CHAPTER

5 ARITHMETIC PROGRESSIONS

Syllabus

➢ *Motivation for studying Arithmetic Progression. Derivation of the n^{th} term and sum of the first n terms of A.P. and their application in solving daily life problems.*

Chapter Analysis

List of Topics	2016			2017			2018
	Delhi	Outside Delhi	Foreign	Delhi	Outside Delhi	Foreign	Delhi & Outside Delhi
Terms of AP	1 Q (2 M) 1 Q (1 M)	1 Q (1 M) 1 Q (2 M)	1 Q (3 M) 1 Q (1 M)		1 Q (1 M) 1 Q (2 M)	1 Q (2 M)	1 Q (1 M)
Sum of AP	1 Q (2 M)	1 Q (3 M)	1 Q (2 M)			1 Q (4 M)	1 Q (2 M)
Sum of the series				1 Q (3 M)		1 Q (3 M)	
Word Problem on AP	1 Q (3 M)			1 Q (3 M) 1 Q (4 M) 1 Q (2 M)	1 Q (3 M) 1 Q (4 M)		1 Q (4 M)

TOPIC-1
To Find n^{th} Term of the Arithmetic Progression

Revision Notes

- An arithmetic progression is a sequence of numbers in which each term is obtained by adding a fixed number d to the preceding term, except the first term.
- The difference between the two successive terms of an A.P. is called the common difference.
- Each number in the sequence of arithmetic progression is called a term of an A.P.
- The arithmetic progression having finite number of terms is called a finite arithmetic progression.
- The arithmetic progression having infinite number of terms is called an infinite arithmetic progression.

- A list of numbers $a_1, a_2, a_3, \ldots\ldots$ is an A.P., if the differences $a_2 - a_1, a_3 - a_2, a_4 - a_3, \ldots$ give the same value *i.e.*, $a_{k+1} - a_k$ is same for all different values of k.
- The general form of an A.P. is $a, a + d, a + 2d, a + 3d, \ldots..$
- If the A.P. $a, a + d, a + 2d, \ldots\ldots\ldots, l$ is reversed to $l, l - d, l - 2d, \ldots\ldots\ldots, a$, the common difference changes to negative of original sequence common difference.

Know the Formulae

- The general term of an A.P. is expressed as :

  $a_n = a + (n - 1)d$. ..... from the starting.

  where, a is the first term and d is the common difference.
- The general term of an A.P. $l, l - d, l - 2d, \ldots\ldots\ldots, a$ is given by :

  $a_n = l + (n - 1)(-d) = l - (n - 1)d$ ..... from the end.

  where, l is the last term, d is the common difference and n is the number of terms.

Know the Terms

- A sequence is defined as an ordered list of numbers.

  The first, second and third terms of a sequence are denoted by t_1, t_2 and t_3 respectively.
- If the terms of sequence are connected with plus (+) or minus (–), the pattern is called a series.

  Example : $2 + 4 + 6 + 8 +$ ..... is a series.
- The sequence of numbers 0, 1, 1, 2, 3, 5, 8, 13,..... was discovered by a famous Italian Mathematician Leonasalo Fibonacci, when he was dealing with the problem of rabbit population.
- If the terms of a sequence or a series are written under specific conditions, then the sequence or series is called a progression.
- If a constant is added or subtracted from each term of an A.P., the resulting sequence is also an A.P.
- If each term of an A.P. is multiplied or divided by a constant, the resulting sequence is also an A.P.
- If the n^{th} term is in linear form *i.e.*, $an + b = a_n$, the sequence is in A.P.
- If the terms are selected at a regular interval, the given sequence is in A.P.
- If three consecutive number a, b and c are in A.P., the sum two numbers is twice the middle number *i.e.*, $2b = a + c$.

Q. Which term of the A.P. 6, 13, 20, 27, ... is 98 more than its 24th term ? U

Sol. **Step I :** The given A.P. is 6, 13, 20, 27, ...

Here first term, $a = 6$

Common difference, $d = 13 - 6 = 7$

Step II : The 24th term, $a_{24} = a + (24 - 1)d$

or, $a_{24} = 6 + 23 \times 7$

$a_{24} = 6 + 161$

$a_{24} = 167$

Step III : Now according to question,

$a_{24} + 98 = a_n$

$167 + 98 = a + (n - 1)d$

$265 = 6 + (n - 1)7$

$259 = (n - 1)7$

$\frac{259}{7} = n - 1$

$37 = n - 1$

or $n = 38$

Hence, 38th term is the required term.

Objective Type Questions (1 mark each)

[A] Multiple Choice Questions :

Q. 1. 30^{th} term of the A. P, : 10, 7, 4,……, is :

(a) 97 **(b) 77**

(c) –77 **(d) –87**

R [NCERT Exemp.]

Sol. Correct option : (c)

Explanation : In the given AP, $a = 10$ and $d = 7 - 10 = -3$

Thus, the 30th term is $t_{30} = 10 + (30-1)(-3) = -77$

Q. 2. 11^{th} term of the A.P., : $-3, -\frac{1}{2}, 2, \ldots$ is :

(a) 28 **(b) 22**

(c) –38 **(d)** $-48\frac{1}{2}$

R [NCERT Exemp.]

Sol. Correct option : (b)

Explanation : In the given A.P., $a = -3$ and $d = -\frac{1}{2} + 3 = \frac{5}{2}$

Thus, the 11th term is $t_{11} = -3 + (11-1)\left(\frac{5}{2}\right) = 22$

Q. 3. In an A.P, if $d = -4$, $n = 7$, $a_n = 4$, then a is;

(a) 6 **(b) 7**

(c) 20 **(d) 28**

R [NCERT Exemp.]

Sol. Correct option : (d)

Explanation : In the given *A.P.*, $d = -4$, $n = 7$, $a_n = 4$

$a_n = a + (n-1)d \Rightarrow 4 = a + (7-1)(-4) \Rightarrow a = 28$

Q. 4. In an A.P, if $a = 3.5$, $d = 0$, $n = 101$, then a_n will be

(a) 0 **(b) 3.5**

(c) 103.5 **(d) 104.5**

R [NCERT Exemp.]

Sol. Correct option : (b)

Explanation : In the given A.P., $a = 3.5$, $d = 0$, $n = 101$

$a_n = a + (n-1)d \Rightarrow a_n = 3.5 + (101-1)0 \Rightarrow a_n = 3.5$

Q. 5. The list of numbers – 10, – 6, – 2, 2,…… is :

(a) an A.P, with $d = -16$

(b) an A.P, with $d = 4$

(c) an A.P, with $d = -4$

(d) not an A.P, U [NCERT Exemp.]

Sol. Correct option : (b)

Explanation : In the given numbers – 10, – 6, – 2, 2,…

$(-6) - (-10) = 4$

$(-2) - (-6) = 4$

$2 - (-2) = 4$

Since, $(-6) - (-10) = (-2) - (-6) = 2 - (-2) = 4$,

thus, the given numbers are in AP with $d = 4$.

Q. 6. The 11^{th} term of the A.P., : $-5, -\frac{5}{2}, 0, \frac{5}{2}, \ldots$ is :

(a) –20 **(b) 20**

(c) –30 **(d) 30**

R [NCERT Exemp.]

Sol. Correct option : (b)

Explanation : In the given A.P.,

$a = -5$, $d = -\frac{5}{2} - (-5) = \frac{5}{2}$, $n = 11$

$t_n = a + (n-1)d \Rightarrow t_{11} = -5 + (11-1)\left(\frac{5}{2}\right) \Rightarrow t_{11} = 20$

Q. 7. The first four terms of an A.P, whose first term is –2 and the common difference is –2, are :

(a) – 2, 0, 2, 4 **(b) – 2, 4, – 8, 16**

(c) – 2, – 4, – 6, – 8 **(d) – 2, – 4, – 8, –16**

U [NCERT Exemp.]

Sol. Correct option : (c)

Explanation : In the given AP, $a = -2$, $d = -2$,

$t_n = a + (n-1)d$

$t_1 = (-2) + (1-1)(-2) = -2$

$t_2 = (-2) + (2-1)(-2) = -4$

$t_3 = (-2) + (3-1)(-2) = -6$

$t_4 = (-2) + (4-1)(-2) = -8$

Q. 8. The 21^{st} term of the A.P, whose first two terms are –3 and 4 is :

(a) 17 **(b) 137**

(c) 143 **(d) –143**

R [NCERT Exemp.]

Sol. Correct option : (b)

Explanation : In the given A.P., $t_1 = -3$ and $t_2 = 4$

$\Rightarrow \quad d = t_2 - t_1 = 4 - (-3) = 7$

$t_n = a + (n-1)d$

$\Rightarrow \quad t_{21} = (-3) + (21-1)(7) = 137$

Q. 9. If the 2^{nd} term of an A.P, is 13 and the 5th term is 25, what is its 7th term?

(a) 30 **(b) 33**

(c) 37 **(d) 38**

R [NCERT Exemp.]

Sol. Correct option : (b)

Explanation : In the given A.P.,, $t_2 = 13$ and $t_5 = 25$

$a + d = 13$

$a + 4d = 25$

Solving these equations, we get $a = 9$ and $d = 4$

Thus,

$t_n = a + (n-1)d$

$\Rightarrow \quad t_7 = 9 + (7-1)4 = 33$

Q. 10. Which term of the A.P, : 21, 42, 63, 84,... is 210?

(a) 9th **(b) 10th**

(c) 11th **(d) 12th**

R [NCERT Exemp.]

Sol. Correct option : (b)

Explanation : In the given A.P., $a = 21$, $d = 42 - 21 = 21$, and $t_n = 210$

Thus,

$$t_n = a + (n-1)d$$

$$\Rightarrow \quad 210 = 21 + (n-1)21$$

$$\Rightarrow \quad n = 10$$

Q. 11. If the common difference of an A.P., is 5, then what is $a_{18} - a_{13}$?

(a) 5 **(b) 20**

(c) 25 **(d) 30**

R [NCERT Exemp.]

Sol. Correct option : (c)

Explanation : In the given A.P., $d = 5$ Thus,

$$a_{18} - a_{13} = a + 17d - a - 12d = 5d = 25$$

Q. 12. What is the common difference of an A.P., in which $a_{18} - a_{14} = 32$?

(a) 8 **(b) – 8**

(c) – 4 **(d) 4**

R [NCERT Exemp.]

Sol. Correct option : (a)

Explanation : In the given A.P., $a_{18} - a_{14} = 32$

Thus,

$$a_{18} - a_{14} = 32$$

$$\Rightarrow \quad a + 17d - a - 13d = 32$$

$$\Rightarrow \quad 4d = 32$$

$$\Rightarrow \quad d = 8$$

Q. 13. Two *A.P*s. have the same common difference. The first term of one of these is –1 and that of the other is – 8. Then the difference between their 4th terms is :

(a) –1 **(b) 8**

(c) 7 **(d) –9**

U [NCERT Exemp.]

Sol. Correct option : (c)

Explanation : Let a_1 and a_2 be the first terms of the two *A.P*s. with the same common difference.

Since $a_1 = -1$ and $a_2 = -8$,

$$t_4 - t'_4 = (-1 + 3d) - (-8 + 3d) = 7$$

Q. 14. If 7 times the 7^{th} term of an AP is equal to 11 times its 11th term, then its 18^{th} term will be :

(a) 7 **(b) 11**

(c) 18 **(d) 0**

R [NCERT Exemp.]

Sol. Correct option : (d)

Explanation : According to question,

$$7t_7 = 11t_{11}$$

$$\Rightarrow \quad 7(a + 6d) = 11(a + 10d)$$

$$\Rightarrow \quad 4a + 68d = 0$$

$$\Rightarrow \quad 4(a + 17d) = 0$$

$$\Rightarrow \quad (a + 17d) = 0$$

$$\Rightarrow \quad t_{18} = 0$$

Q. 15. The 4^{th} term from the end of the A.P., : –11, –8, –5,..., 49 is :

(a) 37 **(b) 40**

(c) 43 **(d) 58**

R [NCERT Exemp.]

Sol. Correct option : (b)

Explanation : In the given *A.P.*, the last term $l = 49$ and common difference $d = -8 + 11 = 3$

4th term from last is $t_4 = 49 - (4-1) \times 3 = 40$

Q. 16. The common difference of the A.P. : $\frac{1}{P}, \frac{1-p}{P}, \frac{1-2p}{P}$,... is :

(a) p **(b) $-p$**

(c) – 1 **(d) 1**

R [NCERT Exemp.]

Sol. Correct option : (c)

Sol. *Explanation :* Common difference,

$$d = \frac{1-p}{p} - \frac{1}{p}$$

$$= \frac{1-p-1}{p}$$

$$= \frac{-p}{p} = -1$$

[B] Very Short Answer Type Questions :

Q. 1. If the nth term of an A.P. – 1, 4, 9, 14, ... is 129. Find the value of *n*.

A [Board Outside Delhi Compt. Set I, II, III 2017]

Sol. Given, $a = -1$ and $d = 4 - (-1) = 5$

$$a_n = -1 + (n-1) \times 5 = 129 \quad ½$$

or, $(n-1)5 = 130$

$(n-1) = 26$

$n = 27$

Hence, 27^{th} term = 129. ½

[CBSE Marking Scheme, 2018]

Q. 2. Which of the term of A.P. 5, 2, –1,..... is – 49 ?

U [Board Term-2, 2012 Set (31)]

Sol. Here, $a = 5$ and $d = -3$

$\because \quad l = a + (n-1)d$

$\therefore \quad -49 = 5 + (n-1)(-3)$

or, $-49 = 5 - 3n + 3$

or, $3n = 49 + 5 + 3$

or, $n = \frac{57}{3} = 19^{th}$ term. **1**

[CBSE Marking Scheme, 2012]

Q. 3. Find the first four terms of an A.P. whose first term is – 2 and common difference is – 2.

U [Board Term-2, 2012 Set (17)]

Sol. $a_1 = -2$,

$a_2 = a_1 + d = -2 + (-2) = -4$

$a_3 = a_2 + d = -4 + (-2) = -6$
$a_4 = a_3 + d = -6 + (-2) = -8$
$\therefore$ First four terms are $-2, -4, -6$ and -8 **1**
[CBSE Marking Scheme, 2012]

Q. 4. Find the tenth term of the sequence $\sqrt{2}, \sqrt{8}, \sqrt{18}, \ldots$. U **[Board Sample paper, 2016]**

Sol. Given sequence is an A.P.

