

# Optical Instruments

## Exercise Solutions

### Solution 1:

Let  $\theta$  be the angle formed by the eyes while looking at tree.

$$\theta = \text{Height/Distance}$$

For trees, A,B,C and D, the value of angle  $\theta$  becomes:

$$\theta_A = 2/50 = 0.04$$

$$\theta_B = 2.5/80 = 0.031$$

$$\theta_C = 1.8/70 = 0.0257 \text{ and}$$

$$\theta_D = 2.8/100 = 0.028$$

Since,  $\theta_A > \theta_B > \theta_C > \theta_D$

The arrangement in decreasing order is given by A, B, D and C.

### Solution 2:

For maximum angular magnification,

Image distance = Least distance of clear vision

$$v = d = -25 \text{ cm}$$

Where d is the distance of clear vision.

Using len's formula-

$$1/f = 1/v - 1/u$$

$$\text{Here } f = 12 \text{ and } v = -25$$

$$\Rightarrow u = -1.8 \text{ cm}$$

The object should be placed 8.1 cm away from the lens.

**Solution 3:**

$$(a) m = 1 + D/f$$

$$\Rightarrow 3 = 1 + 25/f$$

$$\Rightarrow f = 12.5 \text{ cm}$$

Focal length when the image is formed at 25cm is 12.5cm

(b) When the image is formed at infinity

$$m = D/f = 25/12.5 = 2$$

Where  $m$  = magnifying power

**Solution 4:**

Here  $D = 10 \text{ cm}$  and  $f = 10 \text{ cm}$

The maximum angular magnification of the child when the image is formed at distance  $D=10\text{cm}$

$$\Rightarrow m = 1 + D/f$$

$$= 1 + 10/10 = 2$$

Angular magnification is 2.0 when the near point is 10 cm.

**Solution 5:**

Here  $D = 40 \text{ cm}$ ,  $m = 5$  and

Least distance of distinct vision for a normal eye = 25 cm

For relaxed eyes, the image will be formed at infinity.

$$\text{So, } m = D/f = 25/f$$

$$\Rightarrow f = 40 \text{ cm}$$

$$\text{And, } m = D/f = 40/5 = 8$$

The magnifying power of simple microscope for far sighted eye will be 8x.

**Solution 6:**

For eyepiece:

Power of the eyepiece = 5 D

The focal length of the eyepiece:  $f_e = 1/5 = 0.2 \text{ m} = 20 \text{ cm}$

and  $v_e = -25$

Using lens formula,

$$1/f = 1/v - 1/u$$

$$\Rightarrow 1/u_e = 1/-25 - 1/20$$

$$\Rightarrow u_e = 11.11 \text{ cm}$$

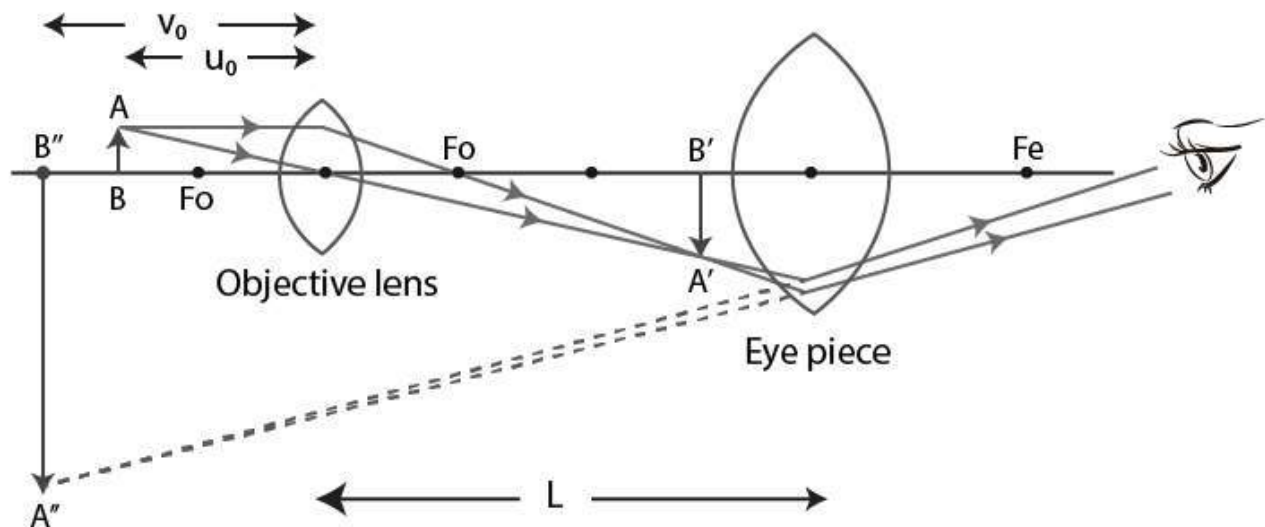
For Objective lens:

