if Sonia is sitting in the 4^{th} column and 1^{st} row, write S(4,1). The teacher's desk is not part of your seating plan. We are treating the teacher just as an observer.

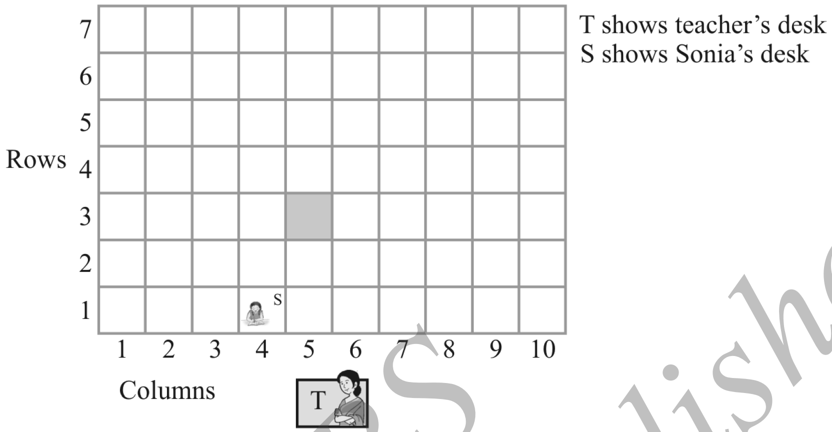
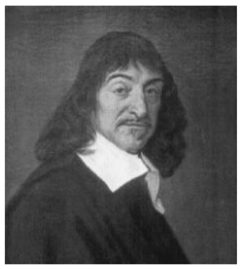


Fig. 9.3

In the discussion above, you observe that position of any object lying in a plane can be represented with the help of two perpendicular lines. In case of 'dot', we require distance of the dot from bottom line as well as from left edge of the paper. In case of seating plan, we require the number of the column and that of the row. This simple idea has far reaching consequences, and has given rise to a very important branch of Mathematics known as *Coordinate Geometry*. In this chapter, we aim to introduce some basic concepts of coordinate geometry. You will study more about these in your higher classes. This study was initially developed by the French philosopher and mathematician *René Déscartes*.

René Déscartes, the great French mathematician of the seventeenth century, liked to lie in bed and think! One day, when resting in bed, he solved the problem of describing the position of a point in a plane. His method was a development of the older idea of latitude and longitude. In honour of Déscartes, the system used for describing the position of a point in a plane is also known as the *Cartesian system.*



René Déscartes (1596 -1650)
Fig. 9.4

EXERCISE 9.1

1. How will you describe the position of a table lamp on your study table to another person?
2. **(Street Plan) :** A city has two main roads which cross each other at the centre of the city. These two roads are along the North-South direction and East-West direction.

All the other streets of the city run parallel to these roads and are 200 m apart. There are 5 streets in each direction. Using 1cm = 200 m, draw a model of the city on your notebook. Represent the roads/streets by single lines.

There are many cross- streets in your model. A particular cross-street is made by two streets, one running in the North - South direction and another in the East - West direction. Each cross street is referred to in the following manner : If the 2^{nd} street running in the North - South direction and 5^{th} in the East - West direction meet at some crossing, then we will call this cross-street (2, 5). Using this convention, find:

(i) how many cross - streets can be referred to as (4, 3).

(ii) how many cross - streets can be referred to as (3, 4).

9.2 Cartesian System

You have studied the *number line* in the chapter on 'Number System'. On the number line, distances from a fixed point are marked in equal units positively in one direction and negatively in the other. The point from which the distances are marked is called the *origin*. We use the number line to represent the numbers by marking points on a line at equal distances. If one unit distance represents the number '1', then 3 units distance represents the number '3', '0' being at the origin. The point in the positive direction at a distance *r* from the origin represents the number *r.* The point in the negative direction at a distance *r* from the origin represents the number –*r*. Locations of different numbers on the number line are shown in Fig. 9.5.

Fig. 9.5

Descartes invented the idea of placing two such lines perpendicular to each other on a plane, and locating points on the plane by referring them to these lines. The perpendicular lines may be in any direction such as in Fig.9.6. But, when we choose

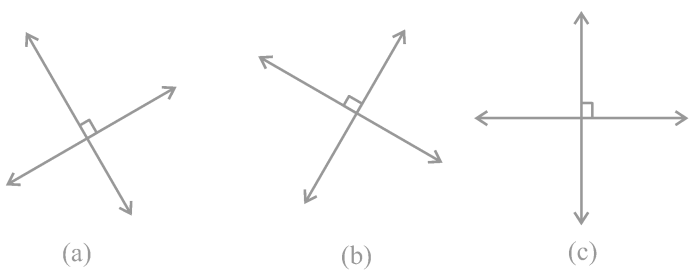
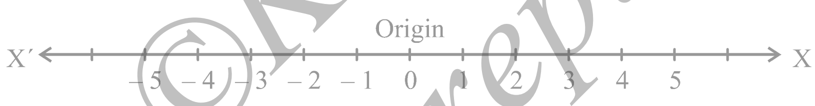


Fig. 9.6

these two lines to locate a point in a plane *in this chapter*, one line will be horizontal and the other will be vertical, as in Fig. 9.6(c).

These lines are actually obtained as follows : Take two number lines, calling them X′X and Y′Y. Place X′X horizontal [as in Fig. 9.7(a)] and write the numbers on it just as written on the number line. We do the same thing with Y′Y except that Y′Y is vertical, not horizontal [Fig. 9.7(b)].



(a) (b)

Fig. 9.7

Combine both the lines in such a way that the two lines cross each other at their zeroes, or origins (Fig. 9.8). The horizontal line X′X is called the x - axis and the vertical line Y′Y is called the y - axis. The point where X′X and Y′Y cross is called the **origin**, and is denoted by O. Since the positive numbers lie on the directions OX and OY, OX and OY are called the **positive directions** of the x - axis and the y - axis, respectively. Similarly, OX′ and OY′ are called the **negative directions** of the x - axis and the y - axis, respectively.

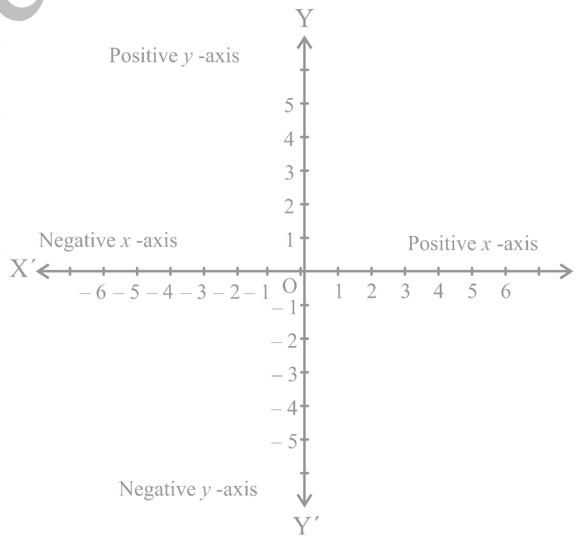


Fig. 9.8

You observe that the axes (plural of the word 'axis') divide the plane into four parts. These four parts are called the *quadrants* (one fourth part), numbered I, II, III and IV anticlockwise from OX (see Fig.9.9). So, the plane consists of the axes and these quadrants. We call the plane, the *Cartesian plane,* or the *coordinate plane,* or the *xy-plane.* The axes are called the *coordinate axes*.

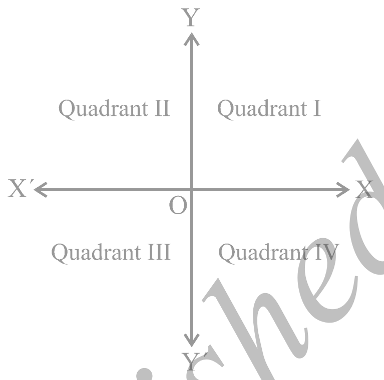


Fig. 9.9

Now, let us see why this system is so basic to mathematics, and how it is useful. Consider the following diagram where the axes are drawn on graph paper. Let us see the distances of the points P and Q from the axes. For this, we draw perpendiculars PM on the x - axis and PN on the y - axis. Similarly, we draw perpendiculars QR and QS as shown in Fig. 9.10.

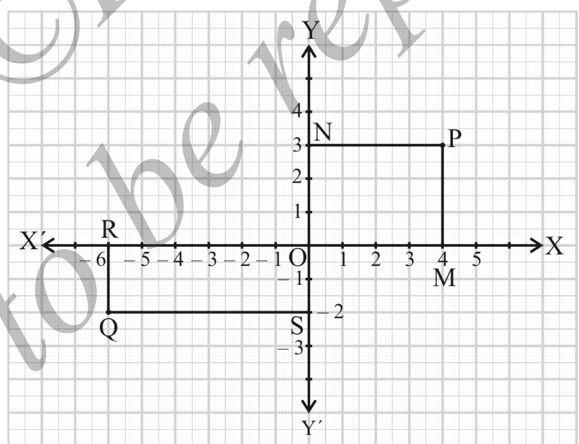


Fig.9.10

You find that

(i) The perpendicular distance of the point P from the y - axis measured along the positive direction of the x - axis is PN = OM = 4 units.