$\sqrt{2}, \sqrt{8}, \sqrt{18}, \ldots$
$= \sqrt{2}, 2\sqrt{2}, 3\sqrt{2} \ldots$

Hence, $a = \sqrt{2}$, $d = \sqrt{2}$ and $n = 10$
$\because$ $a_n = a + (n-1)d$
or, $a_{10} = \sqrt{2} + (10-1)\sqrt{2}$
$= \sqrt{2} + 9\sqrt{2}$
$= 10\sqrt{2}$

Hence, $a_{10} = \sqrt{200}$. **1**

Q. 5. In an A.P., if the common difference (d) = – 4, and the seventh term (a_7) is 4, then find the first term.
U **[Delhi/OD. Set, 2018]**

Sol. Since, $a + 6(-4) = 4$
$\Rightarrow$ $a = 28$ **1**
[CBSE Marking Scheme, 2012]

Detailed Answer :
Given $d = -4$ and $a_7 = 4$
Since, n^{th} term of A.P. is
$a_n = a + (n-1)d$ ½
Then, $a_7 = a + (7-1)d$ $[n = 7]$
$4 = a + 6(-4)$
$a = 4 + 24 = 28$
Hence, first term of an A.P. = 28. ½

Q. 6. Is series $\sqrt{3}, \sqrt{6}, \sqrt{9}, \sqrt{12}, \ldots$ an A.P. ? Give reason.
U **[Board Term-2, 2015]**

Sol. Common difference,
$d = \sqrt{6} - \sqrt{3}$
$= \sqrt{3}\left(\sqrt{2} - 1\right)$
Again, $d = \sqrt{9} - \sqrt{6}$
$= 3 - \sqrt{6}$
$= \sqrt{12} - \sqrt{9} = 2\sqrt{3} - 3$
As common difference are not equal.
Hence, the given series is not in A.P.
[CBSE Marking Scheme, 2015] **1**

Q. 7. What is the next term of an A.P. $\sqrt{7}, \sqrt{28}, \sqrt{63}, \ldots$? U **[Foreign Set I, II, III, 2014]**

Sol. Here, $a = \sqrt{7}$ and $a + d = \sqrt{28}$
$\therefore$ $d = \sqrt{28} - \sqrt{7} = 2\sqrt{7} - \sqrt{7}$
$= \sqrt{7}$
or, Next term $= \sqrt{63} + \sqrt{7}$
or, $= 3\sqrt{7} + \sqrt{7} = 4\sqrt{7}$
or, $= \sqrt{7 \times 16}$ **1**
$= \sqrt{112}$.

Q. 8. Which term of the A.P. 8, 14, 20, 26, ... will be 72 more than its 41st term.
A **[Board Outside Delhi Set-II 2017]**

Sol. Given $a = 8$ and $d = 6$.
Let n^{th} term be 72 more than its 41^{th} term.
$\therefore$ $t_n - t_{41} = 72$
$8 + (n-1)6 - (8 + 40 \times 6) = 72$
$8 + (n-1)6 = 320$
$(n-1)6 = 312$
$n - 1 = 52$
$n = 53$ **1**

AI **Q. 9. Write the n^{th} term of the A.P. $\frac{1}{m}, \frac{1+m}{m}, \frac{1+2m}{m}, \ldots$.**
A **[Board Outside Delhi Compt. Set-I, II, III 2017]**

Sol. We have, $a = \frac{1}{m}$
$d = \frac{1+m}{m} - \frac{1}{m} = 1$
$\therefore$ $a_n = \frac{1}{m} + (n-1)1$
Hence, $a_n = \frac{1}{m} + n - 1 = \frac{1 + (n-1)m}{m}$ **1**

Q. 10. Find the 25th term of the A.P. $-5, \frac{-5}{2}, 0, \frac{5}{2}, \ldots$.
U **[Foreign Set I, II, III, 2015]**

Sol. Here, $a = -5$ and $d = -\frac{5}{2} - (-5) = \frac{5}{2}$
Since, n^{th} term $= a + (n-1)d$
Then, 25^{th} term $= -5 + (25-1) \times \left(\frac{5}{2}\right)$
$= -5 + 60$
$= 55$ **1**
[CBSE Marking Scheme, 2015]

Q. 11. The first three terms of an A.P. are $3y - 1$, $3y + 5$ and $5y + 1$ respectively then find y.
A **[Delhi CBSE Term-2, 2014]**

Sol. Given, $3y - 1$, $3y + 5$ and $5y + 1$ in A.P.
$\therefore$ $(3y + 5) - (3y - 1) = (5y + 1) - (3y + 5)$
or, $3y + 5 - 3y + 1 = 5y + 1 - 3y - 5$
or, $6 = 2y - 4$
or, $2y = 6 + 4$
or, $y = \frac{10}{2}$
$y = 5$ **1**

Q. 12. For what value of k will $k + 9$, $2k - 1$ and $2k + 7$ are the consecutive terms of an A.P. ?

C + A **[Outside Delhi Set II, 2016]**

Sol.

We have-
Three consecutive terms of AP = $k+9, 2k-1, 2k+7$
Then,
$(k+9)(2k+7) = 2(2k-1)$ $\{(a+c = 2b)\}$
$\Rightarrow k+9+2k+7 = 4k-2$
$3k+16 = 4k-2$
$16+2 = 4k-3k$
$18 = k$

[Topper Answer, 2016]

Q. 13. What is the common difference of an A.P. in which $a_{21} - a_7 = 84$? A **[Board Outside Delhi Set-I, II, III, 2017]**

Sol.

4. let a be 1st term and d be the common difference.
$a_{21} - a_7 = 84$
$a + (21-1)d - [a + (7-1)d] = 84$
$a + 20d - a - 6d = 84$
$14d = 84$
$d = 6$
$\therefore$ common difference is 6.

[Topper Answer, 2017] 1

Q. 14. In the A.P. 2, x, 26, then find the value of x.

C + A **[Board Term-2, 2012(13)]**

Sol. Since, 2, x and 26 are in A.P.

$\therefore \quad x - 2 = 26 - x$

or, $\quad 2x = 26 + 2$

or, $\quad x = \frac{28}{2} = 14$ **1**

[CBSE Marking Scheme, 2012]

Q. 15. If 18, a, b, – 3 are in A.P., then find $a + b$.

A **[Board Term-2, 2012 Set (34)]**

Sol. Since 18, a, b, and – 3 are in A.P., Then

$a - 18 = -3 - b$

or, $\quad a + b = -3 + 18$

or, $\quad a + b = 15$ **1**

[CBSE Marking Scheme, 2012]

AI **Q. 16. Find the common difference of the A.P. $\frac{1}{3q}, \frac{1-6q}{3q}, \frac{1-12q}{3q}, \ldots\ldots$.**

A **[Board Term-2, Delhi 2013]**

Sol. Here,

$a = \frac{1}{3q}$ and $a + d = \frac{1-6q}{3q}$

$\therefore \quad d = \frac{1-6q}{3q} - \frac{1}{3q}$

$= \frac{1-6q-1}{3q} = \frac{-6q}{3q} = -2$ **1**

[CBSE Marking Scheme, 2013] 1

Q. 17. Find the first four terms of an A.P. whose first term is $3x + y$ and common difference is $x - y$.

A **[Board Term-2, 2012, Set (25)]**

Sol. Given, $a_1 = 3x + y$

$a_2 = a_1 + d = 3x + y + x - y = 4x$

$a_3 = a_2 + d = 4x + x - y = 5x - y$

and $a_4 = a_3 + d = 5x - y + x - y$

$= 6x - 2y$

So, the four terms are $3x + y$, $4x$, $5x - y$ and $6x - 2y$.

[CBSE Marking Scheme, 2012] 1

Q. 18. Find the 37th term of the A.P. $\sqrt{x}, 3\sqrt{x}, 5\sqrt{x}, \ldots\ldots$.

U **[Board Term-2, 2012 Set (50)]**

Sol. Try yourself, Similar to Q. No. 4 in VSATQ.

Short Answer Type Questions-I (2 marks each)

Q. 1. Find, 100 is a term of the A.P. 25, 28, 31, ... or not. U [Board Term-2, 2012 (12)]

Sol. 25, 28, 31, ... 100. **1**

$a = 25$ and $d = 3$

Let the number of terms be "n".

$\therefore \quad 25 + (n-1) \times 3 = 100$

or, $(n-1) \times 3 = 75$ ½

or, $n = 26$

Hence, 100 is a term of the given A.P. ½

[CBSE Marking Scheme, 2012]

Q. 2. Is 184 a term of the sequence 3, 7, 11, ... ?

U [Board Term-2, 2012 (44)]

Sol. Here, $a = 3$ and $d = 7 - 3 = 11 - 7 = 4$ ½

Since, $a_n = a + (n-1)d$,

Let $a_n = 184$ ½

Then, $184 = 3 + (n-1)\,4$ ½

or, $\frac{181}{4} = n - 1$

or, $45.25 = n - 1$

or, $46.25 = n$, it is not an whole numbers

Hence, 184 is not a term of given A.P. ½

[CBSE Marking Scheme, 2012]

Q. 3. Find the 7th term from the end of A.P. 7, 10, 13, ... 184. [Delhi Set 2014]

A [Board Term-2, 2012 Set (34)]

Sol. Let us write A.P. in reverse order

i.e., 184, ... 13, 10, 7. ½

$d = 7 - 10 = -3$ ½

$a = 184$, $n = 7$

$l_7 = a + 6d$

$l_7 = 184 + 6(-3)$

$= 184 - 18 = 166.$ ½

Hence, 166 is the 7th term from the end. ½

[CBSE Marking Scheme, 2014, 2012]

Q. 4. Which term of the A.P. 3, 12, 21, 30..... will be 90 more than its 50th term.

A [Board Compt. Set-III 2017]

Sol. Given, $a = 3$ and $d = 9$

$\because \quad a_n = a + (n-1)d$ **1**

$\therefore \quad a_{50} = 3 + 49 \times 9 = 444$

Now, $a_n - a_{50} = 90$ ½

$3 + (n-1)9 - 444 = 90$

$(n-1)9 = 90 + 441$

$(n-1) = \frac{531}{9} = 59$

$n = 59 + 1 = 60$ ½

[CBSE Marking Scheme, 2017]

AI **Q. 5. In a certain A.P. 32th term is twice the 12th term. Prove that 70th term is twice the 31st term.**

A [Board Term-2, 2015, 2012, Set 28]

Sol. Let the 1st term be a and common difference be d.

According to the question, $a_{32} = 2a_{12}$

$\therefore \quad a + 31d = 2(a + 11d)$

$a + 31d = 2a + 22d$

$a = 9d$ **1**

Again, $a_{70} = a + 69d$

$= 9d + 69d = 78d$

$\because \quad a_{31} = a + 30d$

$= 9d + 30d = 39d$

Hence, $a_{70} = 2a_{31}$ **Hence Proved. 1**

[CBSE Marking Scheme, 2015]

Q. 6. The 8th term of an A.P. is zero. Prove that its 38th term is triple of its 18th term.

A [Board Term-2, 2012 (28)]

Sol. Given, $a_8 = 0$ or, $a + 7d = 0$ or, $a = -7d$ ½

or, $a_{38} = a + 37d$

or, $a_{38} = -7d + 37d = 30d$ ½

And, $a_{18} = a + 17d$

$= -7d + 17d = 10d$ ½

or, $a_{38} = 30d = 3 \times 10d = 3 \times a_{18}$

$\therefore \quad a_{38} = 3a_{18}$. **Hence Proved.** ½

[CBSE Marking Scheme, 2012]

Q. 7. If five times the fifth term of an A.P. is equal to eight times its eighth term, show that its 13th term is zero. A [Board Term-2, 2012 (13)]

Sol. Let the first term be a and common difference d.

Given $5a_5 = 8a_8$

or, $5(a + 4d) = 8(a + 7d)$ **1**

or, $5a + 20d = 8a + 56d$

or, $3a + 36d = 0$

or, $3(a + 12d) = 0$

or, $a + 12d = 0$

$\therefore \quad a_{13} = 0.$ **1**

[CBSE Marking Scheme, 2012]

Q. 8. The fifth term of an A.P. is 20 and the sum of its seventh and eleventh terms is 64. Find the common difference. A [Foreign Set II, 2015]

Sol. Let the first term be a and common difference be d.

Then, $a + 4d = 20$ ...(i) ½

and $a + 6d + a + 10d = 64$

$a + 8d = 32$ ...(ii)**1**

Solving equations (i) and (ii), we get

Hence, common difference, $d = 3$ ½

[CBSE Marking Scheme, 2015]

Q. 9. The ninth term of an A.P. is – 32 and the sum of its eleventh and thirteenth term is – 94. Find the common difference of the A.P. .

A [Foreign Set III, 2015]

Sol. Try yourself, Similar to Q. 8. in SATQ-I

Q. 10. The seventeenth term of an A.P. exceeds its 10^{th} term by 7. Find the common difference.

A [Board Term-2, 2015, 14]

Sol. Let the first term be a and common difference be d.

Here, $a_{17} = a + 16d$ and $a_{10} = a + 9d$

Given that,

$a + 16d = a + 9d + 7$ **1**

or, $16d - 9d = 7$

or, $7d = 7$

or, $d = 1$

$\therefore$ The common difference, $d = 1$. **1**

[CBSE Marking Scheme, 2015]

Q. 11. The fourth term of an A.P. is 11. The sum of the fifth and seventh terms of the A.P. is 34. Find the common difference. A [Foreign Set I, 2015]

Sol. Try yourself, Similar to Q. 8. in SATQ-I

Q. 12. Find the middle term of the A.P. 213, 205, 197, ... 37. A [Board Term-2, Delhi 2015 (Set II)]

Sol. Here, $a = 213$, $d = 205 - 213 = -8$ and $l = 37$

Let the number of terms be n.

$\because$ $l = a + (n-1)d$

$\therefore$ $37 = 213 + (n-1)(-8)$

or, $37 - 213 = -8(n-1)$

or, $n - 1 = \frac{-176}{-8} = 22$ ½

or, $n = 22 + 1 = 23$ ½

The middle term will be $= \frac{23+1}{2} = 12^{th}$ ½

$\therefore$ $a_{12} = a + (n-1)d$

$= 213 + (12-1)(-8)$

$= 213 - 88$

$= 125$

Thus, the middle term will be 125. ½

[CBSE Marking Scheme, 2015]

AI **Q. 13. The 10th term of an A.P. is – 4 and its 22^{nd} term is (– 16). Find its 38^{th} term.**

A [Board Delhi compt. Set-I, 2017]

Sol. Try yourself, Similar to Q. 14. in SATQ-I

Q. 14. If the 2^{nd} term of an A.P. is 8 and the 5^{th} term is 17, find its 19^{th} term.

A [Board Term-2, 2016 Set HODM40L]

Sol. Let 1^{st} term be a and common difference be d.

