Manipal
Engineering Entrance Exam
Solved Paper 2011

Physics

1. Which of the following statements is wrong?
(a) Voltmeter should have high resistance
(b) Ammeter should have low resistance
(c) Ammeter is placed in parallel across the conductor in a circuit
(d) Voltmeter is placed in parallel across the conductor in a circuit

2. As the temperature of hot junction increases, the thermo emf
(a) always increases
(b) always decreases
(c) may increase or decrease
(d) always remains constant

3. A proton and an α-particle enter a uniform magnetic field perpendicularly with the same speed. If proton takes 25 μs to make 5 revolutions, then the periodic time for the α-particle would be
(a) 50 μs (b) 25 μs
(c) 10 μs (d) 5 μs

4. Two parallel conductors A and B of equal length carry currents i and $10i$, respectively, in the same direction. Then
(a) A and B will repel each other with same force
(b) A and B will attract each other with same force
(c) A will attract B, but B will repel A
(d) A and B will attract each other with different forces

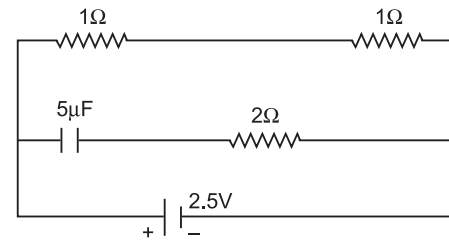
5. An open pipe is suddenly closed at one end with the result that the frequency of third harmonic of the closed pipe is termed to be higher by 100 Hz than the fundamental frequency of the open pipe. The fundamental frequency of the open pipe is
(a) 200 Hz (b) 150 Hz
(c) 100 Hz (d) 250 Hz

6. Two slits 4 mm apart, are illuminated by light of wavelength 6000 Å. What will be the fringe width on a screen placed 2 m from the slits?
(a) 0.12 mm (b) 0.3 mm
(c) 3.0 mm (d) 4.0 mm

7. A thin prism P_1 of angle 4° and refractive index 1.54° is combined with another thin prism P_2 of refractive index 1.72 to produce dispersion without deviation. The angle of P_2 is
(a) 4° (b) 5.33°
(c) 2.6° (d) 3°

8. While viewing a distant object with a telescope suddenly a housefly sits on objective lens. The correct statement is that
(a) housefly will be seen enlarged in image
(b) housefly will be seen reduced in image
(c) intensity of image will be decreased
(d) intensity of image will be increased

9. A capacitor of capacitance 5 μF is connected as shown in the figure. The internal resistance of the cell is 0.5 Ω. The amount of charge on the capacitor plate is



(a) 10 μC (b) 5 μC
(c) 6 μC (d) 10.2 μC

10. A charge of 10^{-9} C is placed on each of the 64 identical drops of radius 2 cm. They are then combined to form a bigger drop. Its potential will be

(a) 7.2×10^3 V (b) 7.2×10^2 V

(c) 1.44×10^2 V (d) 1.44×10^3 V

11. A particle moves in the xy-plane under the action of a force **F** such that the components of its linear momentum **p** at any time t are $p_x = 2\cos t$, $p_y = \sin t$. The angle between **F** and **p** at time t is

(a) 90° (b) 0°

(c) 180° (d) 30°

12. A fighter plane is moving in a vertical circle of radius r. Its minimum velocity at the highest point A of the circle will be

(a) $\sqrt{3gr}$ (b) $\sqrt{2g\,r}$

(c) $\sqrt{g\,r}$ (d) $\sqrt{g\,r/2}$

13. In Millikan's oil drop experiment, a charged drop of mass 1.8×10^{-13} kg is stationary between its plates. The distance between its plates is 0.9 cm and potential difference is 2000 V. The number of electrons in the drop is

(a) 500 (b) 50

(c) 5 (d) 10

14. Decay constant of radium is λ. By a suitable process its compound radium bromide is obtained. The decay constant of radium bromide will be

(a) λ (b) more than λ

(c) less than λ (d) zero

15. Rn decays into Po by emitting an α-particle with half-life of 4 days. A sample contains 6.4×10^{10} atoms of Rn after 12 days, the number of atoms of Rn left in the sample will be

(a) 3.2×10^{10} (b) 0.53×10^{10}

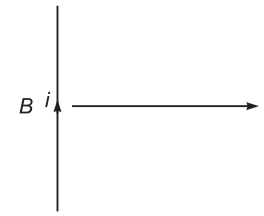
(c) 2.1×10^{10} (d) 0.8×10^{10}

16. The energy gap between the valence and conduction bands of an insulator is about

(a) 0.1 eV (b) 1.0 eV

(c) 5.0 eV (d) zero

17. A conducting wire is moving towards right in a magnetic field B. The direction of induced current in the wire is shown in the figure. The direction of magnetic field will be



(a) in the plane of paper pointing towards right

(b) in the plane of paper pointing towards left

(c) perpendicular to the plane of paper and downwards

(d) perpendicular to the plane of paper and upwards

18. An electron moving with velocity 2×10^{-7} m/s, describes a circle in a magnetic field of strength 2×10^{-2} T. If $\frac{e}{m}$ of electron is 1.76×10^{11} C/kg. Then the diameter of the circle will be

(a) 11 cm (b) 1.1 cm

(c) 1.1 mm (d) 1.1 m

19. In an AC circuit the potential difference and current are represented respectively by $V = 100 \sin (100t)$ volt, $I = 100 \sin \left(100t + \frac{\pi}{3}\right)$ milliampere. The power in the circuit is

(a) 2.5 W (b) 5 W

(c) 10 W (d) 10^4 W

20. Escape velocity from earth's surface is 11 km/s. If radius of a planet is double than that of earth and density is same as that of earth, then the escape velocity from this planet will be

(a) 5.5 km/s (b) 11 km/s

(c) 16.5 km/s (d) 22 km/s

21. The orbital angular momentum of a satellite revolving at a distance r from the centre is L. If the distance is increased to $16r$, then the new angular momentum will be

(a) $16L$ (b) $64L$

(c) $\frac{L}{4}$ (d) $4L$

22. The acceleration of a particle performing SHM is $12\,\text{cm/s}^2$ at a distance of 3 cm from the mean position. Its time period is

(a) 0.5 s (b) 1.0 s

(c) 2.0 s (d) 3.14 s

23. A soap bubble in vacuum has a radius of 3 cm and another soap bubble in vacuum has a radius of 4 cm. If the two bubbles coalesce under isothermal condition, then the radius of the new bubble is
(a) 2.3 cm (b) 4.5 cm
(c) 5 cm (d) 7 cm

24. A 10 cm^3 cube floats in water with a height of 4 cm^3 remaining above the surface. The density of the material from which the cube is made is
(a) 0.6 g cm^{-3} (b) 1.0 g cm^{-3}
(c) 0.4 g cm^{-3} (d) 0.24 g cm^{-3}

25. The heat flows through a rod of length 50 cm and area of cross-section 5 cm^2. Its ends are respectively at 25°C and 125°C. The coefficient of thermal conductivity of the material rod is 0.092 kcal/m°C. The temperature gradient of the rod is
(a) 2°C/ cm (b) 2°C/m
(c) 20°C/cm (d) 20°C/m

26. 5 mol of hydrogen gas is heated from 30°C to 60°C at constant pressure. Heat given to the gas is (given $R = 2$ cal/ mol deg)
(a) 750 cal (b) 630 cal
(c) 1050 cal (d) 1470 cal

27. A fish at a depth of 12 cm in water is viewed by an observer on the bank of a lake. To what height the image of the fish is raised?
(Refractive index of lake water $= \frac{4}{3}$)
(a) 9 cm (b) 12 cm
(c) 3.8 cm (d) 3 cm

28. The rest energy of an electron is 0.511 MeV. The electron is accelerated from rest to a velocity 0.5 *c*. The change in its energy will be
(a) 0.026 MeV (b) 0.051 MeV
(c) 0.079 MeV (d) 0.105 MeV

29. In Millikan's oil drop experiment, an oil drop carrying a charge Q is held stationary by a potential difference 2400 V between the plates. To keep a drop of half the radius stationary, the potential difference had to be made 600V. What is the charge on the second drop?
(a) $\frac{Q}{4}$ (b) $\frac{Q}{2}$
(c) Q (d) $\frac{3Q}{2}$

30. Band spectrum is characteristic of the
(a) atoms
(b) molecules
(c) amorphous solids
(d) crystalline solids

31. As a result of radioactive decay $_{92}U^{238}$ is converted into $_{91}Pa^{234}$. The particles emitted during this decay are
(a) a proton and a neutron
(b) a proton and two α-particles
(c) an α-particle and a β-particle
(d) two β-particles and a proton

32. A *p*-type semiconductor has acceptor levels 57 meV above the valence band. The maximum wavelength of light required to create a hole
(a) 57 Å (b) 57×10^{-3} Å
(c) 217105 Å (d) 11.61×10^{-33} Å

33. In an explosion a body breaks up into pieces of unequal masses. In this case
(a) both parts will have numerically equal momentum
(b) lighter part will have more momentum
(c) heavier part will have more momentum
(d) both parts will have equal kinetic energy

34. A force of 10 N acts on a body of mass 20 kg for 10 s. Change in its momentum is
(a) 5 kg m/s (b) 100 kg m/s
(c) 200 kg m/s (d) 1000 kg m/s

35. In which case does the potential energy decrease?
(a) On compressing a spring
(b) On stretching a spring
(c) On moving a body against gravitational force
(d) On the rising of an air bubble in water

36. A heavy mass is attached to a thin wire and is whirled in a vertical circle. The wire is most likely to break
(a) when the mass is at the highest point of the circle
(b) when the mass is at the lowest point of the circle
(c) when the wire is horizontal
(d) at an angle of $\cos^{-1}\left(\frac{1}{3}\right)$ from the upward vertical

37. The time taken by AC of 50 Hz in reaching from zero to the maximum value is

(a) 50×10^{-3} s (b) 5×10^{-3} s

(c) 1×10^{-3} s (d) 2×10^{-3} s

38. The molar heat capacity of rock salt at low temperature varies with temperature according to Debye's T^3 law

Thus $C = k \dfrac{T^3}{\theta^3}$ where $k = 1940$ Jmol^{-1}K^{-1}

$\theta = 281$ K

Calculate how much heat is required to raise the temperature of 2 moles of rock salt from 10 K to 50 K

(a) 800 J (b) 373 J

(c) 273 J (d) None of these

39. The velocity of an electron in the second orbit of sodium atom (atomic number =11) is v. The velocity of an electron in its fifth orbit will be

(a) v (b) $\dfrac{22}{5}v$

(c) $\dfrac{5}{2}$ (d) $\dfrac{2}{5}v$

40. A charged particle enters a magnetic field H with its initial velocity making an angle of 45° with H. The path of the particle will be

(a) a straight line (b) a circle

(c) an ellipse (d) a helix

41. A 10 μF capacitor is charged to a potential difference of 50V and is connected to another uncharged capacitor in parallel. Now the common potential difference becomes 20V. The capacitance of second capacitor is

(a) 10 μF (b) 20 μF

(c) 30 μF (d) 15 μF

42. An alternating voltage $E = 200\sqrt{2}\sin(100\,t)$ V is connected to a 1 μF capacitor through an AC ammeter. The reading of ammeter is

(a) 10 mA (b) 20 mA

(c) 40 mA (d) 80 mA

43. In a step-up transformer the turn ratio is 1 : 2. A Leclanche cell (emf 1.5 V) is connected across the primary. The voltage across the secondary is

(a) 3.0 V (b) 0.75 V (c) zero (d) 1.5 V

44. If 10% of the current passes through a moving coil galvanometer of resistance 99 Ω. Then the shunt resistance will be

(a) 9.9 Ω (b) 11 Ω

(c) 10 Ω (d) 9 Ω

45. How much energy will necessary for making a body of 500 kg escape from the earth? ($g = 9.8$ m/s^2, radius of the earth $= 6.4 \times 10^6$ m)

(a) about 9.8×10^6 J (b) about 6.4×10^8 J

(c) about 3.1×10^{10} J (d) about 27.4×10^{12} J

46. In a neon discharge tube 2.9×10^{18} Ne^+ ions move to the right each second while 1.2×10^{18} electrons move to the left per second, electrons charge is 1.6×10^{-19} C. The current in the discharge tube is

(a) 1 A, towards right

(b) 0.66 A, towards right

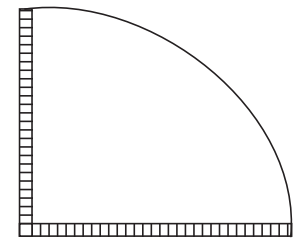
(c) 0.66 A, towards left

(d) zero

47. Water is falling on the blades of a turbine at a rate of 100 kg/s. If the height of the fall is 100 m, the power transferred to the turbine will be approximately

(a) 100 kW (b) 10 kW (c) 1 kW (d) 100 W

48. A metre scale is standing vertically on the earth's surface on one of its end. It now falls on earth without slipping. Find the velocity with which the free end of the scale strikes the earth. ($g = 10$ m/s^2)



(a) 9.8 m/s (b) 5.47 m/s

(c) 4.5 m/s (d) 1 m/s

49. For germanium crystal, the forbidden energy gap in joule is

(a) 1.216×10^{-19} (b) 1.76×10^{-19}

(c) 1.6×10^{-19} (d) zero

50. An electron is accelerated through a potential difference of 200 V. If e/m for the electron be 1.6×10^{11} C/kg, the velocity acquired by the electron will be

(a) 8×10^6 m/s (b) 8×10^5 m/s

(c) 5.9×10^6 m/s (d) 5.9×10^5 m/s

51. At what temperature will the resistance of a copper wire become three times its value at 0°C? (temperature coefficient of resistance for copper = 4×10^{-3}/°C)

(a) 400°C (b) 450°C
(c) 500°C (d) 550°C

52. A coil having an area of $2\,m^2$ is placed in a magnetic field which changes from 1 Wb/m^2 to 4 Wb/m^2 in 2 s. The emf induced in the coil will be

(a) 4 V (b) 3 V
(c) 2 V (d) 1 V

53. A 10 m long copper wire while remaining in the east-west horizontal direction is falling down with a speed of 5.0 m/s. If the horizontal component of the earth's magnetic field = 0.3×10^{-4} Wb/m^2 the emf developed between the ends of the wires is

(a) 0.15 V (b) 1.5 V
(c) 0.15 mV (d) 1.5 mV

54. A current of 1.6 A is passed through a solution of $CuSO_4$. How many Cu^{++} ions are liberated in one minute?

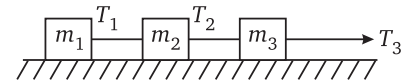
(electronic charge = 1.6×10^{-19} C)

(a) 3×10^{20} (b) 3×10^{19}
(c) 6×10^{20} (d) 6×10^{19}

55. A solenoid has an inductance of 60 H and a resistance of 30 Ω. If it is connected to a 100 V battery, how long will it take for the current to reach $\frac{e-1}{e}$ = 63.2% of its final value?

