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Arithmetic Progression



Let's study.

- Sequence
- n^{th} term of an A.P.
- Arithmetic Progression
- Sum of n terms of an A.P.



Let's learn.

Sequence

We write numbers 1, 2, 3, 4, ... in an order. In this order we can tell the position of any number. For example, number 13 is at 13th position. The numbers 1, 4, 9, 16, 25, 36, 49, ... are also written in a particular order. Here $16 = 4^2$ is at 4th position. similarly, $25 = 5^2$ is at the 5th position; $49 = 7^2$ is at the 7th position. In this set of numbers also, place of each number is determined.



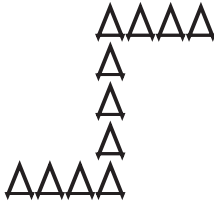
A set of numbers where the numbers are arranged in a definite order, like the natural numbers, is called a **sequence**.

In a sequence a particular number is written at a particular position. If the numbers are written as $a_1, a_2, a_3, a_4, \dots$ then a_1 is first, a_2 is second, ... and so on. It is clear that a_n is at the nth place. A sequence of the numbers is also represented by alphabets f_1, f_2, f_3, \dots and we find that there is a definite order in which numbers are arranged.

When students stand in a row for drill on the playground they form a sequence. We have experienced that some sequences have a particular pattern.

Complete the given pattern

| | | | | | | | | | |
|-------------------|---|---|---|---|--|--|--|--|--|
| Pattern | | | | | | | | | |
| Number of circles | 1 | 3 | 5 | 7 | | | | | |

| | | | | | | | |
|---------------------|-----------------------------------------------------------------------------------|-----------------------------------------------------------------------------------|-----------------------------------------------------------------------------------|--|--|--|--|
| Pattern |  |  |  | | | | |
| Number of triangles | 5 | 8 | 11 | | | | |

Look at the patterns of the numbers. Try to find a rule to obtain the next number from its preceding number. This helps us to write all the next numbers.

See the numbers 2, 11, -6, 0, 5, -37, 8, 2, 61 written in this order.

Here $a_1 = 2$, $a_2 = 11$, $a_3 = -6$, ... This list of numbers is also a sequence. But in this case we cannot tell why a particular term is at a particular position ; similarly we cannot tell a definite relation between the consecutive terms.

In general, only those sequences are studied where there is a rule which determines the next term.

For example (1) 4, 8, 12, 16 ... (2) 2, 4, 8, 16, 32, ...

$$(3) \frac{1}{5}, \frac{1}{10}, \frac{1}{15}, \frac{1}{20} \dots$$

Terms in a sequence

In a sequence, ordered terms are represented as $t_1, t_2, t_3, \dots, t_n \dots$. In general sequence is written as $\{t_n\}$. If the sequence is infinite, for every positive integer n , there is a term t_n .

Activity I : Some sequences are given below. Show the positions of the terms

by t_1, t_2, t_3, \dots

$$(1) 9, 15, 21, 27, \dots \quad \text{Here } t_1 = 9, \quad t_2 = 15, \quad t_3 = 21, \dots$$

$$(2) 7, 7, 7, 7, \dots \quad \text{Here } t_1 = 7, \quad t_2 = \boxed{}, \quad t_3 = \boxed{}, \dots$$

$$(3) -2, -6, -10, -14, \dots \quad \text{Here } t_1 = -2, \quad t_2 = \boxed{}, \quad t_3 = \boxed{}, \dots$$

Activity II : Some sequences are given below. Check whether there is any rule among the terms. Find the similarity between two sequences.

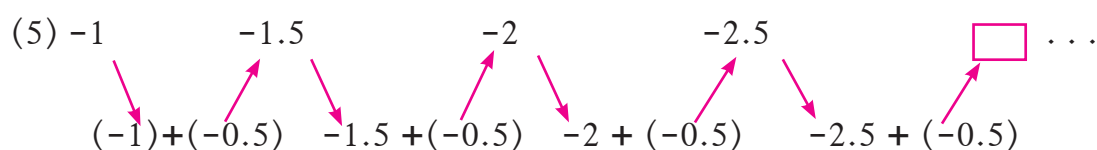
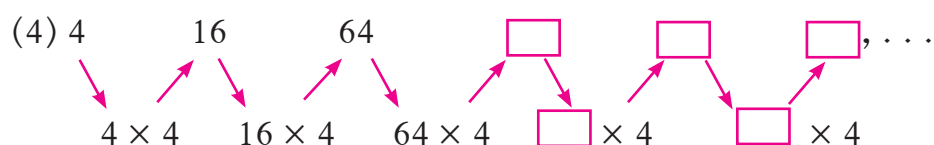
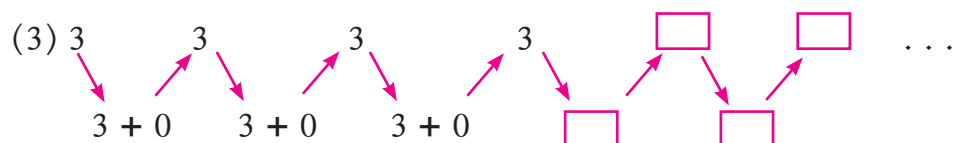
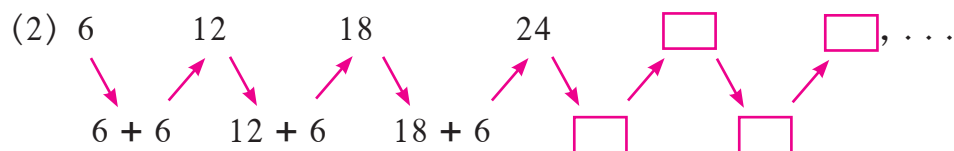
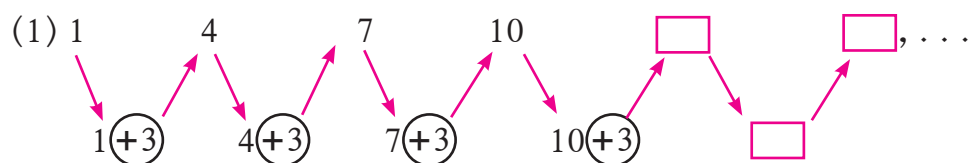
To check the rule for the terms of the sequence look at the arrangements on the next page, and fill the empty boxes suitably.

$$(1) 1, 4, 7, 10, 13, \dots \quad (2) 6, 12, 18, 24, \dots$$

$$(3) 3, 3, 3, 3, \dots \quad (4) 4, 16, 64, \dots$$

$$(5) -1, -1.5, -2, -2.5, \dots \quad (6) 1^3, 2^3, 3^3, 4^3, \dots$$

Let's find the relation in these sequences. Let's understand the thought behind it.