$a_2 = a + d$

$a + d = 8$ ...(i)

and $a_5 = a + 4d$

$a + 4d = 17$ ...(ii)

From (i) and (ii),

$a = 5$ and $d = 3$, **1**

Then, $a_{19} = a + 18d$

$= 5 + 54 = 59$ **1**

[CBSE Marking Scheme, 2016]

Q. 15. If the numbers $x + 3$, $2x + 1$ and $x - 7$ are in A.P, find the value of x. A [Board Term-2 2012 (5)]

Sol. Since, $(2x + 1) - (x + 3) = (x - 7) - (2x + 1)$ **1**

or, $2x + 1 - x - 3 = x - 7 - 2x - 1$

or, $x - 2 = -x - 8$ ½

or, $2x = -6$

or, $x = -3$. ½

[CBSE Marking Scheme, 2012]

Q. 16. Find how many integers between 200 and 500 are divisible by 8.

A [Board Delhi compt. Set-I, II III 2017]

Sol. Integers divisible by 8 are 208, 216, 224, ..., 496. **1**

Which is an A.P.

Given $a = 208$, $d = 8$ and $l = 496$

Let the numbers of terms in A.P. be n.

$\because$ $a_n = a + (n-1)d = l$

$\therefore$ $208 + (n-1)d = 496$

$(n-1)8 = 496 - 208$ ½

$n - 1 = \frac{288}{8} = 36$

$n = 36 + 1 = 37$ ½

Hence, required integers divisible by 8 = 37.

Q. 17. For A.P. show that $a_p + a_{p+2q} = 2a_{p+q}$.

A [Board Term-2, 2012 (1)]

Sol. Let the first term be a and the common difference be d.

$a_p + a_{p+2q} = a + (p-1)d + a + (p + 2q - 1)d$

$= a + pd - d + a + pd + 2qd - d$

$= 2a + 2pd + 2qd - 2d$ **1**

$= 2[a + (p + q - 1)d]$ ...(i) ½

$2a_{p+q} = 2[a + (p + q - 1)d]$ ...(ii) ½

From (i) and (ii), we get

$a_p + a_{p+2q} = 2a_{p+q}$. **Hence proved.**

[CBSE Marking Scheme, 2012]

Q. 18. The fifth term of an A.P. is 26 and its 10th term is 51. Find the A.P.

A [Outside Delhi Compt. Set-II 2017]

Sol. Here, $a_5 = a + 4d = 26$ ...(i) ½

and $a_{10} = a + 9d = 51$ ...(ii) ½

Solving Eqns. (i) and (ii), we get

or, $5d = 25$

$d = 5$ ½

and $a = 6$

Hence, the A.P. is 6, 11, 17 ..... ½

[CBSE Marking Scheme, 2017]

Q. 19. The 4^{th} term of an A.P. is zero. Prove that the 25^{th} term of the A.P. is three times its 11^{th} term.

U [Outside Delhi Set, II 2016]

Sol. Try yourself, Similar to Q. 6. in SATQ-I

Q. 20. Find the 20^{th} term from the last term of the A.P. 3, 8, 13, ... 253. A [CBSE SQP-2018]

Sol. 20^{th} term from the end $= l - (n-1)d$ ½

$= 253 - 19 \times 5$ **1**

$= 158$ ½

[CBSE Marking Scheme, 2016]

Q. 21. If 7 times the 7^{th} term of an A.P. is equal to 11 times its 11^{th} term, then find its 18th term.

A [CBSE SQP-2018] [Foreign Board-2017]

Sol. $7a_7 = 11a_{11}$

$\Rightarrow 7(a + 6d) = 11(a + 10d)$ **1**

$\Rightarrow a + 17d = 0$

$\therefore a_{18} = 0$ **1**

[CBSE Marking Scheme, 2016]

Q. 22. Find whether – 150 is a term of the A.P. 11, 8, 5, 2 ....

A **[Board Delhi Compt. Set-I 2017]**

Sol. Try yourself, Similar to Q. 2. in SATQ-I

Q. 23. Find the number of natural numbers between 101 and 999 which are divisible by both 2 and 5.

A **[CBSE O.D. 2014]**

Sol. The sequence goes like this,

110, 120, 130,....., 990

Since, they have a common difference of 10, they form an A.P. $a = 110$, $a_n = 990$ and $d = 10$ ½

$\because a_n = a + (n-1) \times d$

$\therefore 990 = 110 + (n-1) \times 10$

or, $990 - 110 = (n-1) \times 10$ ½

or, $880 = (n-1) \times 10$

or, $n - 1 = 88$

or, $n = 89$ ½

Hence, there are 89 terms between 101 and 999 which are divisible by both 2 and 5. ½

AI **Q. 24. How many three digit natural numbers are divisible by 7 ?** A **[Board Term-2, 2013]**

Sol. Let A.P. be 105, 112, 119, ..., 994, which is divisible by 7.

Here, $a = 105$, $d = 112 - 105 = 7$ and $a_n = 994$,

Since, $a_n = a + (n-1)d$ ½

or, $994 = 105 + (n-1) \times 7$

or, $889 = (n-1) \times 7$ ½

or, $n - 1 = \frac{889}{7} = 127$ ½

or, $n = 127 + 1 = 128.$ ½

Hence, there will be 128 three digits numbers divisible by 7 in A.P.

Q. 25. How many two digit numbers are divisible by 7 ?

A **[Board Sample paper, 2016]**

Sol. Two digit numbers which are divisible by 7 are 14, 21, 28, ..., 98. ½

It forms an A.P.

Here, $a = 14$, $d = 7$ and $a_n = 98$ ½

Since, $a_n = a + (n-1)d$

$98 = 14 + (n-1)7$ ½

$98 - 14 = 7n - 7$

$84 + 7 = 7n$

or, $7n = 91$

or, $n = 13$ ½

[CBSE Marking Scheme, 2016]

Short Answer Type Questions-II

(3 marks each)

Q. 1. Find the 20th term of an A.P. whose 3rd term is 7 and the seventh term exceeds three times the 3rd term by 2. Also, find its n^{th} term (a_n).

A **[Board Term-2, 2012 (31)]**

Sol. Let the first term be a and the common difference be d.

Given, $a_3 = a + 2d = 7$ ...(i)

According to the problem, $a_7 - 3a_3 = 2$

and, $a_7 - 3 \times 7 = 2$ **1**

or, $a + 6d - 21 = 2$

$a + 6d = 23$ ...(ii)

Solving eq. (i) and (ii),

$d = 4$ and $a = -1$

Now, $a_{20} = a + 19d$ **1**

$= -1 + 76 = 75$

Again, $a_n = a + (n-1)\,d$

$= -1 + 4n - 4$

$= 4n - 5.$

Hence, n^{th} term $= 4n - 5.$ **1**

Q. 2. If 7th term of an A.P. is $\frac{1}{9}$ and 9th term is $\frac{1}{7}$, find 63rd term. U **[Board Term-2, Delhi, 2014]**

Sol. Let the first term be a and the common difference be d of the given A.P.

Given, $a_7 = \frac{1}{9}$

or, $a + 6d = \frac{1}{9}$ ... (i) **1**

and $a_9 = \frac{1}{7}$

or, $a + 8d = \frac{1}{7}$ ... (ii) **1**

On subtracting eqn. (i) from (ii),

$a + 8d - a - 6d = \frac{1}{7} - \frac{1}{9}$

or, $2d = \frac{2}{63}$

or, $d = \frac{1}{63}$

Substituting the value of d in (ii) we get,

$a + 8 \times \frac{1}{63} = \frac{1}{7}$

or, $a = \frac{1}{7} - \frac{8}{63}$

or, $a = \frac{9-8}{63} = \frac{1}{63}$

$\therefore a_{63} = \frac{1}{63} + 62 \times \frac{1}{63} = \frac{1+62}{63}$

or, $a_{63} = \frac{63}{63} = 1$ **1**

Hence, $a_{63} = 1$.

Q. 3. The ninth term of an A.P. is equal to seven times the second term and twelfth term exceeds five times the third term by 2. Find the first term and the common difference.

A **[Board Sample Paper, 2016]**

Sol. Let the first term of A.P. be a and common difference be d.

Given, $a_9 = 7a_2$

or, $a + 8d = 7(a + d)$ ...(i) ½

and $a_{12} = 5a_3 + 2$

Again, $a + 11d = 5(a + 2d) + 2$ ...(ii) **1**

From (i), $a + 8d = 7a + 7d$

$-6a + d = 0$ ...(iii)

From (ii), $a + 11d = 5a + 10d + 2$

$-4a + d = 2$ ...(iv)

Subtracting (iv) from (iii), we get

$-2a = -2$

or, $a = 1$ **1**

From (iii),

$-6 + d = 0$

$d = 6$ ½

Hence, first term = 1 and common difference = 6

[CBSE Marking Scheme, 2016]

Q. 4. Determine an A.P. whose third term is 9 and when fifth term is subtracted from 8th term, we get 6.

A **[Board Term-2, 2015]**

Sol. Let the first term be a and the common difference be d.

Given, $a_3 = 9$ or, $a + 2d = 9$ ....(i)

and $a_8 - a_5 = 6$

or, $(a + 7d) - (a + 4d) = 6$

or, $3d = 6$

or, $d = 2$ ....(ii) **1**

Substituting the value of d in equation (i), we get

or, $a + 2(2) = 9$

or, $a = 5$ **1**

So, A.P. is 5, 7, 9, 11, ... **1**

[CBSE Marking Scheme, 2015]

Q. 5. Divide 56 in four parts in A.P. such that the ratio of the product of their extremes (1st and 4th) to the product of middle (2nd and 3rd) is 5 : 6.

U **[Foreign Set I, 2016]**

Sol. Let the four parts be

$a - 3d, a - d, a + d$ and $a + 3d$.

$\therefore a - 3d + a - d + a + d + a + 3d = 56$

or, $4a = 56$

$a = 14$ **1**

Hence, four parts are $14 - 3d$, $14 - d$, $14 + d$ and $14 + 3d$.

Now, according to question,

$$\frac{(14-3d)(14+3d)}{(14-d)(14+d)} = \frac{5}{6}$$

or, $$\frac{196-9d^2}{196-d^2} = \frac{5}{6}$$

or, $6(196 - 9d^2) = 5(196 - d^2)$

or, $6 \times 196 - 54d^2 = 5 \times 196 - 5d^2$

or, $6 \times 196 - 5 \times 196 = 54d^2 - 5d^2$

or, $(6 - 5) \times 196 = 49d^2$

or, $d^2 = \frac{196}{49} = 4$

or, $d = \pm 2$ **1**

$\therefore$ The four parts are

$\{14 - 3(\pm 2)\}, \{14 - (\pm 2)\}$

Hence, first possible division will be 8, 12, 16 and 20. **1**

and second possible division will be 20, 16, 12 and 8. **1**

Q. 6. The p^{th}, q^{th} and r^{th} terms of an A.P. are a, b and c respectively. Show that $a(q - r) + b(r - p) + c(p - q) = 0$.

U **[Foreign Set II, 2016]**

Sol. Let the first term be a' and the common difference be d.

$a = a' + (p - 1)d$, $b = a' + (q - 1)d$ and $c = a' + (r - 1)d$ **1½**

$a(q - r) = [a' + (p - 1)d][q - r]$

$b(r - p) = [a' + (q - 1)d][r - p]$

and $c(p - q) = [a' + (r - 1)d][p - q]$ ½

$\therefore a(q - r) + b(r - p) + c(p - q) = a'[q - r + r - p + p - q] + d[(p - 1)(q - r) + (q - 1)(r - p) + (r - 1)(p - q)]$ ½

$= a' \times 0 + d[pq - pr + qr - pq + pr - qr + (-q + r - r + p - p + q)] = 0$ **Hence Proved.** ½

[CBSE Marking Scheme, 2016]

Q. 7. The digits of a positive number of three digit number are in A.P. and their sum is 15. The number obtained by reversing the digits is 594 less then the original number. Find the number. A **[Outside Delhi Set II, 2016]**

Sol. Let three digit of 3-digit no be $a-d, a, a+d$.

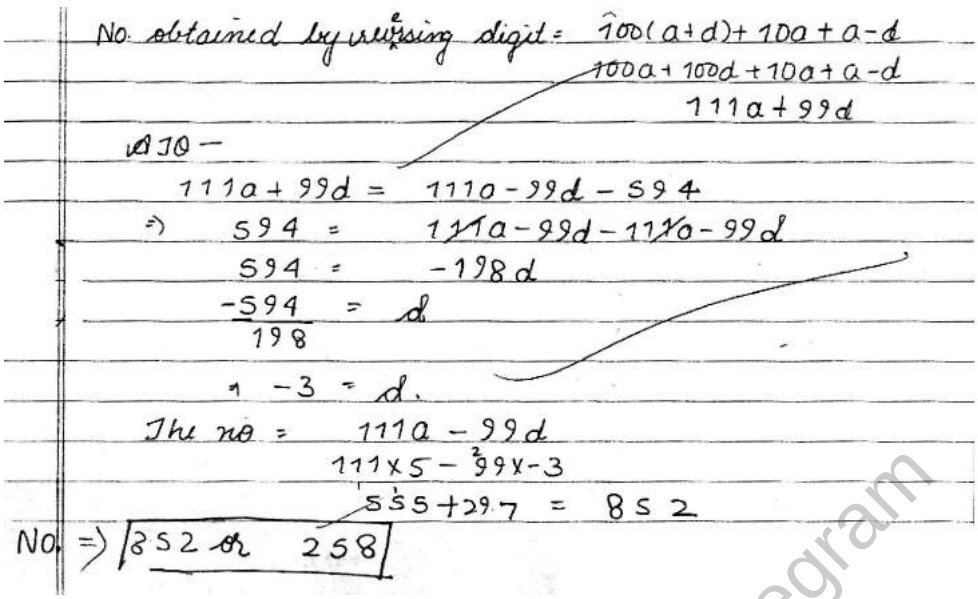
Their sum = 15

$a - d + a + a + d = 15 \Rightarrow 3a = 15 \Rightarrow a = 5$

Required 3 digit no $= 100(a-d) + 10a + a + d$

$100a - 100d + 10a + a + d$

$111a - 99d$

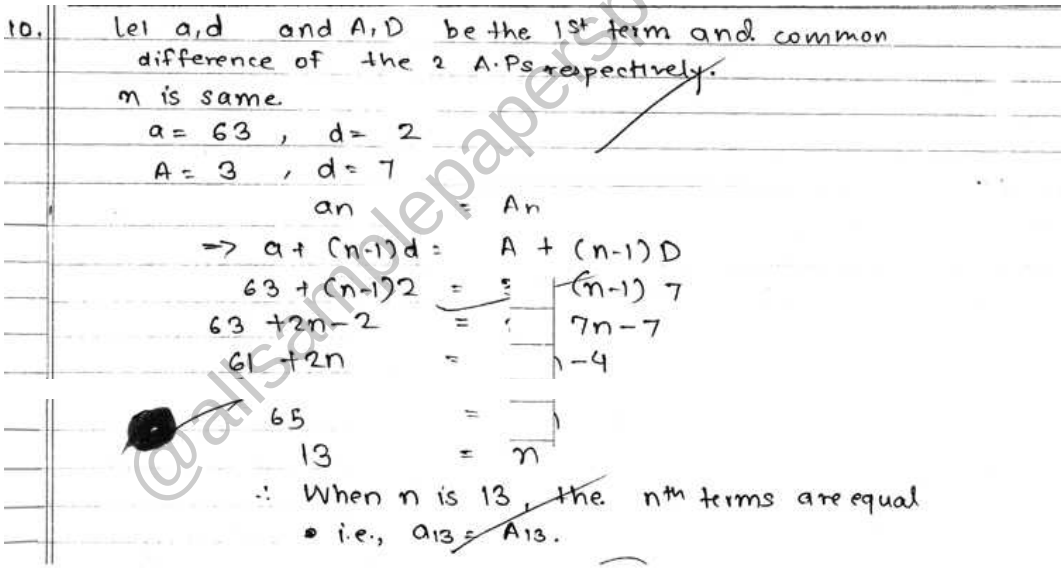


[Topper Answer, 2016] 3

Q. 8. For what value of n, are the n^{th} terms of two A.Ps 63, 65, 67,.... and 3, 10, 17,.... equal ?

C + A [Board Outside Delhi Set III, 2017]

Sol.