(a) 1 s (b) 2 s
(c) e second (d) $2e$ second

56. Three blocks of masses m_1, m_2 are connected by massless strings as shown on a frictionless table. They are pulled with a force $T_3 = 40$ N. If $m_1 = 10$ kg, $m_2 = 6$ kg and $m_3 = 4$ kg, the tension T_2 will be



(a) 20 N (b) 40 N
(c) 10 N (d) 32 N

57. The force constant of weightless spring is 16 N/m. A body of mass 1.0 kg suspended from it is pulled down through 5 cm and then released. The maximum kinetic energy of the system (spring + body) will be

(a) 2×10^{-2} J (b) 4×10^{-2} J
(c) 8×10^{-2} J (d) 16×10^{-2} J

58. A mass M is moving with a constant velocity on a line parallel to the x-axis. Its angular momentum with respect to the origin or z-axis

(a) is zero
(b) remains constant
(c) goes on increasing
(d) goes on decreasing

59. Which of the following pairs is an isobar?

(a) ${}_1H^1$ and ${}_1H^2$ (b) ${}_1H^2$ and ${}_1H^3$
(c) ${}_6C^{12}$ and ${}_6C^{13}$ (d) ${}_{15}P^{30}$ and ${}_{14}Si^{30}$

60. The half-life of the isotope ${}_{11}Na^{24}$ is 15 h.

How much time does it take for $\frac{7}{8}$ th of a sample of this isotope to decay?

(a) 75 h (b) 65 h
(c) 55 h (d) 45 h

Chemistry

1. The largest number of molecules is in

(a) 36 g of water
(b) 28 g of CO_2
(c) 46 g of CH_3OH
(d) 58 g of N_2O_5

2. In the following reaction, which choice has value twice that of the equivalent weight of the oxidising agent?

$$SO_2 + 2H_2O \longrightarrow S + 2H_2O_2$$

(a) 16 (b) 48
(c) 64 (d) 32

3. Orbital is

(a) circular path around the nucleus in which electrons are revolves
(b) space around the nucleus where the probability of finding the electron is maximum
(c) amplitude of electrons wave
(d) None of the above

4. Which of the following sets of quantum number is not permitted?
(a) $n = 3, l = 3, m = 0, s = +\frac{1}{2}$
(b) $n = 3, l = 2, m = +2, s = -\frac{1}{2}$
(c) $n = 3, l = 2, m = -2, s = -\frac{1}{2}$
(d) $n = 3, l = 0, m = 0, s = +\frac{1}{2}$

5. An element M has an atomic mass of 19 and atomic number 9, its ions is represented by
(a) M^+ (b) M^{2+} (c) M^- (d) M^{2-}

6. In S_8 molecules, the type of hybridisation exhibited by sulphur is
(a) sp^2 (b) sp^3
(c) sp (d) sp^3d

7. In the oxyacids of chlorine, Cl—O bond contains
(a) $d\pi - d\pi$ bonding
(b) $d\pi - p\pi$ bonding
(c) $p\pi - p\pi$ bonding
(d) None of the above

8. Anhydrous $AlCl_3$ fume in air due to
(a) oxidation (b) hydrolysis
(c) reduction (d) hydrogenation

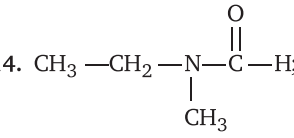
9. The nature of Fe_2O_3 is
(a) acidic (b) basic
(c) amphoteric (d) None of these

10. Concentrated nitric acid (HNO_3) oxidises phosphorus to
(a) H_3PO_4 (b) H_3PO_3
(c) $H_4P_2O_7$ (d) H_3PO_2

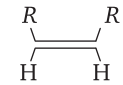
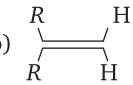
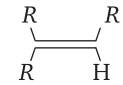
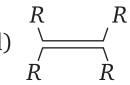
11. Which of the following has the highest nucleophilicity?
(a) F^- (b) OH^-
(c) CH_3^- (d) NH_2^-

12. Number of isomeric primary amines obtained from $C_4H_{11}N$ are
(a) 3 (b) 4
(c) 5 (d) 6

13. Which of the following reaction doesn't support the acidic nature of alkyne?
(a) Reaction with HBr
(b) Reaction with Grignard reagent
(c) Reaction with ammoniacal silver salt
(d) Reaction with metallic sodium

14. 
The IUPAC name of the compound is
(a) N-formyl-N-methyl ethanamide
(b) N-ethyl-N-methyl methanamide
(c) N-methyl-N-oxo ethanamine
(d) None of the above

15. The product of the following reaction,
$C_6H_5COCHBr_2 \xrightarrow{OH^-}$? is
(a) C_6H_5COCHO (b) $C_6H_5COCOOH$
(c) C_6H_5CHO (d) $C_6H_5CHOHCOOH$

16. Which of the following alkenes will react fastest with H_2 under catalytic hydrogenation conditions?
(a)  (b) 
(c)  (d) 

17. Glucose reacts with acetic anhydride to give pentaacetyl derivative. Which of the following is true about that?
(a) that can reduce Fehling or Tollen's reagent
(b) that's soluble in dil. NaOH solution
(c) that consumes one mole of HIO_4
(d) Br_2/H_2O can oxidise

18. Complete hydrolysis of cellulose gives
(a) D-fructose (b) D-ribose
(c) D-glucose (d) L-glucose

19. The saponification value of an oil or fat is measured in terms of
(a) NH_4OH (b) NaOH
(c) C_6H_5OH (d) kOH

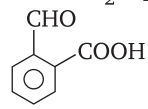
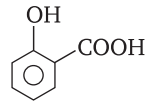
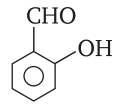
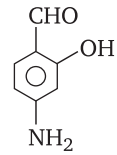
20. N_2 gas is liberated when [HCl + $NaNO_2$] reacts with the following compounds;
A. $CH_3CH_2NH_2$ *B*. urea
C. CH_3CONH_2 *D*. $C_6H_5NH_2$
The answer is
(a) *A, B, C* (b) *B, C, D*
(c) *A, C, D* (d) *A, B, D*

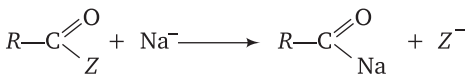
21. Benzaldehyde condenses with N,N-dimethyl aniline in presence of anhydrous $ZnCl_2$ to give
(a) azo dye (b) malachite
(c) michlers ketone (d) buffer yellow

22. Which of the following reactions is given by only primary amines?
(a) Reaction with HNO_2
(b) Reaction with chloroform and alcoholic KOH
(c) Reaction with acetyl chloride
(d) Reaction with Grignard reagent

23. $CH_3COOH \xrightarrow{NH_3} \xrightarrow{\Delta} ?$

The product of the reaction is isomeric with
(a) $CH_2(NH_2)—CHO$ (b) $CH_3CH{=}NOH$
(c) $HCONH—CH_3$ (d) All of these

24. Which one of the following compound gives aspirin on reacting with acetic anhydride in presence of H_2SO_4?
(a)  (b) 
(c)  (d) 

25. 

is fastest when Z is
(a) Cl (b) NH_2
(c) OC_2H_5 (d) $OCOCH_3$

26. Aldehyde not showing cannizzaro's reaction is
(a) paraldehyde (b) chloral
(c) formaldehyde (d) acetaldehyde

27. Both HCHO and CH_3CHO gives similar reactions with all the reagents except
(a) Schiff reagent
(b) Fehling solution
(c) ammoniacal $AgNO_3$
(d) ammonia

28. Which of the following statements is not correct?
(a) All alcohols are miscible with water
(b) Only lower alcohols are miscible with water
(c) All alcohols are not poisonous
(d) Methanol is poisonous

29. Cyclohexanol is
(a) phenol (b) 1° alcohol
(c) 2° alcohol (d) 3° alcohol

30. In the manufacture of ethanol from sugar the enzymes are
(a) diastase and zymase
(b) maltase and zymase
(c) diastase and invertase
(d) invertase and zymase

31. The action of chloral on chlorobenzene gives
(a) BHC (d) DDT
(c) gammexene (d) lindane

32. Which halide will be least reactive in respect to hydrolysis?
(a) vinyl chloride (b) allyl chloride
(c) ethyl chloride (d) *t*-butyl chloride

33. By Wurtz reaction, a mixture of methyl iodide and ethyl iodide gives
(a) butane
(b) ethane
(c) propane
(d) a mixture of above three

34. The cyanide process is used for obtaining
(a) Cu (b) Na
(c) Zn (d) Ag

35. Which of the following ore does not represent the ores of iron?
(a) Cassiterite (b) Limonite
(c) Haematite (d) Magnetite

36. van-Arkel method of purification of metals involves converting the metal to a
(a) volatile stable compound
(b) non-volatile stable compound
(c) volatile unstable compound
(d) None of the above

37. The first element of rare earth metal is
(a) cerium (b) cesium
(c) lanthanide (d) actinide

38. Which of the following transitions involves maximum amount of energy?
(a) $M^-(g) \longrightarrow M(g)$
(b) $M^-(g) \longrightarrow M^+(g)$
(c) $M^+(g) \longrightarrow M^{2+}(g)$
(d) $M^{2+}(g) \longrightarrow M^{3+}(g)$

39. Transition metal with low oxidation number will act as
(a) an oxidising agent (b) a base
(c) an acid (d) None of these

40. Chloride of which of the following element is coloured?
(a) Hg (b) Ag
(c) Co (d) Zn

41. Spiegeleisn is an alloy of
(a) Fe, Co and Cr (b) Fe, Co and Mg
(c) Fe, Mg, and C (d) Fe, C and Mn

42. Which of the following ions form most stable complex compound?
(a) Mn^{2+} (b) Ni^{2+}
(c) Fe^{2+} (d) Cu^{2+}

43. Coordination number of Fe in the complexes $[Fe(CN)_6]^{4-}$, $[Fe(CN)_6]^{3-}$ and $[FeCl_4]^-$ would be respectively
(a) 6, 4, 6 (b) 6, 6, 4
(c) 6, 3, 3 (d) 2, 3, 3

44. Which of the following is a wrong statement?
(a) $Ni(CO)_4$ has zero oxidation number for Ni
(b) $Ni(CO)_4$ has oxidation number +4 for Ni
(c) Ni is metal
(d) CO is gas

45. Rusting of iron is catalysed by which of the following?
(a) Fe (b) Zn
(c) O_2 (d) H^+

46. Standard electrode potential of NHE at 298 K is
(a) 0.05 V (b) 0.10 V
(c) 0.50 V (d) 0.00 V

47. For a cell reaction involving a two electron change, the standard emf of the cell is found to be 0.295 V at 25°C. The equilibrium constant of the reaction at 25°C will be
(a) 10 (b) 1×10^{10}
(c) 1×10^{-10} (d) 10×10^{-2}

48. The ionic conductance of Ba^{2+} and Cl^- are respectively 127 and 76 $ohm^{-1}\text{-}cm^2$ at infinite dilution. The equivalent conductance ($inohm^{-1}$ cm^2) of $BaCl_2$ at infinite dilution will be
(a) 139.5 (b) 203
(c) 279 (d) 101.5

49. Multimolecular colloids are present in
(a) soap solution (b) sol of proteins
(c) sol of gold (d) All of these

50. In physical adsorption, the gas molecules are held by solid surfaces through
(a) strong chemical forces
(b) van der waals' forces
(c) metallic bonds
(d) gravitational forces

51. The osmotic pressure of a 5% (wt./vol.) solution of cane sugar at 150°C is
(a) 3.078 atm (b) 4.078 atm
(c) 5.078 atm (d) 2.45 atm

52. The mole fraction of water in 20% aqueous solution of H_2O_2 is
(a) $\frac{20}{80}$ (b) $\frac{80}{20}$
(c) $\frac{34}{77}$ (d) $\frac{77}{68}$

53. The pH of a solution is increased from 3 to 6, its H^+ ion concentration will be
(a) reduced to half
(b) doubled
(c) reduced by 1000 times
(d) increased by 1000 times

54. Solubility product of $BaCl_2$ is 4×10^{-9}. Its solubility in mol/L would be
(a) 1×10^{-3} (b) 1×10^{-9}
(c) 4×10^{-27} (d) 1×10^{-27}

55. An example of a Lewis acid is
(a) NaCl (b) $MgCl_2$
(c) $AlCl_3$ (d) $SnCl_4$

56. Radioactive decay is a
(a) first order reaction
(b) zero order reaction
(c) second order reaction
(d) third order reaction

57. The rate constant for the first order reaction is 60 s^{-1}. How much time will it take to reduce the concentration of the reactant to $\frac{1}{16}$ th value?
(a) 4.6×10^{-2} s
(b) 4.6×10^{4} s
(c) 4.6×10^{2} s
(d) 4.6×10^{-4} s

58. $(\Delta H - \Delta U)$ for the formation of carbon monoxide (CO) from its elements at 298 K is $(R = 8.314\ \text{JK}^{-1}\ \text{mol}^{-1})$

(a) $-1238.78\ \text{J mol}^{-1}$ (b) $1238.78\ \text{J mol}^{-1}$

(c) $-2477.57\ \text{J mol}^{-1}$ (d) $2477.57\ \text{J mol}^{-1}$

59. An ideal gas at constant temperature and pressure expands, then its

(a) internal energy remains same
(b) internal energy decrease
(c) internal energy increases
(d) entropy first increases and then decreases

60. At which temperature nitrogen under 1.00 atm pressure has the same root mean square speed as that of CO_2 at STP

(a) 0°C (b) 27°C (c) – 99°C (d) – 200°C

Mathematics

1. The value of $\begin{vmatrix} a^2 & b^2 & c^2 \\ (a+1)^2 & (b+1)^2 & (c+1)^2 \\ (a-1)^2 & (b-1)^2 & (c-1)^2 \end{vmatrix}$ is

(a) $3\begin{vmatrix} a^2 & b^2 & c^2 \\ a & b & c \\ 1 & 1 & 1 \end{vmatrix}$ (b) $4\begin{vmatrix} a^2 & b^2 & c^2 \\ a & b & c \\ 1 & 1 & 1 \end{vmatrix}$

(c) $2\begin{vmatrix} a^2 & b^2 & c^2 \\ a & b & c \\ 1 & 1 & 1 \end{vmatrix}$ (d) None of these

2. If $a_1x + b_1y + c_1z = 0$, $a_2x + b_2y + c_2z = 0$, $a_3x + b_3y + c_3z = 0$ and $\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = 0$, then the given system has

(a) one trivial and one non-trivial solution
(b) no solution
(c) one solution
(d) infinite solution

3. Let A be a square matrix all of whose entries are integers. Then, which one of the following is true?

(a) If $\det(A) = \pm 1$, then A^{-1} exists but all its entries are not necessarily integers
(b) If $\det(A) \neq \pm 1$, then A^{-1} exists and all its entries are non-integers
(c) If $\det(A) = \pm 1$, then A^{-1} exists and all its entries are integers
(d) If $\det(A) = \pm 1$, then A^{-1} need not exist

4. Which of the following statements is/are correct?

I. Adjoint of a unit matrix is a unit matrix.
II. $A(\text{adj } A) = (\text{adi } A)\ A = |A|I$
III. Adjoint of a symmetric matrix is symmetric.
IV. Adjoint of a diagonal matrix is a diagonal matrix.

