Chapter 5

Lines and Angles

Introduction to Lines and Angles

Suppose, you are sitting inside a room. If you look around, what do you see? If you observe, we are surrounded by different objects like a table, chair, wall-clock, laptop, water bottle, stairs, etc. And when we look outside, we see objects like trees, roads, electric poles, buildings, and many more things. All these objects are made up of
geometrical figures like lines and angles.

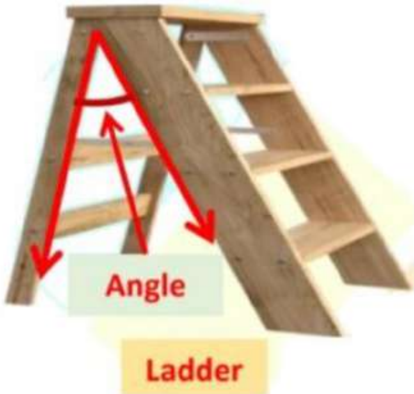
Some real-life examples of the lines and angles

1. If you observe the bamboo plants, they grow in a straight line.
Each bamboo stick resembles a straight line.

2. When we observe the railway tracks, they run in straight lines extending on both directions.

3. If we observe, the adjacent sides of the laptop, they form an angle between them.

4. The two legs of a ladder resemble straight lines. The two legs join at one point and the opening between them is the angle formed by these legs



What do they tell us? Basically, they tell us the orientation of one surface to another. Let us take one more example:

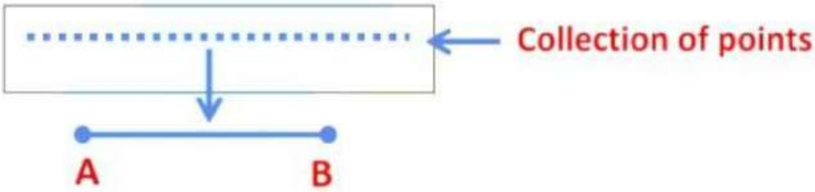
What happens when you open a door?

As soon as you open the door an angle is formed between the edge of the door and the threshold of the panel.

Important Terms:
Point: An exact location that has no size i.e., no length, no width, no depth, the only position is called a point. A point is denoted by a dot (.)

If you take an ink pen and put a dot on paper using that, then that dot represents a point.

Line segment: A collection of points with two fixed endpoints is called a line segment. A line segment AB is denoted by $\overline{AB}$. The length of a line segment is fixed.

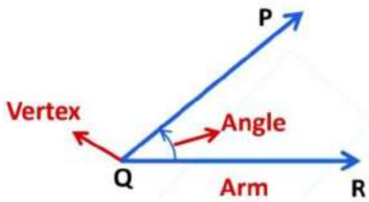


Ray: A part of a line with one fixed point and extends endlessly from the other end is called a ray. Ray AB is denoted by $\overrightarrow{AB}$. The length of a ray is infinite

Line: A-Line is a collection of points going endless in both directions along the straight path. Line AB is denoted by $\overleftrightarrow{AB}$. The length of a line is infinite.

Angles

When two rays originate from a common point then the turn between two rays around the common point or vertex is called the angle between the two rays.



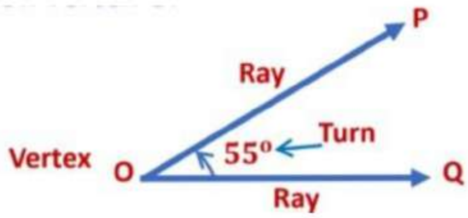
The two rays joining to form an angle are called arms of an angle and the point at which two rays meet to form an angle is called the vertex of the angle.

In the above figure, two rays $\overrightarrow{QP}$ and $\overrightarrow{QR}$ are the arm of an angle which meets at common initial point Q (vertex)and form an ∠PQR. The measure of the angle PQR is written as m∠PQR but instead of writing this we can simply write it as ∠PQR.

Measure of an angle

The amount of turn (rotation) between two rays or lines around their vertex is called measure of an angle. We measure the angles in degrees.