[Topper Answer, 2017] 3

Q. 9. If the tenth term of an A.P. is 52 and the 17th term is 20 more than the 13th term, find A.P.

A [Board Outside Delhi Set-I 2017]

Sol.

$a_{10} = 52$

or, $a + 9d = 52$ ...(i) **1**

Also $a_{17} - a_{13} = 20$

$a + 16d - (a + 12d) = 20$ ½

$4d = 20$

$d = 5$

Substituting, the value d in (i), we get

$a = 7$ **1**

Hence, A.P. = 7, 12, 17, 22 ..... ½

[CBSE Marking Scheme, 2017]

Q. 10. How many three digit numbers are such that when divided by 7, leave a remainder 3 in each case ?

[Board Term-2, 2012 Set (1)]

Sol. The three digit numbers are divided by 7 and leave 3 as remainder are

101, 108, 115, ... 997 **1**

Since these are in A.P. $a = 101$, $d = 7$, $a_n = 997$

$a_n = a + (n-1)d$

$997 = 101 + (n-1)7$

$997 - 101 = 896 = (n-1)7$ ½

$\frac{896}{7} = n - 1$ ½

$\therefore$ $n = 128 + 1 = 129$

Hence, 129 three digit numbers are divided by 7 which leaves remainder is 3. **1**

[CBSE Marking Scheme, 2012]

Q. 11. How many multiples of 4 lie between 11 and 266 ?

[Board Term-2, 2012 Set (21)] 3

Sol. Here, $a = 12$, $l = 264$ and $d = 4$

Let the number of multiples of 4 be n.

Then, $n = \frac{l-a}{d} + 1 = \frac{264-12}{4} + 1$ **1**

$= \frac{252}{4} + 1 = 63 + 1 = 64$ **1**

Hence, there are 64 multiples of 4 that lie between 11 and 266. **1**

Q. 12. Prove that the n^{th} term of an A.P. can not be $n^2 + 1$. Justify your answer. **[Board Term-2 2015]**

Sol. Let n^{th} term of A.P.,

$a_n = n^2 + 1$

Putting the values of $n = 1, 2, 3, ...$, we get

$a_1 = 1^2 + 1 = 2$

$a_2 = 2^2 + 1 = 5$

$a_3 = 3^2 + 1 = 10$ **1**

The obtained sequence

$= 2, 5, 10, 17, ...$

Their common difference

$= a_2 - a_1 = a_3 - a_2 = a_4 - a_3$

or, $5 - 2 \neq 10 - 5 \neq 17 - 10$

$\therefore$ $3 \neq 5 \neq 7$ **1**

Since the common difference are not equal.

Hence, $n^2 + 1$ is not a form of n^{th} term of an A.P. **1**

[CBSE Marking Scheme, 2015]

Long Answer Type Questions

(4 marks each)

Q. 1. The sum of three numbers in A.P. is 12 and sum of their cubes is 288. Find the numbers.

A **[Delhi Set III, 2016]**

Sol. Let the three numbers in A.P. be $a - d$, a and $a + d$.

Then, their sum *i.e.*, $3a = 12$

or, $a = 4$.

Also, $(4 - d)^3 + 4^3 + (4 + d)^3 = 288$ **1**

or, $64 - 48d + 12d^2 - d^3 + 64 + 64 + 48d + 12d^2 + d^3 = 288$

or, $24d^2 + 192 = 288$ **1**

or, $d^2 = 4$

$\therefore$ $d = \pm 2$ **1**

Hence, the numbers are 2, 4 and 6, or 6, 4 and 2. **1**

[CBSE Marking Scheme, 2016]

AI **Q. 2. Find the value of a, b and c such that the numbers a, 7, b, 23 and c are in A.P.** U **[Board Term-2, 2015]**

Sol. Since, a, 7, b, 23 and c are in A.P.

Let the common difference be d

$\therefore$ $a + d = 7$ **.....(i) ½**

and $a + 3d = 23$ **.....(ii) ½**

From (i) and (ii), we get

$a = -1$ and $d = 8$ **1**

Again, $b = a + 2d$

$b = -1 + 2 \times 8$

or, $b = -1 + 16$

or, $b = 15$ **½**

$\therefore$ $c = a + 4d$

$= -1 + 4 \times 8$

$= -1 + 32$

$c = 31$ **½**

$\therefore a = -1$, $b = 15$ and $c = 31$ **1**

[CBSE Marking Scheme, 2015]

Q. 3. An A.P. consists of 50 terms of which 3rd term is 12 and last term is 106. Find the 29th term.

U **[CBSE SQP-2018]**

Sol. Given, $n = 50$, $a_3 = 12$ and $a_{50} = 106$ **1**

Then $a + 2d = 12$ **1**

and $a + 49d = 106$ **1**

On solving, we get $d = 2$ and $a = 8$

$a_{29} = a + 28d$

$= 8 + 28 \times 2 = 64$ **1**

[CBSE Marking Scheme, 2018]

TOPIC-2
Sum of *n* Terms of an Arithmetic Progression

Know the Formulae

- Sum of n terms of an A.P is given by :

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

where, a is the first term, d is the common difference and n is the total number of terms.

- Sum of n terms of an A.P when first and last term is given.

$$S_n = \frac{n}{2}[a + l]$$

where, a is the first term and l is the last term.

➢ The n^{th} term of an A.P is the difference of the sum of first n terms and the sum to first $(n-1)$ terms of it. *i.e.*,

$$a_n = S_n - S_{n-1}.$$

How it is done on GREENBOARD ?

Q. Find the number of terms in the A.P 54, 51, 48, ... whose sum is 513.
Also, give the reason of double answer. U

Sol. **Step I :** The given A.P. is 54, 51, 48, ...
Here $a = 54$, $d = 51 - 54 = -3$
Sum required is 513.

Step II : Applying the sum formula

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$513 = \frac{n}{2}[2 \times 54 + (n-1)(-3)]$$

$$1026 = n[108 - 3n + 3]$$

$$1026 = n[111 - 3n]$$

$$1026 = 111n - 3n^2$$

or, $3n^2 - 111n + 1026 = 0$

or, $3[n^2 - 37n + 342] = 0$

or, $n^2 - 37n + 342 = 0$

Step III : Factorizing the quadratic equation

$n^2 - 19n - 18n + 342 = 0$

$n(n-19) - 18(n-19) = 0$

or, $(n-19)(n-18) = 0$

or, $n = 18$ or 19

Hence, the required number of terms will be 18 or 19.

Objective Type Questions (1 mark each)

[A] Multiple Choice Questions :

Q. 1. The famous mathematician associated with finding the sum of the first 100 natural numbers is :

(a) Pythagoras **(b) Newton**
(c) Gauss **(d) Euclid**

R **[NCERT Exemp.]**

Sol. Correct option : (c)
Explanation : The famous mathematician associated with finding the sum of the first 100 natural numbers is Gauss.

Q. 2. If the first term of an A.P. is –5 and the common difference is 2, then the sum of the first 6 terms is :

(a) 0 **(b) 5**
(c) 6 **(d) 15**

R **[NCERT Exemp.]**

Sol. Correct option : (a)
Explanation : In the given A.P., $a = -5$ and $d = 2$
Thus,

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$\Rightarrow \quad S_6 = \frac{6}{2}[2 \times (-5) + (6-1) \times 2]$$

$$= 0$$

Q. 3. The sum of first 16 terms of the A.P., : 10, 6, 2,... is :

(a) –320 **(b) 320**
(c) –352 **(d) –400**

R **[NCERT Exemp.]**

Sol. Correct option : (a)
Explanation : In the given A.P., $a = 10$, $d = 6 - 10 = -4$
Thus,

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$\Rightarrow S_{16} = \frac{16}{2}[2 \times 10 + (16-1) \times (-4)]$$

$$= -320$$

Q. 4. In an A.P., if $a = 1$, $a_n = 20$ and $S_n = 399$, then n is :

(a) 19 **(b) 21**
(c) 38 **(d) 42**

R **[NCERT Exemp.]**

Sol. Correct option : (c)
Explanation : In the given A.P., $a = 1$, $a_n = 20$ and $S_n = 399$

$$a_n = a + (n-1)d$$

$$\Rightarrow \quad 20 = 1 + (n-1)d$$

$$\Rightarrow (n-1)d = 19$$

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$\Rightarrow \quad 399 = \frac{n}{2}[2 + 19]$$

$$\Rightarrow \quad n = 38$$

Q. 5. The sum of first five multiples of 3 is :

(a) 45 **(b) 55**

(c) 65 **(d) 75**

U [NCERT Exemp.]

Sol. Correct option : (a)

Explanation : In the given AP, $a = 3$, $d = 3$ and $n = 5$

Thus,

$$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$$

$$\Rightarrow \quad S_5 = \frac{5}{2}\left[2\times 3 + (5-1)\times 3\right] = 45$$

Q. 6. The sum of first five positive integers divisible by 6 is :

(a) 180 **(b) 90**

(c) 45 **(d) 30**

R [NCERT Exemp.]

Sol. Correct option : (b)

Explanation : Positive integers divisible by 6 are 6, 12, 18, 24, 30

Since difference is same, its an AP

We need to find sum of first 5 integers

We can use formula

$$S_n = \frac{n}{2}\,(2a + (n-1)\,d)$$

Here, $n = 5, d = 6, a = 6$

$$\therefore \quad S_5 = \frac{5}{2}\,(2\times 6 + (5-1)\times 6)$$

$$S_5 = \frac{5}{2}\,(12 + 24)$$

$$S_5 = \frac{5}{2}\times 36 = 90.$$

[B] Very Short Answer Type Questions :

Q. 1. If *n*th term of an A.P. is (2n + 1), what is the sum of its first three terms ? A [CBSE SQP-2018]

Sol. Since, $a_1 = 3, a_2 = 5$ and $a_3 = 7$ ½

$$S_3 = \frac{3}{2}\,(3 + 7) = 15 \qquad ½$$

[CBSE Marking Scheme, 2018]

Detailed Answer :

$$\because \quad a_n = (2n + 1)$$

$$\therefore \quad a_1 = 2\times 1 + 1 = 3$$

$$l = a_3 = 2\times 3 + 1 = 7$$

Since, $S_n = \frac{n}{2}[a + l]$

Hence, $S_3 = \frac{3}{2}[3 + 7]$

$$S_3 = 15$$

Q. 2. Find the sum of first 16 terms of the A.P. 10, 6, 2, ... A [Board Term-2, 2012, Set (32)]

Sol. Here, $a = 10$, $d = 6 - 10 = -4$ and $n = 16$

$$\because \quad S_n = \frac{n}{2}[2a + (n-1)d]$$

$$\therefore \quad S_{16} = \frac{16}{2}[2\times 10 + (16-1)(-4)]$$

$$= 8[20 + 15\times(-4)]$$

$$= 8[20 - 60]$$

$$= 8\times(-40)$$

$$= -320 \qquad 1$$

[CBSE Marking Scheme, 2012]

Q. 3. What is the sum of five positive integers divisible by 6. [Board Term-2, 2012 Set (23)]

Sol. Here, $a = 6$, $d = 6$ and $n = 5$

$$\because \quad S_n = \frac{n}{2}[2a + (n-1)d]$$

$$\therefore \quad S_5 = \frac{5}{2}[2\times 6 + (5-1)(6)]$$

$$= \frac{5}{2}[12 + 4\times 6]$$

$$= \frac{5}{2}[12 + 24] = \frac{5}{2}[36]$$

$$= 5\times 18 = 90 \qquad 1$$

[CBSE Marking Scheme, 2012]

Q. 4. If the sum of *n* terms of an A.P. is $2n^2 + 5n$, then find the 4^{th} term. A [Board Term-2, 2012 Set (12)]

Sol. Let the sum of first n terms of an A.P. $= S_n$.

Given, $S_n = 2n^2 + 5n$

Now, n^{th} term of A.P. $= S_n - S_{n-1}$

or, $a_n = (2n^2 + 5n) - [2(n-1)^2 + 5(n-1)]$

$= 2n^2 + 5n - [2n^2 - 4n + 2 + 5n - 5]$

$= 2n^2 + 5n - 2n^2 - n + 3$

$a_n = 4n + 3$ ½

Then, 4^{th} term $a_4 = 4\times 4 + 3$

$= 19$ ½

[CBSE Marking Scheme, 2012]

Q. 5. If the sum of first *k* terms of an A.P. is $3k^2 - k$ and its common difference is 6. What is the first term? A [Board Term-2, 2012, Set (44)]

Sol. Let the sum of k terms of A.P. is $S_k = 3k^2 - k$

Now k^{th} term of A.P.

$$a_k = S_k - S_{k-1}$$

$$a_k = (3k^2 - k) - [3(k-1)^2 - (k-1)]$$

$$= 3k^2 - k - [3k^2 - 6k + 3 - k + 1]$$

$$= 3k^2 - k - 3k^2 + 7k - 4$$

$$= 6k - 4 \qquad ½$$

Hence, first term $a = 6\times 1 - 4 = 2$ ½

[CBSE Marking Scheme, 2012]

Short Answer Type Questions-I (2 marks each)

Q. 1. How many terms of the A.P. 65, 60, 55, ... be taken so that their sum is zero ? U **[Delhi Set III, 2016]**

Sol. Here, $a = 65, d = -5$ and $S_n = 0$ ½

Since, $S_n = \frac{n}{2}[2a+(n-1)d]$

Therefore, $\frac{n}{2}[130 + (n-1)(-5)] = 0$ **1**

$\frac{n}{2}[130 - 5n + 5] = 0$

or, $135n - 5n^2 = 0$

or, $n(135 - 5n) = 0$

or, $5n = 135$

or, $n = 27$ as $n \neq 0$ ½

[CBSE Marking Scheme, 2016]

Q. 2. The sum of first 'n' terms of an *A.P.* is given by $S_n = 2n^2 + 3n$. Find the sixteenth term of the A.P. A **[Sample Question Paper 2017]**

Sol. Try yourself, Similar to Q. 4. in VSATQ.