(a) I
(b) II
(c) I or II
(d) All statements are correct

5. The value of $\int \frac{dx}{x\sqrt{1-(\log x)^2}}$ is

(a) $\cos^{-1}(\log x + C)$

(b) $x \log(1 - x^2) + C$

(c) $\frac{1}{2}\cos^{-1}(\log x + C)$

(d) $\sin^{-1}(\log x) + C$

6. $\int \frac{x^2}{(x^2+2)(x^2+3)}dx$ is equal to

(a) $-\sqrt{2}\tan^{-1}\left(\frac{x}{\sqrt{2}}\right) + \sqrt{3}\tan^{-1}\left(\frac{x}{\sqrt{3}}\right) + C$

(b) $\sqrt{2}\tan^{-1}\left(\frac{x}{\sqrt{2}}\right) + \sqrt{3}\tan^{-1}\left(\frac{x}{\sqrt{3}}\right) + C$

(c) $-\sqrt{2}\tan^{-1}x + \sqrt{3}\tan^{-1}x + C$

(d) None of the above

7. The degree and order of the differential equation of the family of all parabolas whose axis is x-axis, are respectively

(a) 2, 1 (b) 1, 2
(c) 3, 2 (d) 2, 3

8. The differential equation satisfied by the family of curve $y = ax\cos\left(\frac{1}{x} + b\right)$, where a, b are parameters, is

(a) $x^2 y_2 + y = 0$ (b) $x^4 y_2 + y = 0$
(c) $xy_2 - y = 0$ (d) $x^4 y_2 - y = 0$

9. Integrating factor of equation

$$(x^2+1)\frac{dy}{dx} + 2xy = x^2 - 1 \text{ is}$$

(a) $\frac{2x}{x^2+1}$ (b) $\frac{x^2-1}{x^2+1}$
(c) x^2+1 (d) None of these

10. The value of $\int_0^{\pi/2} |\sin x - \cos x|\, dx$ is

(a) 0 (b) $2(\sqrt{2}-1)$
(c) $\sqrt{2}-1$ (d) $2(\sqrt{2}+1)$

11. The equation of the circle which passes through the points (2, 3) and (4, 5) and the centre lies on the straight line $y - 4x + 3 = 0$, is
(a) $x^2 + y^2 + 4x - 10y + 25 = 0$
(b) $x^2 + y^2 - 4x - 10y + 16 = 0$
(c) $x^2 + y^2 - 4x - 10y + 25 = 0$
(d) None of the above

12. The number of solutions of $\log_4 (x-1) = \log_2 (x-3)$ is/ are
(a) 0 (b) 1
(c) 2 (d) 3

13. If α, β are the roots of $x^2 - 3x + 1 = 0$, then the equation whose roots are $\frac{1}{\alpha - 2}, \frac{1}{\beta - 2}$, is
(a) $x^2 + x + 1 = 0$ (b) $x^2 - x - 1 = 0$
(c) $x^2 + x - 1 = 0$ (d) None of these

14. If x is real, then the value of $\frac{x^2 + 34x - 71}{x^2 + 2x - 7}$ does not lie between
(a) 8 and –5 (b) –5 and 9
(c) 0 and 9 (d) 5 and 9

15. The square root of $\sqrt{50} + \sqrt{48}$ is
(a) $2^{1/4}(3 + \sqrt{2})$ (b) $2^{1/4}(\sqrt{3} + \sqrt{2})$
(c) $2^{1/4}(2 + \sqrt{2})$ (d) $2^{1/4}(\sqrt{3} + 2)$

16. Let $I_1 = \int_\alpha^{\pi-\alpha} xf(\sin x)\, dx$, $I_2 = \int_{xf(\sin x),\, dx}^{\pi-\alpha} f(\sin x)\, dx$, then I_2 is equal to
(a) $\frac{\pi}{2} I_1$ (b) πI_1
(c) $\frac{2}{\pi} I_1$ (d) $2 I_1$

17. The area of figure bounded by $y = e^x, y = e^{-x}$ and the straight line $x = 1$ is
(a) $\left(e + \frac{1}{e}\right)$ sq unit (b) $\left(e - \frac{1}{e}\right)$ sq unit
(c) $\left(e + \frac{1}{e} - 2\right)$ sq unit (d) $\left(e + \frac{1}{e} + 2\right)$ sq unit

18. The domain and range of $f(x) = \sin^{-1}[x]$ are
(a) $[0,2), \left\{-\frac{\pi}{2}, \frac{\pi}{2}\right\}$
(b) $[-1,2), \left\{-\frac{\pi}{2}, 0, \frac{\pi}{2}\right\}$
(c) $[-1,2), \left\{-\pi, \frac{-\pi}{2}, 0, \frac{\pi}{2}, \pi\right\}$
(d) None of the above

19. If **a**, **b** and **c** are non-collinear vectors such that for some scalars x, y, z, $x\mathbf{a} + x\mathbf{b} + x\mathbf{c} = 0$, then
(a) $x = 0, y = 0, z = 0$
(b) $x \neq 0, y \neq 0, z = 0$
(c) $x = 0, y \neq 0, z \neq 0$
(d) $x \neq 0, y \neq 0, z \neq 0$

20. Let **a**, **b**, **c**, be three vectors such that $\mathbf{c} \neq 0$ and $\mathbf{a} \cdot \mathbf{b} = 2\mathbf{a} \cdot \mathbf{c}, |\mathbf{a}| = |\mathbf{c}| = 1, |\mathbf{b}| = 4$ and $|\mathbf{b} \times \mathbf{c}| = \sqrt{15}$, if $\mathbf{b} - 2\mathbf{c} = \lambda\alpha$. then, λ equals
(a) –1 (b) 1
(c) – 4 (d) 2

21. The image of the point with position vector $\mathbf{i} + 3\mathbf{k}$ in the plane $r \cdot (\mathbf{i} + \mathbf{j} + \mathbf{k}) = 1$ is
(a) $\mathbf{i} - 2\mathbf{j} + \mathbf{k}$ (b) $\mathbf{i} + 2\mathbf{j} - \mathbf{k}$
(c) $-\mathbf{i} - 2\mathbf{j} + \mathbf{k}$ (d) None of these

22. The vectors **c**, **a**, $= x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$ and $\mathbf{b} = \mathbf{j}$ are such that **a**, **b**, **c** form a right handed system, then **c** is
(a) 0 (b) $y\mathbf{j}$
(c) $-z\mathbf{i} + x\mathbf{k}$ (d) $z\mathbf{i} - x\mathbf{k}$

23. The equation of plane passing through a point A (2, –1, 3) and parallel to the vectors $\mathbf{a} = (3, 0, -1)$ and $\mathbf{b} = (-3, 2, 2)$ is
(a) $2x - 3y + 6z + 25 = 0$
(b) $3x - 2y + 6z + 25 = 0$
(c) $2x - 3y + 6z - 25 = 0$
(d) $3x - 2y + 6z - 25 = 0$

24. $\lim_{x \to 1} \frac{(2x-3)(\sqrt{x}-1)}{2x^2 + x - 3}$ is equal to
(a) $-1/10$ (b) $1/10$
(c) $-1/8$ (d) None of these

25. Which is the correct order for a given number α in increasing order?
(a) $\log_2\alpha, \log_3\alpha, \log_e\alpha, \log_{10}\alpha$
(b) $\log_{10}\alpha, \log_3\alpha, \log_e\alpha, \log_2\alpha$
(c) $\log_{10}\alpha, \log_e\alpha, \log_2\alpha, \alpha\log_3\alpha$
(d) None of the above

26. If $x=\frac{\sqrt{5}+\sqrt{2}}{\sqrt{5}-\sqrt{2}}, y=\frac{\sqrt{5}-\sqrt{2}}{\sqrt{5}+\sqrt{2}}$, then $3x^2+4xy-3y^2$ is equal to
(a) $\frac{1}{3}(56\sqrt{10}+12)$ (b) $\frac{1}{3}(56\sqrt{10}-12)$
(c) $\frac{1}{3}(56+12\sqrt{10})$ (d) None of these

27. If $x+y=1$, then $\sum_{r=0}^{n} r^{2\cdot n}C_r\ x^r\ y^{n-r}$ is equal to
(a) nxy (b) $nx\ (x+yn)$
(c) $nx\ (n\ x+y)$ (d) None of these

28. If $Z=f(x+ay)+\phi(x-ay)$, then
(a) $Z_{xx}=Z_{yy}$ (b) $Z_{xx}=a^2\ Z_{yy}$
(c) $Z_{yy}=a^2\ Z_{xx}$ (d) None of these

29. The existence of the unique solution of the system $x+y+z=\lambda$, $5x-y+\mu z=10$, $2x+3y-z=6$ depends on
(a) μ only (b) λ only
(c) λ and μ both (d) neither λ nor μ

30. Let $A=\begin{bmatrix}1 & -1 & 1\\ 2 & 1 & -3\\ 1 & 1 & 1\end{bmatrix}$ and $10B=\begin{bmatrix}4 & 2 & 2\\ -5 & 0 & \alpha\\ 1 & -2 & 3\end{bmatrix}$. If B is the inverse of matrix A, then α is
(a) -1 (b) -2
(c) 2 (d) 5

31. The angle of intersection of ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ and circle $x^2+y^2=ab$ is
(a) $\tan^{-1}\left(\frac{a+b}{ab}\right)$ (b) $\tan^{-1}\left(\frac{a-b}{\sqrt{ab}}\right)$
(c) $\tan^{-1}\left(\frac{a+b}{\sqrt{ab}}\right)$ (d) $\tan^{-1}\left(\frac{a-b}{ab}\right)$

32. If $S=\sum_{n=0}^{\infty}\frac{(\log x)^{2n}}{(2n)!}$, then S is equal to
(a) $x+x^{-1}$ (b) $x-x^{-1}$
(c) $\frac{1}{2}(x+x^{-1})$ (d) None of these

33. The value of $\log 2+2\left(\frac{1}{5}+\frac{1}{3}\cdot\frac{1}{5^3}+\frac{1}{5}\cdot\frac{1}{5^5}+\ldots.\right)$, is
(a) $\log 2+\log 3$ (b) $\log 2+2$
(c) $\frac{1}{2}\log 2$ (d) $\log 3$

34. The equation of parabola whose vertex and focus are (0,4) and (0,2) respectively , is
(a) $y^2-8x=32$ (b) $y^2+8x=32$
(c) $x^2+8y=32$ (d) $x^2-8y=32$

35. The line $x+2y=4$ is translated parallel to itself by 3 unit in the sense of increasing x and then rotated by 30° in the anti-clockwise direction about the point where the shifted line cuts the x-axis. The equation of the line in the new position is
(a) $y=\tan(\theta-30°)(x-4-3\sqrt{5})$
(b) $y=\tan(30°-\theta)(x-4-3\sqrt{5})$
(c) $y=\tan(\theta+30°)(x+4+3\sqrt{5})$
(d) $y=\tan(\theta-30°)(x+4+3\sqrt{5})$

36. The distance between the foci of a hyperbola is double the distance between its vertices and the length of its conjugate axis is 6. The equation of the hyperbola referred to its axes as axes of coordinates are
(a) $3x^2-y^2=3$
(b) $x^2-3y^2=3$
(c) $3x^2-y^2=9$
(d) $x^2-3y^2=9$

37. If the equation $12x^2+7xy-py^2-18x+qy+6=0$ represents a pair of perpendicular straight lines, then
(a) $p=12, q=-1$ (b) $p=-12, q=1$
(c) $p=12, q=1$ (d) $p=1, q=1$

38. If the roots of the equation $\frac{\alpha}{x-\alpha}+\frac{\beta}{x-\beta}=1$ be equal in magnitude but opposite in sign, then $\alpha+\beta$ is equal to
(a) 0 (b) 1
(c) 2 (d) None of these

39. If $2a+3b+6c=0$, then at least one root of the equation $ax^2+bx+c=0$ lies in the interval
(a) (1, 2) (b) (0, 1)
(c) (2, 3) (d) (3, 4)

40. The ratio in which the line $3x + 4y + 2 = 0$ divides the distance between $3x + 4y + 5 = 0$ and $3x + 4y - 5 = 0$, is
(a) 7 : 3
(b) 3 : 7
(c) 2 : 3
(d) None of the above