(ii) The perpendicular distance of the point P from the x - axis measured along the positive direction of the y - axis is PM = ON = 3 units.

(iii) The perpendicular distance of the point Q from the y - axis measured along the negative direction of the x - axis is OR = SQ = 6 units.

(iv) The perpendicular distance of the point Q from the x - axis measured along the negative direction of the y - axis is OS = RQ = 2 units.

Now, using these distances, how can we describe the points so that there is no confusion?

We write the coordinates of a point, using the following conventions:

(i) The x - *coordinate* of a point is its perpendicular distance from the y - axis measured along the x -axis (positive along the positive direction of the x - axis and negative along the negative direction of the x - axis). For the point P, it is + 4 and for Q, it is – 6. The x - coordinate is also called the *abscissa*.

(ii) The y - *coordinate* of a point is its perpendicular distance from the x - axis measured along the y - axis (positive along the positive direction of the y - axis and negative along the negative direction of the y - axis). For the point P, it is + 3 and for Q, it is –2. The y - coordinate is also called the *ordinate*.

(iii) In stating the coordinates of a point in the coordinate plane, the x - coordinate comes first, and then the y - coordinate. We place the coordinates in brackets.

Hence, the coordinates of P are (4, 3) and the coordinates of Q are (– 6, – 2).

Note that the coordinates describe a point in the plane *uniquely.* (3, 4) is not the same as (4, 3).

Example 1 : See Fig. 9.11 and complete the following statements:

(i) The abscissa and the ordinate of the point B are _ _ _ and _ _ _, respectively. Hence, the coordinates of B are (_ _, _ _).

(ii) The x-coordinate and the y-coordinate of the point M are _ _ _ and _ _ _, respectively. Hence, the coordinates of M are (_ _, _ _).

(iii) The x-coordinate and the y-coordinate of the point L are _ _ _ and _ _ _, respectively. Hence, the coordinates of L are (_ _, _ _).

(iv) The x-coordinate and the y-coordinate of the point S are _ _ _ and _ _ _, respectively. Hence, the coordinates of S are (_ _, _ _).

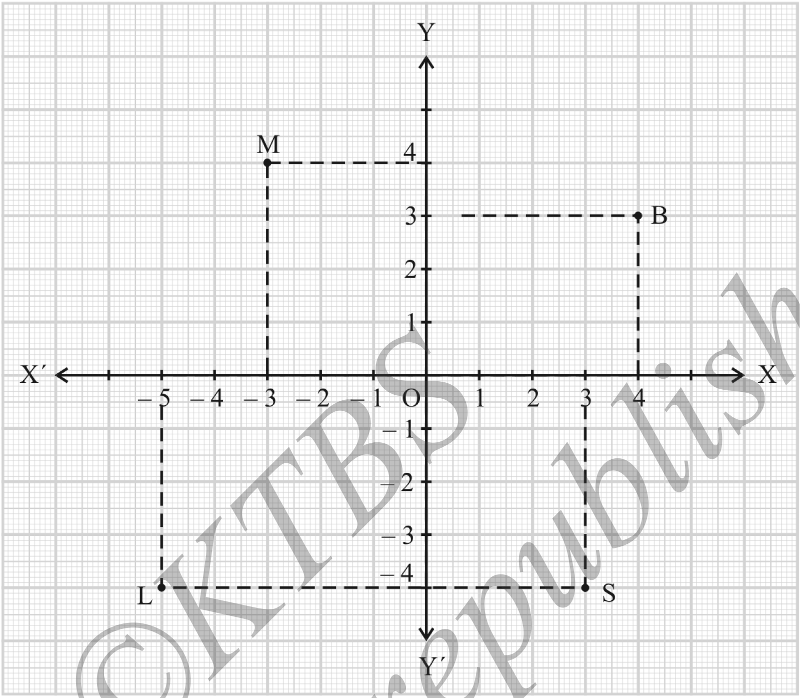


Fig. 9.11

Solution : (i) Since the distance of the point B from the *y* - axis is 4 units, the *x* - coordinate or abscissa of the point B is 4. The distance of the point B from the *x* - axis is 3 units; therefore, the *y* - coordinate, i.e., the ordinate, of the point B is 3. Hence, the coordinates of the point B are (4, 3).

As in (i) above :

(ii) The *x* - coordinate and the *y* - coordinate of the point M are –3 and 4, respectively. Hence, the coordinates of the point M are (–3, 4).

(iii) The *x* - coordinate and the *y* - coordinate of the point L are –5 and – 4, respectively. Hence, the coordinates of the point L are (–5, – 4).

(iv) The *x* - coordinate and the *y*- coordinate of the point S are 3 and – 4, respectively. Hence, the coordinates of the point S are (3, – 4).

Example 2 : Write the coordinates of the points marked on the axes in Fig. 9.12.

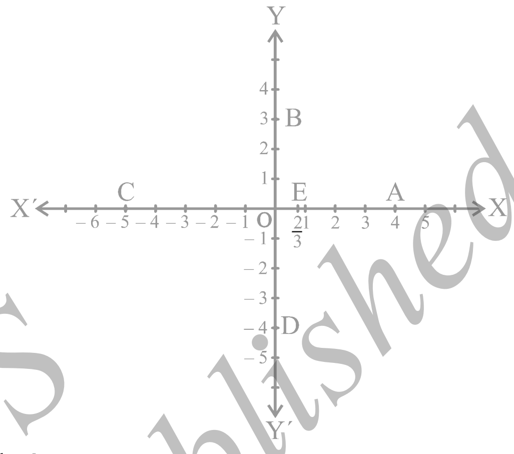


Fig. 9.12

Solution : You can see that :

(i) The point A is at a distance of + 4 units from the y - axis and at a distance zero from the x - axis. Therefore, the x - coordinate of A is 4 and the y - coordinate is 0. Hence, the coordinates of A are (4, 0).

(ii) The coordinates of B are (0, 3). Why?

(iii) The coordinates of C are (– 5, 0). Why?

(iv) The coordinates of D are (0, – 4). Why?

(v) The coordinates of E are $\left(\frac{2}{3}, 0\right)$. Why?

Since every point on the x - axis has no distance (zero distance) from the x - axis, therefore, the y - coordinate of every point lying on the x - axis is always zero. Thus, the coordinates of any point on the x - axis are of the form (x, 0), where x is the distance of the point from the y - axis. Similarly, the coordinates of any point on the y - axis are of the form (0, y), where y is the distance of the point from the x - axis. Why?

What are the coordinates of the ***origin*** **O**? *It has zero distance from both the axes so that its abscissa and ordinate are both zero. Therefore, the coordinates of the origin are* **(0, 0)**.

In the examples above, you may have observed the following relationship between the signs of the coordinates of a point and the quadrant of a point in which it lies.

(i) If a point is in the 1st quadrant, then the point will be in the form (+, +), since the 1st quadrant is enclosed by the positive x - axis and the positive y - axis.

(ii) If a point is in the 2nd quadrant, then the point will be in the form (–, +), since the 2nd quadrant is enclosed by the negative x - axis and the positive y - axis.

(iii) If a point is in the 3rd quadrant, then the point will be in the form (–, –), since the 3rd quadrant is enclosed by the negative x - axis and the negative y - axis.

(iv) If a point is in the 4th quadrant, then the point will be in the form (+, –), since the 4th quadrant is enclosed by the positive x - axis and the negative y - axis (see Fig. 9.13).

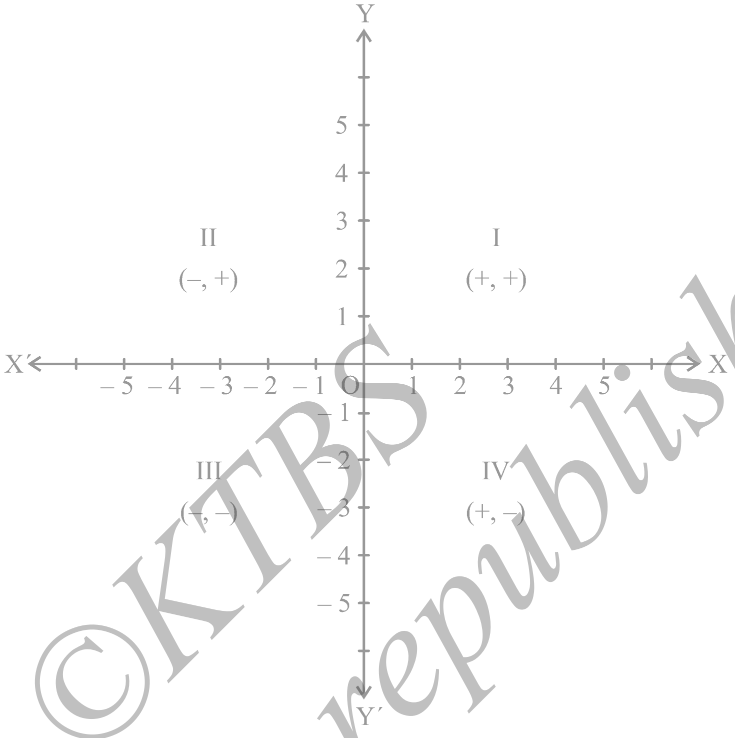


Fig. 9.13

Remark : The system we have discussed above for describing a point in a plane is only a convention, which is accepted all over the world. The system could also have been, for example, the ordinate first, and the abscissa second. However, the whole world sticks to the system we have described to avoid any confusion.