(6) 1^3 , 2^3 , 3^3 , ...

Here in the sequences (1), (2), (3), (5), the similarity is that next term is obtained by adding a particular number to the previous number. Each of these sequences is called an Arithmetic Progression.

Sequence (4) is not an arithmetic progression. In this sequence the next term is obtained by multiplying the previous term by a particular number. This type of sequences is called a Geometric Progression.

Sequence (6) is neither arithmetic progression nor geometric progression.

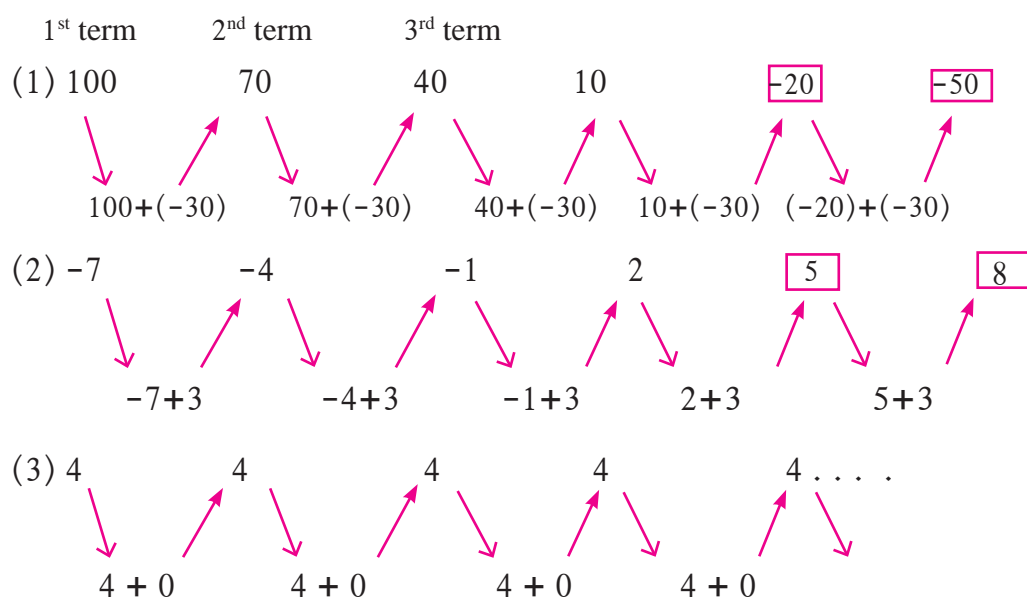
This year we are going to study arithmetic progression.

Arithmetic Progression

Some sequences are given below. For every sequence write the next three terms.

(1) 100, 70, 40, 10, ... (2) -7, -4, -1, 2, ... (3) 4, 4, 4, ...

In the given sequences, observe how the next term is obtained.



In each sequence above, every term is obtained by adding a particular number in the previous term. The difference between two consecutive terms is constant.

The difference in ex. (i) is negative, in ex. (ii) it is positive and in ex. (iii) it is zero.

If the difference between two consecutive terms is constant then it is called the common difference and is generally denoted by letter d .

In the given sequence if the difference between two consecutive terms $(t_{n+1} - t_n)$ is constant then the sequence is called Arithmetic Progression (A.P.). In this sequence $t_{n+1} - t_n = d$ is the common difference.

In an A.P. if first term is denoted by a and common difference is d then,

$$t_1 = a, \quad t_2 = a + d$$

$$t_3 = (a + d) + d = a + 2d$$

A.P. having first term as a and common difference d is

$$a, (a + d), (a + 2d), (a + 3d), \dots$$

Let's see some examples of A.P.

Ex.(1) Arifa saved ₹ 100 every month. In one year the total amount saved after every month is as given below.

| Month | I | II | III | IV | V | VI | VII | VIII | IX | X | XI | XII |
|------------|-----|-----|-----|-----|-----|-----|-----|------|-----|------|------|------|
| Saving (₹) | 100 | 200 | 300 | 400 | 500 | 600 | 700 | 800 | 900 | 1000 | 1100 | 1200 |

The numbers showing the total saving after every month are in A.P.

Ex. (2) Pranav borrowed ₹ 10000 from his friend and agreed to repay ₹ 1000 per month. So the remaining amount to be paid in every month will be as follows.

| No. of month | 1 | 2 | 3 | 4 | 5 | ... | ... | ... |
|-----------------------|--------|-------|-------|-------|-----|-------|-------|-----|
| Amount to be paid (₹) | 10,000 | 9,000 | 8,000 | 7,000 | ... | 2,000 | 1,000 | 0 |

Ex. (3) Consider the table of 5, that is numbers divisible by 5.

5, 10, 15, 20, . . . 50, 55, 60, is an infinite A.P.

Ex (1) and (2) are finite A.P. while (3) is an infinite A.P.



Let's remember!

- (1) In a sequence if difference $(t_{n+1} - t_n)$ is constant then the sequence is called an arithmetic progression.
- (2) In an A.P. the difference between two consecutive terms is constant and is denoted by d .
- (3) Difference d can be positive, negative or zero.
- (4) In an A.P. if the first term is a , and common difference is d then the terms in the sequence are $a, (a + d), (a + 2d), \dots$

Activity : Write one example of finite and infinite A.P. each.

Solved examples

Ex. (1) Which of the following sequences are A.P ? If it is an A.P, find next two terms.

- (i) 5, 12, 19, 26, ... (ii) 2, -2, -6, -10, ...
(iii) 1, 1, 2, 2, 3, 3, ... (iv) $\frac{3}{2}, \frac{1}{2}, -\frac{1}{2}, \dots$

Solution : (i) In this sequence 5, 12, 19, 26, . . . ,

First term = $t_1 = 5$, $t_2 = 12$, $t_3 = 19, \dots$

$$t_2 - t_1 = 12 - 5 = 7$$

$$t_3 - t_2 = 19 - 12 = 7$$

Here first term is 5 and common difference which is constant is $d = 7$

\therefore This sequence is an A.P.

Next two terms in this A.P. are $26 + 7 = 33$ and $33 + 7 = 40$.

Next two terms in given A.P. are 33 and 40

(ii) In the sequence 2, -2, -6, -10, . . . ,

$$t_1 = 2, \quad t_2 = -2, \quad t_3 = -6, \quad t_4 = -10 \dots$$

$$t_2 - t_1 = -2 - 2 = -4$$

$$t_3 - t_2 = -6 - (-2) = -6 + 2 = -4$$

$$t_4 - t_3 = -10 - (-6) = -10 + 6 = -4$$

From this difference between two consecutive terms that is $t_n - t_{n-1} = -4$

$\therefore d = -4$, which is constant. \therefore It is an A.P.