41. If the circle $x^2 + y^2 + 2gx + 2fy + c = 0$ is touched by $y = x$ at P such that $OP = 6\sqrt{2}$, then the value of c is
(a) 36 (b) 72
(c) 144 (d) None of these

42. The coefficient of x^4 in the expansion of $(1 + x + x^2 + x^3)^n$ is
(a) ${}^nC_4$
(b) ${}^nC_4 + {}^nC_2$
(c) ${}^nC_4 + {}^nC_2 + {}^nC_4 \cdot {}^nC_2$
(d) ${}^nC_4 + {}^nC_2 + {}^nC_1 \cdot {}^nC_2$

43. $1 + \frac{1+x}{2!} + \frac{1+x+x^2}{3!} + \frac{1+x+x^2+x^3}{4!} + \ldots$ is equal to
(a) $\frac{e^x + 1}{x - 1}$ (b) $\frac{e^x + 1}{x + 1}$
(c) $\frac{e^x - e}{x + 1}$ (d) $\frac{e^x - e}{x - 1}$

44. The inequation $n! > 2^{n-1}$ is true for
(a) for all $n \in N$
(b) for all $n > 1$
(c) for all $n > 2$
(d) None of the above

45. Let $A = \{(x, y) : y = e^x, x \in R\}$ and $B = \{(x, y) : y = e^{-x}, x \in R\}$. Then,
(a) $A \cap B = \phi$ (b) $A \cap B \neq \phi$
(c) $A \cup B = R^2$ (d) None of these

46. In ΔABC, $\frac{b-c}{r_1} + \frac{c-a}{r_2} + \frac{a-b}{r_3}$ is equal to
(a) 0 (b) abc
(c) $a + b + c$ (d) $ab + bc + ca$

47. $ABCD$ is a rectangular field . A vertical lamp post of height 12 m stands at the corner A. If the angle of elevation of its top from B is 60° and from C is 45°, then the area of the field is
(a) $48\sqrt{2}$ m^2 (b) $12\sqrt{2}$ m^2
(c) 48 m^2 (d) $12\sqrt{3}$ m^2

48. The function $f(x) = \max\{(1 - x), (1 + x), 2\}$, $x \in (-\infty, \infty)$ is equivalent to
(a) $f(x) = \begin{cases} 1 + x, & x \leq -1 \\ 2, & -1 < x < 1 \\ 1 - x, & x \geq 1 \end{cases}$
(b) $f(x) = \begin{cases} 1 - x, & x \leq -1 \\ 2, & -1 < x < 1 \\ 1 + x, & x \geq 1 \end{cases}$
(c) $f(x) = \begin{cases} 1 - x, & x \leq -1 \\ 1, & -1 < x < 1 \\ 1 + x, & x \geq 1 \end{cases}$
(d) None of the above

49. The number of values of x in the interval $[0, 5\pi]$ satisfying the equation $3\sin^2 x - 7 \sin x + 2 = 0$ is
(a) 0 (b) 5 (c) 6 (d) 10

50. $\left(1 + \cos\frac{\pi}{8}\right)\left(1 + \cos\frac{3\pi}{8}\right)\left(1 + \cos\frac{5\pi}{8}\right)\left(1 + \cos\frac{7\pi}{8}\right)$ is equal to
(a) $\frac{1}{2}$ (b) $\frac{1}{8}$
(c) $\cos\frac{\pi}{8}$ (d) $\frac{1+\sqrt{2}}{2\sqrt{2}}$

51. The number of solutions of the equation $2\sin^{-1}\sqrt{x^2 - x + 1} + \cos^{-1}\sqrt{x^2 - x} = \frac{3\pi}{2}$ is
(a) 0 (b) 2
(c) 4 (d) ∞

52. Each side of a square subtends an angle of 60° at the top of a tower h metre high standing in the centre of the square. If a is the length of each side of the square, then
(a) $2a^2 = h^2$ (b) $2h^2 = a^2$
(c) $3a^2 = 2h^2$ (d) $2h^2 = 3a^2$

53. The value of $\cos^{-1}\left(\cos\frac{7\pi}{6}\right)$ is
(a) $\frac{7\pi}{6}$ (b) $\frac{5\pi}{3}$
(c) $\frac{5\pi}{6}$ (d) $\frac{13\pi}{6}$

54. If $n \in N$, then $|\sin nx|$
(a) $\leq n|\sin x|$ (b) $\geq n|\sin x|$
(c) $= n|\sin x|$ (d) None of these

55. In ΔABC, if $a = 30, b = 24, = 18$, then r_3 is equal to
(a) 15 (b) 18 (c) 36 (d) 12

56. The circumcentre of a triangle formed by the lines $xy + 2x + 2y + 4 = 0$ and $x + y + 2 = 0$, is
(a) $(0, -1)$ (b) $(-1, 0)$
(c) $(1, 1)$ (d) $(-1, -1)$

57. One diagonal of a square is along the line $8x - 15y = 0$ and one of its vertex is $(1, 2)$. Then, the equation of the sides of the square passing through this vertex are
(a) $23x + 7y = 9, 7x + 23y = 53$
(b) $23x - 7y - 9 = 0, 7x + 23y - 53 = 0$
(c) $23x - 7y + 9 = 0, 7x + 23y + 53 = 0$
(d) None of the above

58. The two curves $x^3 - 3xy^2 + 2 = 0$ and $3x^2 y - y^3 = 2$
(a) cut at right angles
(b) touch each other
(c) cut at an angle $\pi/3$
(d) cut at an angle $\pi/4$

59. If $\frac{2x+3}{(x+1)(x-3)} = \frac{a}{x+1} + \frac{b}{x-3}$, then $a + b$ is equal to
(a) 1 (b) 2
(c) 9/4 (d) $-1/4$

60. $\lim_{n\to\infty} \frac{1}{n}\left[\sec^2\frac{\pi}{4n} + \sec^2\frac{2\pi}{4n} + \ldots + \sec^2\frac{n\pi}{4n}\right]$ is equal to
(a) $\frac{4}{\pi}$ (b) $\frac{2}{\pi}$
(c) $\frac{3}{\pi}$ (d) $\frac{5}{\pi}$

61. If $f(x) = |\cos x|$ and $g(x) = [x]$, then $gof(x)$ is equal to
(a) $|\cos[x]|$ (b) $|\cos x|$
(c) $[|\cos x|]$ (d) $|[\cos x]|$

62. If $\lim_{x\to\infty}\left(1 + \frac{a}{x} + \frac{b}{x^2}\right)^{2x} = e^2$, then
(a) $a = 1, b = 2$ (b) $a = 2, b = 1$
(c) $a = 1, b \in R$ (d) None of these

63. The area bounded by the curve $y^2 = 2x + 1$ and the straight line $x - y - 1 = 0$ is given by
(a) $\frac{9}{2}$ sq units (b) $\frac{43}{6}$ sq units
(c) $\frac{35}{6}$ sq units (d) None of these

64. The value of $f(0)$, so that the function $f(x) = \frac{2 - (256 - 7x)^{1/8}}{(5x + 3x)^{1/5} - 2}$, $x \neq 0$ is continuous everywhere is given by
(a) -1 (b) 1
(c) 2^6 (d) None of these

65. The set of points where the function $f(x) = x|x|$ is differentiable, is
(a) $(-\infty, \infty)$ (b) $(-\infty, 0) \cup (0, \infty)$
(c) $(0, \infty)$ (d) $[0, \infty)$

66. If $f(x) = |x|^{|\sin x|}$, then $f'\left(-\frac{\pi}{4}\right)$ is equal to
(a) $\left(\frac{\pi}{4}\right)^{1/\sqrt{2}}\left(\frac{\sqrt{2}}{2}\log\frac{4}{\pi} - \frac{2\sqrt{2}}{\pi}\right)$
(b) $\left(\frac{\pi}{4}\right)^{1/\sqrt{2}}\left(\frac{\sqrt{2}}{2}\log\frac{4}{\pi} + \frac{2\sqrt{2}}{\pi}\right)$
(c) $\left(\frac{\pi}{4}\right)^{1/\sqrt{2}}\left(\frac{\sqrt{2}}{2}\log\frac{\pi}{4} - \frac{2\sqrt{2}}{\pi}\right)$
(d) $\left(\frac{\sqrt{2}}{2}\log\frac{\pi}{4} + \frac{2\sqrt{2}}{\pi}\right)$

67. A circle touches the x-axis and also touches the circle which centre at $(0, 3)$ and radius 2. The locus of the centre of the circle is
(a) a parabola (b) a circle
(c) an ellipse (d) a hyperbola

68. The circle on focal radii of a parabola as diameter touches
(a) the x-axis
(b) the tangent at the vertex
(c) the directrix
(d) None of the above

69. The equation of the ellipse whose axes are parallel to the coordinate axes having its centre at the point $(2, -3)$ and focus at $(3, -3)$ and one vertex at $(4, -3)$ is
(a) $\frac{(x-2)^2}{4} + \frac{(y+3)^2}{3} = 1$
(b) $\frac{(x+2)^2}{3} + \frac{(y+3)^2}{4} = 1$
(c) $\frac{(x+2)^2}{4} + \frac{(y+3)^2}{3} = 1$
(d) None of the above

70. The diameter of $16x^2 - 9y^2 = 144$ which is conjugate to $x = 2y$ is

(a) $y = \frac{32x}{9}$ (b) $x = \frac{16}{9}y$

(c) $y = \frac{16}{9}x$ (d) None of these

71. If $\frac{2x+3}{(x+1)(x-3)} = \frac{a}{x+1} + \frac{b}{x-3}$, then $a + b$ is equal to

(a) 1
(b) 2
(c) 9/4
(d) –1/4

72. If $\frac{e^x + 2}{(e^x - 1)(2e^x - 3)} = -\frac{3}{e^x - 1} + \frac{B}{2e^x - 3}$, then B is equal to

(a) 1 (b) 3
(c) 5 (d) 7

73. $\frac{3\sqrt{2}}{\sqrt{6}+\sqrt{3}} - \frac{4\sqrt{3}}{\sqrt{6}+\sqrt{2}} + \frac{\sqrt{6}}{\sqrt{3}+\sqrt{2}}$ is equal to

(a) $5\sqrt{2}$ (b) $3\sqrt{2}$
(c) $2\sqrt{3}$ (d) 0

74. If $\log_3 2$, $\log_3 (2^x - 5)$ and $\log_3\left(2^x - \frac{7}{2}\right)$ are in AP, then x is equal to

(a) $1, \frac{1}{2}$ (b) $1, \frac{1}{3}$

(c) $1, \frac{3}{2}$ (d) None of these

75. If the roots of the cubic equation $ax^3 + bx^2 + cx + d = 0$ are in GP, then

(a) $c^3a = b^3d$ (b) $ca^3 = bd^3$
(c) $a^3b = c^3d$ (d) $ab^3 = cd^3$

76. In the group $G = (\{1, 2, 3, 4\}, \times_5)$ the solution of $2^{-1} \times (3 \times_5 x) = 4$ is

(a) 1 (b) 2
(c) 3 (d) None of these

77. Negation of "Ram is in Class X or Rashmi is in Class XII" is

(a) Ram is not in Class X but Rashmi is in Class XII
(b) Ram is not in Class X but Rashmi is in Class XII
(c) Either Ram is not in Class X or Rashmi is not in class XII
(d) None of the above

78. The inverse of the proportion $(p \wedge \sim q) \Rightarrow r$ is

(a) $\sim r \Rightarrow \sim p \vee q$ (b) $\sim p \vee q \Rightarrow \sim r$
(c) $r \Rightarrow p \vee \sim q$ (d) None of these

79. The least remainder when 17^{30} is divided by 5, is

(a) 2 (b) 1
(c) 4 (d) 3

80. The solution of differential equation $dy - \sin x \sin y\, dx = 0$ is

(a) $e^{\cos x} \cdot \tan\frac{y}{2} = C$ (b) $e^{\cos x} \cdot \tan y = C$

(c) $\cos x \cdot \tan y = C$ (d) $\cos x \cdot \sin y = C$

General English and Aptitude

Directions (Q. Nos. 1-4) *Choose the alternative which can be substituted for the given group of words.*

1. A person concerned with practical results and values

(a) plagiarist (b) realist
(c) pragmatist (d) fundamentalist

2. Member of a band of robbers

(a) dacoit (b) brigand
(c) thief (d) pirate

3. A person without manners or polish is

(a) rustic (b) naive
(c) boorish (d) barbarian

4. A speech by an actor at the end of a play

(a) epilogue (b) monologue
(c) duologue (d) prologue

Directions (Q. Nos. 5-8) *Choose the most suitable alternative to fill in the blank.*

5. The teacher ordered Kamal to leave the room and ..... him to return.

(a) stopped (b) rufused
(c) forabade (d) challenged

6. I hope you must have ..... by now that failures are the stepping stones to success.

(a) known (b) felt
(c) decided (d) realised

7. The tyrant …… anyone whom he regarded as a rival.
(a) massacred (b) killed
(c) exterminated (d) slaughtered

8. In a little publicised deal, Pepsi Cola has …… the entire soft drink market in Afghanistan.
(a) conquered
(b) swallowed
(c) captured
(d) occupied

Directions (Q. Nos. 9-12) *Choose the alternative which is nearest in meaning to the word given in capital letters.*

9. DILETTANTE
(a) Opponent (b) Specialist
(c) Amateur (d) Expert

10. FLAK
(a) Adventure (b) Advice
(c) Criticism (d) Praise

11. HOODLUM
(a) Pioneer (b) Criminal
(c) Devotee (d) Scholar

12. SPASMODIC
(a) Continuous (b) Gradual
(c) Intermittent (d) Spontaneous

Directions (Q. Nos. 13-16) *Choose the alternative which is opposite in meaning of the word given in capital letters.*

13. SHAME
(a) Glorify (b) Exalt
(c) Dignify (d) Enshrine

14. RESCUE
(a) Extricate (b) Waver
(c) Bind (d) Desert

15. AGONY
(a) Pleasure (b) Bliss
(c) Ecstasy (d) Fear

16. REQUISITE
(a) Dispensable (b) Random
(c) Inappropriate (d) Chaotic

Directions (Q. Nos. 17-20) *In each of the following questions, a sentence is given with a part of it italicised. Choose the alternative which is an improvement upon this italicised part. If the sentence is correct as it is, choose (d) as your answer.*