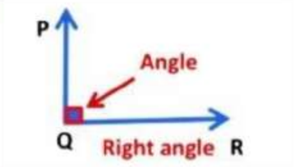
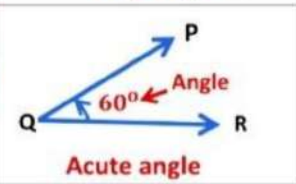
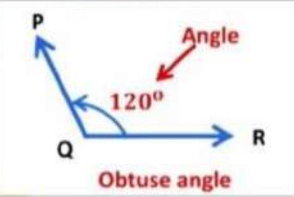
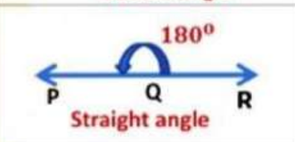
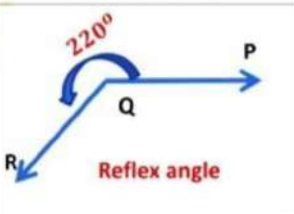
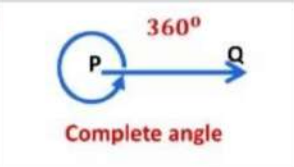
Example: (i)In we see that turn between two $\overrightarrow{OQ}$ rays $\overrightarrow{OP}$ and is 55° with common vertex O.
∠POQ = 55°.



(ii) In the clock we see the turn between hour hand and minute hand of a clock is 90° from the center.

Types of Angles

Classification of Angles (based on their measure)

An angle whose measure is exactly 90^0 is called a right angle. In the adjacent figure, $\angle PQR = 90^0$	
An angle whose measure is more than 0^0 but less than 90^0 is called an acute angle. In the adjacent figure, $0^0 < \angle PQR < 90^0$	
An angle whose measure is more than 90^0 but less than 180^0 is called an obtuse angle. In the adjacent figure, $\angle PQR = 120^0$ $180^0 > \angle PQR > 90^0$	
An angle whose measure exactly 180^0 is called a straight angle. In the adjacent figure, $\angle PQR = 180^0$	
An angle whose measure is more than 180^0 but less than 360^0 is called a reflex angle. In the adjacent figure, $\angle PQR = 220^0$ $360^0 > \angle PQR > 180^0$	
An angle whose measure is exactly 360^0 is called a complete angle.	

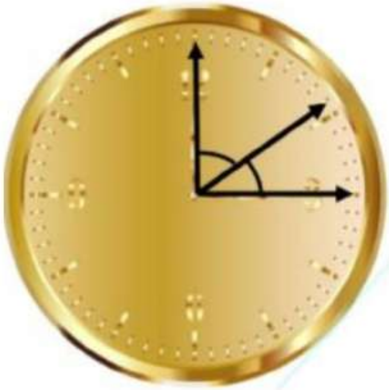
Complementary Angles

Related Angles

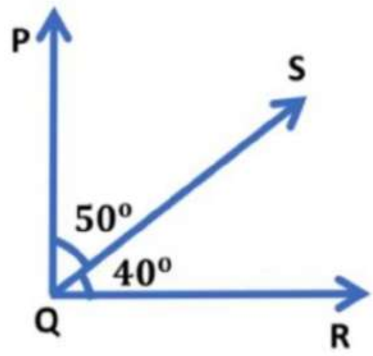
Complementary Angles– Any two angles are said to be complementary if the sum of their measure is equal to 90°.

.Example: Clock

The hour-hand and the minute-hand of the clock form a pair of complementary angles.

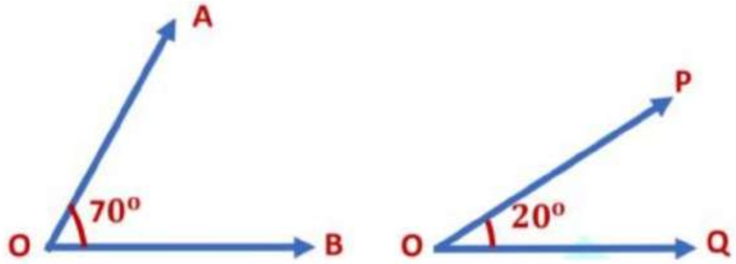


$55° + 35° = 90°$.



Here,
$\angle PQS + \angle SQR = 50° + 40° = 90°$
In the above figure, we see that the sum of two angles is 90°.
Hence, ∠PQS and ∠SQR are complementary angles. And ∠PQS and ∠SQR are said to be complements of each other.

Example: Are the given angles complementary?



In the given figure,
$\angle AOB = 70°$and $\angle POQ = 20°$

$\angle AOB + \angle POQ = 70° + 20° = 90°$
Therefore, ∠AOB and ∠POQ are complementary angles.

Example: Find the complement angle of 55°
Let the complement of 55° is A°

$$A° + 55° = 90°$$

$$A° + 55° - 55° = 90° - 55°$$
$$A° = 90° - 55°$$
$$A° = 35°$$

Hence, the complementary angle of 55° is 35°.

Identify whether the following pairs of angles are complementary or not?

(i) 10°, 80°
We know, any two angles are said to be complementary if the sum of their measure is equal to 90°.

Sum of the measure of the given angles = $10° + 80° = 90°$

These angles are complementary.