Next two terms in this A.P. are $(-10) + (-4) = -14$ and $(-14) + (-4) = -18$

(iii) In the sequence 1, 1, 2, 2, 3, 3, . . . ,

$$t_1 = 1, \quad t_2 = 1, \quad t_3 = 2, \quad t_4 = 2, \quad t_5 = 3, \quad t_6 = 3 \dots$$

$$t_2 - t_1 = 1 - 1 = 0 \quad t_3 - t_2 = 2 - 1 = 1$$

$$t_4 - t_3 = 2 - 2 = 0 \quad t_3 - t_2 \neq t_2 - t_1$$

In this sequence difference between two consecutive terms is not constant.

\therefore This sequence is not an A.P.

(iv) In the sequence $\frac{3}{2}, \frac{1}{2}, -\frac{1}{2}, -\frac{3}{2}, \dots$,

$$t_1 = \frac{3}{2}, \quad t_2 = \frac{1}{2}, \quad t_3 = -\frac{1}{2}, \quad t_4 = -\frac{3}{2} \dots$$

$$t_2 - t_1 = \frac{1}{2} - \frac{3}{2} = -\frac{2}{2} = -1$$

$$t_3 - t_2 = -\frac{1}{2} - \frac{1}{2} = -\frac{2}{2} = -1$$

$$t_4 - t_3 = -\frac{3}{2} - (-\frac{1}{2}) = -\frac{3}{2} + \frac{1}{2} = -\frac{2}{2} = -1$$

Here the common difference $d = -1$ which is constant.

\therefore Given sequence is an A.P. Let's find next two terms of this A.P.

$$-\frac{3}{2} - 1 = -\frac{5}{2}, \quad \frac{5}{2} - 1 = -\frac{7}{2}$$

\therefore Next two terms are $-\frac{5}{2}$ and $-\frac{7}{2}$

Ex. (2) The first term a and common difference d are given. Find first four terms of A.P.

(i) $a = -3, d = 4$ (ii) $a = 200, d = 7$

(iii) $a = -1, d = -\frac{1}{2}$ (iv) $a = 8, d = -5$

Solution : (i) Given $a = -3, d = 4$

$$t_1 = -3$$

$$t_2 = t_1 + d = -3 + 4 = 1$$

$$t_3 = t_2 + d = 1 + 4 = 5$$

$$t_4 = t_3 + d = 5 + 4 = 9$$

\therefore A.P. is $= -3, 1, 5, 9, \dots$

(iii) $a = -1, d = -\frac{1}{2}$

$$a = t_1 = -1$$

$$t_2 = t_1 + d = -1 + \left(-\frac{1}{2}\right) = -\frac{3}{2}$$

$$t_3 = t_2 + d = -\frac{3}{2} + \left(-\frac{1}{2}\right) = -\frac{4}{2} = -2$$

$$t_4 = t_3 + d = -2 + \left(-\frac{1}{2}\right)$$

$$= -2 - \frac{1}{2} = -\frac{5}{2}$$

\therefore A.P. is $= -1, -\frac{3}{2}, -2, -\frac{5}{2}, \dots$

(ii) Given $a = 200, d = 7$

$$a = t_1 = 200$$

$$t_2 = t_1 + d = 200 + 7 = 207$$

$$t_3 = t_2 + d = 207 + 7 = 214$$

$$t_4 = t_3 + d = 214 + 7 = 221$$

\therefore A.P. is $= 200, 207, 214, 221, \dots$

(iv) $a = 8, d = -5$

$$a = t_1 = 8$$

$$t_2 = t_1 + d = 8 + (-5) = 3$$

$$t_3 = t_2 + d = 3 + (-5) = -2$$

$$t_4 = t_3 + d = -2 + (-5) = -7$$

$8, 3, -2, -7, \dots$

\therefore A.P. is $= 8, 3, -2, -7, \dots$

Practice Set 3.1

1. Which of the following sequences are A.P. ? If they are A.P. find the common difference .

(1) $2, 4, 6, 8, \dots$ (2) $2, \frac{5}{2}, 3, \frac{7}{3}, \dots$ (3) $-10, -6, -2, 2, \dots$

(4) $0.3, 0.33, .0333, \dots$ (5) $0, -4, -8, -12, \dots$ (6) $-\frac{1}{5}, -\frac{1}{5}, -\frac{1}{5}, \dots$

(7) $3, 3 + \sqrt{2}, 3 + 2\sqrt{2}, 3 + 3\sqrt{2}, \dots$ (8) $127, 132, 137, \dots$

2. Write an A.P. whose first term is a and common difference is d in each of the following.

(1) $a = 10, d = 5$ (2) $a = -3, d = 0$ (3) $a = -7, d = \frac{1}{2}$

(4) $a = -1.25, d = 3$ (5) $a = 6, d = -3$ (6) $a = -19, d = -4$

3. Find the first term and common difference for each of the A.P.

(1) 5, 1, -3, -7, ... (2) 0.6, 0.9, 1.2, 1.5, ...

(3) 127, 135, 143, 151, ... (4) $\frac{1}{4}, \frac{3}{4}, \frac{5}{4}, \frac{7}{4}, \dots$



Let's think.

- Is 5, 8, 11, 14, ... an A.P. ? If so then what will be the 100th term?
Check whether 92 is in this A.P.? Is number 61 in this A.P.?



Let's learn.

n^{th} term of an A. P.

In the sequence 5, 8, 11, 14, ... the difference between two consecutive terms is 3. Hence, this sequence is an A.P.

Here the first term is 5. If 3 is added to 5 we get the second term 8. Similarly to find 100th term what should be done?

| | | |
|------------|-------------|--------------------|
| First term | Second term | Third term ... |
| Number 5, | $5 + 3 = 8$ | $8 + 3 = 11 \dots$ |

In this way reaching upto 100th term will be time consuming. Let's see if we can find any formula for it.

| | | | | | | | |
|----------------------------------|----------------------------------|----------------------------------|----------------------------------|-----|----------------------------------|--------------------------------------------|-----|
| 5 | 8 | 11 | 14 | ... | ... | ... | ... |
| 5 | $5 + 1 \times 3$ | $5 + 2 \times 3$ | $5 + 3 \times 3$ | ... | $5 + (n - 1) \times 3$ | $5 + n \times 3$ | ... |
| 1 st term t_1 | 2 nd term t_2 | 3 rd term t_3 | 4 th term t_4 | ... | n^{th} term t_n | $(n + 1)^{\text{th}}$ term t_{n+1} | ... |

Generally in the A.P. t_1, t_2, t_3, \dots . If first term is a and common difference is d ,

$$t_1 = a$$

$$t_2 = t_1 + d = a + d = a + (2 - 1)d$$

$$t_3 = t_2 + d = a + d + d = a + 2d = a + (3 - 1)d$$

$$t_4 = t_3 + d = a + 2d + d = a + 3d = a + (4 - 1)d$$

We get $t_n = a + (n - 1)d$.