EXERCISE 9.2

1. Write the answer of each of the following questions:

(i) What is the name of horizontal and the vertical lines drawn to determine the position of any point in the Cartesian plane?

(ii) What is the name of each part of the plane formed by these two lines?

(iii) Write the name of the point where these two lines intersect.

2. See Fig.9.14, and write the following:

(i) The coordinates of B.

(ii) The coordinates of C.

(iii) The point identified by the coordinates (–3, –5).

(iv) The point identified by the coordinates (2, – 4).

(v) The abscissa of the point D.

(vi) The ordinate of the point H.

(vii) The coordinates of the point L.

(viii) The coordinates of the point M.

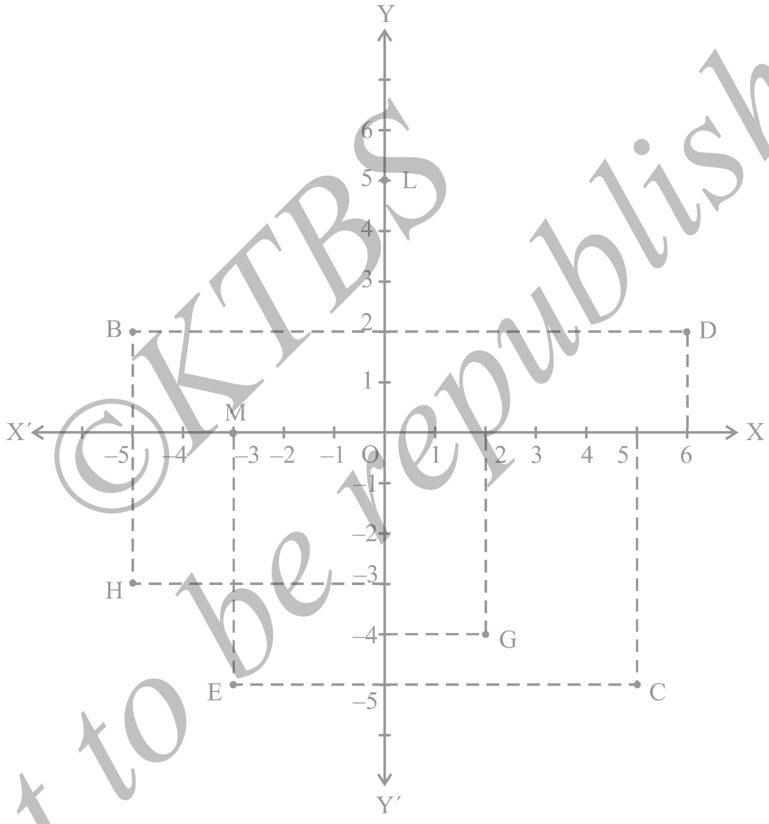


Fig. 9.14

9.3 Plotting a Point in the Plane if its Coordinates are Given

Uptil now we have drawn the points for you, and asked you to give their coordinates. Now we will show you how we place these points in the plane if we know its coordinates. We call this process "plotting the point".

Let the coordinates of a point be (3, 5). We want to plot this point in the coordinate plane. We draw the coordinate axes, and choose our units such that one centimetre represents one unit on both the axes. The coordinates of the point (3, 5) tell us that the distance of this point from the y - axis along the positive x - axis is 3 units and the distance of the point from the x - axis along the positive y - axis is 5 units. Starting from the origin O, we count 3 units on the positive x - axis and mark the corresponding point as A. Now, starting from A, we move in the positive direction of the y - axis and count 5 units and mark the corresponding point as P (see Fig.9.15). You see that the distance of P from the y - axis is 3 units and from the x - axis is 5 units. Hence, P is the position of the point. Note that P lies in the 1st quadrant, since both the coordinates of P are positive. Similarly, you can plot the point Q (5, – 4) in the coordinate plane. The distance of Q from the x - axis is 4 units along the negative y - axis, so that its y - coordinate is – 4 (see Fig.9.15). The point Q lies in the 4th quadrant. Why?

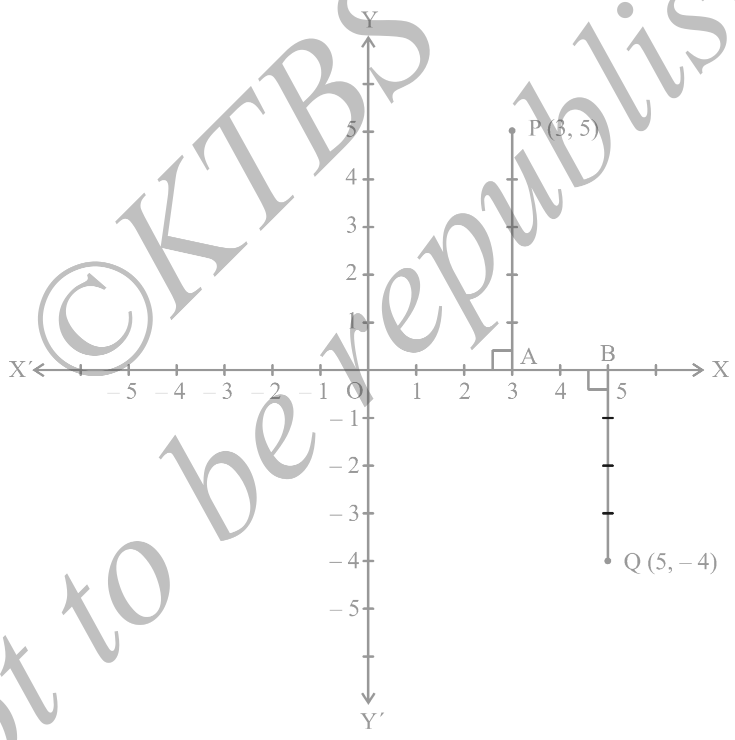


Fig. 9.15

Example 3 : Locate the points (5, 0), (0, 5), (2, 5), (5, 2), (–3, 5), (–3, –5), (5, –3) and (6, 1) in the Cartesian plane.

Solution : Taking 1cm = 1unit, we draw the x - axis and the y - axis. The positions of the points are shown by dots in Fig.9.16.

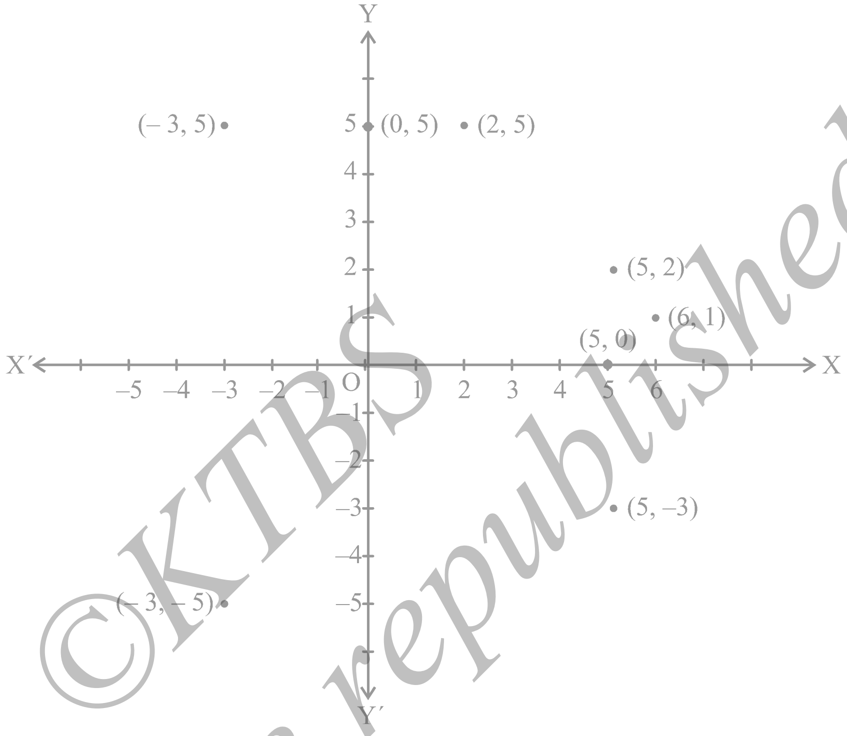


Fig. 9.16

Note : In the example above, you see that (5, 0) and (0, 5) are not at the same position. Similarly, (5, 2) and (2, 5) are at different positions. Also, (–3, 5) and (5, –3) are at different positions. By taking several such examples, you will find that, if $x \neq y$, **then the position of (x, y) in the Cartesian plane is different from the position of (y, x)**. So, if we interchange the coordinates x and y, the position of (y, x) will differ from the position of (x, y). This means that the order of x and y is important in (x, y). Therefore, (x, y) is called an ordered pair. The ordered pair $(x, y) \neq$ ordered pair (y, x), if $x \neq y$. Also $(x, y) = (y, x)$, if $x = y$.

Example 4 : Plot the following ordered pairs (x, y) of numbers as points in the Cartesian plane. Use the scale 1cm = 1 unit on the axes.

x	– 3	0	– 1	4	2
y	7	–3.5	– 3	4	– 3

Solution : The pairs of numbers given in the table can be represented by the points (– 3, 7), (0, –3.5), (– 1, – 3), (4, 4) and (2, – 3). The locations of the points are shown by dots in Fig.9.17.