(ii)45°, 40°
We know, two angles are said to be complementary if the sum of their measure is equal to 90°.

Sum of the measure of angles = $45° + 40° = 85°$
∴ These angles are not complementary angles.

Supplementary Angles

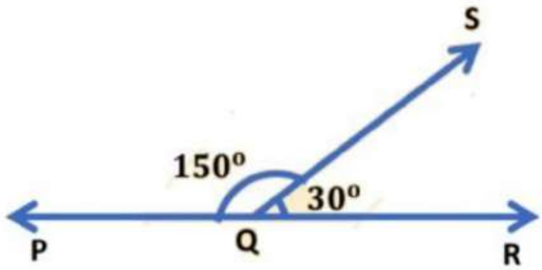
Two angles are said to be supplementary if the sum of their measure is equal to 180°

Example: Clock:

The two angles formed by the hands of the above clock are supplementary. The measure of two angles 120° and 60° are given and when we add up that angles we get 180°.

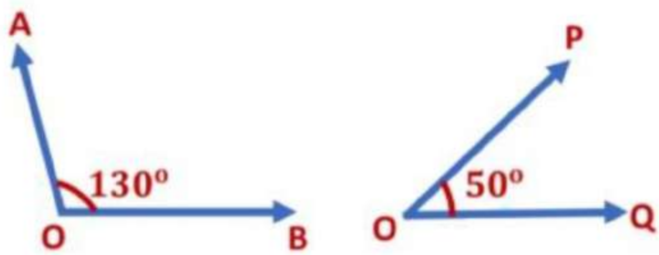
$120° + 60° = 180°$.

Hence, we can say that they are supplementary angles or supplements of each other.



Here,
$\angle PQS + \angle SQR = 150° + 30° = 180°$
In the above figure, we see that the sum of two angles is 180°
.
Hence, ∠PQS and ∠SQR are supplementary angles and ∠PQS and ∠SQR are said to be supplements of each other.

Example: The following ∠AOB and ∠POQ are supplementary angles or not?



In the given figure,

$\angle AOB + \angle POQ = 130° + 50° = 180°$

$\angle AOB + \angle POQ = 180°$

$\therefore$ $\angle AOB$ and $\angle POQ$ are supplementary angles. Or

$\angle AOB$ and $\angle POQ$ are said to be supplements of each other.

Example: Find the angle which equals to its supplement
Let the measure of the angle = $x°$ and

The measure of its supplement = $x°$

We know, two angles are said to be supplementary if the sum of their measure is equal to 180°

Therefore,

$$x° + x° = 180°$$

$$2x° = 180°$$

$$x° = 90°$$

Hence, the measure of the required angle is 90°.

Example: Two supplementary angles are in the ratio of 3: 2.Find the angles.

Let the two supplementary angles be 3x and 2x,

$$3x + 2x = 180°$$

A pair of angles are said to be supplementary if the sum of the angles is equal to 180°.

$$5x = 180°$$

$$\Rightarrow x = \frac{180}{5}° = 36°$$

$$x = 36°$$

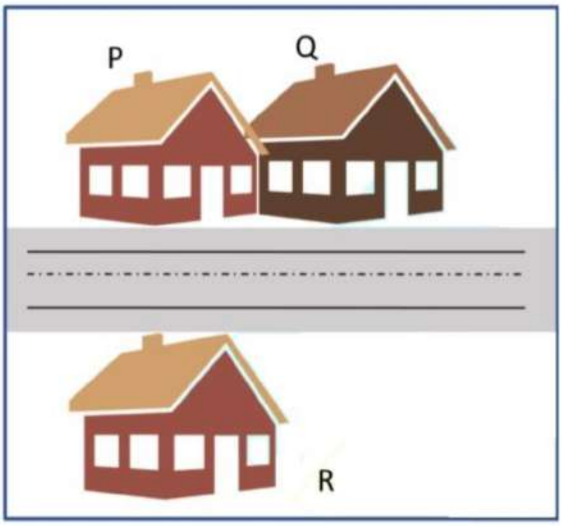
$$3x = 3 \times 36° = 108°$$

$$2x = 2 \times 36° = 72°$$

Therefore, the two supplementary angles are 108° and 72°.

Adjacent Angle

Before discussing the adjacent angle, let us understand the meaning of the word ‘adjacent’.
So, what do you observe in the following picture?

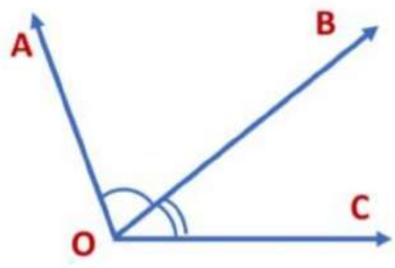


In the above picture, we see that house Q is next to house P also house R is next to house P but if we see the picture more clearly we see that house P and house Q share their compound wall with each other. House P and house R do not share their wall with each other. There has to be something common to be called adjacent. So, house Q is adjacent to house P.