Using the above formula we can find the 100th term of the A.P. 5, 8, 11, 14, . . .

Here $a = 5$ $d = 3$

$$t_n = a + (n - 1)d$$

$$\therefore t_{100} = 5 + (100 - 1) \times 3$$

$$= 5 + 99 \times 3$$

$$= 5 + 297$$

$$t_{100} = 302$$

100th tem of this A.P. is 302.

Let's check whether 61 is in this A.P. To find the answer we use the same formula.

$$t_n = a + (n - 1)d$$

$$t_n = 5 + (n - 1) \times 3$$

If 61 is n^{th} term means t_n , then

$$\therefore 61 = 5 + 3n - 3$$

$$= 3n + 2$$

$$\therefore 3n = 59$$

$$\therefore n = \frac{59}{3}$$

But then, n is not a natural number.

\therefore 61 is not in this A.P.



Let's think.

Kabir's mother keeps a record of his height on each birthday. When he was one year old, his height was 70 cm, at 2 years he was 80 cm tall and 3 years he was 90 cm tall. His aunt Meera was studying in the 10th class. She said, "it seems like Kabir's height grows in Arithmetic Progression". Assuming this, she calculated how tall Kabir will be at the age of 15 years when he is in the 10th ! She was shocked to find it. You too assume that Kabir grows in A.P. and find out his height at the age of 15 years.

Solved examples

Ex. (1) Find t_n for following A.P. and then find 30th term of A.P.

3, 8, 13, 18, ...

Solution : Given A.P. 3, 8, 13, 18, ...

Here $t_1 = 3, t_2 = 8, t_3 = 13, t_4 = 18, \dots$

$$d = t_2 - t_1 = 8 - 3 = 5$$

We know that $t_n = a + (n - 1)d$

$$\therefore t_n = 3 + (n - 1) \times 5 \quad \because a = 3, d = 5$$

$$\therefore t_n = 3 + 5n - 5$$

$$\therefore t_n = 5n - 2$$

$$\begin{aligned} \therefore 30^{\text{th}} \text{ term} &= t_{30} = 5 \times 30 - 2 \\ &= 150 - 2 = 148 \end{aligned}$$

Ex. (2) Which term of the following A.P. is 560?

2, 11, 20, 29, ...

Solution : Given A.P. 2, 11, 20, 29, ...

Here $a = 2, d = 11 - 2 = 9$

n^{th} term of this A.P. is 560.

$$t_n = a + (n - 1)d$$

$$\begin{aligned} \therefore 560 &= 2 + (n - 1) \times 9 \\ &= 2 + 9n - 9 \end{aligned}$$

$$\therefore 9n = 567$$

$$\therefore n = \frac{567}{9} = 63$$

$\therefore 63^{\text{rd}}$ term of given A.P. is 560.

Ex. (3) Check whether 301 is in the sequence 5, 11, 17, 23, ... ?

Solution : In the sequence 5, 11, 17, 23, ...

$t_1 = 5, t_2 = 11, t_3 = 17, t_4 = 23, \dots$

$$t_2 - t_1 = 11 - 5 = 6$$

$$t_3 - t_2 = 17 - 11 = 6$$

\therefore This sequence is an A.P.

First term $a = 5$ and $d = 6$

If 301 is n^{th} term, then.

$$t_n = a + (n - 1)d = 301$$

$$\begin{aligned} \therefore 301 &= 5 + (n - 1) \times 6 \\ &= 5 + 6n - 6 \end{aligned}$$

$$\therefore 6n = 301 + 1 = 302$$

$$\therefore n = \frac{302}{6}. \text{ But it is not an integer.}$$

$\therefore 301$ is not in the given sequence.

Ex. (4) How many two digit numbers are divisible by 4?

Solution : List of two digit numbers divisible by 4 is

12, 16, 20, 24, ..., 96.

Let's find how many such numbers are there.

$$t_n = 96, \quad a = 12, \quad d = 4$$

From this we will find the value of n .

$$t_n = 96, \therefore \text{By formula,}$$

$$\begin{aligned} 96 &= 12 + (n - 1) \times 4 \\ &= 12 + 4n - 4 \end{aligned}$$

$$\therefore 4n = 88$$

$$\therefore n = 22$$

\therefore There are 22 two digit numbers divisible by 4.

Ex. (5) – The 10th term and the 18th term of an A.P. are 25 and 41 respectively then find 38th term of that A.P., similarly if n^{th} term is 99. Find the value of n .

Solution : In the given A.P. $t_{10} = 25$ and $t_{18} = 41$.

We know that, $t_n = a + (n - 1)d$

$$\therefore t_{10} = a + (10 - 1)d$$

$$\therefore 25 = a + 9d \dots \text{(I)}$$

Similarly $t_{18} = a + (18 - 1)d$

$$\therefore 41 = a + 17d \dots \text{(II)}$$

$$25 = a + 9d \dots \text{From (I) .}$$

$$a = 25 - 9d.$$

Substituting this value in equation II.

$$\therefore \text{Equation (II) } a + 17d = 41$$

$$\therefore 25 - 9d + 17d = 41$$

$$8d = 41 - 25 = 16$$

$$d = 2$$

Substituting $d = 2$ in equation I.

$$a + 9d = 25$$

$$\therefore a + 9 \times 2 = 25$$

$$\therefore a + 18 = 25$$

$$\therefore a = 7$$

$$\text{Now, } t_n = a + (n - 1)d$$

$$\therefore t_{38} = 7 + (38 - 1) \times 2$$

$$t_{38} = 7 + 37 \times 2$$

$$t_{38} = 7 + 74$$

$$t_{38} = 81$$

If n^{th} term is 99, then to find value of n .

$$t_n = a + (n - 1)d$$

$$99 = 7 + (n - 1) \times 2$$

$$99 = 7 + 2n - 2$$

$$99 = 5 + 2n$$

$$\therefore 2n = 94$$

$$\therefore n = 47$$

\therefore In the given progression 38th term is 81 and 99 is the 47th term.

Practice Set 3.2

1. Write the correct number in the given boxes from the following A. P.

(i) 1, 8, 15, 22, . . .

Here $a = \square$, $t_1 = \square$, $t_2 = \square$, $t_3 = \square$,

$t_2 - t_1 = \square - \square = \square$

$t_3 - t_2 = \square - \square = \square \therefore d = \square$

(ii) 3, 6, 9, 12, . . .

Here $t_1 = \square$, $t_2 = \square$, $t_3 = \square$, $t_4 = \square$,

$t_2 - t_1 = \square$, $t_3 - t_2 = \square \therefore d = \square$

(iii) -3, -8, -13, -18, . . .