Power of the objective lens = 25 D

The focal length of the objective lens is  $m = 1/f_0$

$$\Rightarrow f_0 = 1/25 = 0.04 \text{ m} = 4 \text{ cm}$$

Now,



$L$  = length of the compound microscope  
 $u_0$  = object distance and  
 $v_0$  = image distance for the objective lens.

So, image distance for the objective lens:  $v_0 = L - u_0$

$$\Rightarrow v_0 = 30 - 11.11 = 18.89 \text{ cm}$$

[Since the image formed is real and inverted]

Using lens formula,

$$1/f_0 = 1/v_0 - 1/u_0$$

$$\Rightarrow 1/u_0 = 1/18.89 - 1/4$$

$$\text{or } u_0 = -5.07 \text{ cm}$$

Now, maximum magnifying power of the compound microscope,  $m$  :

$$m = (v_0/u_0) (1 + D/f_e)$$

$$= (-18.89/-5.07)[1 + 25/20]$$

$$= 8.3831$$

Therefore, the maximum magnifying power of the compound microscope is 8.3831.

**Solution 7:**

For the given compound microscope

$f_o = 1 \text{ cm}$ ,  $D = 24 \text{ cm}$  and  $f_e = 6 \text{ cm}$

For eye piece,  $v_e = -24 \text{ cm}$  and  $f_e = 6 \text{ cm}$

Now, using mirror formula,

$$1/v_e - 1/u_e = 1/f_e$$

$$\Rightarrow 1/u_e = 1/(-24) - 1/6 = -5/24$$

$$\Rightarrow u_e = -4.8 \text{ cm}$$

(a) When the separation between the objective and the eyepiece =  $9.8 \text{ cm}$   
for the objective lens, the image will be at a distance

$$v_o = 9.8 - 4.8 = 5 \text{ cm}$$

using mirror formula,  $1/f_o = 1/v_o - 1/u_o$

$$\Rightarrow 1/u_o = 1/5 - 1 = -4/5$$

$$\text{or } u_o = -1.25 \text{ cm}$$

Now, Magnifying power of the compound microscope:

$$m = (v_o/u_o) (1 + D/f_e)$$

$$= (5/1.25)[1 + 24/6]$$

$$= 20$$

(b) When the separation between the objective and the eyepiece is 11.8 cm

$$v_o = 11.8 - 4.8 = 7 \text{ cm}$$

Again from mirror formula,

$$1/7 - 1/u_o = 1$$

$$\Rightarrow u_o = -1.16 \text{ cm}$$

Now, Magnifying power of the compound microscope is

$$m = (v_o/u_o) (1 + D/f_e)$$

$$= (7/1.16)[1 + 24/6]$$

$$= 30$$

Hence, magnification lies in the range of 20-30.

**Solution 8:**

The focal length of the eyepiece :  $f_e = 1/10 = 10 \text{ cm}$

$D = 25 \text{ cm}$

Separation between objective and eyepiece = 20 cm

For eyepiece:

Power of the eyepiece = 10 D

To distinguish the two points having minimum separation, the magnifying power should be maximum-Image distance,

$$v_e = -25 \text{ cm and focal length} = f_e = 10 \text{ cm}$$

$$1/u_e = 1/v_e - 1/f_e = 1/-25 - 1/10 = -7/50 \text{ cm}$$

$$\Rightarrow u_e = -50/7 \text{ cm}$$

The image distance for the objective lens should be

$$v_o = 20 - 50/7 = 90/7 \text{ cm}$$

For Objective lens:

The focal length of the objective lens:  $f_o = 1/5$

$$1/u_o = 1/v_o - 1/f_o = 7/90 - 1/5 = -11/90 \text{ cm}$$

$$u_o = -90/11 \text{ cm}$$

Now, max magnification of the compound microscope :

$$m = (-v_o/u_o) (1 + D/f_e)$$

$$= 11/7 \times 3.5$$

$$= 5.5$$

Hence, minimum separation eye can distinguish  $[0.22/5.5 = 0.04] = 0.04 \text{ mm}$ .