17. Old habits *die hardly*.
(a) die much hardly
(b) die hard
(c) die too hard
(d) No improvement

18. The teacher asked, "*Why you are late*?"
(a) Why you were late
(b) Why late you are
(c) Why are you late
(d) No improvement

19. It is ten years since I *have begun* living here.
(a) begun (b) had begun
(c) began (d) No improvement

20. The various practices and norms for bank's transactions *are laid down* by the Reserve Bank of India.
(a) are laid up (b) are led down
(c) are lead up (d) No improvement

Directions (Q. Nos. 21-25) *In the following questions, choose the option which shows common feature in the relationship given in each question.*

21. Knot : Watt : Fathom
(a) The terms are connected with rope
(b) They are units of measurement
(c) The terms are used by sailors
(d) The terms are used for installing electricity

22. Majlis : Diet : Knesset
(a) These are parliaments of countries
(b) These are old names of certain countries
(c) These are foreign languages
(d) These are the names of foods eaten in different countries

23. Kanha : Periyar : Dachigam
(a) These are mountain peaks
(b) These are famous lagoons
(c) These are hill stations
(d) These are animal sanctuaries

24. Stirrup : Anvil : Drum
(a) These are used by folk artists
(b) The items are used by riders
(c) They are parts of ear
(d) These are musical instruments

25. Kennedy : Indira : Palme
(a) They were Prime Ministers
(b) They were very popular among children
(c) They were Presidents.
(d) All of them were assassinated

Directions (Q. Nos. 26-30) *In the following questions, choose the group of words that shows the same relationship as given in the question.*

26. Chair : Door : Stick
(a) Book : Pen : Notebook
(b) Statue : Brick : Pitcher
(c) Mason : Carpenter : Cobbler
(d) Tomato : Potato : Brinjal

27. Music : Guitar : Performer
(a) Trick : Rope : Acrobat
(b) Dance : Tune : Instrument
(c) Food : Recipe : Cook
(d) Patient : Medicine : Doctor

28. Hair : Brush : Wig
(a) Cement : Brick : Building
(b) Paper : Pen : Pencil
(c) Iron : Hammer : Axe
(d) Bread : Butter : Milk

29. Village : City : Suburb
(a) Footpath : Road : Highway
(b) Pillow : Quilt : Bed
(c) Flowers : Garden : Park
(d) Pen : Pencil : Colour

30. Smile : Laugh : Cry
(a) Frown : Anger : Temper
(b) Sit : Sleep : Play
(c) Touch : Catch : Release
(d) Morning : Night : Day

31. Which river of India is called Vridha Ganga?
(a) Krishna (b) Godavari
(c) Cauveri (d) Narmada

32. The capital of Tanzania is
(a) Nairobi (b) Lusaka
(c) Kaupala (d) Dar-e-Salaam

33. During which decade did the population of India record a negative growth rate?
(a) 1921-31 (b) 1911-21
(c) 1941-51 (d) 1931-41

34. Marx belonged to
(a) Germany (b) Holland
(c) France (d) Britain

35. Who is the author of 'A River Sutra'?
(a) VS Naipaul (b) Nirad C Chaudhri
(c) Gita Mehta (d) Vikram Seth

36. Who is known as the 'Father of Indian Missile Technology'?
(a) Dr UR Rao
(b) Dr APJ Abdul Kalam
(c) Dr Chidambaram
(d) Dr Homi Bhabha

37. The art and science of map making is called
(a) remote sensing
(b) cartography
(c) photogrammetry
(d) mapping

38. WTO basically promotes
(a) financial support
(b) global peace
(c) unilateral trade
(d) multilateral trade

39. The forest in Sunderban is called
(a) scrub jungle (b) mangrove
(c) deciduous forest (d) tundra

40. Indian Standard Time relates to
(a) 75.5° E longitude
(b) 82.5° E longitude
(c) 90.5° E longitude
(d) 0° longitude

Answers

Physics

1. (c)	**2.** (b)	**3.** (c)	**4.** (b)	**5.** (a)	**6.** (b)	**7.** (d)	**8.** (c)	**9.** (a)	**10.** (a)
11. (a)	**12.** (c)	**13.** (b)	**14.** (a)	**15.** (d)	**16.** (c)	**17.** (c)	**18.** (b)	**19.** (a)	**20.** (d)
21. (d)	**22.** (d)	**23.** (c)	**24.** (a)	**25.** (a)	**26.** (c)	**27.** (d)	**28.** (c)	**29.** (b)	**30.** (b)
31. (c)	**32.** (c)	**33.** (d)	**34.** (b)	**35.** (d)	**36.** (b)	**37.** (b)	**38.** (c)	**39.** (d)	**40.** (d)
41. (d)	**42.** (b)	**43.** (c)	**44.** (b)	**45.** (c)	**46.** (b)	**47.** (a)	**48.** (b)	**49.** (a)	**50.** (a)
51. (c)	**52.** (b)	**53.** (d)	**54.** (a)	**55.** (b)	**56.** (d)	**57.** (a)	**58.** (b)	**59.** (d)	**60.** (d)

Chemistry

1. (a)	**2.** (d)	**3.** (b)	**4.** (a)	**5.** (c)	**6.** (b)	**7.** (b)	**8.** (b)	**9.** (c)	**10.** (a)
11. (c)	**12.** (b)	**13.** (a)	**14.** (b)	**15.** (a)	**16.** (a)	**17.** (b)	**18.** (c)	**19.** (d)	**20.** (a)
21. (b)	**22.** (b)	**23.** (c)	**24.** (b)	**25.** (a)	**26.** (d)	**27.** (d)	**28.** (a)	**29.** (c)	**30.** (d)
31. (b)	**32.** (a)	**33.** (d)	**34.** (d)	**35.** (a)	**36.** (a)	**37.** (a)	**38.** (d)	**39.** (a)	**40.** (c)
41. (d)	**42.** (d)	**43.** (b)	**44.** (b)	**45.** (d)	**46.** (d)	**47.** (b)	**48.** (a)	**49.** (c)	**50.** (b)
51. (c)	**52.** (c)	**53.** (c)	**54.** (a)	**55.** (c)	**56.** (a)	**57.** (a)	**58.** (a)	**59.** (a)	**60.** (c)

Mathematics

1. (b)	**2.** (d)	**3.** (c)	**4.** (d)	**5.** (d)	**6.** (a)	**7.** (a)	**8.** (b)	**9.** (c)	**10.** (b)
11. (c)	**12.** (b)	**13.** (b)	**14.** (d)	**15.** (b)	**16.** (c)	**17.** (c)	**18.** (b)	**19.** (a)	**20.** (c)
21. (c)	**22.** (d)	**23.** (c)	**24.** (a)	**25.** (b)	**26.** (a)	**27.** (c)	**28.** (c)	**29.** (a)	**30.** (d)
31. (b)	**32.** (c)	**33.** (d)	**34.** (c)	**35.** (a)	**36.** (c)	**37.** (c)	**38.** (a)	**39.** (b)	**40.** (b)
41. (b)	**42.** (d)	**43.** (d)	**44.** (c)	**45.** (b)	**46.** (a)	**47.** (a)	**48.** (b)	**49.** (c)	**50.** (b)
51. (b)	**52.** (b)	**53.** (c)	**54.** (a)	**55.** (d)	**56.** (d)	**57.** (b)	**58.** (a)	**59.** (b)	**60.** (a)
61. (c)	**62.** (c)	**63.** (d)	**64.** (d)	**65.** (a)	**66.** (a)	**67.** (a)	**68.** (b)	**69.** (a)	**70.** (a)
71. (b)	**72.** (d)	**73.** (d)	**74.** (d)	**75.** (a)	**76.** (a)	**77.** (d)	**78.** (b)	**79.** (c)	**80.** (a)

General English and Aptitude

1. (c)	**2.** (b)	**3.** (a)	**4.** (a)	**5.** (c)	**6.** (d)	**7.** (c)	**8.** (c)	**9.** (c)	**10.** (d)
11. (b)	**12.** (c)	**13.** (b)	**14.** (d)	**15.** (c)	**16.** (a)	**17.** (b)	**18.** (c)	**19.** (c)	**20.** (d)
21. (b)	**22.** (a)	**23.** (d)	**24.** (c)	**25.** (d)	**26.** (b)	**27.** (a)	**28.** (c)	**29.** (d)	**30.** (b)
31. (b)	**32.** (d)	**33.** (b)	**34.** (a)	**35.** (c)	**36.** (b)	**37.** (b)	**38.** (d)	**39.** (b)	**40.** (b)

Hints & Solutions

Physics

1. Ammeter is always connected in series with circuit.

3. Time period of proton $T_p = \dfrac{25}{5} = 5\,\mu s$

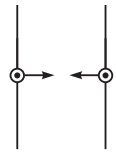
By using $T = \dfrac{2\pi m}{qB}$

$$\Rightarrow \quad \frac{T_\alpha}{T_p} = \frac{m_\alpha}{m_p} \times \frac{q_p}{q_\alpha} = \frac{4m_p}{m_p} \times \frac{q_p}{2q_p} = 2$$

$$\Rightarrow \quad T_\alpha = 2T_p = 10\,\mu s$$

4. The magnitude of force acting on length l of each conductor due to other is

$$\frac{F}{l} = \frac{\mu_0}{2\pi} \frac{i_1 i_2}{r}$$



Given $i_1 = i, i_2 = 10\,i$

$$\therefore \quad \frac{F}{l} = \frac{\mu_0}{2\pi} \frac{10 i^2}{r}$$

From Fleming's left hand rule, this is force of attraction.

5. Fundamental frequency of open pipe

$$n_1 = \frac{v}{2l}$$

Frequency of third harmonic of closed pipe

$$n_1 = \frac{3v}{4l}$$

$$\therefore \quad \frac{3v}{4l} - \frac{v}{2l} = 100$$

$$\frac{v}{4l} = 100$$

$$\frac{1}{2}\left(\frac{v}{2l}\right) = 100$$

$$\therefore \quad n_1 = 200 \text{ Hz}$$

6. Separation of slits $d = 4 \text{ mm} = 4 \times 10^{-3}$ m

Wavelength $\lambda = 6000 \text{ Å} = 6 \times 10^{-7}$ m

Distance of screen $D = 2$ m

Fringe width $\beta = \dfrac{\lambda D}{d}$

$$= \frac{6 \times 10^{-7} \times 2}{4 \times 10^{-3}}$$

$$= 3 \times 10^{-4} \text{ m}$$

$$= 0.3 \text{ mm}$$

7. Angle of deviation $\delta = (n-1)\,A$.

Dispersion produced by both the prism's will be equal.

$$(n_1 - 1)\,A_1 = (n_2 - 1)\,A_2$$

$$\Rightarrow \quad A_2 = \frac{(n_1 - 1)\,A_1}{(n_2 - 1)} = \frac{(1.54 - 1)}{(1.72 - 1)} = 3^\circ$$

Hence, the angle of prism P_2 is 3°.

8. If housefly sits on objective lens, then the intensity of image will decrease, because some of light rays will be stopped by housefly but housefly will not be seen.

9. We know that capacitor offers infinite resistance for DC source.

Current taken from the cell

$$i = \frac{E}{R + r}$$

$$\therefore \quad i = \frac{2.5}{1 + 1 + 0.5} = 1 \text{ A}$$

Potential drop across capacitor

$$V = E - ir$$

$$= 2.5 - 1 \times 0.5 = 2 \text{ V}$$

The current in 2 Ω resistor is zero.

Charge on capacitor

$$Q = CV$$

$$= 5 \times 2 = 10\,\mu C$$

10. Volume of bigger drop = volume of 64 small drops

i.e., $\frac{4}{3}\pi R^3 = 64 \times \frac{4}{3}\pi r^3$

or $R = 4r$

Potential on small drop

$$V_1 = \frac{1}{4\pi\varepsilon_0}\frac{q}{r}$$

Potential on bigger drop

$$V_2 = \frac{1}{4\pi\varepsilon_0}\frac{64q}{R}$$

$$V_2 = \frac{1}{4\pi\varepsilon_0}\frac{64q}{4r}$$

$$= \frac{9\times10^9 \times 64 \times 10^{-9}}{4\times2\times10^{-2}}$$

$$= 7.2\times10^3 \text{ V}$$

11. $p_x = 2\cos t$, $p_y = 2\sin t$,

$\therefore \quad \mathbf{p} = p_x\,\mathbf{i} + p_y\,\mathbf{j}$

$= 2\cos t\,\mathbf{i} + 2\sin t\,\mathbf{j}$

Force = rate of change of momentum

$$= \frac{d\,\mathbf{p}}{dt}$$

$\Rightarrow \quad \mathbf{F} = -2\sin t\,\mathbf{i} + 2\cos t\,\mathbf{j}$

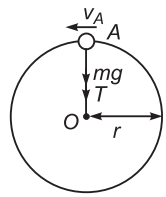
$$\therefore \cos\theta = \frac{\mathbf{F}\cdot\mathbf{p}}{|\mathbf{F}|\cdot|\mathbf{p}|}$$

$$= \frac{(-2\sin t\,\mathbf{i} + 2\cos t\,\mathbf{j})\,(2\cos t\,\mathbf{i} + 2\sin t\,\mathbf{j})}{\sqrt{4\sin^2 t + 4\cos^2 t}\sqrt{4\cos^2 t + 4\sin^2 t}}$$

$\Rightarrow \quad \cos\theta = 0$

$\Rightarrow \quad \theta = 90°$

12. When a body tied to the end of a string is revolved in a vertical circle, the speed of the body is different at different points of the circular paths.