Q. 3. Find the sum of first 8 multiples of 3. A **[Delhi/OD Set-2018]**

Sol. Here, $S = 3 + 6 + 9 + 12 + ... + 24$

$= 3(1 + 2 + 3 + ... + 8)$ **1**

$= 3 \times \frac{8 \times 9}{2}$

$= 108$ **1**

[CBSE Marking Scheme, 2018]

Detailed Answer :

First 8 multiples of 3 are 3, 6, 9, 12, 15, 18, 21 and 24.

Then, $S = 3 + 6 + 9 + 12 + 15 + 18 + 21 + 24$ ½

These numbers are in A.P.

where $a = 3, d = 3$ and $n = 8$ **1**

Since, $S_n = \frac{n}{2}[2a + (n-1)d]$

$\therefore \quad S_8 = \frac{8}{2}[2 \times 3 + (8-1)3]$

$S_8 = 4[6 + 21]$

$S_8 = 4 \times 27 = 108$

Thus, sum of first 8 multiples of 3 is 108. ½

Q. 4. In an A.P., if $S_5 + S_7 = 167$ and $S_{10} = 235$, then find the A.P., where S_n denotes the sum of first n terms. A **[Outside Delhi CBSE Board, Term-2 2015, Set I, II, III]**

Sol. $S_n = \frac{n}{2}[2a+(n-1)d]$

Given, $S_5 + S_7 = 167$

Hence, $\frac{5}{2}(2a+4d) + \frac{7}{2}(2a+6d) = 167$

or, $24a + 62d = 334$

or $12a + 31d = 167$ ....(i) ½

Given, $S_{10} = 235$

or, $5(2a + 9d) = 235$

or $2a + 9d = 47$ ...(ii) ½

Solving (i) and (ii), wet get

$a = 1$ and $d = 5$ ½

Hence A.P. $= 1, 6, 11, ...$ ½

[CBSE Marking Scheme, 2015]

Q. 5. Find the sum of sixteen terms of an A.P. – 1, – 5, – 9, ... U **[Board Term-2, 2012 Set (8)]**

Sol. Here, $a_1 = -1, a_2 = -5$ and $d = -4$

$\because \quad S_n = \frac{n}{2}[2a+(n-1)d]$ ½

$\therefore \quad S_{16} = \frac{16}{2}[2\times(-1)+(16-1)(-4)]$ ½

$= 8[-2-60] = 8(-62)$

$= -496$ **1**

[CBSE Marking Scheme, 2012]

Q. 6. If the n^{th} term of an A.P. is $7 - 3n$, find the sum of twenty five terms. U **[Board Term-2, 2012 Set (16)]**

Sol. Here $n = 25$ and $a_n = 7 - 3n$

Taking $n = 1, 2, 3, ...$, we get

$a_1 = 7 - 3 \times 1 = 4$

$a_2 = 7 - 3 \times 2 = 1$

and $a_3 = 7 - 3 \times 3 = -2$ ½

$\therefore$ Given A.P. is 4, 1, – 2, ...

Here, $a = 4$ and $d = 1 - 4 = -3$ ½

Since, $S_n = \frac{n}{2}[2a+(n-1)d]$ ½

Now, $S_{25} = \frac{25}{2}[2\times4+(25-1)(-3)]$

$= \frac{25}{2}[8+24(-3)]$

$= \frac{25}{2}(8-72)$

$= -800$ ½

[CBSE Marking Scheme, 2012]

AI **Q. 7. If the 1st term of a series is 7 and 13^{th} term is 35. Find the sum of 13 terms of the sequence.** U **[Board Term-2, 2012 Set (36)]**

Sol. Here $a = 7, a_{13} = 35$

Since, $a_n = a + (n-1)d$

$\therefore \quad a_{13} = a + 12d$

or, $35 = 7 + 12d$

or $d = \frac{7}{3}$

Again, $S_n = \frac{n}{2}[2a + (n-1)d]$

$S_{13} = \frac{13}{2}[2\times 7 + 12\times\left(\frac{7}{3}\right)]$ **1**

$= \frac{13}{2}[14 + 28]$

$= \frac{13}{2}\times 42 = 273$ **1**

[CBSE Marking Scheme, 2012]

Q. 8. If S_n denotes the sum of n terms of an A.P. whose common difference is d and first term is a, find $S_n - 2S_{n-1} + S_{n-2}$. A **[Board Term-2, 2011 (A1)]**

Sol. Since, $T_n = S_n - S_{n-1}$ ½

and $T_{n-1} = S_{n-1} - S_{n-2}$ ½

Hence, $S_n - 2S_{n-1} + S_{n-2} = S_n - S_{n-1} - S_{n-1} + S_{n-2}$

$= (S_n - S_{n-1}) - (S_{n-1} - S_{n-2})$ ½

$= T_n - T_{n-1} = d.$ ½

[CBSE Marking Scheme, 2011]

Q. 9. The sum of first n terms of an A.P. is $5n - n^2$. Find the n^{th} term of the A.P.

A **[Foreign Set I, II, III, 2014]**

Sol. Let the sum of first n terms of A.P. $= S_n$

Given, $S_n = 5n - n^2$

Now, n^{th} term of A.P. $= S_n - S_{n-1}$ **1**

or, $a_n = (5n - n^2) - [5(n-1) - (n-1)^2]$

$= 5n - n^2 - [5n - 5 - (n^2 + 1 - 2n)]$

$= 5n - n^2 - (5n - 5 - n^2 + 1 + 2n)$

$= 5n - n^2 - 7n + 6 + n^2$

$= -2n + 6$

or, $a_n = -2(n - 3)$

$\therefore$ n^{th} term, $a_n = -2(n-3)$. **1**

Q. 10. The first and last term of an A.P. are 5 and 45 respectively. If the sum of all its terms is 400, find its common difference. **[Delhi Set 2014]**

A **[Board Term-2, 2012 Set (19)]**

Sol. $a = 5$ and $l = 45$ (given)

$\therefore$ $45 = 5 + (n-1)d$

or, $(n-1)d = 40$ ....(i)

Given, $S_n = 400$ **1**

Since, $S_n = \frac{n}{2}(a + l)$

$\therefore$ $400 = \frac{n}{2}(5 + 45)$

or, $800 = 50\,n$

or, $n = 16$

$\therefore$ From (i),

$(n-1)d = 40$

$\therefore$ $15d = 40$

or, $d = \frac{40}{15} = \frac{8}{3}$ **1**

[CBSE Marking Scheme, 2012]

Q. 11. If the sum of the first 7 terms of an A.P. is 49 and that of the first 17 terms is 289, find the sum of its first n terms. A **[Board Foreign Set-II 2012]**

Sol. $\because$ $S_n = \frac{n}{2}[2a + (n-1)d]$

$\therefore$ $S_7 = \frac{7}{2}(2a + 6d) = 49$

or, $a + 3d = 7$ ...(i) ½

Again, $S_{17} = \frac{17}{2}(2a + 16d) = 289$

or, $a + 8d = 17$ ...(ii) ½

On subtracting (i) from (ii), we get

or, $5d = 10$ or, $d = 2$

and $a = 1$

Therefore, $S_n = \frac{n}{2}[2\times 1 + (n-1)2]$ ½

$= \frac{n}{2}[2 + 2n - 2] = n^2$

Hence, sum of first n terms $= n^2$ ½

[CBSE Marking Scheme, 2011]

Q. 12. How many terms of the A.P. $-6, \frac{-11}{2}, -5, -\frac{9}{2}$..... are needed to give their sum zero.

A **[Board outside Delhi compt. Set-III, 2017]**

Sol. Given $a = -6$ and $d = -\frac{11}{2} - (-6) = \frac{1}{2}$

Since, $S_n = \frac{n}{2}[2a + (n-1)d]$

Let sum of n terms be zero.

$\therefore$ $S_n = \frac{n}{2}\left[2\times -6 + (n-1)\frac{1}{2}\right] = 0$ ½

or, $\frac{n}{2}\left[-12 + \frac{n}{2} - \frac{1}{2}\right] = 0$

or, $\frac{n}{2}\left[\frac{n}{2} - \frac{25}{2}\right] = 0$

or, $n^2 - 25n = 0$ **1+½**

$n(n - 25) = 0$

$n = 25$ as $n \neq 0$

Hence, terms are needed $= 25$.

Q. 13. In an A.P. of 50 terms, the sum of the first 10 terms is 210 and the sum of its last 15 terms is 2565. Find the A.P. A **[Board Foreign Set-III 2017]**

Sol. Given, $S_{10} = 210$

Since, $S_n = \frac{n}{2}[2a + (n-1)d]$

or, $\frac{10}{2}(2a+9d) = 210$ ½

or, $2a + 9d = 42$ ...(i)

Since, $a_{36} = a + 35d$

and $a_{50} = a + 49d$

Hence, Sum of last 15 terms $= \frac{15}{2}(a+35d+a+49d)$

or, $\frac{15}{2}(2a+84d) = 2565$ ½

or, $a + 42d = 171$ ...(ii) ½

On solving (i) and (ii), we get

$a = 3$ and $d = 4$ ½

Hence, given A.P. = 3, 7, 11 .....

[CBSE Marking Scheme, 2017]

Q. 14. Reshma wanted to save at least ₹ 6,500 for sending her daughter to school next year (after 12 months). She saved ₹ 450 in the first month and raised her savings by ₹ 20 every next month. How much will she be able to save in next 12 months ? Will she be able to send her daughter to the school next year ?

A; E **[Foreign Set I, II, III, 2016]**

Sol. Here $a = ₹\,450$, $d = ₹\,20$, $n = 12$

$S_n = \frac{n}{2}[2a + (n-1)d]$

$S_{12} = \frac{12}{2}[2 \times 450 + 11 \times 20]$

$= 6[1120] = 6720 > 6500$ **2**

∴ Reshma will be able to send her daughter to school **[CBSE Marking Scheme, 2016]**

Short Answer Type Questions-II

(3 marks each)

AI **Q. 1. In an A.P. the sum of first n terms is $\frac{3n^2}{2} + \frac{13n}{2}$. Find the 25th term.** A **[Board Sample Paper, 2016]**

Sol. $S_n = \frac{3n^2+13n}{2}$

$a_n = S_n - S_{n-1}$

or, $a_{25} = S_{25} - S_{24}$

$= \frac{3(25)^2+13(25)}{2} - \frac{3(24)^2+13(24)}{2}$ **1**

$= \frac{1}{2}\{3(25^2-24^2) + 13(25-24)\}$ **1**

$= \frac{1}{2}(3 \times 49 + 13) = 80$ **1**

[CBSE Marking Scheme, 2016]

Q. 2. The sum of first n terms of three arithmetic progressions are S_1, S_2 and S_3 respectively. The first term of each A.P. is 1 and common differences are 1, 2 and 3 respectively. Prove that $S_1 + S_3 = 2S_2$. A **[O.D. Set III, 2016]**

Sol. Since, $S_1 = 1 + 2 + 3 + ... + n.$

$S_2 = 1 + 3 + 5 + ...$upto n terms

and $S_3 = 1 + 4 + 7 + ...$upto n terms

or, $S_1 = \frac{n(n+1)}{2}$ ½

Also, $S_2 = \frac{n}{2}[2 \times 1 + (n-1)2]$

$= \frac{n}{2}[2n] = n^2$ ½

and $S_3 = \frac{n}{2}[2 \times 1 + (n-1)3]$

$= \frac{n(3n-1)}{2}$ ½

Now, $S_1 + S_3 = \frac{n(n+1)}{2} + \frac{n(3n-1)}{2}$ ½

$= \frac{n[n+1+3n-1]}{2}$

$= \frac{n[4n]}{2}$

$= 2n^2 = 2S_2$ **Hence Proved. 1**

Q. 3. If S_n denotes, the sum of the first n terms of an A.P. prove that $S_{12} = 3(S_8 - S_4)$.

A **[Delhi CBSE Board, 2015, Set I]**

Sol. Let a be the first term and d be the common difference.

Since, $S_n = \frac{n}{2}[2a+(n-1)d]$

$S_{12} = 6[2a + 11d]$

$= 12a + 66d$ ...(i) **1**

$S_8 = 4[2a + 7d] = 8a + 28d$ ½

and $S_4 = 2[2a + 3d] = 4a + 6d$ ½

Then, $3(S_8 - S_4) = 3[(8a + 28d) - (4a + 6d)]$

$= 3[4a + 22d] = 12a + 66d$

From equation (i) and (ii), $S_{12} = 3(S_8 - S_9)$ **1**

[CBSE Marking Scheme, 2015]

Q. 4. The 14th term of an A.P. is twice its 8th term. If the 6th term is – 8, then find the sum of its first 20 terms. A **[Outside Delhi CBSE Board, 2015, Set I]**

Sol. Let first term be a and common difference be d.

Here, $a_{14} = 2a_8$

or, $a + 13d = 2(a + 7d)$

$a + 13d = 2a + 14d$

$a = -d$ ...(i) ½

Again, $a_6 = -8$

or, $a + 5d = -8$ ...(ii) ½

Solving (i) and (ii), we get

$a = 2, d = -2$ ½

$$S_{20} = \frac{20}{2}[2\times2+(20-1)(-2)] \quad ½$$

$$= 10[4 + 19 \times (-2)]$$

$$= 10(4 - 38)$$

$$= 10 \times (-34) = -340 \quad \mathbf{1}$$

[CBSE Marking Scheme, 2015]

AI **Q. 5. If the ratio of the sums of first n terms of two A.P.'s is $(7n + 1) : (4n + 27)$, find the ratio of their m^{th} terms.** A [O.D. Set I, 2016]

Sol. Let a and A be the first terms and d and D be the common difference of two A.P.'s

Then, according to question,

$$\frac{S_n}{S'_n} = \frac{\frac{n}{2}[2a+(n-1)d]}{\frac{n}{2}[2A+(n-1)D]} = \frac{7n+1}{4n+27} \quad \mathbf{1}$$

or, $$\frac{2a+(n-1)d}{2A+(n-1)D} = \frac{7n+1}{4n+27}$$

or, $$\frac{a+(\frac{n-1}{2})d}{A+(\frac{n-1}{2})D} = \frac{7n+1}{4n+27} \quad ...(i)\ ½$$

Putting $\frac{n-1}{2} = m-1$ or, $n = 2m-1$ **1**

From equ. (i) $\frac{a+(m-1)d}{A+(m-1)D} = \frac{7(2m-1)+1}{4(2m-1)+27}$

Hence, $\frac{a_m}{A_m} = \frac{14m-6}{8m+23}$ ½

Q. 6. If the sum of the first n terms of an A.P. is $\frac{1}{2}[3n^2 + 7n]$, then find its n^{th} term. Hence write its 20^{th} term.