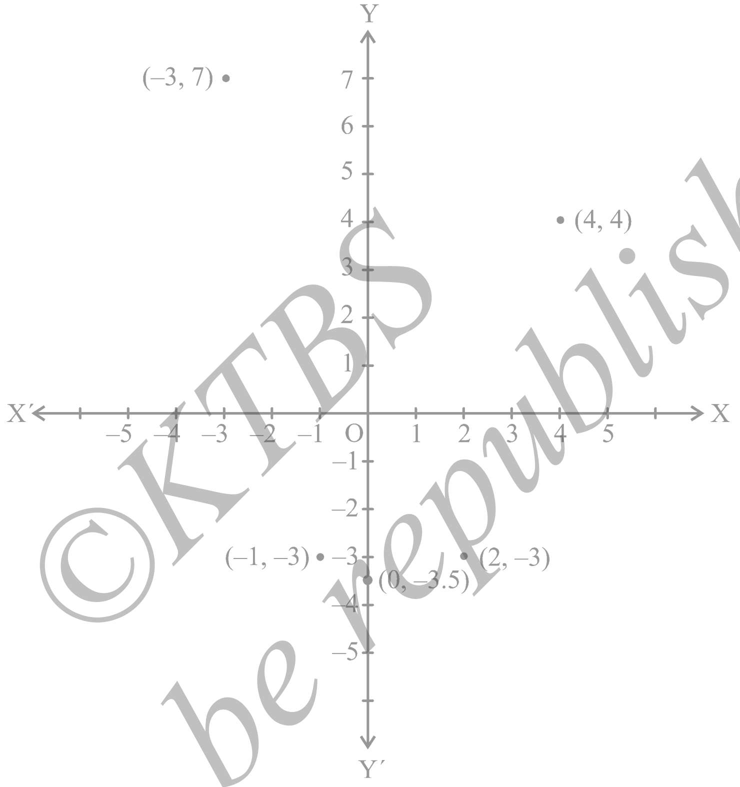


Fig. 9.17

Activity 2 : *A game for two persons* (Requirements: two counters or coins, graph paper, two dice of different colours, say red and green):

Place each counter at (0, 0). Each player throws two dice simultaneously. When the first player does so, suppose the red die shows 3 and the green one shows 1. So, she moves her counter to (3, 1). Similarly, if the second player throws 2 on the red and 4 on the green, she moves her counter to (2, 4). On the second throw, if the first player throws 1 on the red and 4 on the green, she moves her counter from (3, 1) to (3 + 1, 1 + 4), that is, adding 1 to the x - coordinate and 4 to the y - coordinate of (3, 1).

The purpose of the game is to arrive first at (10, 10) without overshooting, i.e., neither the abscissa nor the ordinate can be greater than 10. Also, a counter should not coincide with the position held by another counter. For example, if the first player's counter moves on to a point already occupied by the counter of the second player, then the second player's counter goes to (0, 0). If a move is not possible without overshooting, the player misses that turn. You can extend this game to play with more friends.

Remark : Plotting of points in the Cartesian plane can be compared to some extent with drawing of graphs in different situations such as Time-Distance Graph, Side-Perimeter Graph, etc which you have come across in earlier classes. In such situations, we may call the axes, t-axis, d-axis, s-axis or p-axis, etc. in place of the x and y axes.

EXERCISE 9.3

1. In which quadrant or on which axis do each of the points (– 2, 4), (3, – 1), (– 1, 0), (1, 2) and (– 3, – 5) lie? Verify your answer by locating them on the Cartesian plane.
2. Plot the points (x, y) given in the following table on the plane, choosing suitable units of distance on the axes.

x	– 2	–1	0	1	3
y	8	7	– 1.25	3	– 1

9.4 Summary

In this chapter, you have studied the following points :

1. To locate the position of an object or a point in a plane, we require two perpendicular lines. One of them is horizontal, and the other is vertical.
2. The plane is called the Cartesian, or coordinate plane and the lines are called the coordinate axes.
3. The horizontal line is called the x -axis, and the vertical line is called the y - axis.
4. The coordinate axes divide the plane into four parts called quadrants.
5. The point of intersection of the axes is called the origin.
6. The distance of a point from the y - axis is called its x-coordinate, or abscissa, and the distance of the point from the x-axis is called its y-coordinate, or ordinate.
7. If the abscissa of a point is x and the ordinate is y, then (x, y) are called the coordinates of the point.
8. The coordinates of a point on the x-axis are of the form $(x, 0)$ and that of the point on the y-axis are $(0, y)$.
9. The coordinates of the origin are (0, 0).
10. The coordinates of a point are of the form (+ , +) in the first quadrant, (–, +) in the second quadrant, (–, –) in the third quadrant and (+, –) in the fourth quadrant, where + denotes a positive real number and – denotes a negative real number.
11. If $x \neq y$, then $(x, y) \neq (y, x)$, and $(x, y) = (y, x)$, if $x = y$.

CHAPTER 10

LINEAR EQUATIONS IN TWO VARIABLES

The principal use of the Analytic Art is to bring Mathematical Problems to Equations and to exhibit those Equations in the most simple terms that can be.

*—**Edmund Halley***

10.1 Introduction

In earlier classes, you have studied linear equations in one variable. Can you write down a linear equation in one variable? You may say that $x + 1 = 0$, $x + \sqrt{2} = 0$ and $\sqrt{2}\, y + \sqrt{3} = 0$ are examples of linear equations in one variable. You also know that such equations have a unique (i.e., one and only one) solution. You may also remember how to represent the solution on a number line. In this chapter, the knowledge of linear equations in one variable shall be recalled and extended to that of two variables. You will be considering questions like: Does a linear equation in two variables have a solution? If yes, is it unique? What does the solution look like on the Cartesian plane? You shall also use the concepts you studied in Chapter 3 to answer these questions.

10.2 Linear Equations

Let us first recall what you have studied so far. Consider the following equation:

$$2x + 5 = 0$$

Its solution, i.e., the root of the equation, is $-\frac{5}{2}$. This can be represented on the number line as shown below:

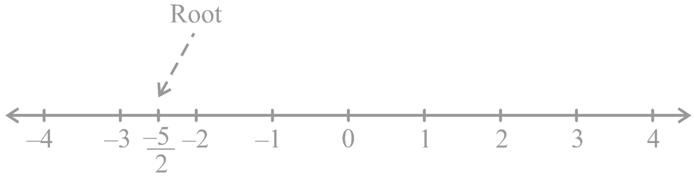


Fig. 10.1

While solving an equation, you must always keep the following points in mind:

The solution of a linear equation is not affected when:

(i) the same number is added to (or subtracted from) both the sides of the equation.

(ii) you multiply or divide both the sides of the equation by the same non-zero number.

Let us now consider the following situation:

In a One-day International Cricket match between India and Sri Lanka played in Nagpur, two Indian batsmen together scored 176 runs. Express this information in the form of an equation.

Here, you can see that the score of neither of them is known, i.e., there are two unknown quantities. Let us use x and y to denote them. So, the number of runs scored by one of the batsmen is x, and the number of runs scored by the other is y. We know that

$$x + y = 176,$$

which is the required equation.

This is an example of a linear equation in two variables. It is customary to denote the variables in such equations by x and y, but other letters may also be used. Some examples of linear equations in two variables are:

$$1.2s + 3t = 5,\ p + 4q = 7,\ \pi u + 5v = 9 \text{ and } 3 = \sqrt{2}\,x - 7y.$$

Note that you can put these equations in the form $1.2s + 3t - 5 = 0$, $p + 4q - 7 = 0$, $\pi u + 5v - 9 = 0$ and $\sqrt{2}\,x - 7y - 3 = 0$, respectively.

So, any equation which can be put in the form $ax + by + c = 0$, where a, b and c are real numbers, and a and b are not both zero, is called a *linear equation in two variables*. This means that you can think of many many such equations.

Example 1 : Write each of the following equations in the form $ax + by + c = 0$ and indicate the values of a, b and c in each case:

(i) $2x + 3y = 4.37$ (ii) $x - 4 = \sqrt{3}\,y$ (iii) $4 = 5x - 3y$ (iv) $2x = y$

Solution : (i) $2x + 3y = 4.37$ can be written as $2x + 3y - 4.37 = 0$. Here $a = 2$, $b = 3$ and $c = -4.37$.

(ii) The equation $x - 4 = \sqrt{3}\,y$ can be written as $x - \sqrt{3}\,y - 4 = 0$. Here $a = 1$, $b = -\sqrt{3}$ and $c = -4$.

(iii) The equation $4 = 5x - 3y$ can be written as $5x - 3y - 4 = 0$. Here $a = 5$, $b = -3$ and $c = -4$. Do you agree that it can also be written as $-5x + 3y + 4 = 0$? In this case $a = -5$, $b = 3$ and $c = 4$.

(iv) The equation $2x = y$ can be written as $2x - y + 0 = 0$. Here $a = 2$, $b = -1$ and $c = 0$.

Equations of the type $ax + b = 0$ are also examples of linear equations in two variables because they can be expressed as

$$ax + 0.y + b = 0$$

For example, $4 - 3x = 0$ can be written as $-3x + 0.y + 4 = 0$.