The resultant of tension (T) and weight (mg) provides the necessary centripetal force

i.e., $T + mg = \frac{mv_A^2}{r}$

Tension (T) becomes zero at the points of minimum velocity, therefore

$\therefore \quad 0 + mg = \frac{mv_A^2}{r}$

$\Rightarrow \quad v_A = \sqrt{gr}$

13. Drop is stationary, *i.e.*,

weight of drop = electric force

i.e., $mg = qE$

or $mg = q\frac{V}{d}$

or $mg = ne\frac{V}{d}$

$\therefore \quad n = \frac{mgd}{eV}$

$$= \frac{1.8\times10^{-13}\times10\times0.9\times10^{-2}}{1.6\times10^{-19}\times2000}$$

$n = 50$

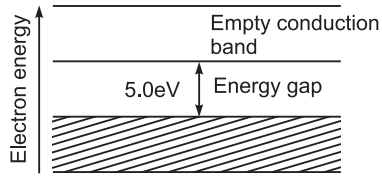
14. Many nuclei are radioactive. This means that they are unstable like radium and will eventually decay by emitting as particle transforming the nucleus into another nucleus. A chain of decays take place until a stable nucleus is reached. In the compound radium bromide, radium is radioactive while bromide is not, hence decay constant of compound is same as that of radium that is λ.

15. Remaing amount $N = N_0\left(\frac{1}{2}\right)^n$

$$N = 6.4\times10^{10}\left(\frac{1}{2}\right)^{12/4}$$

$$= 0.8\times10^{10} \text{ atoms}$$

16. An insulator has its valence band filled with electrons and its conduction band is empty. The energy gap is large approximately 5.0 eV, so it is not possible for ordinary process to provide enough energy for valence electrons to break out of their valence band and jump into the conduction band where they might carry electric current.



17. According to Fleming's right hand rule, magnetic field will be perpendicular to the plane of paper and act downward.

18. The force exerted on electron moving with velocity v is

$$F = evB \sin\theta$$

where $\theta = 90°$

$$\therefore \quad F_B = evB \qquad \ldots(i)$$

Centripetal force is given by

$$F_C = \frac{mv^2}{r} \qquad \ldots(ii)$$

Equating Eqs. (i) and (ii), we get

$$r = \frac{mv^2}{eB}$$

Diameter $d = 2r = \dfrac{2v}{(e/m)B}$

$$\Rightarrow \quad d = \frac{2 \times 2 \times 10^7}{1.76 \times 10^{11} \times 2 \times 10^{-2}}$$

$$= 1.1 \text{ cm}$$

19. Given, $V = 100 \sin 100t$

$$i = 100 \sin\left(100t + \frac{\pi}{3}\right)$$

Comparing with standard forms

$$V = V_0 \sin\omega t$$

$$I = I_0 \sin(\omega t + \phi)$$

$$\therefore \quad V_0 = 100 \text{ V}$$

$$I_0 = 100 \text{ mA} = 100 \times 10^{-3} \text{ A}$$

$$\phi = \frac{\pi}{3}$$

Power $P = \dfrac{V_0 I_0}{2} \cos\phi$

$$\therefore \quad P = \frac{100 \times 100 \times 10^{-3}}{2} \cos\frac{\pi}{3} = 2.5 \text{ W}$$

20. Escape velocity $v_e = \sqrt{\dfrac{2GM}{R}}$

$$= R\sqrt{\frac{8}{3}G\rho}$$

$$\therefore \quad v \propto R$$

We can say, if radius of a planet is double than that of earth then escape velocity from this planet will be 22 km/s.

21. $L = mvr = m\sqrt{\dfrac{GM}{r}}\, r = m\sqrt{GMr}$

$$\therefore \quad L \propto \sqrt{r}$$

$$\Rightarrow \quad \frac{L_2}{L_1} = \sqrt{\frac{r_2}{r_1}}$$

$$= \sqrt{\frac{16r}{r}} = 4$$

$$\Rightarrow \quad L_2 = 4L$$

22. Acceleration $a = \omega^2 y$

$$a = \left(\frac{2\pi}{T}\right)^2 y$$

$$\Rightarrow \quad T^2 = \frac{4\pi^2 y}{a}$$

$$\Rightarrow \quad T = 2\pi\sqrt{\frac{y}{a}}$$

$$= 2\pi\sqrt{\frac{3}{12}}$$

$$T = 2\pi \times \frac{1}{2} = 3.14 \text{ s}$$

23. $r = \sqrt{r_1^2 + r_2^2} = \sqrt{9 + 16} = \sqrt{25} = 5$ cm.

24. $\dfrac{\text{Volume of cube submerged}}{\text{Total volume}}$

$$= \frac{\text{density of cube material}}{\text{density of water}}$$

$$\Rightarrow \quad \frac{10 - 4}{10} = \frac{d}{1}$$

$$\therefore \quad d = \frac{6}{10} = 0.6 \text{ gcm}^{-3}$$

25. Temperature gradient of the rod

$$\frac{\Delta\theta}{\Delta x} = \frac{\theta_2 - \theta_1}{l}$$

$$\therefore \quad \frac{\Delta\theta}{\Delta x} = \frac{125-25}{50}$$

$$= \frac{100}{50} = 2^\circ \text{C/cm}$$

26. $Q = nC_p \Delta T$

For hydrogen

$$C_p = \frac{7}{2}R$$

So, $Q = n\frac{7}{2}R\ \Delta T$

$$= 5 \times \frac{7}{2} \times 2 \times 30$$

$$= 1050 \text{ cal}$$

27. Apparent rise $= d\left(1 - \frac{1}{{}_a\mu_w}\right)$

$$= 12 \times \left(1 - \frac{3}{4}\right)$$

$$= 12 \times \frac{1}{4} = 3 \text{ cm}$$

28. According to Einstein's equation relation between rest energy and moving energy is

$$E = E_0\left(1 - \frac{v^2}{c^2}\right)^{-1/2}$$

Given, $v = 0.5\,c$

$$\therefore \quad E = E_0\left(1 - \frac{(0.5\,c)^2}{c^2}\right)^{-1/2}$$

$$E = E_0(1 - 0.25)^{-1/2}$$

$$= \frac{E_0}{\sqrt{0.75}}$$

Change in energy $\Delta E = E - E_0$

$$= \frac{E_0}{\sqrt{0.75}} - E_0$$

$$= E_0\left(\frac{1}{\sqrt{0.75}} - 1\right)$$

$$= 0.511\left(\frac{1}{\sqrt{0.75}} - 1\right)$$

$$= 0.079 \text{ MeV}$$

29. Force on charge, $F = QE$

or $\quad F = \frac{QV}{d} \quad (\because V = Ed)$

For drop to be stationary,

weight of drop = force due to charge

i.e., $\quad mg = \frac{QV}{d}$

For two drops, $\frac{Q_1}{Q_2} \cdot \frac{V_1}{V_2} = \frac{m_1}{m_2}$

or $\quad \frac{Q_1}{Q_2} \cdot \frac{V_1}{V_2} = \frac{\frac{4}{3}\pi r_1^3 \rho}{\frac{4}{3}\pi r_2^3 \rho}$

or $\quad \frac{Q_2}{Q_1} = \frac{r_2^3}{r_1^3} \times \frac{V_1}{V_2}$

$\therefore \quad \frac{Q_2}{Q} = \frac{(r/2)^3}{r^3} \times \frac{2400}{600}$

or $\quad Q_2 = \frac{Q}{2}$

30. Band spectrum is defined as a molecular spectrum consisting of numerous closely spaced, often unresolved emission lines or absorption lines which occur across a limited range of frequencies. Each line represents an increment of energy due to a change in the rotational state of the molecule. Bands caused by titanium oxide, zirconium and carbon compounds are characteristic of low temperature stars.

31. Decrease in mass number = 238 – 234 = 4

No. of α-particles emitted $= \frac{4}{4} = 1$

Decrease in atomic number due to emission of 1 α-particle is 2, but now atomic number is 91, hence number of β-particles = 91 + 2 – 92 = 1.

32. $E = 57 \text{ meV} = 57 \times 10^{-3} \text{ eV}$

$$\therefore \quad \lambda = \frac{12375}{E} = \frac{12375}{57 \times 10^{-3}}$$

$$= 217.105 \times 10^3 \text{ Å}$$

$$\lambda = 217105 \text{ Å}$$

33. From law of conservation of linear momentum *i.e.,*

final momentum = initial momentum

$$m_1 v_1 + m_2 v_2 = 0 \qquad \ldots(i)$$

where m_1 and m_2 are masses of pieces.

(initial momentum is zero because body is stationary)

From Eq. (i), $m_1 v_1 = -m_2 v_2$

So, numerical value of momentum of both pieces will be equal.

34. $dp = F \times dt$

$$= 10 \times 10 = 100 \text{ kg-m/s}$$

36. The tension at the highest point is zero, but tension at the lowest point is 6 times the weight of the body. Hence, wire is most likely to break at the lowest point.

37. The time taken by AC in reaching from zero to maximum value is

$$t = \frac{1}{4} = \frac{1}{4f}$$

$$= \frac{1}{50 \times 4} = 5 \times 10^{-3} \text{ s}$$

38. $dQ = nCdT$

$$dQ = nk \frac{T^3}{\theta^3} dT$$

$$Q = \frac{nk}{\theta^3} \int_{T_1}^{T_2} T^3 dT$$

$$= \frac{nK}{\theta^3} \left(\frac{T_2^4 - T_1^4}{4} \right)$$

$$= \frac{2 \times 1940 \times (50^4 - 10^4)}{(281)^3 \times 4}$$

$$= 272.79 \text{ J} = 273 \text{ J}$$

39. Velocity of electron in nth orbit

$$v_n = \frac{Ze^2}{2\varepsilon_0 nh}$$

$$\Rightarrow \quad v \propto \frac{1}{n}$$

$$v_1 = v, n_1 = 2, n_2 = 5$$

$$\therefore \quad \frac{v}{v_2} = \frac{n_2}{n_1}$$

$$\therefore \quad \frac{v}{v_2} = \frac{5}{2}$$

$$\Rightarrow \quad v_2 = \frac{2v}{5}$$

40. If charged particle enters in magnetic field making an angle less than 90°, then its path will be a helix.

41. $C_1 = 10\,\mu\text{F}, V = 50$ V

Charge on first capacitor

$$q = C_1 V$$

$$= 10 \times 50 = 500\,\mu\text{C}$$

Common potential $V = \dfrac{\text{total charge}}{\text{total capacitance}}$

$$\therefore \quad 20 = \frac{500}{C_1 + C_2}$$

or $$20 = \frac{500}{10 + C_2}$$

or $$C_2 = 15\,\mu\text{F}$$

42. Given $E = 200\sqrt{2} \sin(100t)$ ...(i)

Comparing Eq. (i) with

$$E = E_0 \sin \omega t$$

Peak voltage $E_0 = 200\sqrt{2}, \omega = 100$

$$C = 1\mu\text{F} = 1 \times 10^{-6} \text{ F}$$

Reading of ammeter, $i = \dfrac{E_{\text{rms}}}{X_C}$

$$= \frac{E_0 \omega_C}{\sqrt{2}} = \frac{E_{\text{rms}}}{1/\omega C} = \frac{E_0 \omega C}{\sqrt{2}}$$

$$= \frac{200\sqrt{2} \times 100 \times 1 \times 10^{-6}}{\sqrt{2}}$$

$$= 20 \times 10^{-3} \text{ A}$$

$$= 20 \text{ mA}$$

43. Transformer works only in alternating current. Primary coil is connected to Lechlanche cell *i. e.,* connected by DC source, so voltage across secondary will be zero.

44. Resistance of galvanometer $G = 99\ \Omega$

Resistance of shunt $S = \dfrac{i_g G}{i - i_g}$

$$= \frac{10 \times 99}{100 - 10} = 11\ \Omega$$

45. Given $m = 500$ kg, $g = 9.8$ m/s^2

AND $R_e = 6.4 \times 10^6$ m

Energy required to scape

$$E = mg\,R_e$$
$$= 500 \times 9.8 \times 6.4 \times 10^6$$
$$= 3.1 \times 10^{10} \text{ J}$$

46. Direction of current is opposite to the direction of flow of electron and in the same direction of positive ion.

Total current $I = I_{Ne} + I_{electron}$

$$= (2.9 \times 10^{18} + 1.2 \times 10^{18})\, e$$
$$= 4.1 \times 10^{18} \times 1.6 \times 10^{-19}$$
$$= 0.66 \text{ A}$$

Direction of current will be the direction of flow of Ne^+ ion *i. e.* towards right.

47. Power $= \dfrac{\text{work}}{\text{time}}$

$$= \frac{mgh}{t} = \left(\frac{m}{t}\right) gh$$

Given, $\dfrac{m}{t} = 100$ kg/s, $h = 100$ m, $g = 10$ m/s^2

$\therefore$ Power $= 100 \times 10 \times 100$

$= 100$ kW

48. Velocity of free end

$$v = \sqrt{3gh}$$
$$= \sqrt{3 \times 10 \times 1}$$
$$= 5.47 \text{ m/s}$$

49. Forbidden energy gap for germanium crystal

$$= 0.76 \text{ eV}$$
$$= 0.76 \times 1.6 \times 10^{-19} \text{ J}$$
$$= 1.216 \times 10^{-19} \text{ J}$$

50. $V = 200$ V, $\dfrac{e}{m} = 1.6 \times 10^{11}$ C /kg

Kinetic energy of electron $= eV$

i.e, $\dfrac{1}{2} mv^2 = eV$

or $v^2 = \dfrac{2e}{m} V$

or $v = \sqrt{2\dfrac{e}{m} V}$

$$= \sqrt{2 \times 1.6 \times 10^{11} \times 200}$$
$$= 8 \times 10^6 \text{ m/s}$$

51. From definition of temperature resistance coefficient

$$R_t = R_0\,(1 + \alpha t)$$
$$3R_0 = R_0\,(1 + 4 \times 10^{-3}\, t)$$
$$3 = 1 + 4 \times 10^{-3}\, t$$
$$4 \times 10^{-3}\, t = 2$$
$$t = \frac{2}{4 \times 10^{-3}}$$
$$= 500^\circ \text{C}$$

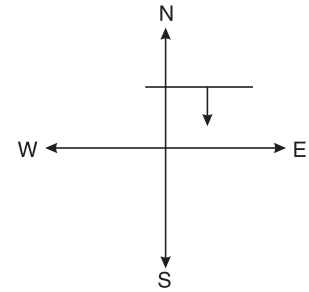
52. Induced emf $e = nA \dfrac{dB}{dt}$

$A =$ area of coil $= 2m^2$

and $n = 1$

$$e = \frac{1 \times 2 \times (4-1)}{2} = 3 \text{ V}$$

53. Given, $l = 10$ m, $v = 5$ m/s



and $H = 0.3 \times 10^{-4}$ Wb/m^2

Induced emf $e = Hvl$

$$= 0.3 \times 10^{-4} \times 5 \times 10$$
$$= 1.5 \times 10^{-3} \text{ V}$$
$$= 1.5 \text{ m V}$$

54. $i = 1.6$ A, $t = 1$ minute $= 60$ s

Charge on one $Cu^{++} = 2e$

Let $n\,Cu^{++}$ are liberated from

$$Q = ne$$

$$it = n(2e)$$

$$1.6 \times 60 = n(2 \times 1.6 \times 10^{-19})$$

$$n = 3 \times 10^{20} \text{ ions}$$

55. Given, $L = 60$ H

$$R = 30\ \Omega$$

$$V = 100 \text{ V}$$

In L-R circuit current to reach 63.2% of its final value in time constant.