**Solution 9:**

For the given compound microscope:  $f_o = 0.5 \text{ cm}$  and tube length =  $6.5 \text{ cm}$

$$m \text{ (Magnifying power)} = 100$$

When the image is formed at infinity, image formed by the objective lens is real and it should lie on the focus point of the eyepiece.

$$v_o + f_e = 6.5 \text{ cm} \dots(1)$$

For normal adjustment, the magnifying power

$$m = v_o/u_o (D/f_e)$$

$$[\text{As, } v_o/u_o = 1 - v_o/f_o]$$

$$\Rightarrow m = -1 - (v_o/f_o)(D/f_e)$$

$$\Rightarrow 100 = -1 - (v_o/0.5)(25/f_e)$$

$$\Rightarrow 2v_o - 4 f_e = 1 \dots\dots(2)$$

Solving (1) and (2), we get

$$f_e = 2 \text{ cm and } v_o = 4.5$$

The focal length of the eyepiece is  $2 \text{ cm}$  when the image formed is at infinity.

**Solution 10:**

Here,  $f_o = 1.0 \text{ cm}$ ,  $f_e = 5 \text{ cm}$ ,  $u_o = 0.5 \text{ cm}$  and  $v_e = 30 \text{ cm}$



Using lens formula for objective lens:

$$1/f_o = 1/v_o - 1/u_o$$

on substituting the values,

$$\Rightarrow v_o = -1 \text{ cm}$$

Objective lens forms a virtual image at the side same as that of the object at a distance of 1 cm from the objective lens.

This image acts as a virtual object for the eyepiece.

For eyepiece,

$$1/f_e = 1/v_e - 1/u_e$$

Here  $v_e = 30$  and  $f_e = 5$  (given)

$$\Rightarrow u_e = -6 \text{ cm}$$

Separation between the objective and the eyepiece is  $(6 - 1) 5 \text{ cm}$

**Solution 11:**

The focal length of the objective lens :

$$f_o = 1/25 \text{ D} = 0.04 \text{ m} = 4 \text{ cm and}$$

$$f_e = 1/20 \text{ D} = 0.05 \text{ m} = 5 \text{ cm}$$

(a) Instrument must be a microscope.

As, focal length of objective is less than the eyepiece,  $f_o < f_e$

(b) length of the Tube,  $l = 25 \text{ cm}$

Distance between image and eye lens,  $v_e = 25 \text{ cm}$

We know, length of the tube,  $l = v_o + f_e$

$$\Rightarrow v_o = 20 \text{ cm}$$

Now, using lens formula:

$$1/f_o = 1/v_o - 1/u_o$$

on substituting the values,

$$\Rightarrow u_o = 5 \text{ cm}$$

Now, For the microscope, the angular magnification  $m$ :

$$m = (v_o/u_o) (1 + D/f_e) = 20/5 \times 25/5 = 20 \text{ cm}$$

### **Solution 12:**

For the astronomical telescope in normal adjustment.  $m = 50$ ,  $L = 102 \text{ cm}$

Where  $m$  is magnifying power and  $L$  is length of the tube

Let  $f_o$  and  $f_e$  focal length of objective and eyepiece respectively,

$$m = f_o/f_e$$

$$\Rightarrow f_o = 50 f_e \dots(1) \text{ and}$$

$$L = f_o + f_e = 102 \text{ cm} \dots(2)$$

Solving (1) and (2) we get

$$f_e = 2 \text{ cm} = 0.02 \text{ m} \text{ and } f_o = 100 \text{ cm} = 1 \text{ m}$$