Here $t_3 = \square$, $t_2 = \square$, $t_4 = \square$, $t_1 = \square$,

$t_2 - t_1 = \square$, $t_3 - t_2 = \square \therefore a = \square, d = \square$

(iv) 70, 60, 50, 40, . . .

Here $t_1 = \square$, $t_2 = \square$, $t_3 = \square$, . . .

$\therefore a = \square, d = \square$

2. Decide whether following sequence is an A.P., if so find the 20th term of the progression.

-12, -5, 2, 9, 16, 23, 30, . . .

3. Given Arithmetic Progression 12, 16, 20, 24, . . . Find the 24th term of this progression.

4. Find the 19th term of the following A.P.

7, 13, 19, 25, . . .

5. Find the 27th term of the following A.P.

9, 4, -1, -6, -11, . . .

6. Find how many three digit natural numbers are divisible by 5.

7. The 11th term and the 21st term of an A.P. are 16 and 29 respectively, then find the 41th term of that A.P.

8. 11, 8, 5, 2, . . . In this A.P. which term is number -151?

9. In the natural numbers from 10 to 250, how many are divisible by 4?

10. In an A.P. 17th term is 7 more than its 10th term. Find the common difference.

The Wise Teacher

Once upon a time, there lived a king. He appointed two teachers Tara and Meera to teach horse riding for one year to his children Yashwantraje and Geetadevi. He asked both of them how much salary they wanted.

Tara said, "Give me 100 gold coins in first month and every month increase the amount by 100 gold coins." Meera said, "Give me 10 gold coins in the first month and every month just double the amount of the previous month."

The king agreed. After three months Yashwantraje said to his sister, "My teacher is smarter than your teacher as she had asked for more money." Geetadevi said, "I also thought the same, I asked Meeratai about it. She only smiled and said compare the salaries after 8 months. I calculated their 9 months salaries. You can also check."

| Months | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
|----------------|-----|-----|-----|-----|-----|-----|-----|------|------|----|----|----|
| Tara's salary | 100 | 200 | 300 | 400 | 500 | 600 | 700 | 800 | 900 | - | - | - |
| Meera's salary | 10 | 20 | 40 | 80 | 160 | 320 | 640 | 1280 | 2560 | - | - | - |

Complete the above table.

Tara's salary 100, 200, 300, 400, . . . is in A.P.

$$t_1 = 100, \quad t_2 = 200, \quad t_3 = 300, \dots \quad t_2 - t_1 = 100 = d$$

Common difference is 100.

Meera's salary 10, 20, 40, 80, . . . is not an A.P. Reason is $20 - 10 = 10$, $40 - 20 = 20$, $80 - 40 = 40$ So the common difference is not constant.

But here each term is double the preceding term.

$$\text{Here } \frac{t_2}{t_1} = \frac{20}{10} = 2, \quad \frac{t_3}{t_2} = \frac{40}{20} = 2, \quad \frac{t_4}{t_3} = \frac{80}{40} = 2$$

$\therefore \frac{t_{n+1}}{t_n}$ The ratio of a term and its preceding term is constant. This type of progression is called a geometric progression. Notice that if ratio $\frac{t_{n+1}}{t_n}$ is greater than 1, then geometric progression will increase faster than arithmetic progression.

If the ratio is smaller than 1, note how the geometric progression changes.

This year we are going to study Arithmetic Progression only. We have seen how to find the n^{th} term of an A.P. Now we are going to see how to find the sum of the first n terms.

Quick Addition

Three hundred years ago there was a single teacher school in Germany. The teacher was Buttner and he had an assistant Johann Martin Bortels. He used to teach alphabets to the children and sharpen their pencils. Buttner was a strict teacher. One day he wanted to do some work and wanted peace in the class, so he tried to occupy all students with a lengthy addition. They were asked to add all intergers from 1 to 100. In few minutes one slate was slammed on the floor. He looked at Carl Gauss and asked, "I asked you to add all integers from 1 to 100. Why did you keep the slate down? Don't you want to do it ?"

Carl Gauss said, "I have done the addition."

The teacher asked, "How did you do it so quickly? You wouldn't have written all the numbers ! What is the answer ?"

Carl Gauss said, "Five thousand fifty"

Teacher was so surprised and asked him, 'How do you find the answer?'

Carl Gauss explained his quick addition method:

| | | | | | | | | | | |
|--------------------------|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| Nos. in increasing order | 1 | 2 | 3 | 4 | ... | ... | ... | ... | ... | 100 |
| | 1 | 2 | 3 | 4 | ... | ... | ... | ... | ... | 100 |
| Nos. in decreasing order | 100 | 99 | 98 | 97 | ... | ... | ... | ... | ... | 1 |
| | 100 | 99 | 98 | 97 | ... | ... | ... | ... | ... | 1 |
| Sum | 101 | 101 | 101 | 101 | ... | ... | ... | ... | ... | 101 |

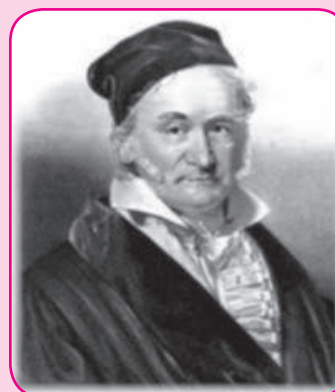
The sum of each pair is 101. This sum occurs 100 times so 101×100 is the product needed. It is 10100. In this 1 to 100 are counted two times. Therefore, half of 10100 is 5050 and sum of 1 to 100 is 5050. The teacher appreciated his work.

Now using this method of Gauss, let's find sum of n terms of an A.P.

Johann Friedrich Carl Gauss

30th April 1777 – 23rd February 1855.

Carl Gauss was a great German mathematician, He was born in Braunschweig, he was the only son of uneducated parents. He showed a glimpse of his intelligence in Buttner's school. After some years, Buttner's helper, Johann Martin Bartels and Gauss became friends. Together, they published a book on Algebra. Bartels made the other people realise the extra ordinary intelligence of Gauss.





Let's learn.

Sum of first n terms of an A. P.