Equation of current in L-R circuit

$$I = I_0(1 - e^{-Rt/L})$$

Required time = time constant

$$= \frac{L}{R} = \frac{60}{30} = 2 \text{ s}$$

56. $T_2 = \dfrac{(m_1 + m_2)T_3}{m_1 + m_2 + m_3}$

$$= \frac{(10 + 6) \times 40}{10 + 6 + 4}$$

$$= \frac{16 \times 40}{20}$$

$$= 32 \text{ N}$$

57. Maximum KE of the system

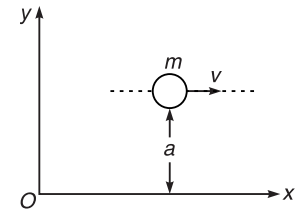
= Maximum PE of the system

$$= \frac{1}{2}kx^2$$

$$= \frac{1}{2} \times 16 \times (5 \times 10^{-2})^2$$

$$= 2 \times 10^{-2} \text{ J}$$

58. Angular momentum of particle w.r.t. origin



= linear momentum × perpendicular distance of line of action of linear momentum from origin

$= mv \times a = mva$ = constant

59. Isobars are the elements which have same mass number. So, ${}_{15}P^{30}$ and ${}_{14}Si^{30}$ are isobars.

60. Let initial amount be N_0.

Remaining amount $N = N_0 - \dfrac{7N_0}{8} = \dfrac{N_0}{8}$

$$\therefore \quad N = N_0\left(\frac{1}{2}\right)^n$$

$$\Rightarrow \quad \frac{N_0}{8} = N_0\left(\frac{1}{2}\right)^n$$

$$\Rightarrow \quad \left(\frac{1}{2}\right)^3 = \left(\frac{1}{2}\right)^n$$

$$\Rightarrow \quad n = 3$$

$$\therefore \quad \frac{t}{T_{1/2}} = 3$$

$$\Rightarrow \quad \frac{t}{15} = 3 \quad \text{or} \quad t = 15 \times 3$$

$$t = 45 \text{ h}$$

Chemistry

1. 36 g of water $= \dfrac{36}{18} = 2$ mol

28 g of $CO_2 = \dfrac{28}{44} = 0.64$ mol

46 g of $CH_3OH = \dfrac{46}{32} = 1.44$ mol

58 g of $N_2O_5 = \dfrac{58}{108} = 0.54$ mol

2. $SO_2 + H_2O \longrightarrow 3S + 2H_2O$

Eq. wt. of SO_2 in the reaction

$$= \frac{\text{molar mass of } SO_2}{\text{change in oxidation state}} = \frac{64}{4} = 16$$

Twice of 16 will be 32.

3. An atomic orbital is a three dimensional region of definite shape around the nucleus where the probability of finding the electron is maximum. Therefore, an atom has both characteristic energy and a characteristic shape.

4. For n, $l = 0$ to $(n - 1)$

Therefore, when $n = 3$, $l \neq 3$

5. Configuration of an element ${}_9M = 1s^2, 2s^2, 2p^5$. Hence, its ion will be M^- because it can easily gain one electron to have octet.

6. In S_8 molecules, each sulphur is sp^3 hybridised with two lone pairs of electrons occupying two of the hybrid orbitals and reducing the S—S—S bond angle to 105°.

7. In oxyacids of chlorine, HClO, $HClO_2$, $HClO_3$ and $HClO_4$, the central Cl atom is sp^3 hybridised. The half filled unhybridised *d*-orbitals of Cl and *p*-orbitals of oxygen overlap to form π-bonds.

8. Anhydrous $AlCl_3$ is highly hygroscopic and hence gives fumes of hydrogen chloride by the hydrolysis of moist air.

$$\underset{\text{moist air}}{AlCl_3 + 3H_2O} \longrightarrow Al(OH)_3 + 3HCl$$

9. Fe_2O_3 is amphoteric in nature as it reacts with both acid and alkali.

$$\underset{\text{acid}}{Fe_2O_3 + 6HCl} \longrightarrow 2FeCl_3 + 3H_2O$$

$$\underset{\text{alkali}}{Fe_2O_3 + Na_2CO_3} \longrightarrow 2NaFeO_2 + CO_2$$

10. $P_4 + 20HNO_3 \longrightarrow 4H_3PO_4 + 20NO_2 + 4H_2O$

11. $\overline{C}H_3$ is the best nucleophile because carbon has the least electronegativity among key atoms of the given options.

12. Four isomeric primary amines are obtained from $C_4H_{11}N$ are possible.

(i) $CH_3CH_2CH_2NH_2$

(ii) $CH_3-\underset{\displaystyle CH_3}{\underset{|}{CH}}-CH_2-NH_2$

(iii) $CH_3-\underset{\displaystyle CH_3}{\overset{\displaystyle CH_3}{\overset{|}{\underset{|}{C}}}}-NH_2$

(iv) $\overset{-}{C}H_3-\overset{-}{C}H_2-\underset{\displaystyle NH_2}{\underset{|}{CH}}-CH_3$

13. Reaction of alkyne with HBr supports it addition nature.

14. $CH_3-CH_2-\underset{\displaystyle CH_3}{\underset{|}{N}}-\overset{\displaystyle O}{\overset{\|}{C}}-H$

N-ethyl-N-methyl methanamide.

15. $C_6H_5COCHBr_2 \xrightarrow{OH^-} C_6H_5-\overset{\displaystyle O}{\overset{\|}{C}}-\underset{\displaystyle OH}{\overset{\displaystyle OH}{\overset{\curvearrowleft}{\underset{\curvearrowright}{CH}}}} \xrightarrow[-H_2O]{} C_6H_5-\overset{\displaystyle O}{\overset{\|}{C}}-CHO$

16. During catalytic hydrogenation, the hydrogens are transferred from the catalyst to the same side of the double bond. Evidently, smaller the number of *R* substituents, lesser is the steric hinderance and hence faster is the rate of hydrogenation. Thus, option (a) with two *R* groups on the same side of the molecule is correct.

17. Pentacetyl derivative can undergo basic hydrolysis.

18. Cellulose is a polysaccharide, composed of D-glucose units which are joined by β-glycosidic linkages. On complete hydrolysis cellulose produces D-glucose.

$$(C_6H_{10}O_5)_n + nH_2O \xrightarrow{H^+} \underset{\text{D-glucose}}{nC_6H_{12}O_6}$$

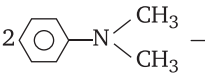
19. Saponification value is the number of mg of KOH required to neutralise the fatty acid resulting from the complete hydrolysis of 1 g of oil or fat.

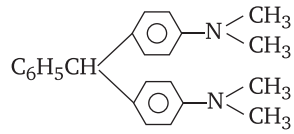
20. $CH_3CH_2NH_2 \xrightarrow{NaNO_2 + HCl} CH_3CH_2OH + N_2 + H_2O + NaCl$

$NH_2CONH_2 \xrightarrow{NaNO_2 + HCl} 2N_2 + H_2O + CO_2 + NaCl$

$CH_3CONH_2 \xrightarrow{NaNO_2 + HCl} CH_3COOH + N_2 + H_2O + NaCl$

$C_6H_5NH_2 \xrightarrow{NaNO_2 + HCl} C_6H_5N_2^+Cl^- + H_2O + NaCl$

21. $C_6H_5CHO + 2$  $\xrightarrow[-H_2O]{\text{Anhy. } ZnCl_2}$



malachite green

22. When a primary amine reacts with chloroform in presence of alco. KOH, it gives *iso*-cyanide which has abnoxious odour. This reaction is known as carbylamine reaction and it is given only by primary amines.

$$\underset{\text{primary amine}}{RHN_2} + CHCl_3 + 3KOH \xrightarrow[\text{alco.}]{\Delta} RNC + 3KCl + 3H_2O$$

23. $CH_3COOH \xrightarrow{NH_3} CH_3COONH_4 \xrightarrow[-H_2O]{\Delta}$

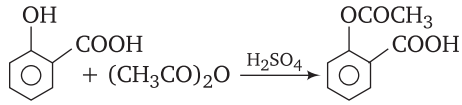
$\underset{\text{acetic acid}}{CH_3COOH}$ $\underset{\text{acetamide}}{CH_3CO\ NH_2}$

The isomers of CH_3CONH_2 are

$\underset{\text{(I)}}{NH_2CH_2CHO}$, $\underset{\text{(II)}}{CH_3—CH=NOH}$,

$\underset{\text{(III)}}{H—CONH—CH_3}$

24. Salicylic acid gives aspirin on reaction with acetic anhydride in presence of H_2SO_4.



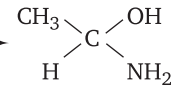
salicylic acid aspirin

25. Cl^- is the best leaving group being the weakest nucleophile out of $\overline{NH_2}$, Cl^-, $O^-C_2H_5$ and CH_3COO^-.

26. Acetaldehyde does not show cannizzaro's reaction due to the presence of α-atom.

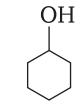
27. HCHO and CH_3CHO give different reactions with NH_3.

$\underset{\text{formaldehyde}}{6HCHO} + 4NH_3 \longrightarrow \underset{\text{Urotropine}}{(CH_2)_6N_4} + 6H_2O$

$\underset{\text{acetaldehyde}}{CH_3CHO} + NH_3 \longrightarrow$ 

acetaldehyde ammonia

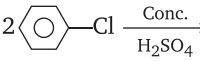
28. Solubility of alcohols in water decreases with increase in molecular masses.

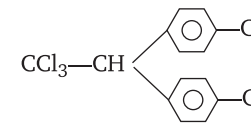
29. 

cyclohexanol
2° alcohol

30. $C_{12}H_{22}O_{11} + H_2O \xrightarrow{\text{Invertase}} \underset{\text{glucose}}{C_6H_{12}O_6} + \underset{\text{fructose}}{C_6H_{12}O_6}$

$C_6H_{12}O_6 \xrightarrow{\text{Zymase}} 2C_2H_5OH + 2CO_2$

31. $\underset{\text{Chloral}}{CCl_3CHO} + 2$  $\xrightarrow[H_2SO_4]{\text{Conc.}}$



DDT

32. Vinyl chloride is least reactive towards hydrolysis because of non-reactivity of chlorine atom due to resonance stabilization.

33. Wurtz reaction

$$CH_3I + 2Na + IC_2H_5 \xrightarrow[-2NaI]{\text{Dry ether}} C_2H_6 + C_3H_8 + C_4H_{10}$$

34. Cyanide process is used in the extraction of silver and gold.

35. SnO_2—Cassiterite, Fe_3O_4—Magnetite

Fe_2O_3—Haematite, $Fe_2O_3 \cdot 3H_2O$—Limonite

36. $Ti + 2I_2 \xrightarrow{500\ K} \underset{\substack{\text{volatile stable}\\ \text{compound}}}{TiI_4} \xrightarrow{1700\ K}$

$\underset{\text{pure metal}}{Ti} + 2I_2$

37. Cerium is the first element of rare earth metals.

38. $M^{2+} \longrightarrow M^{3+}$, after the removal of $2e^-$, the nuclear charge per electron increases, due to which high energy is required to remove $3e^-$.

39. Transition metal which have low oxidation number show the oxidising nature because of great tendency to lose the electron.

40. In $CoCl_3$, Co is in + 3 state

$Co^{3+} = 1s^2, 2s^2\ 2p^6, 3s^2\ 3p^6\ 3d^6, 4s^\circ$

4 unpaired electrons, so it will be coloured.

41. Spiegeleisn is an alloy of Fe, C and Mn.

42. The magnitude of stability constants for some divalent metal ions of the first transition series with oxygen or nitrogen donor ligands increases in the order

$Mn^{2+} < Fe^{2+} < Co^{2+} < Ni^{2+} < Cu^{2+} < Zn^{2+}$

43. Coordination number is equal to total number of ligands in a complex.

44. In $[Ni(CO)_4]$, Ni has zero oxidation number.

45. Rusting of iron is catalysed by H^+ ions.

46. At 298 K, standard electrode potential of NHE is 0.00 V.

47. $\Delta G^\circ = -nFE^\circ$

$$\Delta G^\circ = -2.303\,RT\,\log K_c$$

$$nFE^\circ = 2.303\,RT\,\log K_c$$

$$\log K_c = \frac{nFE^\circ}{2.303RT} = \frac{2 \times 96500 \times 0.295}{2.303 \times 8.314 \times 298}$$

$$\log K_c = 9.97$$

$$K_c = 1 \times 10^{10}$$

48. Equivalence conductance = conducting power of all the ion produced by one gram equivalent of an electrolyte.

Since, Eq. wt. of $BaCl_2 = \frac{\text{mol. wt.}}{2}$ then

$$\lambda^\infty_{BaCl_2} = \frac{1}{2}\lambda^\infty\ Ba^{2+} + \lambda^\infty Cl^-$$

$$= \frac{127}{2} + 76$$

$$= 139.5\ \text{ohm}^{-1}\text{cm}^2$$

49. In multimolecular colloids, colloidal size particles or molecules are built by the aggregation of ions or smaller particles. *eg*. gold sol.