Now, we can apply the same concept to study the adjacent angle.

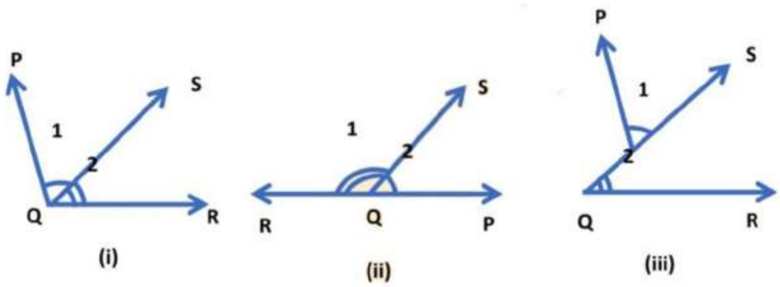
Two angles are adjacent, if

(i) They have a common vertex
(ii) They have a common arm
(iii) But no common interior point



Here, ∠AOB and ∠BOC have a common vertex O. They have a common arm OB.
But no common interior point. Also, they have a non-common arm OA and OC.
Hence, ∠AOB and ∠BOC are adjacent angles.

Example: In the following figure angles marked with 1 and 2 are they adjacent? If not give a reason for that.



In figure number (i) we see

∠PQS and ∠SQR have a common arm QS.
They have a common vertex Q.
They do not have a common interior point.
Hence, ∠PQS and ∠SQR are adjacent angles

In figure number (ii) we see

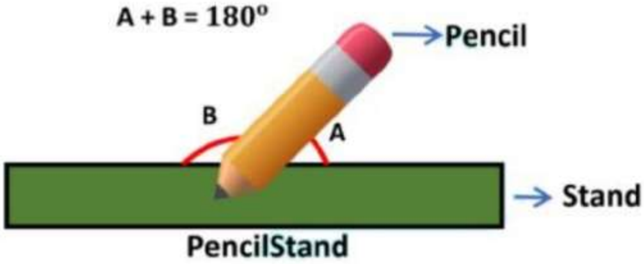
∠RQS and ∠SQP have a common arm QS.
They have a common vertex Q.
They do not have a common interior point
Hence, ∠RQS and ∠SQP are adjacent angles

In figure number (iii) we see

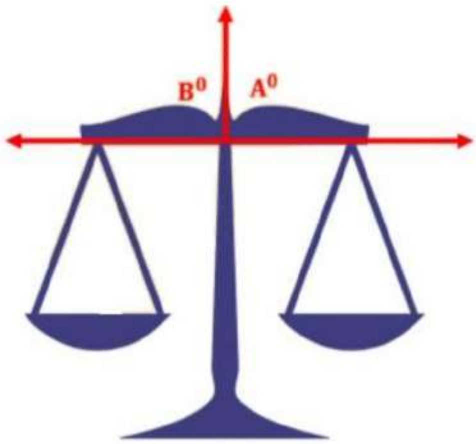
∠PTS and ∠SQR have a common arm QS.
They do not have a common vertex.
They do not have a common interior point.

The above figure does not satisfy all the conditions for being adjacent angles. Hence, ∠PTS and ∠SQR are not adjacent angles.

Linear Pair of Angles



In the above picture we see, a pencil is placed on the stand.
Suppose the pencil represents a ray. So, ∠A is formed between the pencil stand and pencil. ∠B is formed between pencil and pencil stand back again. Both the angles formed have a common vertex. So, ∠A and ∠B form a straight angle. Therefore, $\angle A + \angle B = 180°$. Hence, they form a linear pair.
Now, what do you observe in the following figures

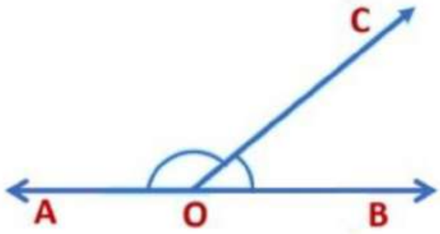


In the above figure, we see that the beam is kept on the beam support. Suppose the beam be a ray.

In the given figure beam support is a straight line. So, ∠A and ∠B form a straight angle. Therefore, $\angle A + \angle B = 180°$. Hence, this is also a form of linear pair.

Now, we can apply the same concept to study the linear pair.