Arithmetic Progression $a, a + d, a + 2d, a + 3d, \dots, a + (n - 1)d$

In this progression a is the first term and d is the common difference. Let's write the sum of first n terms as S_n .

$$S_n = [a] + [a + d] + \dots + [a + (n-2)d] + [a + (n-1)d]$$

Reversing the terms and rewriting the expression again,

$$S_n = [a + (n-1)d] + [a + (n-2)d] + \dots + [a + d] + [a]$$

On adding,

$$2S_n = [a + a + (n-1)d] + [a + d + a + (n-2)d] + \dots + [a + (n-2)d + a + d] + [a + (n-1)d + a]$$

$$2S_n = [2a + (n-1)d] + [2a + (n-1)d] + \dots + [2a + (n-1)d] \dots n \text{ times.}$$

$$\therefore 2S_n = n [2a + (n-1)d]$$

$$\therefore S_n = \frac{n}{2} [2a + (n-1)d] \quad \text{or} \quad S_n = na + \frac{n(n-1)}{2} d$$

Ex. Let's find the sum of first 100 terms of A.P. 14, 16, 18, \dots

$$\text{Here } a = 14, \quad d = 2, \quad n = 100$$

$$S_n = \frac{n}{2} [2a + (n-1)d]$$

$$\therefore S_{100} = \frac{100}{2} [2 \times 14 + (100-1) \times 2]$$

$$= 50 [28 + 198]$$

$$= 50 \times 226 = 11300$$

\therefore Sum of first 100 terms of given A.P. is 11,300



Let's remember!

For the given Arithmetic Progression, if first term is a and common difference is d then

$$t_n = [a + (n-1)d]$$

$$S_n = \frac{n}{2} [2a + (n-1)d] = na + \frac{n(n-1)}{2} d$$

Let's find one more formula for sum of first n terms.

In the A.P. $a, a + d, a + 2d, a + 3d, \dots, a + (n - 1)d$

First term $= t_1 = a$ and n^{th} term is $[a + (n - 1)d]$

$$\text{Now } S_n = \frac{n}{2} [\underline{a} + \underline{a + (n-1)d}]$$

$$\therefore S_n = \frac{n}{2} [t_1 + t_n] = \frac{n}{2} [\text{First term} + \text{last term}]$$

🔗🔗🔗 Solved examples 🔗🔗🔗

Ex. (1) Find the sum of first n natural numbers.

Solution : First n natural numbers are $1, 2, 3, \dots, n$.

Here $a = 1, d = 1, n^{\text{th}}$ term $= n$

$$\therefore S_n = 1 + 2 + 3 + \dots + n$$

$$S_n = \frac{n}{2} [\text{First term} + \text{last term}] \dots \dots \text{(by the formula)}$$

$$= \frac{n}{2} [1 + n]$$

$$= \frac{n(n+1)}{2}$$

$$\therefore \text{Sum of first } n \text{ natural number is } \frac{n(n+1)}{2}.$$

Ex. (2) Find the sum of first n even natural numbers.

Solution : First n even natural numbers are $2, 4, 6, 8, \dots, 2n$.

$t_1 = \text{First term} = 2, t_n = \text{last term} = 2n$

Method I

$$= \frac{n}{2} [t_1 + t_n]$$

$$= \frac{n}{2} [2 + 2n]$$

$$= \frac{n}{2} \times 2 (1 + n)$$

$$= n (1 + n)$$

Method II

$$S_n = 2 + 4 + 6 \dots + 2n$$

$$= 2(1 + 2 + 3 + \dots + n)$$

$$= \frac{2[n(n+1)]}{2}$$

$$= n (1 + n)$$

Method III

$$S_n = \frac{n}{2} [2a + (n-1)d]$$

$$= \frac{n}{2} [2 \times 2 + (n-1)2]$$

$$= \frac{n}{2} [4 + 2n - 2]$$

$$= \frac{n}{2} [2 + 2n]$$

$$= \frac{n}{2} \times 2 (1 + n)$$

$$= n (1 + n)$$

$$\therefore \text{Sum of first } n \text{ even natural numbers is } n (1 + n).$$

Ex. (3) Find the sum of first n odd natural numbers.

Solution : First n natural numbers

$$1, 3, 5, 7, \dots, (2n - 1).$$

$$a = t_1 = 1 \text{ and } t_n = (2n - 1), d = 2$$

Method I

$$\begin{aligned} S_n &= \frac{n}{2} [t_1 + t_n] \\ &= \frac{n}{2} [1 + (2n - 1)] \\ &= \frac{n}{2} [1 + 2n - 1] \\ &= \frac{n}{2} \times 2n \\ &= n^2 \end{aligned}$$

Method II

$$\begin{aligned} S_n &= \frac{n}{2} [2a + (n-1)d] \\ &= \frac{n}{2} [2 \times 1 + (n-1) \times 2] \\ &= \frac{n}{2} [2 + 2n - 2] \\ &= \frac{n}{2} \times 2n \\ &= n^2 \end{aligned}$$

Method III

$$\begin{aligned} &1 + 3 + \dots + 2n-1 \\ &= (1 + 2 + 3 + \dots + 2n) \\ &\quad - (2 + 4 + 6 + \dots + 2n) \\ &= \frac{2n(2n+1)}{2} - \frac{2n(n+1)}{2} \\ &= (2n^2 + n) - (n^2 + n) \\ &= n^2 \end{aligned}$$

Ex. (4) Find the sum of all odd numbers from 1 to 150.

Solution : 1 to 150 all odd numbers are 1, 3, 5, 7, \dots , 149.

Which is an A.P.

Here $a = 1$ and $d = 2$. First let's find how many odd numbers are there from 1 to 150, so find the value of n , if $t_n = 149$

$$t_n = a + (n - 1)d$$

$$149 = 1 + (n - 1)2 \quad \therefore 149 = 1 + 2n - 2$$

$$\therefore n = 75$$

Now let's find the sum of these 75 numbers $1 + 3 + 5 + \dots + 149$.

$$a = 1 \text{ and } d = 2, n = 75$$

Method I $S_n = \frac{n}{2} [2a + (n-1)d]$

$$S_n = \boxed{}$$

$$S_n = \boxed{} \times \boxed{}$$

$$S_n = \boxed{}$$

Method II $S_n = \frac{n}{2} [t_1 + t_n]$

$$S_n = \frac{75}{2} [1 + 149]$$

$$S_n = \boxed{} \times \boxed{}$$

$$S_n = \boxed{}$$

Practice Set 3.3

1. First term and common difference of an A.P. are 6 and 3 respectively ; find S_{27} .

$$a = 6, d = 3, S_{27} = ?$$

$$S_n = \frac{n}{2} [\square + (n-1) d]$$

$$S_{27} = \frac{27}{2} [12 + (27-1) \square]$$

$$= \frac{27}{2} \times \square$$

$$= 27 \times 45 = \square$$

2. Find the sum of first 123 even natural numbers.
3. Find the sum of all even numbers between 1 and 350.
4. In an A.P. 19th term is 52 and 38th term is 128, find sum of first 56 terms.
5. Complete the following activity to find the sum of natural numbers between 1 and 140 which are divisible by 4.

Between 1 and 140, natural numbers divisible by 4

4, 8, , 136

How many numbers ? $\therefore n = \square$

$n = \square, a = \square, d = \square$

$$t_n = a + (n-1)d$$

$$136 = \square + (n-1) \times \square$$

$$n = \square \rightarrow S_n = \frac{n}{2} [2a + (n-1)d]$$

$$S_{\square} = \frac{\square}{2} [\quad] = \square$$

Sum of numbers from 1 to 140, which are divisible by 4 =

- 6.★ Sum of first 55 terms in an A.P. is 3300, find its 28th term.