Now,

$$\text{Power of objective lens} = 1/f_o = 1 \text{ D and}$$

$$\text{Power of eye piece lens} = 1/f_e = 1/0.02 = 50 \text{ D}$$

**Solution 13:**

Focal length of the objective:  $f_o + f_e = L$

Where  $L$  = Length of the tube and  $f_e$  = Focal length of the eyepiece

Here  $L = 100$  cm and  $f_o = 10$  cm (given)

$$\Rightarrow f_o = L - f_e = 90 \text{ cm}$$

Now, Magnifying power of the telescope:  $m = f_o/f_e = 9$

**Solution 14:**

Focal length of the objective:  $f_o - f_e = L$

[The image will be formed at infinity]

Where  $L$  = Length of the tube and  $f_o$  = Focal length of the objective lens of the telescope

Here  $L = 27$  cm and  $f_o = 30$  cm (given)

$$\Rightarrow f_o = L + f_e = 90 \text{ cm}$$

As concave eyepiece lens used in Galilean telescope  $\Rightarrow f_e = f_o - L = 30 - 27 = 3$  cm

**Solution 15:**

For the far sighted person,  $u = -20$  cm,  $v = -50$  cm

$$\Rightarrow 1/f = 1/v - 1/u$$

$$\Rightarrow 1/f = 3/100$$

$$\Rightarrow f = 100/3 \text{ cm} = 1/3 \text{ m}$$

The power of the lens is 3D

**Solution 16:**

For the near sighted person, so,  $u = \text{infinity}$ ,  $v = -200 \text{ cm} = -2 \text{ m}$

$$\Rightarrow \frac{1}{f} = \frac{1}{v} - \frac{1}{u}$$

$$\Rightarrow \frac{1}{f} = -\frac{1}{2} = -0.5$$

$$\Rightarrow f = \frac{100}{3} \text{ cm} = \frac{1}{3} \text{ m}$$

The power of the lens is  $-0.5 \text{ D}$ .

**Solution 17:**

A person wears glasses of power  $-25 \text{ D}$ .

So, person must be near sighted, so,  $u = \text{infinity}$ ,  $v = \text{far point}$

we know,  $m = \frac{1}{f} \Rightarrow f = \frac{1}{m}$

$$\text{or } f = \frac{1}{(-2.5)} = -0.4 \text{ m} = -40 \text{ cm}$$

using lens formula:

$$\Rightarrow \frac{1}{f} = \frac{1}{v} - \frac{1}{u}$$

$$\Rightarrow \frac{1}{v} = \frac{1}{(-40)}$$

$$\Rightarrow v = -40$$

The far point for the near-sighted person is  $40 \text{ cm}$ .

**Solution 18:**

A professor read a greeting card received on his 50th birthday with + 2.5 D glasses keeping the card 25 cm away. (Given)

After 10 years,

$u = -50$  cm,  $f = (1/2.5)$  D = 40 cm and  $v =$  near point

Now,  $1/f = 1/v - 1/u$

$$\Rightarrow 1/v = 1/200$$

$\Rightarrow$  near point is 200 cm

To read the farewell letter at a distance of 25 cm,  
 $u = -25$  cm,  $v = 200$  cm

Using lens formula:

$$f = 2/9 \text{ m}$$

So, required power of the lens =  $p = 1/f = 4.5$  D

**Solution 19:**

We know, lens formula,  $1/f = 1/v - 1/u$

(a) When the eye lens is fully relaxed,  $u = \text{infinity}$

The retina is 2 cm behind the eye-lens, so  $v = 2$  cm = 0.02 m

Using lens formula:  $f = 0.02$

Now, power of the eye lens =  $p = 1/f = 50$  D

(b) When the eye lens is strained:

Distance of object,  $u = -25$  cm =  $-0.25$  m

and  $v = 2$  cm = 0.02 m

Using lens formula:

$$f = 1/54$$

Now, power of the eye lens =  $p = 1/f = 54$  D

**Solution 20:**

We know, lens formula,  $1/f = 1/v - 1/u$

Near point of the child =  $u = 10 \text{ cm} = 0.1$

Far point of the child =  $100 \text{ cm}$

Distance of the retina from the eye lens =  $v = 2 \text{ cm} = 0.02 \text{ m}$

Using lens formula:  $f = 1/60 \text{ m}$

Now, Power of the lens when the near sight is  $10 \text{ cm} = p = 1/f = 60 \text{ D}$