A [Delhi CBSE Board Term-2, 2015 (Set II)]

Sol.

$$S_n = \frac{1}{2}[3n^2 + 7n]$$

$$S_1 = \frac{1}{2}[3 \times (1)^2 + 7(1)]$$

$$= \frac{1}{2}[3 + 7]$$

$$= \frac{1}{2} \times 10 = 5 \quad ½$$

$$S_2 = \frac{1}{2}[3(2)^2 + 7 \times 2]$$

$$= \frac{1}{2}[12 + 14]$$

$$= \frac{1}{2} \times 26$$

$$= 13 \quad ½$$

$$a_1 = S_1 = 5 \quad ½$$

$$a_2 = S_2 - S_1 = 13 - 5 = 8 \quad ½$$

$$d = a_2 - a_1 = 8 - 5 = 3 \quad ½$$

Now, A.P. is 5, 8, 11, ...

n^{th} term, $a_n = a + (n-1)d$

$= 5 + (n-1)3 = 3n + 2$

Hence, $a_{20} = 3 \times 20 + 2$

$a_{20} = 62$ **1**

[CBSE Marking Scheme, 2012]

Q. 7. In an A.P., if the 12^{th} term is -13 and the sum of its first four terms is 24, find the sum of its first ten terms. A [Foreign Set I, II, 2015]

Sol. Let the first term be a and the common difference be d.

Given, $a_{12} = a + 11d = -13$ ...(i) ½

Since, $S_n = \frac{n}{2}[2a+(n-1)d]$

Hence, $S_4 = 2[2a + 3d] = 24$

$2a + 3d = 12$ ...(ii) **1**

Multiplying (i) by 2 and subtracting (ii) from it, we get

$(2a + 22d) - (2a + 3d) = -26 - 12$

$19d = -38$

$d = -2$ ½

Putting the value of d in (i), we get

$a + 11 \times -2 = -13$

$a = -13 + 22$

$a = 9$ ½

Now, $S_n = \frac{n}{2}[2a+(n-1)d]$

$S_{10} = \frac{10}{2}(2\times9+9\times-2)$

$= 5 \times (18 - 18)$

$= 0$ ½

Hence, $S_{10} = 0$

[CBSE Marking Scheme, 2015]

Q. 8. The tenth term of an A.P., is -37 and the sum of its first six terms is -27. Find the sum of its first eight terms. A [Foreign Set III, 2015]

Sol. Try yourself, Similar to Q. 7 in SATQ-II

Q. 9. Find the sum of first seventeen terms of A.P. whose 4^{th} and 9^{th} terms are -15 and -30 respectively.

A [Board Term-2, 2014]

Sol. Let the first term be a

and common difference be d. $[\because a_n = a + (n-1)d]$

$a_4 = a + 3d = -15$ .... (i)

and $a_9 = a + 8d = -30$ .... (ii)

Subtracting eqn (i) from eqn (ii), we get

$(a + 8d) - (a + 3d) = -30 - (-15)$

$5d = -15$

$d = \frac{-15}{5} = -3$ **1**

From (i), $a + 3d = -15$

$a + 3(-3) = -15$

$a = -15 + 9 = -6$ **1**

Again $S_{17} = \frac{17}{2}[2\times(-6)+(17-1)(-3)]$

$= \frac{17}{2}[-12+16\times(-3)]$

$= \frac{17}{2}[-12-48]$

$= \frac{17}{2}[-60] = 17\times(-30)$

$= -510$ **1**

[CBSE Marking Scheme, 2014]

Q. 10. The common difference of an A.P. is –2. Find its sum, if first term is 100 and last term is –10.

A **[Board Term-2 2014]**

Sol. Given, $a = 100$, $d = -2$ and $a_n = -10$ **1**

Using, $a_n = a + (n-1)d$

or, $-10 = 100 + (n-1)(-2)$

or, $-10 = 100 - 2n + 2$

or, $2n = 112$

or, $n = 56$

$\because$ Here, 56^{th} term is – 10.

$\therefore$ Number of terms in A.P. is 56.

$\therefore$ $S_n = \frac{n}{2}(a+l)$ **1**

$S_{56} = \frac{56}{2}(100-10)$

$= \frac{56}{2}(90)$

$= 56 \times 45$

or, $S_n = 2520$. **1**

[CBSE Marking Scheme, 2014]

Q. 11. The 16^{th} term of an A.P. is five times its third term. If its 10^{th} term is 41, then find the sum of its first fifteen terms.

A **[Outside Delhi CBSE, 2015, Set II]**

Sol. Try yourself, Similar to Q. 4 in SATQ-II

Q. 12. The 13^{th} term of an A.P. is four times its 3^{rd} term. If the fifth term is 16, then find the sum of its first ten terms. A **[Outside Delhi, 2015 Set III]**

Sol. Try yourself, Similar to Q. 4 in SATQ-II

Q. 13. The n^{th} term of an A.P. is given by $(-4n + 15)$. Find the sum of first 20 terms of this A.P.

A **[Board Term-2, 2013]**

Sol. Try yourself, Similar to Q. 6 in SATQ-I

Q. 14. The sum of first 7 terms of an A.P. is 63 and sum of its next 7 terms is 161. Find 28^{th} term of A.P.

[Foreign Set I, II, III, 2014]

Sol. Since, $S_n = \frac{n}{2}[2a + (n-1)d]$

Given, $S_7 = 63$

Hence, $S_7 = \frac{7}{2}[2a + 6d] = 63$

or, $2a + 6d = 18$ .... (i) **1**

Now, sum of 14 terms is

$S_{14} = S_{\text{first 7}} + S_{\text{next 7}}$

$= 63 + 161$

$= 224.$

$\therefore$ $\frac{14}{2}[2a + 13d] = 224$

or, $2a + 13d = 32$ .... (ii) **1**

On subtracting (i) from (ii), we get

$(2a + 13d) - (2a + 6d) = 32 - 18$

$7d = 14$ or, $d = 2$

Putting the value of d in (i), we get

$a = 3$

Since, $a_n = a + (n-1)d$

$\therefore$ $a_{28} = 3 + 2 \times (27)$

$= 57$

$\therefore$ The 28^{th} term $= 57$. **1**

Q. 15. The sum of first n terms of an A.P. is given by $S_n = 3n^2 - 4n$. Determine the A.P. and the 12^{th} term.

[Delhi CBSE Term-2, 2014]

[Board Term-2, 2012 Set (13)]

Sol. Try yourself, Similar to Q. 6 in SATQ-II

Q. 16. Find the sum of all two digit natural numbers which are divisible by 4.

A **[Delhi Comp.t Set-II, 2017]**

Sol. First two digit multiple of 4 is 12 and last is 96

So, $a = 12$, $d = 4$ and $l = 96$

Let n^{th} term be last term = 96 **1**

$\therefore$ $a_n = a + (n-1)d = l$

$12 + (n-1)4 = 96$

$n - 1 = 21$

$n = 21 + 1 = 22$ **1**

Now, $S_{22} = \frac{22}{2}[12+96]$

$= 11 \times 108$

$= 1188$ **1**

[CBSE Marking Scheme, 2017]

Q. 17. Find the sum of the following series.

5 + (– 41) + 9 + (– 39) + 13 + (– 37) + 17 + ... + (– 5) + 81 + (– 3) A **[Board foreign Set-I, 2017]**

Sol. The series can be written as

$(5 + 9 + 13 + ... + 81) + [(-41) + (-39) + (-37) + (-35) ... (-5) + (-3)]$

For the series $(5 + 9 + 13 + ... + 81)$ ½

$a = 5$

$d = 4$

and $a_n = 81$

Then, $a_n = 5 + (n-1)\,4 = 81$

or, $(n-1)4 = 76$ ½

$n = 20$

$S_n = \frac{20}{2}(5+81) = 860$

For series $(-41) + (-39) + (-37) + ... + (-5) + (-3)$

$a_n = -3$ ½

$a = -41$

$d = 2$

Then, $a_n = -41 + (n-1)(2)$

$\therefore$ $n = 20$

$S_n = \frac{20}{2}[-41 + (-3)] = -440$ ½

Hence, the Sum of the series = 860 – 440
= 420 **1**

[CBSE Marking Scheme, 2017]

AI **Q. 18. Find the sum of n terms of the series**

$$\left(4-\frac{1}{n}\right)+\left(4-\frac{2}{n}\right)+\left(4-\frac{3}{n}\right)+.....$$

A **[CBSE Board Delhi Set-I, II, III 2017]**

Sol. Let sum of n term be S_n

$\therefore\ S_n = \left[4-\frac{1}{n}\right]+\left[4-\frac{2}{n}\right]+\left[4-\frac{3}{n}\right]+.....$ up to n terms **1**

or, (4 + 4 + 4 + ... up to n terms)

$$-\left(\frac{1}{n}+\frac{2}{n}+\frac{3}{n}+..... \text{ up to } n \text{ term}\right)$$

or, (4 + 4 + 4 + ... up to n terms)

$$-\frac{1}{n}(1+2+3+..... \text{ up to } n \text{ terms})$$

or, $4n-\frac{1}{n}\times\frac{n(n+1)}{2}$ ½ + **1**

or, $4n-\frac{n+1}{2} = \frac{7n-1}{2}$

Hence, sum of n terms $= \frac{7n-1}{2}$ ½

[CBSE Marking Scheme, 2017]

Q. 19. Find the sum of the integers between 100 and 200 that are divisible by 6.

[Board Term-2, 2012 Set (5)] 3

Sol. The series as per question is 102, 108, 114, ..., 198 which is an A.P.

Given, $a = 102$, $d = 6$ and $l = 198$

Then $198 = 102 + (n-1)6$ ½

or, $\frac{96}{6} = n-1$

or, $n = 17$ ½

$S_n = \frac{n}{2}(a+l)$ ½

$\therefore\ S_{17} = \frac{17}{2}[102+198]$

or, $S_{17} = \frac{17}{2}\times 300 = 17\times 150 = 2550$. ½

[CBSE Marking Scheme, 2012]

Q. 20. If the sum of the first 14 terms of an A.P. is 1050 and its first term is 10, find its 20th term.

A **[Board Outside Delhi Compt. Set-III 2017]**

Sol. Given, $a = 10$, and $S_{14} = 1050$

Let the common difference of the A.P. be d. ½

Since, $S_n = \frac{n}{2}[2a+(n-1)d]$

$\therefore\ S_{14} = \frac{14}{2}[2\times 10+(14-1)d]$

$= 1050$ ½

$20+13d = \frac{1050}{7} = 150$

$13d = 130$

$d = \frac{130}{13} = 10$ **1**

$a_n = a + (n-1)d$

$a_{20} = 10 + 19 \times 10 = 200$ **1**

Hence, $a_{20} = 200$.

[CBSE Marking Scheme, 2017]

Q. 21. Find the number of terms of the A.P. – 12, – 9, – 6, ..., 21. If 1 is added to each term of this A.P., then find the sum of all the terms of the A.P. thus obtained. **[Board Term-2, 2013] 3**

Sol. – 12, – 9, – 6, ..., 21 is the given A.P., then

$a = -12$, $d = -9-(-12) = 3$

$a_n = a + (n-1)d$ and $a_n = 21$ **1**

or, $21 = -12 + (n-1)\times 3$

or, $21 + 12 = (n-1)\times 3$

or, $33 = (n-1)\times 3$

or, $n-1 = 11$

or, $n = 11+1$

or, $n = 12$

Now, if 1 is added to each term we have a new A.P. with

– 12 + 1, – 9 + 1, – 6 + 1, ..., 21 + 1

i.e., – 11, – 8, – 5 , ... 22

Now we have $a = -11$, $d = 3$ and $l = 22$ **1**

and $n = 12$

$\therefore$ Sum of this obtained A.P.

or, $S_{12} = \frac{12}{2}[-11+22]$

$= 6 \times 11 = 66$

Hence the sum of new A.P. = 66. **1**

Q. 22. Find the sum of all odd numbers between 0 and 50.

[Delhi Compt. Set-III 2017]

Sol. Given, 1 + 3 + 5 + 7 + ... + 49

Let, total odd numbers of terms be n. **1**

$a_n = 1 + (n-1)\times 2 = 49$

$(n-1)\times 2 = 49-1 = 48$

$n-1 = 24$

$n = 24+1 = 25$ **1**

$S_{25} = \frac{25}{2}(1+49)$

$= 25 \times 25 = 625$

Hence, sum of odd numbers between 0 and 50 = 625 **1**

Q. 23. Find the sum of first 15 multiples of 8.

[Delhi Compt. Set-I 2017]

Sol. First term of given A.P. be 8 and common difference be 8. Then,

$S_n = \frac{n}{2}[2a+(n-1)d]$ ½

Therefore, $S_{15} = \frac{15}{2}[2\times 8+(15-1)8]$ ½

$= \frac{15}{2}[16+112]$ **1**

$$= \frac{15}{2} \times 128 = 960$$

Hence, the sum of 15 terms = 960. **1**

AI Q. 24. If m^{th} term of A.P. is $\frac{1}{n}$ and n^{th} term is $\frac{1}{m}$, find the sum of first mn terms. [CBSE Board Set-I, II 2017]

Sol. Let first term of given A.P. be a and common difference be d.

$\therefore \quad a_m = a + (m-1)d = \frac{1}{n}$ ...(i) ½

and $a_n = a + (n-1)d = \frac{1}{m}$ ...(ii) ½

On subtracting (ii) from (i) we get

$$(m-n)d = \frac{1}{n} - \frac{1}{m} = \frac{m-n}{mn} \quad \textbf{1}$$

or, $d = \frac{1}{mn}$

and $a = \frac{1}{mn}$

Now $S_{mn} = \frac{mn}{2}\left(2 \cdot \frac{1}{mn} + (mn-1)\frac{1}{mn}\right)$

$= \frac{mn}{2}\left(\frac{2}{mn} + \frac{mn}{mn} - \frac{1}{mn}\right)$

$S_{mn} = \frac{mn}{2}\left[\frac{1}{mn} + 1\right]$

$= \frac{1}{2}[mn+1]$

Hence, the sum of first mn terms $= \frac{1}{2}[mn+1]$. **1**

Q. 25. How many terms of an A.P. 9, 17, 25, ... must be taken to give a sum of 636 ? [Outside Delhi Set-III, 2017]

Sol.

19. $a = 9$, $d = 8$, $S_n = 636$.