- 7★ In an A.P. sum of three consecutive terms is 27 and their product is 504, find the terms.
(Assume that three consecutive terms in A.P. are $a - d$, a , $a + d$.)
- 8★ Find four consecutive terms in an A.P. whose sum is 12 and sum of 3rd and 4th term is 14.
(Assume the four consecutive terms in A.P. are $a - d$, a , $a + d$, $a + 2d$.)
- 9★ If the 9th term of an A.P. is zero then show that the 29th term is twice the 19th term.



Let's learn.

Application of A.P.

Ex. (1) A mixer manufacturing company manufactured 600 mixers in 3rd year and in 7th year they manufactured 700 mixers. If every year there is same growth in the production of mixers then find (i) Production in the first year (ii) Production in 10th year (iii) Total production in first seven years.

Solution : Addition in the number of mixers manufactured by the company per year is constant therefore the number of production in successive years is in A.P.

(i) Let's assume that company manufactured t_n mixers in the n^{th} year then as per given information,

$$t_3 = 600, t_7 = 700$$

We know that $t_n = a + (n-1)d$

$$t_3 = a + (3-1)d$$

$$a + 2d = 600 \dots (I)$$

$$t_7 = a + (7-1)d$$

$$t_7 = a + 6d = 700$$

$$a + 2d = 600 \therefore \text{Substituting } a = 600 - 2d \text{ in equation (II),}$$

$$600 - 2d + 6d = 700$$

$$4d = 100 \therefore d = 25$$

$$a + 2d = 600 \therefore a + 2 \times 25 = 600$$

$$a + 50 = 600 \therefore a = 550$$

\therefore Production in first year was 550.

$$(ii) t_n = a + (n-1)d$$

$$t_{10} = 550 + (10-1) \times 25$$

$$= 550 + 225$$

Production in 10th year was 775.

(iii) For finding total production in first 7 years let's use formula for S_n .

$$S_n = \frac{n}{2} [2a + (n-1)d]$$

$$S_n = \frac{7}{2} [1100 + 150] = \frac{7}{2} [1250] = 7 \times 625 = 4375$$

Total production in first 7 years is 4375 mixers.

Ex. (2) Ajay sharma repays the borrowed amount of ₹ 3,25,000 by paying ₹ 30500 in the first month and then decreases the payment by ₹ 1500 every month. How long will it take to clear his amount?

Solution : Let the time required to clear the amount be n months. The monthly payment decreases by ₹ 1500. Therefore the payments are in A.P.

First term = $a = 30500$, $d = -1500$

Amount = $S_n = 3,25,000$

$$S_n = \frac{n}{2} [2a + (n-1)d]$$

$$3,25,000 = \frac{n}{2} [2 \times 30500 + (n-1)d]$$

$$= \frac{n}{2} [2 \times 30500 - 1500n + 1500]$$

$$3,25,000 = 30500n - 750n^2 + 750n$$

$$\therefore 750n^2 - 31250n + 325000 = 0$$

divide both sides by 250.

$$\therefore 3n^2 - 125n + 1300 = 0$$

$$\therefore 3n^2 - 60n - 65n + 1300 = 0$$

$$\therefore 3n(n-20) - 65(n-20) = 0$$

$$\therefore (n-20)(3n-65) = 0$$

$$\therefore n-20 = 0, 3n-65 = 0$$

$$\therefore n = 20 \text{ or } n = \frac{65}{3} = 21\frac{2}{3}$$

In an A.P. n is a natural number.

$$\therefore n \neq \frac{65}{3} \quad \therefore n = 20$$

(Or, after 20 months, $S_{20} = 3,25,000$ then the total amount will be repaid. It is not required to think about further period of time.)

\therefore To clear the amount 20 months are needed.

Ex. (3) Anvar saves some amount every month. In first three months he saves ₹ 200, ₹ 250 and ₹ 300 respectively. In which month will he save ₹ 1000?

Solution: Saving in first month ₹ 200; Saving in second month ₹ 250;

200, 250, 300, . . . this is an A.P.

Here $a = 200$, $d = 50$, Let's find n using t_n formula and then find S_n .

$$\begin{aligned}t_n &= a + (n-1)d \\&= 200 + (n-1)50 \\&= 200 + 50n - 50\end{aligned}$$

$$1000 = 150 + 50n$$

$$150 + 50n = 1000$$

$$50n = 1000 - 150$$

$$50n = 850$$

$$\therefore n = 17$$

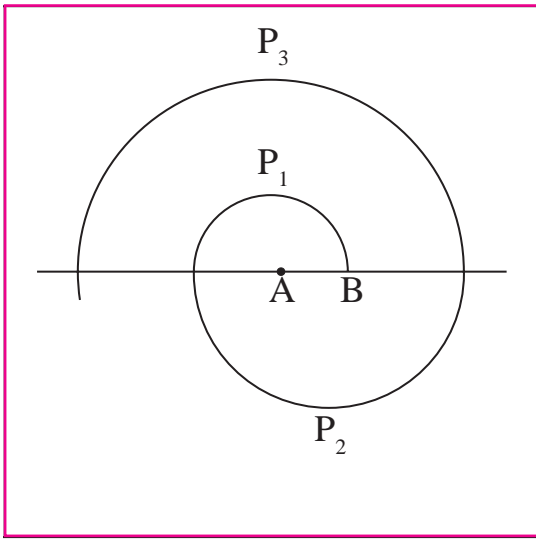
In the 17th month he will save ₹ 1000.

Let's find that in 17 months how much total amount is saved.

$$\begin{aligned}S_n &= \frac{n}{2} [2a + (n-1)d] \\&= \frac{17}{2} [2 \times 200 + (17-1) \times 50] \\&= \frac{17}{2} [400 + 800] \\&= \frac{17}{2} [1200] \\&= 17 \times 600 \\&= 10200\end{aligned}$$

In 17 months total saving is ₹ 10200.

Ex. (4) As shown in the figure, take point A on the line and draw a half circle P_1 of



radius 0.5 with A as centre. It intersects given line in point B. Now taking B as centre draw a half circle P_2 of radius 1 cm which is on the other side of the line.