When, the near point of the child is  $100 \text{ cm} = 1 \text{ m}$ , so,  $u = -1 \text{ m}$

Using lens formula:  $1/f = 1/(0.02) - 1/-1 = 51$

$\Rightarrow f = 1/51$

Now, Power of the lens when near point  $1 \text{ m}$  is  $p = 1/f = 51 \text{ D}$

So, the range of power of the eyes is  $+60 \text{ D}$  to  $+51 \text{ D}$

**Solution 21:** The separation of the glass from the eye is  $1 \text{ cm}$

For the near-sighted person,  $u = \text{infinity}$

Distance of the image from glass is  $v = \text{Distance between image and eye} - \text{distance between the glass and the eye}$

$= 25 - 1 = 24 \text{ cm} = 0.24 \text{ m}$

For glass,  $u = \text{infinity}$  and  $v = -24 \text{ cm} = -0.24 \text{ m}$

Using lens formula,  $1/f = 1/v - 1/u = 1/(-0.24) = -4.2 \text{ D}$

**Solution 22:**

Near point of the person =  $100 \text{ cm}$

It is required to read at  $20 \text{ cm}$  in the following two cases:

(a) When the contact lens is used,

$$u = -20 \text{ cm} = -0.2 \text{ m and } v = -100 \text{ cm} = -1 \text{ m}$$

Using lens formula,  $1/f = 1/v - 1/u$

$$\Rightarrow 1/f = 1/-1 - 1/(-0.2) = 4$$

Now, power of the contact lens =  $p = 1/f = 4\text{D}$

(b) When the person uses spectacles at 2 cm from the eyes:

$$u = -(20 - 2) = -18 \text{ cm} = -0.18 \text{ m and } v = -100 \text{ cm} = -1 \text{ m}$$

Using lens formula,  $1/f = 1/v - 1/u = 9/41$

Now, power of spectacles =  $p = 1/f = 4.55 \text{ D}$

### **Solution 23:**

Focal lens of the glasses =  $f = 1/p = (1/1.5) \text{ m}$

As,

Power of the glasses to have normal vision from 25 cm onwards,  $p = +1.5 \text{ D}$ . (given)

and  $u = -25 \text{ cm} = -0.25 \text{ m}$

Using lens formula .  $1/f = 1/v - 1/u$

$$\Rightarrow 1/v = 1/f + 1/u = 1/1.5 + 1/(-0.25) = 2.5$$

$$\Rightarrow v = 0.4 \text{ m} = 40 \text{ cm}$$

When she is not wearing glasses, the near point is 40 cm. The focal length of magnifying

glass at that point becomes:  $f = 1/20 = 0.05 \text{ m} = 5 \text{ cm}$

(a) The maximum magnifying power with glass:

$$m = 1 + D/f$$

[Given  $D = 25 \text{ cm}$ ]

$$= 1 + 25/5$$

$$= 6$$

(b) Without the glasses,  $D = 40 \text{ cm}$

$$m = 1 + D/f = 1 + 40/5 = 9$$

### **Solution 24:**

A lady cannot see objects closer than 40 cm from the left eye and closer than 100 cm from the right eye.

For the left glass lens:

Least distance of clear vision,  $u = -25 \text{ cm}$

The lady cannot see objects closer than 40cm i.e.  $v = -40 \text{ cm}$

Using lens formula,  $1/f = 1/v - 1/u$

$$\Rightarrow 1/f = 3/200$$

$$\Rightarrow f = 66.6 \text{ cm}$$



Hence, the focal length of the left eye glass lens is 66.6 cm

For the right eye glass:

The lady cannot see objects closer than 40cm =  $v = -100$  cm

Least distance of clear vision =  $u = -25$  cm

Using lens formula:

$$1/f = 1/v - 1/u$$

$$\Rightarrow 1/f = 3/100$$

$$\Rightarrow f = 33.3 \text{ cm}$$

Hence, the focal length of the right eye glass lens is 33.3 cm

(a) For an astronomical telescope, eye piece lens should have smaller focal length. so she should use right lens,  $f = 33.3$  cm, as the eye piece lens.

(b) With relaxed eye,

$$f_o = 200/3 \text{ cm} \text{ and } f_e = 100/3 \text{ cm}$$

$$\text{So, magnification} = m = f_o/f_e = 2$$