Example 2 : Write each of the following as an equation in two variables:

(i) $x = -5$ (ii) $y = 2$ (iii) $2x = 3$ (iv) $5y = 2$

Solution : (i) $x = -5$ can be written as $1.x + 0.y = -5$, or $1.x + 0.y + 5 = 0$.

(ii) $y = 2$ can be written as $0.x + 1.y = 2$, or $0.x + 1.y - 2 = 0$.

(iii) $2x = 3$ can be written as $2x + 0.y - 3 = 0$.

(iv) $5y = 2$ can be written as $0.x + 5y - 2 = 0$.

EXERCISE 10.1

1. The cost of a notebook is twice the cost of a pen. Write a linear equation in two variables to represent this statement.

   (Take the cost of a notebook to be ₹ x and that of a pen to be ₹ y).

2. Express the following linear equations in the form $ax + by + c = 0$ and indicate the values of a, b and c in each case:

   (i) $2x + 3y = 9.3\overline{5}$ (ii) $x - \frac{y}{5} - 10 = 0$ (iii) $-2x + 3y = 6$ (iv) $x = 3y$

   (v) $2x = -5y$ (vi) $3x + 2 = 0$ (vii) $y - 2 = 0$ (viii) $5 = 2x$

10.3 Solution of a Linear Equation

You have seen that every linear equation in one variable has a unique solution. What can you say about the solution of a linear equation involving two variables? As there are two variables in the equation, a solution means a pair of values, one for x and one for y which satisfy the given equation. Let us consider the equation $2x + 3y = 12$. Here, $x = 3$ and $y = 2$ is a solution because when you substitute $x = 3$ and $y = 2$ in the equation above, you find that

$$2x + 3y = (2 \times 3) + (3 \times 2) = 12$$

This solution is written as an ordered pair (3, 2), first writing the value for x and then the value for y. Similarly, (0, 4) is also a solution for the equation above.

On the other hand, (1, 4) is not a solution of $2x + 3y = 12$, because on putting $x = 1$ and $y = 4$ we get $2x + 3y = 14$, which is not 12. Note that (0, 4) is a solution but not (4, 0).

You have seen at least two solutions for $2x + 3y = 12$, i.e., (3, 2) and (0, 4). Can you find any other solution? Do you agree that (6, 0) is another solution? Verify the same. In fact, we can get many many solutions in the following way. Pick a value of your choice for x (say $x = 2$) in $2x + 3y = 12$. Then the equation reduces to $4 + 3y = 12$, which is a linear equation in one variable. On solving this, you get $y = \frac{8}{3}$. So $\left(2, \frac{8}{3}\right)$ is another solution of $2x + 3y = 12$. Similarly, choosing $x = -5$, you find that the equation becomes $-10 + 3y = 12$. This gives $y = \frac{22}{3}$. So, $\left(-5, \frac{22}{3}\right)$ is another solution of $2x + 3y = 12$. So there is no end to different solutions of a linear equation in two variables. That is, *a linear equation in two variables has infinitely many solutions.*

Example 3 : Find four different solutions of the equation $x + 2y = 6$.

Solution : By inspection, $x = 2$, $y = 2$ is a solution because for $x = 2$, $y = 2$

$$x + 2y = 2 + 4 = 6$$

Now, let us choose $x = 0$. With this value of x, the given equation reduces to $2y = 6$ which has the unique solution $y = 3$. So $x = 0$, $y = 3$ is also a solution of $x + 2y = 6$. Similarly, taking $y = 0$, the given equation reduces to $x = 6$. So, $x = 6$, $y = 0$ is a solution of $x + 2y = 6$ as well. Finally, let us take $y = 1$. The given equation now reduces to $x + 2 = 6$, whose solution is given by $x = 4$. Therefore, (4, 1) is also a solution of the given equation. So four of the infinitely many solutions of the given equation are:

$$(2, 2), (0, 3), (6, 0) \text{ and } (4, 1).$$

Remark : Note that an easy way of getting a solution is to take $x = 0$ and get the corresponding value of y. Similarly, we can put $y = 0$ and obtain the corresponding value of x.

Example 4 : Find two solutions for each of the following equations:

(i) $4x + 3y = 12$

(ii) $2x + 5y = 0$

(iii) $3y + 4 = 0$

Solution : (i) Taking $x = 0$, we get $3y = 12$, i.e., $y = 4$. So, (0, 4) is a solution of the given equation. Similarly, by taking $y = 0$, we get $x = 3$. Thus, (3, 0) is also a solution.
(ii) Taking $x = 0$, we get $5y = 0$, i.e., $y = 0$. So (0, 0) is a solution of the given equation.

Now, if you take $y = 0$, you again get (0, 0) as a solution, which is the same as the earlier one. To get another solution, take $x = 1$, say. Then you can check that the corresponding value of y is $-\frac{2}{5}\cdot$ So $\left(1, -\frac{2}{5}\right)$ is another solution of $2x + 5y = 0$.

(iii) Writing the equation $3y + 4 = 0$ as $0.x + 3y + 4 = 0$, you will find that $y = -\frac{4}{3}$ for any value of x. Thus, two solutions can be given as $\left(0, -\frac{4}{3}\right)$ and $\left(1, -\frac{4}{3}\right)$.

EXERCISE 10.2

1. Which one of the following options is true, and why?
   $y = 3x + 5$ has
   (i) a unique solution, (ii) only two solutions, (iii) infinitely many solutions
2. Write four solutions for each of the following equations:
   (i) $2x + y = 7$ (ii) $\pi x + y = 9$ (iii) $x = 4y$
3. Check which of the following are solutions of the equation $x - 2y = 4$ and which are not:
   (i) (0, 2) (ii) (2, 0) (iii) (4, 0)
   (iv) $\left(\sqrt{2}, 4\sqrt{2}\right)$ (v) (1, 1)
4. Find the value of k, if $x = 2$, $y = 1$ is a solution of the equation $2x + 3y = k$.

10.4 Graph of a Linear Equation in Two Variables

So far, you have obtained the solutions of a linear equation in two variables algebraically. Now, let us look at their geometric representation. You know that each such equation has infinitely many solutions. How can we show them in the coordinate plane? You may have got some indication in which we write the solution as pairs of values. The solutions of the linear equation in Example 3, namely,

$$x + 2y = 6 \quad (1)$$

can be expressed in the form of a table as follows by writing the values of y below the corresponding values of x :

Table 1

x	0	2	4	6	. . .
y	3	2	1	0	. . .

In the previous chapter, you studied how to plot the points on a graph paper. Let us plot the points (0, 3), (2, 2), (4, 1) and (6, 0) on a graph paper. Now join any two of these points and obtain a line. Let us call this as line AB (see Fig. 10.2).

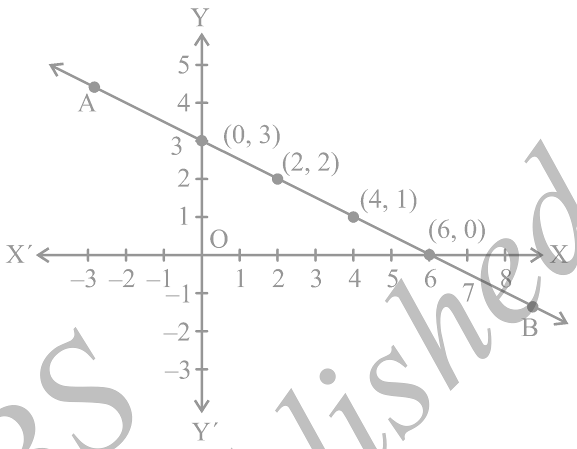


Fig. 10.2

Do you see that the other two points also lie on the line AB? Now, pick another point on this line, say (8, –1). Is this a solution? In fact, $8 + 2(–1) = 6$. So, (8, –1) is a solution. Pick any other point on this line AB and verify whether its coordinates satisfy the equation or not. Now, take any point not lying on the line AB, say (2, 0). Do its coordinates satisfy the equation? Check, and see that they do not.

Let us list our observations:

1. Every point whose coordinates satisfy Equation (1) lies on the line AB.
2. Every point (a, b) on the line AB gives a solution $x = a$, $y = b$ of Equation (1).
3. Any point, which does not lie on the line AB, is not a solution of Equation (1).

So, you can conclude that every point on the line satisfies the equation of the line and every solution of the equation is a point on the line. In fact, a linear equation in two variables is represented geometrically by a line whose points make up the collection of solutions of the equation. This is called the *graph* of the linear equation. So, to obtain the graph of a linear equation in two variables, it is enough to plot two points corresponding to two solutions and join them by a line. However, it is advisable to plot more than two such points so that you can immediately check the correctness of the graph.

Remark : The reason that a, degree one, polynomial equation $ax + by + c = 0$ is called a *linear* equation is that its geometrical representation is a straight line.

Example 5 : Given the point (1, 2), find the equation of a line on which it lies. How many such equations are there?

Solution : Here (1, 2) is a solution of a linear equation you are looking for. So, you are looking for any line passing through the point (1, 2). One example of such a linear equation is $x + y = 3$. Others are $y – x = 1$, $y = 2x$, since they are also satisfied by the coordinates of the point (1, 2). In fact, there are infinitely many linear equations which are satisfied by the coordinates of the point (1, 2). Can you see this pictorially?