Now again taking A as centre draw a half circle P_3 of radius 1.5 cm. If we draw half circles like this having radius 0.5 cm, 1 cm, 1.5 cm, 2 cm, we get a figure of spiral shape. Find the length of such spiral shaped figure formed by 13 such half circles. ($\pi = \frac{22}{7}$)

Solution : Semi circumferences P_1, P_2, P_3, \dots are drawn by taking centres A, B, A, B,... It is given that radius of the first circle is 0.5 cm. The radius of the second circle is 1.0 cm,... From this information we will find $P_1, P_2, P_3, \dots P_{13}$.

$$\text{Length of the first semi circumference} = P_1 = \pi r_1 = \pi \times \frac{1}{2} = \frac{\pi}{2}$$

$$P_2 = \pi r_2 = \pi \times 1 = \pi$$

$$P_3 = \pi r_3 = \pi \times 1.5 = \frac{3}{2} \pi$$

The lengths are P_1, P_2, P_3, \dots , and the numbers $\frac{1}{2} \pi, 1 \pi, \frac{3}{2} \pi, \dots$ are in A.P.

Here $a = \frac{1}{2} \pi, d = \frac{1}{2} \pi$, From this let's find S_{13} .

$$S_n = \frac{n}{2} [2a + (n-1)d]$$

$$S_{13} = \frac{13}{2} \left[2 \times \frac{\pi}{2} + (13-1) \times \frac{1}{2} \pi \right]$$

$$= \frac{13}{2} [\pi + 6 \pi]$$

$$= \frac{13}{2} \times 7 \pi =$$

$$= \frac{13}{2} \times 7 \times \frac{22}{7}$$

$$= 143 \text{ cm.}$$

\therefore The total length of spiral shape formed by 13 semicircles is 143 cm.

Ex. (5) In the year 2010 in the village there were 4000 people who were literate. Every year the number of literate people increases by 400. How many people will be literate in the year 2020?

Solution :

| | | | | | |
|-----------------|------|------|------|-----|----------------------|
| Year | 2010 | 2011 | 2012 | ... | 2020 |
| Literate People | 4000 | 4400 | 4800 | ... | <input type="text"/> |

$$a = 4000, \quad d = 400 \quad n = 11$$

$$\begin{aligned}
 t_n &= a + (n-1)d \\
 &= 4000 + (11-1)400 \\
 &= 4000 + 4000 \\
 &= 8000
 \end{aligned}$$

In year 2020, 8000 people will be literate.

Ex. (6) In year 2015, Mrs. Shaikh got a job with salary ₹ 1,80,000 per year. Her employer agreed to give ₹ 10,000 per year as increment. Then in how many years will her annual salary be ₹ 2,50,000?

Solution :

| | | | | |
|------------|----------------------|-----------------------|----------------------|-----|
| Year | First Year (2015) | Second Year (2016) | Third Year (2017) | ... |
| Salary (₹) | [1,80,000] | [1,80,000 + 10,000] | | ... |

$$a = 1,80,000 \quad d = 10,000 \quad n = ? \quad t_n = 2,50,000 \text{ ₹}$$

$$t_n = a + (n-1)d$$

$$2,50,000 = 1,80,000 + (n-1) \times 10,000$$

$$(n-1) \times 10000 = 70,000$$

$$(n-1) = 7$$

$$n = 8$$

In the 8th year her annual salary will be ₹ 2,50,000.

Practice Set 3.4

- ### Problem Set - 3

1. Choose the correct alternative answer for each of the following sub questions.

- (4) For an given A.P. $t_7 = 4$, $d = -4$ then $a = \dots$
 (A) 6 (B) 7 (C) 20 (D) 28
- (5) For an given A.P. $a = 3.5$, $d = 0$, $n = 101$, then $t_n = \dots$
 (A) 0 (B) 3.5 (C) 103.5 (D) 104.5
- (6) In an A.P. first two terms are $-3, 4$ then 21^{st} term is \dots
 (A) -143 (B) 143 (C) 137 (D) 17
- (7) If for any A.P. $d = 5$ then $t_{18} - t_{13} = \dots$
 (A) 5 (B) 20 (C) 25 (D) 30
- (8) Sum of first five multiples of 3 is. \dots
 (A) 45 (B) 55 (C) 15 (D) 75
- (9) $15, 10, 5, \dots$ In this A.P. sum of first 10 terms is \dots
 (A) -75 (B) -125 (C) 75 (D) 125
- (10) In an A.P. 1^{st} term is 1 and the last term is 20. The sum of all terms is $= 399$ then $n = \dots$
 (A) 42 (B) 38 (C) 21 (D) 19
2. Find the fourth term from the end in an A.P. $-11, -8, -5, \dots, 49$.
3. In an A.P. the 10^{th} term is 46, sum of the 5^{th} and 7^{th} term is 52. Find the A.P.
4. The A.P. in which 4^{th} term is -15 and 9^{th} term is -30 . Find the sum of the first 10 numbers.
5. Two A.P.'s are given $9, 7, 5, \dots$ and $24, 21, 18, \dots$. If n^{th} term of both the progressions are equal then find the value of n and n^{th} term.
6. If sum of 3^{rd} and 8^{th} terms of an A.P. is 7 and sum of 7^{th} and 14^{th} terms is -3 then find the 10^{th} term.
7. In an A.P. the first term is -5 and last term is 45. If sum of all numbers in the A.P. is 120, then how many terms are there? What is the common difference?
8. Sum of 1 to n natural numbers is 36, then find the value of n .

9. Divide 207 in three parts, such that all parts are in A.P. and product of two smaller parts will be 4623.
10. There are 37 terms in an A.P., the sum of three terms placed exactly at the middle is 225 and the sum of last three terms is 429. Write the A.P.
- 11★ If first term of an A.P. is a , second term is b and last term is c , then show that sum of all terms is $\frac{(a+c)(b+c-2a)}{2(b-a)}$.
- 12★ If the sum of first p terms of an A.P. is equal to the sum of first q terms then show that the sum of its first $(p+q)$ terms is zero. ($p \neq q$)
- 13★ If m times the m^{th} term of an A.P. is equal to n times n^{th} term then show that the $(m+n)^{\text{th}}$ term of the A.P. is zero.
14. ₹ 1000 is invested at 10 percent simple interest. Check at the end of every year if the total interest amount is in A.P. If this is an A.P. then find interest amount after 20 years. For this complete the following activity.

$$\text{Simple interest} = \frac{P \times R \times N}{100}$$

$$\text{Simple interest after 1 year} = \frac{1000 \times 10 \times 1}{100} = \square$$

$$\text{Simple interest after 2 year} = \frac{1000 \times 10 \times 2}{100} = \square$$

$$\text{Simple interest after 3 year} = \frac{\square \times \square \times \square}{100} = 300$$

According to this the simple interest for 4, 5, 6 years will be 400, \square , \square respectively.

From this $d = \square$, and $a = \square$

Amount of simple interest after 20 years

$$t_n = a + (n-1)d$$

$$t_{20} = \square + (20-1) \square$$

$$t_{20} = \square$$

Amount of simple interest after 20 years is = \square



□□□