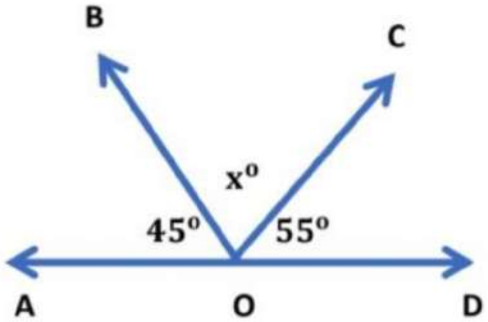
A linear pair is a pair of adjacent angles whose non-common arms are opposite rays and sum of these adjacent angles is 180°.



In the above figure,

$\angle AOC$, $\angle BOC$ are adjacent angles as O is the common vertex, OC is the common arm, and their non-common arms (OA and OB) are two opposite rays and the sum of the angles $\angle AOC$, $\angle BOC$ is 180°. Therefore, $\angle AOC$ and $\angle BOC$ form a linear pair. Thus, we can say that $\angle AOC + \angle BOC = 180°$.

Example: Find the value of x in the following figure



In the given figure,

$\angle AOB$,$\angle BOC$ and $\angle COD$ form a linear pair.

We know the sum of a linear pair of angles is 180°.

Therefore,

$$\angle AOB + \angle BOC + \angle COD = 180°$$

$$45° + x° + 55° = 180°$$

$$100° + x° = 180°$$

Subtracting 100 from both sides

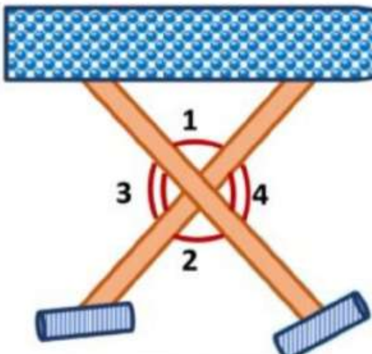
$$100° + x° - 100° = 180° - 100°$$

$$x° = 80°$$

Hence, the value of x in the figure is 80°.

Vertically Opposite Angles

Before discussing vertically opposite angles we see some real-life examples. What do you observe in the following figures?



Iron stand

In the figure of the iron stand, we see the legs of the iron stand intersects each other at the common vertex and four angles are formed.

Here, $\angle 1 + \angle 3 = 180°$ [These angles form a linear pair]......(i)

And, $\angle 2 + \angle 3 = 180°$ [These angles form a linear pair].......(ii)

From, (i) and (ii)

$\angle 1 + \angle 3 = \angle 2 + \angle 3$

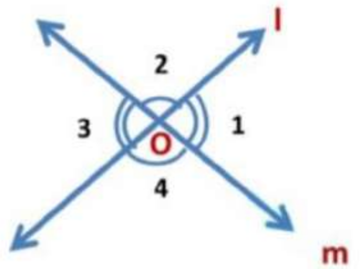
$\Rightarrow \angle 1 = \angle 2$

Here, $\angle 1$ and $\angle 2$ are vertically opposite angles.

Similarly, $\angle 3 = \angle 4$ and they are vertically opposite angles.

Therefore, when any two lines intersect, they form two pairs of angles without any common arm. These angles are called vertically opposite angles.

Theorem: When two lines intersect each other, then the vertically opposite angles are equal



Let line l and m, which intersect at O, making angles $\angle 1$, $\angle 2$, $\angle 3$, and $\angle 4$.

$\angle 1$ is vertically opposite to $\angle 3$ and $\angle 2$ is vertically opposite to $\angle 4$

To Prove: $\angle 1 = \angle 3$ and $\angle 2 = \angle 4$
Now,
$\angle 1 + \angle 2 = 180°$ ....Linear pair of angles
$\angle 1 = 180° - \angle 2$ ... (i)
$\angle 2 + \angle 3 = 180°$ ....Linear pair of angles
$\angle 3 = 180° - \angle 2$ ... (ii)
Therefore ,
From (i) and (ii)

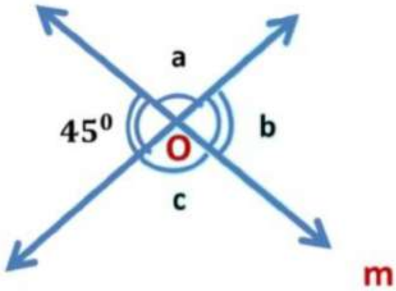
$$\angle 1 = \angle 3$$

Similarly, we can prove that $\angle 2 = \angle 4$,
$\angle 2 + \angle 3 = 180°$ ....Linear pair of angles
$\angle 2 = 180° - \angle 3$ ...(iii)
$\angle 3 + \angle 4 = 180°$ ....Linear pair of angles
$\angle 4 = 180° - \angle 3$ ...(iv)
Therefore,
From (iii) and (iv)

$$\angle 2 = \angle 4$$

From the above explanation, we see that two pairs of vertically opposite angles are equal.