Example 6 : Draw the graph of $x + y = 7$.

Solution : To draw the graph, we need at least two solutions of the equation. You can check that $x = 0$, $y = 7$, and $x = 7$, $y = 0$ are solutions of the given equation. So, you can use the following table to draw the graph:

Table 2

x	0	7
y	7	0

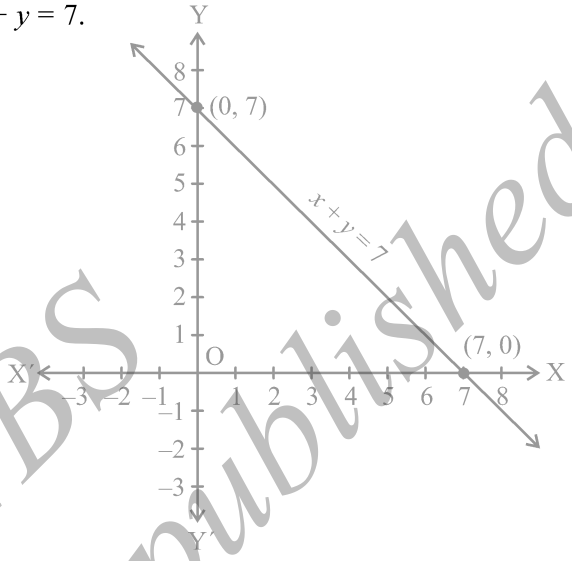


Fig. 10.3

Draw the graph by plotting the two points from Table 2 and then by joining the same by a line (see Fig. 10.3).

Example 7 : You know that the force applied on a body is directly proportional to the acceleration produced in the body. Write an equation to express this situation and plot the graph of the equation.

Solution : Here the variables involved are force and acceleration. Let the force applied be y units and the acceleration produced be x units. From ratio and proportion, you can express this fact as $y = kx$, where k is a constant. (From your study of science, you know that k is actually the mass of the body.)

Now, since we do not know what k is, we cannot draw the precise graph of $y = kx$. However, if we give a certain value to k, then we can draw the graph. Let us take $k = 3$, i.e., we draw the line representing $y = 3x$.

For this we find two of its solutions, say (0, 0) and (2, 6) (see Fig. 10.4).

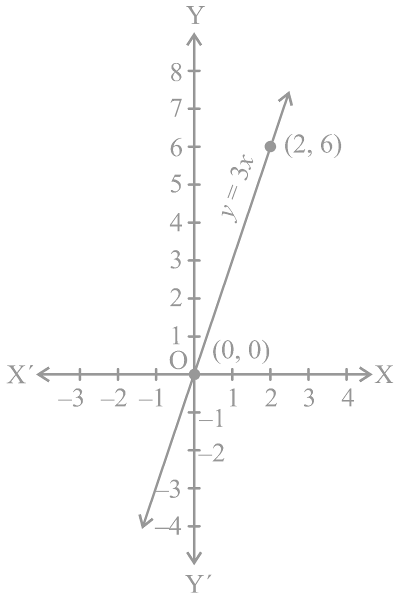


Fig. 10.4

From the graph, you can see that when the force applied is 3 units, the acceleration produced is 1 unit. Also, note that (0, 0) lies on the graph which means the acceleration produced is 0 units, when the force applied is 0 units.

Remark : The graph of the equation of the form $y = kx$ is a line which always passes through the origin.

Example 8 : For each of the graphs given in Fig. 10.5 select the equation whose graph it is from the choices given below:

(a) For Fig. 10.5 (i),

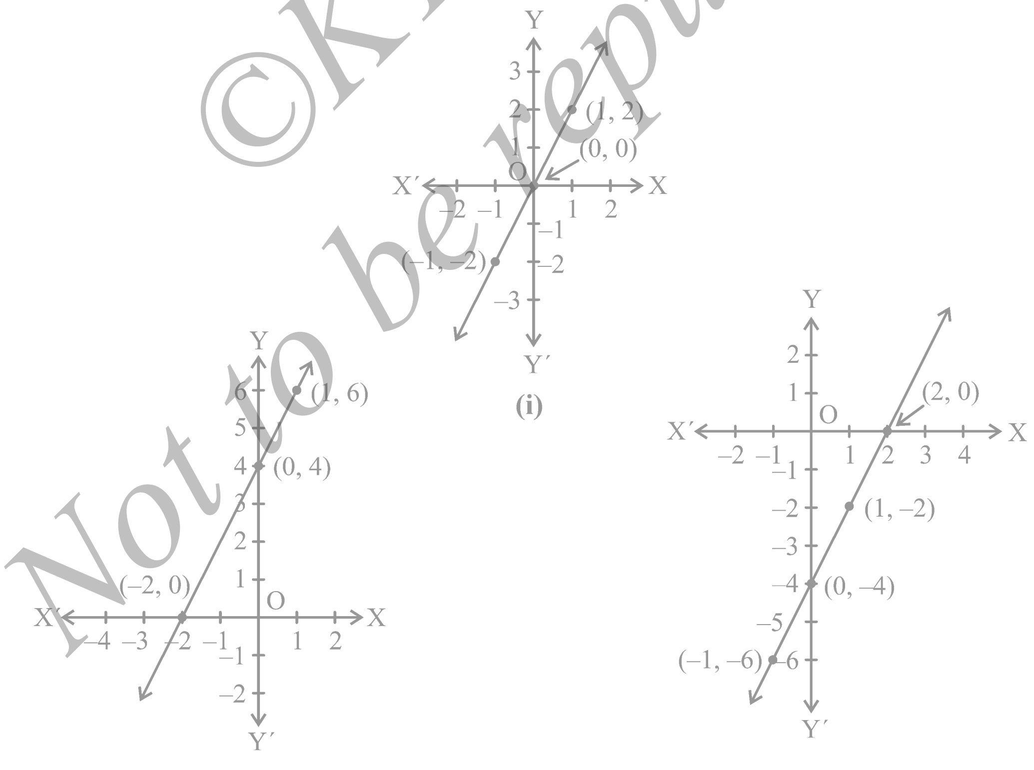
(i) $x + y = 0$ (ii) $y = 2x$ (iii) $y = x$ (iv) $y = 2x + 1$

(b) For Fig. 10.5 (ii),

(i) $x + y = 0$ (ii) $y = 2x$ (iii) $y = 2x + 4$ (iv) $y = x – 4$

(c) For Fig. 10.5 (iii),

(i) $x + y = 0$ (ii) $y = 2x$ (iii) $y = 2x + 1$ (iv) $y = 2x – 4$



(i) (ii) (iii)

Fig. 10.5

Solution : (a) In Fig. 10.5 (i), the points on the line are (–1, –2), (0, 0), (1, 2). By inspection, $y = 2x$ is the equation corresponding to this graph. You can find that the y-coordinate in each case is double that of the x-coordinate.

(b) In Fig. 10.5 (ii), the points on the line are (–2, 0), (0, 4), (1, 6). You know that the coordinates of the points of the graph (line) satisfy the equation $y = 2x + 4$. So, $y = 2x + 4$ is the equation corresponding to the graph in Fig. 10.5 (ii).

(c) In Fig. 10.5 (iii), the points on the line are (–1, –6), (0, –4), (1, –2), (2, 0). By inspection, you can see that $y = 2x – 4$ is the equation corresponding to the given graph (line).

EXERCISE 4.3

1. Draw the graph of each of the following linear equations in two variables:
   (i) $x + y = 4$ (ii) $x – y = 2$ (iii) $y = 3x$ (iv) $3 = 2x + y$
2. Give the equations of two lines passing through (2, 14). How many more such lines are there, and why?
3. If the point (3, 4) lies on the graph of the equation $3y = ax + 7$, find the value of a.
4. The taxi fare in a city is as follows: For the first kilometre, the fare is ₹ 8 and for the subsequent distance it is ₹ 5 per km. Taking the distance covered as x km and total fare as Rs y, write a linear equation for this information, and draw its graph.
5. From the choices given below, choose the equation whose graphs are given in Fig. 10.6 and Fig. 10.7.

For Fig. 10. 6

(i) $y = x$
(ii) $x + y = 0$
(iii) $y = 2x$
(iv) $2 + 3y = 7x$

For Fig. 10.7

(i) $y = x + 2$
(ii) $y = x – 2$
(iii) $y = –x + 2$
(iv) $x + 2y = 6$

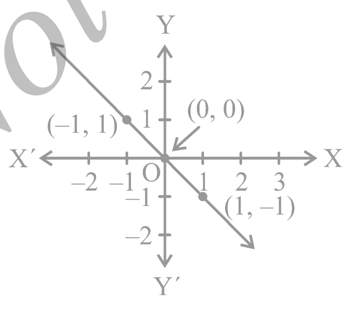


Fig. 10.6

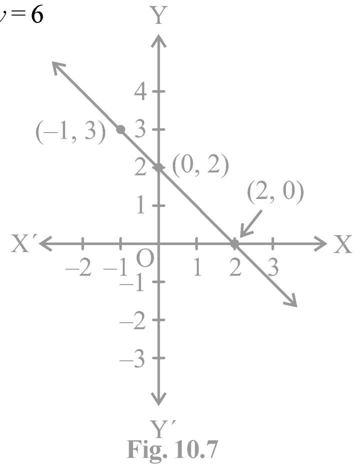


Fig. 10.7

6. If the work done by a body on application of a constant force is directly proportional to the distance travelled by the body, express this in the form of an equation in two variables and draw the graph of the same by taking the constant force as 5 units. Also read from the graph the work done when the distance travelled by the body is

(i) 2 units (ii) 0 unit

7. Yamini and Fatima, two students of Class IX of a school, together contributed ₹ 100 towards the Prime Minister's Relief Fund to help the earthquake victims. Write a linear equation which satisfies this data. (You may take their contributions as ₹ x and ₹ y.) Draw the graph of the same.