$S_n = \frac{n}{2}[2a + (n-1)d]$

$636 = \frac{n}{2}[18 + (n-1)8]$

$636 = n[9 + (n-1)4]$

$636 = n(9 + 4n - 4)$

$636 = n(5 + 4n)$

$636 = 5n + 4n^2$

$4n^2 + 5n - 636 = 0$

$4n^2 + 53n - 48n - 636 = 0$

~~n & $4n+5$~~

$4n^2 - 48n + 53n - 636 = 0$.

$4n(n-12) + 53(n-12) = 0$

$(4n+53)(n-12) = 0$

$\therefore n = \frac{-53}{4}$ or 12.

as n is a natural number, $n = 12$

$\therefore$ 12 terms are required to give sum 636.

[Topper Answer, 2017] 3

Q. 26. How many terms of the A.P. $-6, -\frac{11}{2}, -5, \ldots\ldots$ are needed to give the sum – 25 ? Explain the reason for double answer.

[Board Term-2, 2012 Set (13)]

Sol. A.P. is $-6, -\frac{11}{2}, -5, \ldots\ldots$

Then, $a = -6$

$d = -\frac{11}{2} + \frac{6}{1} = \frac{1}{2}$ ½

and $S_n = -25$

or, $S_n = \frac{n}{2}[2a + (n-1)d]$

Hence, $-25 = \frac{n}{2}\left[-12 + (n-1) \times \frac{1}{2}\right]$ **1**

or, $-50 = n\left[\frac{-24 + (n-1)}{2}\right]$

or, $-100 = n[n-25]$

or, $n^2 - 25n + 100 = 0$ **1**

or, $(n-20)(n-5) = 0$

or, $n = 20, 5$

or, $S_{20} = S_5$

Two answers $\because$ a is negative and d is positive and the sum of the terms from 5^{th} to 20^{th} is zero. ½

[CBSE Marking Scheme, 2012]

Q. 27. If S_1, S_2 and S_3 be the sum of n, $2n$ and $3n$ terms respectively of an A.P. Prove that $S_3 = 3(S_2 - S_1)$.

[Board Term-2, 2012 Set (59)]

Sol. Let 'a' be the first term and 'd' be the common difference.

Then, $S_1 = \frac{n}{2}[2a + (n-1)d]$

$S_2 = \frac{2n}{2}[2a + (2n-1)d]$

and $S_3 = \frac{3n}{2}[2a + (3n-1)d]$ 1½

Again,

$3(S_2 - S_1) = 3\left[\frac{2n}{2}[2a + (2n-1)d] - \frac{n}{2}[2a + (n-1)d]\right]$

$= 3\left[\frac{n}{2}[4a + 2(2n-1)d] - [2a + (n-1)d]\right]$

$= 3\left[\frac{n}{2}(4a + 4nd - 2d - 2a - nd + d)\right]$ 1

$= 3\left[\frac{n}{2}(2a + 3nd - d)\right]$

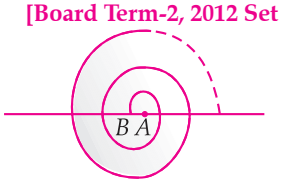
$= \frac{3n}{2}[2a + (3n-1)d]$

$= S_3$ [Hence proved] ½

[CBSE Marking Scheme, 2012]

Q. 28. A spiral is made up of successive semi-circles with centres alternately at A and B starting with A, of radii 1 cm, 2 cm, 3 cm,.... as shown in the figure. What is the total length of spiral made up of eleven consecutive semi-circles ? (Use $\pi = 3.14$)

[Board Term-2, 2012 Set (50); [NCERT]



Sol. Let r_1, r_2,..... be the radii of semi-circles and l_1, l_2, ... be the lengths of circumferences of semi-circles, then

$l_1 = \pi r_1 = \pi(1) = \pi$ cm

$l_2 = \pi r_2 = \pi(2) = 2\pi$ cm

$l_3 = \pi r_3 = 3\pi$ cm

.....

.....

$l_{11} = 11\pi$ cm

$\therefore$ Total length of spiral $= l_1 + l_2 + ...+ l_{11}$ 1

$= \pi + 2\pi + 3\pi + ... + 11\pi$

$= \pi(1 + 2 + 3 + ... + 11)$

$= \pi \times \frac{11 \times 12}{2}$ 1

$= 66 \times 3.14$

$= 207.24$ cm. 1

[CBSE Marking Scheme, 2012]

Q. 29. In an A.P. if sum of its first n terms is $3n^2 + 5n$ and its k^{th} term is 164, find the value of k.

[CBSE Comptt Set-I, II, III 2018]

Sol. Here, $S_n = 3n^2 + 5n$

$\Rightarrow$ $S_1 = 3.1^2 + 5.1 = 8 = a_1$ ½

$S_2 = 3.2^2 + 5.2 = 22 = a_1 + a_2$

$a_2 = 22 - 8 = 14 \Rightarrow d = 6$ 1

$a_k = 164 \Rightarrow 8 + (k-1)6 = 164$ ½

$\Rightarrow$ $k = 27$ 1

[CBSE Marking Scheme, 2018]

Q. 30. Aditi required ₹ 2500 after 12 weeks to send her daughter to school. She saved ₹ 100 in the first week and increased her weekly saving by ₹ 20 every week. Find whether she will be able to send her daughter after 12 weeks.

A; E [Delhi CBSE Term-2, 2015 Set I, II, III]

Sol. Here, required money is ₹ 2500

a = saving in 1^{st} week = ₹ 100

d = difference in weekly saving = ₹ 20

A.P. formed by saving,

According to the question,

Sequence is 100, 120, 140, ... upto 12 terms

$\because$ $S_n = \frac{n}{2}[2a + (n-1)d]$

$\therefore$ $S_{12} = \frac{12}{2}[2 \times 100 + (12-1) \times 20]$

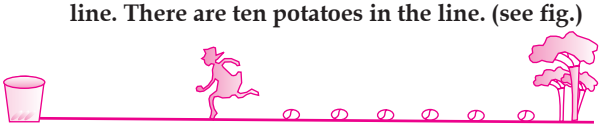
or, $= 6[200 + 11 \times 20]$

or, $= 6[200 + 220]$

or, $= 6 \times 420 = 2520$ 3

She will be able to send her daughter to school after 12 weeks. [CBSE Marking Scheme, 2015]

Q. 31. In a potato race, a bucket is placed at the starting point, which is 5 m from the first potato and the other potatoes are placed 3 m apart in a straight line. There are ten potatoes in the line. (see fig.)



Each competitor starts from the bucket, picks up the nearest potato, runs back with it, drops it in the bucket, runs back to pick up the next potato, runs to the bucket to drop it in, and she continues in the same way until all the potatoes are in the bucket.

(i) What is the total distance the competitor has to run ?

(ii) Which mathematical concept is used in the above problem ? A; E [NCERT]

Sol. (i) Since, distance between the first potato and the bucket = 5 m

and also there are 10 potatoes which are 3m apart.

Distance covered by the competitor in first pick

$= 2 \times 5 = 10$ m

Distance covered by the competitor in second pick

$= 2 \times (5 + 3 \times 1) = 2 \times 8$

$= 16$ m ½

Distance covered by the competitor in third pick

$= 2 \times (5 + 3 \times 2)$

$= 2 \times (5 + 6) = 22$ m

Similarly, distance covered by the competitor in 10th pick

$= 2 \times (5 + 3 \times 9)$

$= 2 \times (5 + 27) = 64$ m

Therefore, the sequence becomes,

10, 16, 22, ..., 64

Let S be the total distance covered by the competitor.

i.e.,

$S = 10 + 16 + 22 + ... + 64$

Here,

$a = 10, d = 16 - 10 = 6, n = 10, l = 64$

Now, $S_n = \frac{n}{2}[a + l]$

$\therefore \quad S_{10} = \frac{10}{2}[10 + 64] = 5(74)$

$= 370$ m **2**

Hence, the total distance covered by the competitor = 370 m.

(ii) Arithmetic Progression. **1**

Q. 32. In a school, students thought of planting trees in and around the school to reduce air pollution. It was decided that the number of trees that each section of each class will plant, will be the same as the class, in which they are studying, *e.g.*, a section of Class I will plant 1 tree, a section of Class II will plant 2 trees and so on till Class XII. There are three sections of each class.

(i) How many trees will be planted by the students ?

(ii) Which mathematical concept is used in the above problem ? A; E **[Delhi CBSE, Term-2 2014]**

Sol. (i) Since, each section of class plants the same number of trees as the class number and there are three sections of each class.

$\therefore$ Total number of trees planted by the students

$= 3[1 + 2 + ... + 12]$

$= 3\left[\frac{12}{2}[2 \times 1 + (12 - 1) \times 1]\right]$

$= 3[6(2 + 11)]$

$= 18 \times 13 = 234$ **2**

$\therefore$ Students planted 234 trees.

(ii) Arithmetic Progression. **1**

Long Answer Type Questions (4 marks each)

Q. 1. The minimum age of children to be eligible to participate in a painting competition is 8 years. It is observed that the age of youngest boy was 8 years and the ages of rest of participants are having a common difference of 4 months. If the sum of ages of all the participants is 168 years, find the age of eldest participant in the painting competition. A **[Board Sample Paper, 2016]**

Sol. Here, $a = 8, d = 4$ months $= \frac{1}{3}$ years and $S_n = 168$ ½

Since $S_n = \frac{n}{2}[2a + (n-1)d]$

Hence, $168 = \frac{n}{2}\left[2(8) + (n-1)\frac{1}{3}\right]$ ½

$n^2 + 47n - 1008 = 0$ **1**

or, $n^2 + 63n - 16n - 1008 = 0$

or, $(n - 16)(n + 63) = 0$

or, $n = 16$ or $n = -63$

$n = 16$

(n cannot be negative So – 63 rejected) **1**

Thus, the age of the eldest participant $= a + 15d$ $= 13$ years **[CBSE Marking Scheme, 2016]** **1**

Q. 2. A thief runs with a uniform speed of 100 m/minute. After one minute a policeman runs after, the thief to catch him. He goes with a speed of 100 m/minute in the first minute and increases his speed by 10 m/minute every succeeding minute. After how many minutes the policeman will catch the thief. A **[Delhi Set I, II, 2016]**

Sol. Let total time to catch the thief be n minutes.

Then, total distance covered by thief $= (100n)$ metres ½

Total distances to be covered by policeman $= 100 + 110 + 120 + ... + (n - 1)$ terms ½

$\therefore \quad 100n = \frac{n-1}{2}[200 + (n - 2)10]$ **1**

$n^2 - 3n - 18 = 0$ ½

$(n - 6)(n + 3) = 0$ ½

or, $n = 6$ ½

Policeman takes 6 minutes to catch the thief. ½

[CBSE Marking Scheme, 2016]

Q. 3. If S_n denotes the sum of first n terms of an A.P., prove that, $S_{30} = 3(S_{20} - S_{10})$. U **[Delhi 2015 Set III, Foreign Set I, II, III, 2014]**

Sol. Try yourself, Similar to Q. 3. in SATQ-II

Q. 4. The sum of first 20 terms of an A.P. is 400 and sum of first 40 terms is 1600. Find the sum of its first 10 terms. A **[Board Term-2, 2015]**

Sol. Let the first term be a and common difference be d and sum of first 20 terms be S_{20}.

$$S_{20} = \frac{20}{2}(2a+19d)$$

or, $400 = \frac{20}{2}(2a+19d)$

or, $400 = 10\,[2a + 19d]$

or $2a + 19d = 40$ ...(i) **1**

Also, $S_{40} = \frac{40}{2}(2a+39d)$

or, $1600 = 20\,[2a + 39d]$

or $2a + 39d = 80$ ...(ii) **1**

From (i) and (ii), we get

$a = 1$ and $d = 2$

Then, $S_{10} = \frac{10}{2}[2\times1+(10-1)(2)]$ **1**

$= 5\,[2 + 9 \times 2]$

$= 5\,[2 + 18]$

$= 5 \times 20 = 100$ **1**

[CBSE Marking Scheme, 2015]

Q. 5. Find $\left(4-\frac{1}{n}\right)+\left(7-\frac{2}{n}\right)+\left(10-\frac{3}{n}\right)+$ ..... **upto** n **terms.** A **[Board Term-2, 2015]**

Sol. Try yourself, Similar to Q. 18. in SATQ-II

Q. 6. Find the 60th term of the A.P. 8, 10, 12, ..., if it has a total of 60 terms and hence find the sum of its last 10 terms.

A **[Outside Delhi, CBSE Board, 2015 Set I, II]**

Sol. Given $a = 8$ and $d = 10–8 = 2$

$\because$ $a_n = a + (n-1)d$

$\therefore$ $a_{60} = 8 + (60-1)2$

$= 8 + 59 \times 2 = 126$ **1**

and $a_{51} = 8 + 50 \times 2$

$= 8 + 100 = 108$

Sum of last 10 terms $= a_{51} + a_{52} + ... + a_{60}$ **1**

Here, $a = 108$ and $d = 2$ ½

$S_{10} = \frac{10}{2}[2\times108+(10-1)2]$

$= 5(216 + 18)$

$= 5 \times 234 = 1170$ **1**

Hence, sum of last 10 terms = 1170. ½

[CBSE Marking Scheme, 2015]

Q. 7. An arithmetic progression 5, 12, 19, ... has 50 terms. Find its last term. Hence find the sum of its last 15 terms.

A **[Outside, Delhi CBSE Board, 2015, Set III]**

Sol. We have, $a = 5$, $d = 12 – 5 = 7$ and $n = 50$

$\therefore$ $a_{50} = 5 + (50-1)7$

$= 5 + 49 \times 7 = 348$ **1**

Also the first term of the A.P. of last 15 terms be a_{36}

$a_{36} = 5 + 35 \times 7$

$= 5 + 245 = 250$ **1**

Now, sum of last 15 terms

$\therefore$ $S_n = \frac{15}{2}[2\times250+(15-1)7]$

$= \frac{15}{2}(500+14\times7)$

$= \frac{15}{2}\times598$

$= 4485$

Hence, sum of last 15 terms = 4485 **2**

[CBSE Marking Scheme, 2015]

Q. 8. Find the middle term of the sequence formed by all three-digit numbers which leave a remainder 3, when divided by 4. Also find the sum of all numbers on both sides of the middle terms separately. U **[Foreign Set I, 2015]**

Sol. The three digit numbers which leaves 3 as remainder when divided by 4 are :

103, 107, 111, ... 999 ½

Now, the first number $a = 103$, last number $l = 999$ and common difference = 4

Let the number of terms in this sequence be n.