Example: Find the value of a, b, c in the following figure



Here,
$b = 45°$ ...[Vertically opposite angles]
$b + a = 180°$ ...[Linear pair of angles]
$\Rightarrow 45° + a = 180°$
$a = 180° - 45°$
$a = 135°$
We know, $a = c$ ... [Vertically opposite angles]
Hence,

$a = c = 135°$

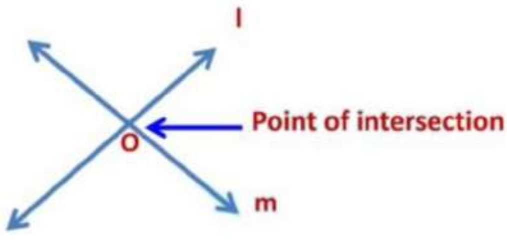
Therefore,

$a = c = 135°, b = 45°$

Angles Formed by a Transversal Line

Pairs of Lines

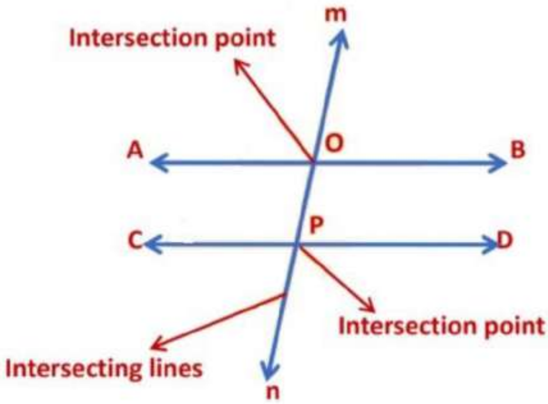
Intersecting lines- Two lines are said to be intersecting when they cross each other at one point only and the point at which they intersect is called the point of intersection.



Here, two lines *l* and m intersect each other at point O and the point O is called the point of intersection.

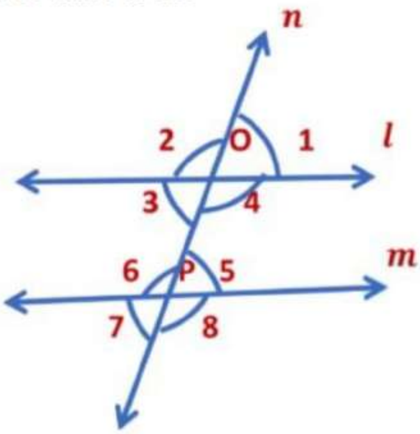
Transversal lines

A line that intersects two or more lines in a plane at distinct points is called a transversal line.



Here, line ***mn*** intersects two lines AB and CD at two distinct points O and P respectively. Hence, line mn is called the transversal line and points O and P are called the points of intersection.

Angles made by a Transversal

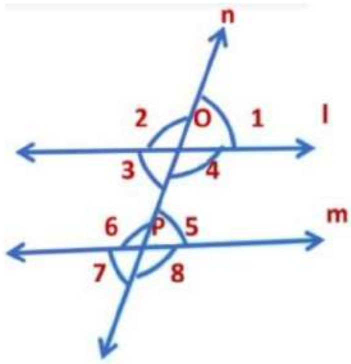


Here, the two lines l and m are intersected by a transversal n at points O and P respectively. We see that four angles are formed at each point O and P, namely ∠1, ∠2, ∠3, ∠4, ∠5, ∠6, ∠7, and ∠8.

Interior Angles: The angles whose arms include the segment OP are called interior angles	∠3, ∠4, ∠5 and ∠6
Exterior Angles: The angles whose arms do not include the segment OP are called exterior angles	∠1, ∠2, ∠7 and ∠8

Corresponding Angles – The angles on the same side of a transversal are known as corresponding angles if both lies either above or below the lines.	∠1 &∠5, ∠2 &∠6, ∠4 &∠8, ∠3 &∠7
Alternate Interior Angles – The pairs of interior angles on opposite sides of the transversal are called alternate interior angles.	∠4 &∠6, ∠3 &∠5
Alternate Exterior Angles – The pairs of exterior angles on opposite sides of the transversal are called alternate exterior angles.	∠1 &∠7, ∠2&∠8
Interior angles on the same side of the transversal – They are also called co-interior angles.	∠4 &∠5, ∠3 &∠6

Example: In the figure given below, l || m and $\angle 1 = 53°$. Find $\angle 6$ and $\angle 7$.