8. In countries like USA and Canada, temperature is measured in Fahrenheit, whereas in countries like India, it is measured in Celsius. Here is a linear equation that converts Fahrenheit to Celsius:

$$F = \left(\frac{9}{5}\right)C + 32$$

(i) Draw the graph of the linear equation above using Celsius for x-axis and Fahrenheit for y-axis.

(ii) If the temperature is 30°C, what is the temperature in Fahrenheit?

(iii) If the temperature is 95°F, what is the temperature in Celsius?

(iv) If the temperature is 0°C, what is the temperature in Fahrenheit and if the temperature is 0°F, what is the temperature in Celsius?

(v) Is there a temperature which is numerically the same in both Fahrenheit and Celsius? If yes, find it.

10.5 Equations of Lines Parallel to the *x*-axis and *y*-axis

You have studied how to write the coordinates of a given point in the Cartesian plane. Do you know where the points (2, 0), (–3, 0), (4, 0) and (n, 0), for any real number n, lie in the Cartesian plane? Yes, they all lie on the x-axis. But do you know why? Because on the x-axis, the y-coordinate of each point is 0. In fact, every point on the x-axis is of the form (x, 0). Can you now guess the equation of the x-axis? It is given by $y = 0$. Note that $y = 0$ can be expressed as $0.x + 1.y = 0$. Similarly, observe that the equation of the y-axis is given by $x = 0$.

Now, consider the equation $x - 2 = 0$. If this is treated as an equation in one variable x only, then it has the unique solution $x = 2$, which is a point on the number line. However, when treated as an equation in two variables, it can be expressed as

$x + 0.y - 2 = 0$. This has infinitely many solutions. In fact, they are all of the form $(2, r)$, where r is any real number. Also, you can check that every point of the form $(2, r)$ is a solution of this equation. So as, an equation in two variables, $x - 2 = 0$ is represented by the line AB in the graph in Fig. 10.8.

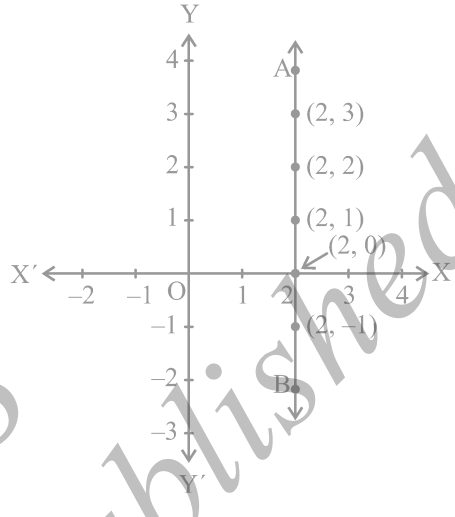


Fig. 10.8

Example 9 : Solve the equation $2x + 1 = x - 3$, and represent the solution(s) on (i) the number line,

(ii) the Cartesian plane.

Solution : We solve $2x + 1 = x - 3$, to get

$$2x - x = -3 - 1$$

i.e., $$x = -4$$

(i) The representation of the solution on the number line is shown in Fig. 10.9, where $x = -4$ is treated as an equation in one variable.

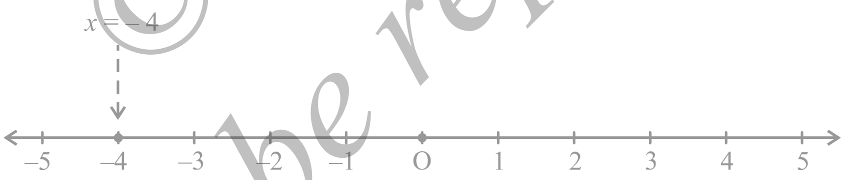


Fig. 10.9

(ii) We know that $x = -4$ can be written as $x + 0.y = -4$

which is a linear equation in the variables x and y. This is represented by a line. Now all the values of y are permissible because $0.y$ is always 0. However, x must satisfy the equation $x = -4$. Hence, two solutions of the given equation are $x = -4$, $y = 0$ and $x = -4$, $y = 2$.

Note that the graph AB is a line parallel to the y-axis and at a distance of 4 units to the left of it (see Fig. 10.10).

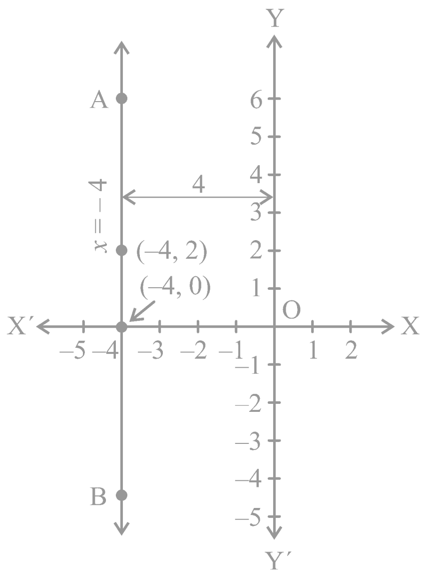


Fig. 10.10

Similarly, you can obtain a line parallel to the x-axis corresponding to equations of the type

$$y = 3 \quad \text{or} \quad 0.x + 1.y = 3$$

EXERCISE 10.4

1. Give the geometric representations of $y = 3$ as an equation
   (i) in one variable
   (ii) in two variables
2. Give the geometric representations of $2x + 9 = 0$ as an equation
   (i) in one variable
   (ii) in two variables

10.6 Summary

In this chapter, you have studied the following points:

1. An equation of the form $ax + by + c = 0$, where a, b and c are real numbers, such that a and b are not both zero, is called a linear equation in two variables.
2. A linear equation in two variables has infinitely many solutions.
3. The graph of every linear equation in two variables is a straight line.
4. $x = 0$ is the equation of the y-axis and $y = 0$ is the equation of the x-axis.
5. The graph of $x = a$ is a straight line parallel to the y-axis.
6. The graph of $y = a$ is a straight line parallel to the x-axis.
7. An equation of the type $y = mx$ represents a line passing through the origin.
8. Every point on the graph of a linear equation in two variables is a solution of the linear equation. Moreover, every solution of the linear equation is a point on the graph of the linear equation.

CHAPTER 11

AREAS OF PARALLELOGRAMS AND TRIANGLES

11.1 Introduction

In Chapter 2, you have seen that the study of Geometry, originated with the measurement of earth (lands) in the process of recasting boundaries of the fields and dividing them into appropriate parts. For example, a farmer *Budhia* had a triangular field and she wanted to divide it equally among her two daughters and one son. Without actually calculating the area of the field, she just divided one side of the triangular field into three equal parts and joined the two points of division to the opposite vertex. In this way, the field was divided into three parts and she gave one part to each of her children. Do you think that all the three parts so obtained by her were, in fact, equal in area? To get answers to this type of questions and other related problems, there is a need to have a relook at areas of plane figures, which you have already studied in earlier classes.

You may recall that the part of the plane enclosed by a simple closed figure is called a *planar region* corresponding to that figure. The magnitude or measure of this planar region is called its *area*. This magnitude or measure is always expressed with the help of a number (in some unit) such as 5 cm^2, 8 m^2, 3 hectares etc. So, we can say that area of a figure is a number (in some unit) associated with the part of the plane enclosed by the figure.

We are also familiar with the concept of congruent figures from earlier classes and from Chapter 5. *Two figures are called congruent, if they have the same shape and the same size*. In other words, if two figures A and B are congruent (see Fig. 11.1) , then using a tracing paper,

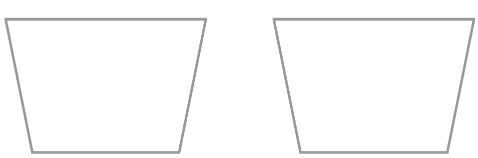


Fig. 11.1

you can superpose one figure over the other such that it will cover the other completely. So *if two figures A and B are congruent, they must have equal areas*. However, the converse of this statement is *not true*. In other words, *two figures having equal areas need not be congruent*. For example, in Fig. 11.2, rectangles ABCD and EFGH have equal areas (9×4 cm^2 and 6×6 cm^2) but clearly they are not congruent. (Why?)