$\therefore$ $l = a + (n-1)d$

or, $103 + (n-1)4 = 999$

$(n-1) = \frac{896}{4} = 224$

$n = 224 + 1 = 225$ ½

Middle term $= \frac{225+1}{2}$

$= 113^{\text{th}}$ term

$a_{113} = 103 + 112 \times 4 = 551$ ½

and $a_{112} = 551 – 4 = 547$ ½

Sum of first 112 terms $= \frac{112}{2}(103\times2+111\times4)$

$= 56 \times 650$

$= 36400$

and $a_{114} = 551 + 4 = 555$ **1**

The sum of last 112 terms $= \frac{112}{2}(2\times555+111\times4)$

$= 56(1110 + 444)$

$= 56 \times 1554$

$= 87024$ **1**

[CBSE Marking Scheme, 2015]

Q. 9. Find the middle term of the sequence formed all numbers between 9 and 95, which leave a remainder 1 when divided by 3. Also find the sum of the numbers on both sides of the middle term separately. U **[Foreign Set II, 2015]**

Sol. The sequence is 10, 13, ..., 94. **1**

$94 = 10 + (n-1)\,3$

or, $n = 29$ ½

Therefore, $\frac{29+1}{2} = 15^{th}$ term is the middle term

½

Middle term = $10 + 14 \times 3 = 52$ **1**

Sum of first 14 terms = $\frac{14}{2}[20 + 13 \times 3] = 413$

½

$S_n = \frac{n}{2}[2a + (n-1)d]$

Sum of the last 14 terms = $\frac{14}{2}[110 + 13 \times 3]$

$= 1043$ ½

[CBSE Marking Scheme, 2015]

Q. 10. Find the middle term of the sequence formed by all three-digit numbers which leave a remainder 5 when divided by 7. Also, find the sum of all numbers on both sides of the middle term separately. **[Foreign Set III, 2014, 2015]**

Sol. Try yourself, Similar to Q. 8. in LATQ.

Q. 11. If the sum of first n terms of an A.P. is given by $S_n = 3n^2 + 4n$. Determine the A.P. and the n^{th} term.

U **[Board Term-2, 2014]**

Sol. Try yourself, Similar to Q. 6. in SATQ-II

Q. 12. The sum of the 3rd and 7th terms of an A.P. is 6 and their product is 8. Find the sum of first 20 terms of the A.P. A **[Board Term-2, 2012 Set (21)]**

Sol. Given, $a_3 + a_7 = 6$ and $a_3 \times a_7 = 8$

or, $2a + 8d = 6$ and $(a + 2d)(a + 6d) = 8$. **1**

or, $a + 4d = 3$ or, $a = 3 - 4d$.

and $(a + 2d)(a + 6d) = 8$

Substituting the value of $a = (3 - 4d)$, we get

$(3 - 4d + 2d)(3 - 4d + 6d) = 8$

or, $(3 + 2d)(3 - 2d) = 8$ or, $9 - 4d^2 = 8$ **1**

$4d^2 = 1$ or, $d^2 = \frac{1}{4}$

or, $d = \pm\frac{1}{2}$

Case (i) : $d = \frac{1}{2}$ and, $a = 1$; **1**

$S_n = \frac{n}{2}[2a + (n-1)d]$

Then, $S_{20} = \frac{20}{2}\left[2 + \frac{19}{2}\right]$

$S_{20} = 115$

Case (ii) : $d = -\frac{1}{2}$ and $a = 5$; **1**

Then, $S_{20} = \frac{20}{2}\left[2 \times 5 + 19 \times \left(-\frac{1}{2}\right)\right]$

$= 10\left[10 - \frac{19}{2}\right] = 5$

[CBSE Marking Scheme, 2012]

AI **Q. 13. A sum of ₹ 280 is to be used towards four prizes. If each prize after the first is ₹ 20 less than its preceding prize, find the value of each of the prizes.** A **[Board Term-2, 2012 (44)]**

Sol. Let 1st prize be ₹ x

∴ The series in A.P. is

$x, x - 20, x - 40, x - 60, ...$ ½

Then $a = x$, $d = -20$, $S_n = 280$ and $n = 4$ ½

Since, $S_n = \frac{n}{2}[2a + (n-1)d]$

Hence, $S_4 = \frac{4}{2}[2x + 3(-20)]$ **1**

$280 = 2[2x - 60]$

$140 = 2x - 60$

Thus, $2x = 200$ or, $x = 100$ **1**

The prizes are ₹ 100, ₹ 80, ₹ 60, ₹ 40. **1**

Q. 14. In a garden bed, there are 23 rose plants in the first row, 21 are in the 2nd, 19 in 3rd row and so on. There are 5 plants in the last row. How many rows are there of rose plants ? Also find the total number of rose plants in the garden.

A **[Board Term-2, 2012 (1)]**

Sol. The number of rose plants in the 1st, 2nd, 3rd rows ..... are 23, 21, 19, ..., 5.

$a = 23$, $d = -2$ and $a_n = 5$

∴ $a_n = a + (n-1)d$ **1**

or, $5 = 23 + (n-1)(-2)$

or, $n = 10$ **2**

Total number of rose plants in the flower bed,

$S_n = \frac{n}{2}[2a + (n-1)d]$

$S_{10} = 5(46 - 18)$

$= 140.$ **1**

[CBSE Marking Scheme, 2012]

Q. 15. A sum of ₹ 1890 is to be used to give seven cash prizes to students of a school for their overall academic performance. If each prize is ₹ 50 less than its preceding prize, find the value of each of the prizes. A **[Board Term-2, 2012 (5)]**

Sol. Try yourself, Similar to Q. 3. in SATQ-II

Q. 16. If the sum of first m terms of an A.P. is same as the sum of its first n terms ($m \neq n$), show that the sum of its first ($m + n$) terms is zero.

[Board Term-2, 2012 (12)]

Sol. Let 1st term of series be a, common difference be d, then ½

$S_m = S_n$

or, $\frac{m}{2}[2a + (m-1)d] = \frac{n}{2}[2a + (n-1)d]$ **1**

or, $2a(m-n) + \{(m^2 - n^2) - (m-n)\}d = 0$ ½

or, $2a(m-n) + \{(m-n)(m+n) - (m-n)\}d = 0$

or, $(m-n)[2a + (m+n-1)d] = 0$

or, $2a + (m+n-1)d = 0$, $[\because m - n \neq 0]$

$\therefore \quad S_{m+n} = \frac{m+n}{2}[2a+(m+n-1)d]$ **1**

$= \frac{m+n}{2} \times 0 = 0.$ **1**

Hence Proved.

[CBSE Marking Scheme, 2012]

Q. 17. A man repays a loan of ₹ 3250 by paying ₹ 20 in the first month and then increases the payment by ₹ 15 every month. How long will it take him to clear the loan ? A **[Board Term-2, 2012 Set (34)]**

Sol. Here, $a = 20, d = 15$

and $S_n = 3250$

Then, $S_n = \frac{n}{2}[2a+(n-1)d]$

or, $3250 = \frac{n}{2}[2a+(n-1)\times 15]$ **1**

or, $3250 \times 2 = n[40 + 15n - 15]$

or, $6500 = n[25 + 15n]$

or, $1300 = n[5 + 3n]$

or, $3n^2 + 5n - 1300 = 0$

or, $3n^2 + 65n - 60n - 1300 = 0$

or, $n(3n + 65) - 20(3n + 65) = 0$

or, $(n - 20)(3n + 65) = 0$ **1**

Either $n = -\frac{65}{3}$, (not possible) **1**

or $n = 20$

Thus, Man will repay loan in 20 months. **1**

[CBSE Marking Scheme, 2012]

AI **Q. 18. If $1 + 4 + 7 + 10 ..... + x = 287$, find the value of x.** A **[Board Foreign Set-I 2017]**

Sol. Given, $a = 1$ and $d = 3$.

Let number of terms in the series be n.

$\therefore \quad S_n = \frac{n}{2}[2a+(n-1)d]$ **1**

$\therefore \quad \frac{n}{2}[2\times 1+(n-1)3] = 287$

or, $\frac{n}{2}[2+(3n-3)] = 287$

or, $3n^2 - n = 574$

or, $3n^2 - n - 574 = 0$

or, $3n^2 - 42n + 41n - 574 = 0$

or, $3n(n - 14) + 41(n - 14) = 0$

or, $(n - 14)(3n + 41) = 0$ **1½**

$n = 14$

or, $n = -\frac{41}{3}$, it is not possible

Thus, the 14th term is x,

$\therefore \quad a + (n-1)d = x$

or, $1 + 13 \times 3 = x$

Hence, $x = 40.$ **1½**

Q. 19. Find the sum of first 24 terms of an A.P. whose nth term given by $a_n = 3 + 2n$.

A **[Board Outside Delhi Comptt. Set I, II, III 2017]**

Sol. Try yourself, Similar to Q. 6. in SATQ-I

Q. 20. The ratio of the sums of first m and first n terms of an A.P. is $m^2 : n^2$. Show that the ratio of its m^{th} and n^{th} terms is $(2m - 1):(2n - 1)$.

[CBSE Board Delhi Set I 2017]

Sol. Let first term of given A.P. be a and common difference be d also sum of first m and first n terms be S_m and S_n respectively.

$\therefore \quad \frac{S_m}{S_n} = \frac{m^2}{n^2}$ **1**

or, $\frac{\frac{m}{2}[2a+(m-1)d]}{\frac{n}{2}[2a+(n-1)d]} = \frac{m^2}{n^2}$

or, $\frac{2a+(m-1)d}{2a+(n-1)d} = \frac{m^2}{n^2}\times\frac{n}{m} = \frac{m}{n}$ **1**

or, $m(2a + (n - 1)d) = n[2a + (m - 1)d]$ **1**

$d = 2a$

Now $\frac{a_m}{a_n} = \frac{a+(m-1)d}{a+(n-1)d}$

$= \frac{a+(m-1)\times 2a}{a+(n-1)\times 2a}$

or, $\frac{a+2ma-2a}{a+2na-2a} = \frac{2ma-a}{2na-a} = \frac{a(2m-1)}{a(2n-1)}$ **1**

$= 2m - 1 : 2n - 1$ **Proved.**

AI **Q. 21. If the p^{th} term of an A.P. is $\frac{1}{q}$ and q^{th} term is $\frac{1}{p}$.**

Prove that the sum of first pq term of the A.P. is $\left[\frac{pq+1}{2}\right]$. **[CBSE Board Delhi Set III 2017]**

Sol. Let first term and common difference of given A.P. be a and d respectively.

$\therefore \quad a_p = a + (p-1)d = \frac{1}{q}$ ...(i) **1**

and $a_q = a + (q-1)d = \frac{1}{p}$ ...(ii) **1**

Solving (i) and (ii) we get

$a = \frac{1}{pq}$ and $d = \frac{1}{pq}$ **1**

$\therefore \quad S_{pq} = \frac{pq}{2}\left[2\times\frac{1}{pq}+(pq-1)\frac{1}{pq}\right]$

$= \frac{pq+1}{2}$ **1**

[CBSE Marking Scheme, 2017]

Q. 22. If the ratio of the 11th term of an A.P. to its 18th term is 2 : 3, find the ratio of the sum of the first five term to the sum of its first 10 terms.

[Delhi Compt. Set I, II, III 2017]

Sol. Since, $\frac{a_{11}}{a_{18}} = \frac{a+10d}{a+17d} = \frac{2}{3}$

or, $2(a + 17d) = 3(a + 10d)$ **1**

$a = 4d$ ...(i)

Now, $\frac{S_5}{S_{10}} = \frac{\frac{5}{2}(2a+4d)}{\frac{10}{2}[2a+9d]}$ ½

Putting the value of $a = 4d$, we get **1**

or, $\frac{S_5}{S_{10}} = \frac{\frac{5}{2}(8d+4d)}{5(8d+9d)}$ **1**

$\frac{12d}{34d} = \frac{6}{17}$ ½

Hence, $S_5 : S_{10} = 6:17$.

Q. 23. An A.P. consists of 37 terms. The sum of the three middle most terms is 225 and the sum of the last three terms is 429. Find the A.P. **[HOTS]**
[Sample Paper 2017]

Sol. Let the middle most terms of the A.P. be $(a - d)$, a and $(a + d)$.

Given, $a - d + a + a + d = 225$

or, $3a = 225$ **1**

or, $a = 75$ ½

and the middle term $= \frac{37+1}{2} = 19^{th}$ term

$\therefore$ A.P. is

$(a - 18d)$, ..., $(a - 2d)$, $(a - d)$, a, $(a + d)$, $(a + 2d)$, ... , $(a + 18d)$ **1**

Sum of last three terms

$(a + 18d) + (a + 17d) + (a + 16d) = 429$

or, $3a + 51d = 429$

or, $225 + 51d = 429$ or, $d = 4$ ½

First term, $a_1 = a - 18d = 75 - 18 \times 4 = 3$.

$a_2 = 3 + 4 = 7$

Hence, A.P. = 3, 7, 11, ... , 147. **1**

Q. 24. The sum of four consecutive numbers in an A.P. is 32 and the ratio of the product of the first and the last term to the product of two middle terms is 7 : 15. Find the numbers. **[CBSE Delhi/OD Set-2018]**

Sol. Let the four consecutive terms of A.P. be $(a - 3d)$, $(a - d)$, $(a + d)$ and $(a + 3d)$. ½

By given conditions

$a - 3d + a - d + d + a + 3d = 32$

$\Rightarrow 4a = 32 \Rightarrow a = 8$ ½

And $\frac{(a-3d)(a+3d)}{(a-d)(a+d)} = \frac{7}{15}$ **1**

$\frac{a^2-9d^2}{a^2-d^2} = \frac{7}{15}$

$d^2 = 4$

$d = \pm 2$ **1**

Hence, the numbers are 2, 6, 10 and 14 or 14, 10, 6 and 2. **1**

[CBSE Marking Scheme, 2018]

Q. 25. If the ratio of the sum of the first n terms of two A.Ps is $(7n + 1) : (4n + 27)$, then find the ratio of their 9th terms. A **[Outseide Delhi Set III 2017]**

Sol.

Let a, d and A, D be the 1st term and common difference of the 2 APs respectively.

Then,

$$\frac{\frac{n}{2}[2a + (n-1)d]}{\frac{n}{2}[2A + (n-1)D]} = \frac{7n+1}{4n+27}$$

$$\frac{2a + (n-1)d}{2A + (n-1)D} = \frac{7n+1}{4n+27}$$

Replacing n by 17 in both LHS and RHS,

$$\frac{2a + (17-1)d}{2A + (17-1)D} = \frac{7(17)+1}{4(17)+27}$$

$$\frac{2a+16d}{2A+16D} = \frac{119+1}{68+27}$$

$$\frac{2(a+8d)}{2(A+8D)} = \frac{120}{95}$$

as $a + (n-1)d = a_n$,

$$\frac{a_9}{A_9} = \frac{24}{19}$$

$\therefore$ ratio of 9th terms is 24:19

[Topper's Answer, 2017] 4

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