We have,
$\angle 1 = \angle 3$ [Vertically opposite angles]

$\angle 3 = \angle 7$ [Corresponding angles]

$$\therefore \angle 1 = \angle 7$$

$$\rightarrow \angle 7 = 53° \because \angle 1 = 53° \text{ (Given)}$$

$\angle 6 + \angle 7 = 180°$ [Linear pair]

$$\angle 6 + 53° = 180°$$

$$\angle 6 + 53° - 53° = 180° - 53°$$

$$\angle 6 = 127°$$

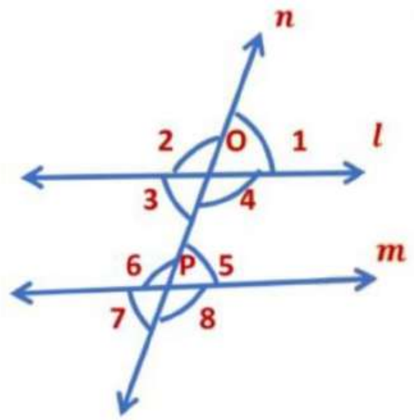
Thus, $\angle 6 = 127°$ and $\angle 7 = 53°$

Transversal of Parallel Lines

If two lines lying in a same plane do not intersect, when produced on either side, then such lines are said to be parallel to each other.

Here, lines ***l*** and m are parallel to each other.

Transversal of parallel lines
Here, lines l and m are parallel to each other and transversal n intersects line l and m at point O and P respectively.



When the two parallel lines l and m are cut by a transversal n, then obtained the following relations:

(i) When a transversal intersects two parallel lines, then each pair of alternate interior angles are equal.

When line n intersects two parallel lines l and m, then we see that each pair of alternate interior angles is equal.

$\therefore \angle 3 = \angle 5, \angle 4 = \angle 6$

(ii) When a transversal intersects two parallel lines, each pair of alternate exterior angles are equal.

When line n intersects two parallel lines l and m, then we see that each pair of alternate exterior angles is equal.

$\therefore \angle 2 = \angle 8, \angle 1 = \angle 7$

(iii) When a transversal intersects two parallel lines, each pair of corresponding angles are equal.

When line n intersects two parallel lines l and m, then we see that each pair of corresponding angles is equal.

$\therefore \angle 3 = \angle 7, \angle 2 = \angle 6, \angle 1 = \angle 5$, and $\angle 4 = \angle 8$

(iv) When a transversal intersects two parallel lines, then each pair of interior angles on the same side of the transversal are supplementary

When line n intersects two parallel lines l and m, then we see that each pair of interior angles on the same side of the transversal are supplementary.

In the above figure,

$\angle 3 = \angle 4$ .....Linear pair of angles

We know sum of the linear pair of angles is $180°$

$\therefore \angle 3 + \angle 4 = 180°$

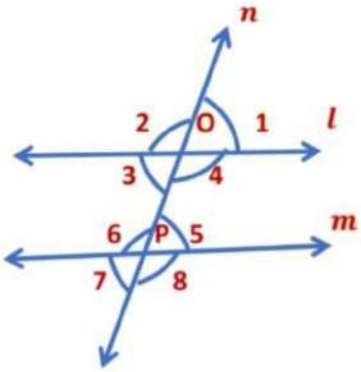
But, $\angle 4 = \angle 6$ ...Pair of alternate interior angles

Therefore, we can say that

$\therefore \angle 3 + \angle 6 = 180°$

Similarly, $\angle 4 + \angle 5 = 180°$

Example: In the given figure l || m, $\angle 1 = 55°$. Find $\angle 5, \angle 6$, and $\angle 7$.



We have,

$\angle 1 = \angle 5$ .....Corresponding angles

$\therefore \angle 5 = 55°$ [$\because \angle 1 = 55°$]

$\angle 5 = \angle 7$ .....Vertically opposite angles

$\therefore \angle 7 = 55°$ [$\because \angle 5 = 55°$]

Now,

$\angle 6 + \angle 7 = 180°$.....Linear pair of angles

$\angle 6 + 55° = 180°$

$\angle 6 + 55° - 55° = 180° - 55°$

$\angle 6 = 180° - 55°$

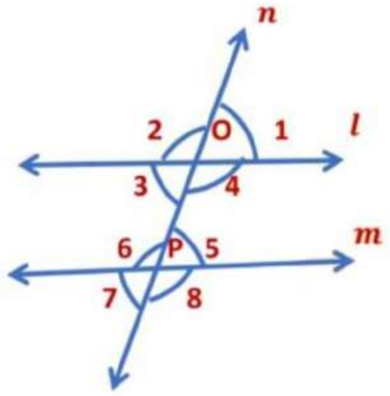
$\angle 6 = 125°$

Thus, $\angle 5 = 55°$, $\angle 6 = 125°$and $\angle 7 = 55°$

Checking for parallel lines

Some special pairs of angles can be used to test if the lines are parallel or not.