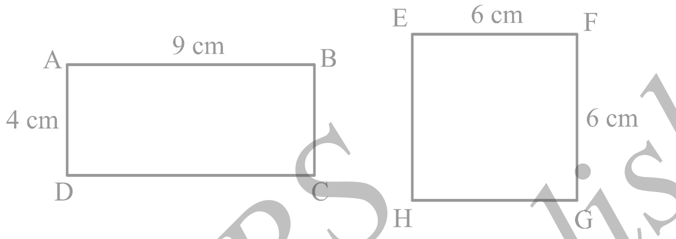


Fig. 11.2

Now let us look at Fig. 11.3 given below:

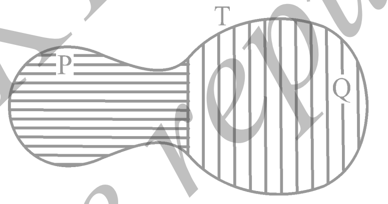


Fig. 11.3

You may observe that planar region formed by figure T is made up of two planar regions formed by figures P and Q. You can easily see that

Area of figure T = Area of figure P + Area of figure Q.

You may denote the area of figure A as ar(A), area of figure B as ar(B), area of figure T as ar(T), and so on. Now you can say that *area of a figure is a number* (*in some unit*) *associated with the part of the plane enclosed by the figure with the following two properties:*

(1) *If* A *and* B *are two congruent figures, then ar*(A) = ar(B);

and (2) *if a planar region formed by a figure* T *is made up of two non-overlapping planar regions formed by figures* P *and* Q*, then* ar(T) = ar(P) + ar(Q).

You are also aware of some formulae for finding the areas of different figures such as rectangle, square, parallelogram, triangle etc., from your earlier classes. In this chapter, attempt shall be made to consolidate the knowledge about these formulae by studying some relationship between the areas of these geometric figures under the condition when they lie on the same base and between the same parallels. This study will also be useful in the understanding of some results on 'similarity of triangles'.

11.2 Figures on the Same Base and Between the Same Parallels

Look at the following figures:

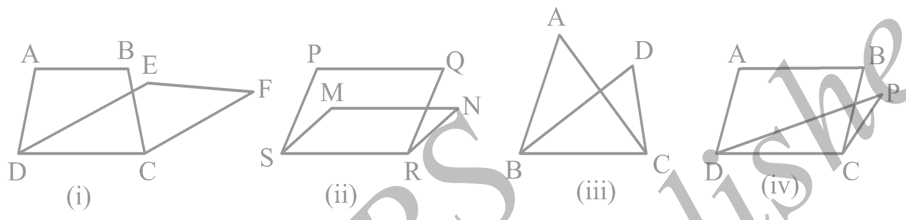


Fig. 11.4

In Fig. 11.4(i), trapezium ABCD and parallelogram EFCD have a common side DC. We say that trapezium ABCD and parallelogram EFCD *are on the same base* DC. Similarly, in Fig. 11.4 (ii), parallelograms PQRS and MNRS are on the same base SR; in Fig. 11.4(iii), triangles ABC and DBC are on the same base BC and in Fig. 11.4(iv), parallelogram ABCD and triangle PDC are on the same base DC.

Now look at the following figures:

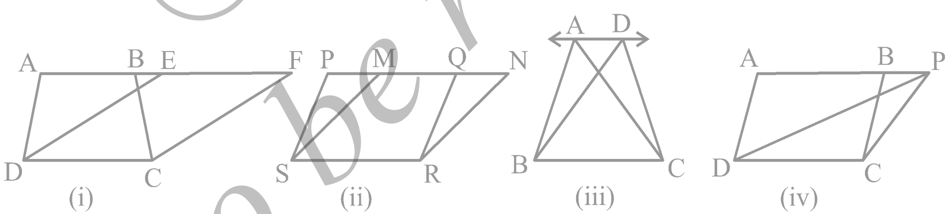


Fig. 11.5

In Fig. 11.5(i), clearly trapezium ABCD and parallelogram EFCD are on the *same base* DC. In addition to the above, the vertices A and B (of trapezium ABCD) opposite to base DC and the vertices E and F (of parallelogram EFCD) opposite to base DC lie on a line AF parallel to DC. We say that trapezium ABCD and parallelogram EFCD are on *the same base* DC *and between the same parallels* AF and DC. Similarly, parallelograms PQRS and MNRS are on the same base SR and between the same parallels PN and SR [see Fig.11.5 (ii)] as vertices P and Q of PQRS and vertices M and N of MNRS lie on a line PN parallel to base SR.In the same way, triangles ABC and DBC lie on the same base BC and between the same parallels AD and BC [see Fig. 11.5 (iii)] and parallelogram ABCD and triangle PCD lie on the same base DC and between the same parallels AP and DC [see Fig. 11.5(iv)].

So, *two figures are said to be on the same base and between the same parallels, if they have a common base (side) and the vertices (or the vertex) opposite to the common base of each figure lie on a line parallel to the base.*

Keeping in view the above statement, you cannot say that Δ PQR and Δ DQR of Fig. 11.6(i) lie between the same parallels l and QR. Similarly, you cannot say that

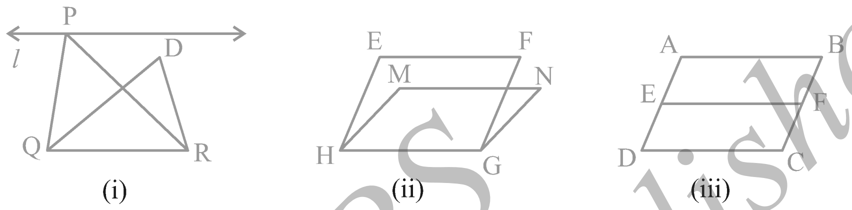


Fig. 11.6

parallelograms EFGH and MNGH of Fig. 11.6(ii) lie between the same parallels EF and HG and that parallelograms ABCD and EFCD of Fig. 11.6(iii) lie between the same parallels AB and DC (even though they have a common base DC and lie between the parallels AD and BC). So, it should clearly be noted that *out of the two parallels, one must be the line containing the common base.* Note that ΔABC and ΔDBE of Fig. 11.7(i) are not on the common base. Similarly, ΔABC and parallelogram PQRS of Fig. 11.7(ii) are also not on the same base.

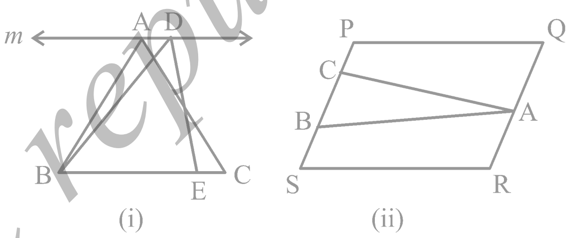


Fig. 11.7

EXERCISE 11.1

1. Which of the following figures lie on the same base and between the same parallels. In such a case, write the common base and the two parallels.

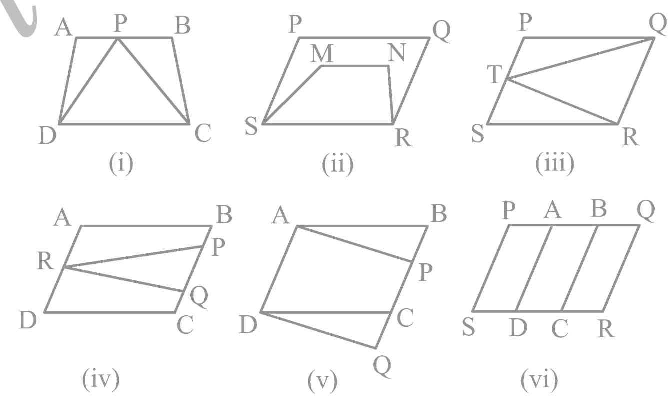


Fig. 11.8

11.3 Parallelograms on the same Base and Between the same Parallels

Now let us try to find a relation, if any, between the areas of two parallelograms on the same base and between the same parallels. For this, let us perform the following activities:

Activity 1 : Let us take a graph sheet and draw two parallelograms ABCD and PQCD on it as shown in Fig. 11.9.

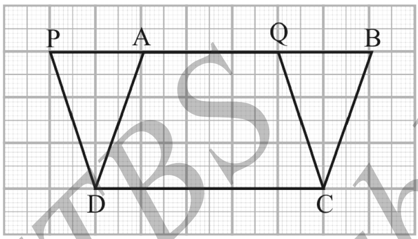


Fig. 11.9

The above two parallelograms are on the same base DC and between the same parallels PB and DC. You may recall the method of finding the areas of these two parallelograms by counting the squares.

In this method, the area is found by counting the number of complete squares enclosed by the figure, the number of squares a having more than half their parts enclosed by the figure and the number of squares having half their parts enclosed by the figure. The squares whose less than half parts are enclosed by the figure are ignored. You will find that areas of both the parallelograms are (approximately) $15cm^2$. Repeat this activity* by drawing some more pairs of parallelograms on the graph sheet. What do you observe? Are the areas of the two parallelograms different or equal? If fact, they are equal. So, this may lead you to conclude that *parallelograms on the same base and between the same parallels are equal in area*. However, remember that this is just a verification.

Activity 2 : Draw a parallelogram ABCD on a thick sheet of paper or on a cardboard sheet. Now, draw a line-segment DE as shown in Fig. 11.10.

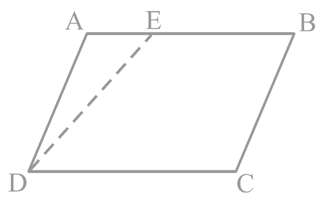


Fig. 11.10

*This activity can also be performed by using a Geoboard.