(i) When a transversal intersects two parallel lines, such that if any pair of corresponding angles are equal, then the lines are parallel.

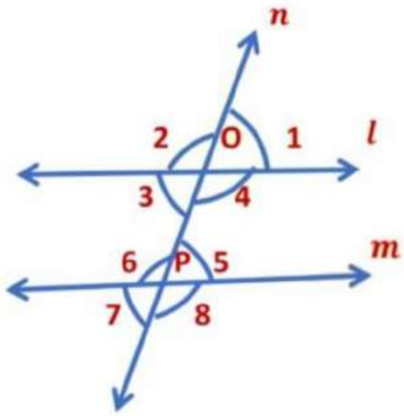


In the given figure, transversal n intersects two lines l and m in such a way that,

$\angle 3 = \angle 7$, $\angle 2 = \angle 6$, $\angle 1 = \angle 5$, and $\angle 4 = \angle 8$ ...(Pairs of corresponding angles are equal)

Hence, we can say that lines are parallel.

(ii) When a transversal intersects two parallel lines, such that if any pair of alternate interior angles are equal, the lines have to be parallel.

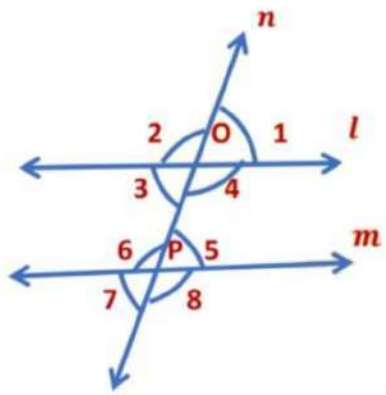


In the given figure, transversal n intersects two lines l and m in such a way that,

$\angle 3 = \angle 5$, $\angle 4 = \angle 6$ ... (Alternate interior angles are equal)

Hence, we can say that lines are parallel.

(iii) When transversal intersects two parallel lines, such that if any pair of alternate exterior angles are equal, the lines have to be parallel

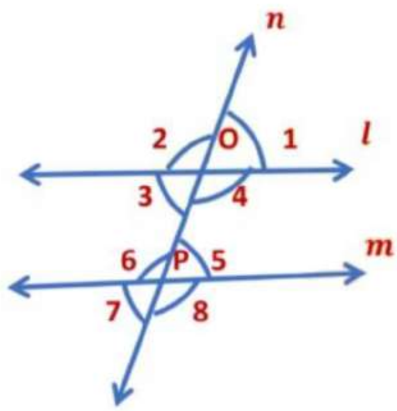


In the given figure, transversal n intersects two lines l and m in such a way that,

$\angle 1 = \angle 7$, $\angle 2 = \angle 8$ ... (Alternate exterior angles are equal)

Hence, we can say that lines are parallel.

(i) When transversal intersects two parallel lines, such that if any pair of interior angles on the same side of the transversal are supplementary, the lines have to be parallel.



In the given figure, transversal n intersects two lines l and m in such a way that,

$\angle 3$, $\angle 6$ and $\angle 4$, $\angle 5$ ...Pairs of co-interior angles or angles on the same sides of the transversal

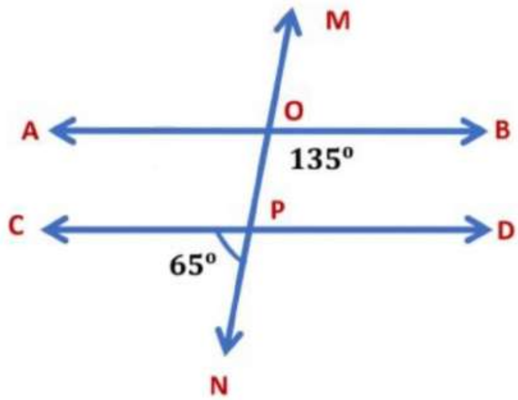
Hence,

$\angle 3 + \angle 6 = 180°$ and

$\angle 4 + \angle 5 = 180°$

Hence, we can say that lines are parallel.

Example: Find whether AB || CD.



In the given figure,

$\angle CPN = \angle OPD = 65°$...Vertically opposite angles

$\angle BOP + \angle OPD = 180°$

Thus, the sum of co-interior angle is 180°

Hence,

$135° + \angle OPD = 180°$

$\angle OPD = 180° - 135°$

$\angle OPD = 45°$

Therefore, AB || CD