50. Physical adsorption involves physical forces *i.e*., van der Waals' forces.

51. $C = \frac{5}{342} \times \frac{1}{100} \times 1000 = \frac{50}{342}\ \text{mol L}^{-1}$

$$\pi = \frac{50}{342} \times 0.082 \times 423$$

$$= 5.07\ \text{atm}$$

52. Mole fraction of $H_2O = \frac{80/18}{\frac{80}{18} + \frac{20}{34}} = \frac{34}{77}$

53. When pH = 3 then $[H^+] = 10^{-3}$ M. After that we increased the pH from 3 to 6 then $[H^+] = 10^{-6}$ M means reduced 1000 times.

54. $BaCl_2 \rightleftharpoons \underset{s}{\overset{2+}{Ba}} + \underset{2s}{2Cl^-}$ $K_{sp} = 4s^3$,

$$s^3 = \frac{4 \times 10^{-9}}{4} = 10^{-3}$$

$$s = 10^{-3}\ \text{M}$$

55. $AlCl_3$ in a lewis acid because it is a electron pair acceptor, its central atom have a vacant *d*-orbital.

56. Radioactive decay is a first order reaction.

57. $t = \frac{2.303}{k}\log\frac{[A]_0}{[A]}$

$$= \frac{2.303}{60} = \log\frac{a}{\frac{a}{16}} = \frac{2.303}{60}\log 16$$

$$= \frac{2.303}{60} \times 1.204$$

$$= 0.0462\ \text{s} = 4.6 \times 10^{-2}\ \text{s}$$

58. $C_{(s)} + \frac{1}{2}O_{2(g)} \longrightarrow CO_{(s)}$ $(\Delta H - \Delta U) = -\Delta nRT$

$$= -\frac{1}{2} \times 8.314 \times 298 = -1238.78$$

59. At constant temperature and pressure, internal energy of ideal gas remains unaffected.

60. Let RMS speed of nitrogen at T K be u and is equal to that of CO_2 at STP

$$u = \sqrt{\frac{3RT}{28}} = \sqrt{\frac{3R \times 273}{44}}$$

$$T = \frac{273 \times 28}{44} = 173.73\ \text{K} = -99.27^\circ\text{C}$$

Mathematics

1. Let $\Delta = \begin{vmatrix} a^2 & b^2 & c^2 \\ (a+1)^2 & (b+1)^2 & (c+1)^2 \\ (a-1)^2 & (b-1)^2 & (c-1)^2 \end{vmatrix}$

Applying $R_2 \to R_2 - R_3$

$$= 4\begin{vmatrix} a^2 & b^2 & c^2 \\ a & b & c \\ (a-1)^2 & (b-1)^2 & (c-1)^2 \end{vmatrix}$$

Applying $R_3 \to R_3 - (R_1 - 2R_2)$

$$= 4\begin{vmatrix} a^2 & b^2 & c^2 \\ a & b & c \\ 1 & 1 & 1 \end{vmatrix}$$

2. Given system has infinite solution.

3. As $\det(A) = \pm 1$, A^{-1} exists and

$$A^{-1} = \frac{1}{\det(A)}(\text{adj } A) = \pm \text{ adj } A$$

All entries in adj (A) are integers.

$\therefore A^{-1}$ has integer entries.

4. All statements are correct.

5. Let $I = \int \frac{dx}{x\sqrt{1-(\log x)^2}}$

Put $\log x = t$

$\Rightarrow \frac{1}{x}dx = dt$

$\therefore I = \int \frac{dt}{\sqrt{1-t^2}} = \sin^{-1} t + C$

$= \sin^{-1}(\log x) + C$

6. $\int \frac{x^2}{(x^2+2)(x^2+3)}dx = \int \left(\frac{3}{x^2+3} - \frac{2}{x^2+2}\right)dx$

$= \frac{3}{\sqrt{3}}\tan^{-1}\left(\frac{x}{\sqrt{3}}\right) - \frac{2}{\sqrt{2}}\tan^{-1}\left(\frac{x}{\sqrt{2}}\right) + C$

$= \sqrt{3}\tan^{-1}\left(\frac{x}{\sqrt{3}}\right) - \sqrt{2}\tan^{-1}\left(\frac{x}{\sqrt{2}}\right) + C$

7. Let the equation of parabola whose axis is x- axis, is

$$y^2 = \pm 4a(x-h)$$

On differentiating w.r.t. x, we get

$$2yy_1 = \pm 4a$$

$\Rightarrow \quad yy_1 = \pm 2a$

Again differentiating, we get

$$(y_1)^2 + yy_2 = 0$$

Hence, order and degree are respectively 1 and 2.

8. Given that, $y = ax\cos\left(\frac{1}{x} + b\right)$

On differentiating w.r.t. x, we get

$$y_1 = a\left[\cos\left(\frac{1}{x}+b\right) - x\sin\left(\frac{1}{x}+b\right)\left(-\frac{1}{x^2}\right)\right]$$

$$\Rightarrow y_1 = a\left[\cos\left(\frac{1}{x}+b\right) + \frac{1}{x}\sin\left(\frac{1}{x}+b\right)\right]$$

Again differentiating, we get

$$y_2 = a\left[-\sin\left(\frac{1}{x}+b\right)\left(-\frac{1}{x^2}\right) - \frac{1}{x^2}\sin\left(\frac{1}{x}+b\right) - \frac{1}{x^3}\cos\left(\frac{1}{x}+b\right)\right]$$

$\Rightarrow \quad y_2 = -\frac{ax}{x^4}\cos\left(\frac{1}{x}+b\right)$

$\Rightarrow \quad x^4 y_2 + y = 0$

9. Given differential equation can be rewritten as

$$\frac{dy}{dx} + \frac{2x}{1+x^2}y = \frac{x^2-1}{x^2+1}$$

Here, $P = \frac{2x}{1+x^2}$ and $Q = \frac{x^2-1}{x^2+1}$

$\therefore \quad \text{IF} = \int e^{\frac{2x}{1+x^2}dx} = e^{\log(1+x^2)}$

$= 1 + x^2$

10. $\int_0^{\pi/2} |\sin x - \cos x|\, dx$

$= \int_0^{\pi/4} -(\sin x - \cos x)\, dx + \int_{\pi/4}^{\pi/2} -(\sin x - \cos x)\, dx$

$$= -[-\cos x - \sin x]_0^{\pi/4} + [-\cos x - \sin x]_{\pi/4}^{\pi/2}$$

$$= -\left[-\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}} + 1\right] + \left[-0 - 1 + \left(\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}\right)\right]$$

$$= \sqrt{2} - 1 - 1 + \sqrt{2}$$

$$= 2(\sqrt{2} - 1)$$

11. Let the centre of circle be (h, k). Then

$$\sqrt{(h-2)^2 + (k-3)^2} = \sqrt{(h-4)^2 + (k-5)^2} \quad \ldots(i)$$

and $k - 4h + 3 = 0$ ...(ii)

From Eq. (i)

$$-4h - 6k + 8h + 10k = 16 + 25 - 4 - 9$$

$$\Rightarrow \quad h + k - 7 = 0 \quad \ldots(iii)$$

On solving Eqs. (ii) and (iii), we get the centre of circle (2, 5).

Now, radius $= \sqrt{(2-2)^2 + (5-3)^2} = 2$

$\therefore$ Required equation of circle is

$$(x-2)^2 + (y-5)^2 = 2^2$$

$$\Rightarrow \quad x^2 + y^2 - 4x - 10y + 25 = 0$$

12. Given that, $\log_4 (x-1) = \log_2 (x-3)$

$$\therefore \quad x - 1 = (x-3)^2$$

$$\Rightarrow \quad x^2 - 7x + 10 = 0$$

$$\Rightarrow \quad (x-5)(x-2) = 0$$

$$\Rightarrow \quad x = 2, 5$$

$\Rightarrow$ $x = 5$, but $x = 2 \log_2 (x-3)$ is not satisfied.

13. Since, α and β are the roots of $x^2 - 3x + 1 = 0$, then

$$\alpha + \beta = 3 \text{ and } \alpha\beta = 1$$

Now, $$S = \frac{1}{\alpha - 2} + \frac{1}{\beta - 2}$$

$$= \frac{\alpha + \beta - 4}{\alpha\beta - 2(\alpha + \beta) + 4}$$

$$= \frac{3 - 4}{1 - 2 \cdot 3 + 4} = 1$$

and $$P = \frac{1}{\alpha - 2} \times \frac{1}{\beta - 2}$$

$$= \frac{1}{\alpha\beta - 2(\alpha + \beta - 2)}$$

$$= 1$$

$\therefore$ Required equation is

$$x^2 - Sx + P = 0$$

$$\Rightarrow \quad x^2 - x - 1 = 0$$

14. Let $y = \dfrac{x^2 + 34x - 71}{x^2 + 2x - 7}$

$$\Rightarrow \quad x^2(y-1) + 2(y-17)x + (71 - 7y) = 0$$

For real values of x, its discriminant, $D \geq 0$

$$\therefore \quad 4(y-17)^2 - 4(y-1)(71-7y) \geq 0$$

$$\Rightarrow \quad (y^2 - 34y + 289) - (71y - 7y^2 - 71 + 7y) \geq 0$$

$$\Rightarrow \quad y^2 - 14y + 45 \geq 0$$

$$\Rightarrow \quad (y-5)(y-9) \geq 0$$

$$\Rightarrow \quad y \leq 5 \text{ and } y \geq 9$$

Hence, y does not lie between 5 and 9

15. Now, $\sqrt{50} + \sqrt{48} = 5\sqrt{2} + 4\sqrt{3}$

$$= \sqrt{2}[5 + 2 \cdot \sqrt{2} \cdot \sqrt{3}]$$

$$= \sqrt{2}(\sqrt{3} + \sqrt{2})^2$$

$$\therefore \quad \sqrt{\sqrt{50} + \sqrt{48}} = 2^{1/4}(\sqrt{3} + \sqrt{2})$$

16. Since, $I_1 = \int_a^{\pi - a} x\, f(\sin x)\, dx$

$$= \int_a^{\pi - a} (\pi - x)\, f(\sin(\pi - x))\, dx$$

$$= \int_a^{\pi - a} (\pi - x)\, f(\sin x)\, dx$$

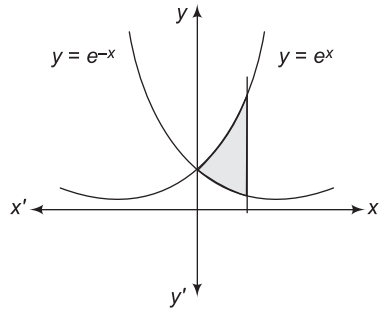
$$= \int_a^{\pi - a} \pi f(\sin x)\, dx - I_1$$

$$\Rightarrow \quad 2I_1 = \pi I_2$$

$$\Rightarrow \quad I_2 = \frac{2}{\pi} I_1$$

17. Required area $= \int_0^1 (e^x - e^{-x})\, dx$

$$= [e^x + e^{-x}]_0^1$$



$$= \left(e + \frac{1}{e} - 2\right) \text{ sq unit}$$

18. $f(x) = \sin^{-1}[x]$ is defined, if $-1 \le [x] \le 1$

$\Rightarrow \quad [x] = \{-1, 0, 1\}$

$\therefore \quad x \in [-1, 2)$

$\therefore$ Range is $\{\sin^{-1}(-1), \sin^{-1} 0, \sin^{-1} 1\}$

$$= \left\{-\frac{\pi}{2}, 0, \frac{\pi}{2}\right\}$$

19. Since, $x\,\mathbf{a} + y\mathbf{b} + z\mathbf{c} = 0$

Also, $\mathbf{a}$, $\mathbf{b}$ and $\mathbf{c}$ are non-collinear vectors, then $x = y = z = 0$

20. Let angle between $\mathbf{b}$ and $\mathbf{c}$ is α, then

$$|\mathbf{b} \times \mathbf{c}| = \sqrt{15} \text{ (given)}$$

$\Rightarrow \quad |\mathbf{b}||\mathbf{c}| \sin\alpha = \sqrt{15}$

$\Rightarrow \quad \sin\alpha = \frac{\sqrt{15}}{4}$

$\therefore \quad \cos\alpha = \frac{1}{4}$

Also, $\mathbf{b} - 2\mathbf{c} = \lambda\mathbf{a}$

$\Rightarrow \quad (\mathbf{b} - 2\mathbf{c})^2 = \lambda^2 (\mathbf{a})^2$

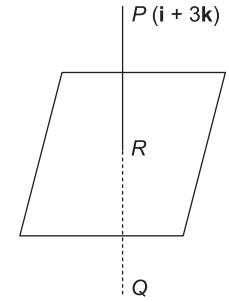
$\Rightarrow \quad |\mathbf{b}|^2 + 4|\mathbf{c}|^2 - 4\,\mathbf{b} \cdot \mathbf{c} = \lambda^2 |\mathbf{a}|^2$

$\Rightarrow \quad 16 + 4 - 4\{|\mathbf{b}||\mathbf{c}| \cos\alpha\} = \lambda^2$

$\Rightarrow \quad \lambda^2 = 16$

$\Rightarrow \quad \lambda = \pm 4$

21. Let Q be the image of the point $P(\mathbf{i} + 3\mathbf{k})$ in the plane $\mathbf{r} \cdot (\mathbf{i} + \mathbf{j} + \mathbf{k}) = 1$. Then, PQ is normal to the plane. Since, PQ passes through P and in normal to the given plane, therefore equation of PQ is $\mathbf{r} = (\mathbf{i} + 3\mathbf{k}) + \lambda(\mathbf{i} + \mathbf{j} + \mathbf{k})$



Since, Q lies on the line PQ, so let the position vector of Q be $(\mathbf{i} + 3\mathbf{k}) + \lambda(\mathbf{i} + \mathbf{j} + \mathbf{k})$ *i.e.*, $(1 + \lambda)\,\mathbf{i} + \lambda\,\mathbf{j} + (3 + \lambda)\,\mathbf{k}$.

Since, R is the mid-point of PQ, therefore position vector of R is

$$\frac{(1 + \lambda)\,\mathbf{i} + \lambda\,\mathbf{j} + (3 + \lambda)\,\mathbf{k} + \mathbf{i} + 3\mathbf{k}}{2}$$

$$= \left(\frac{\lambda + 2}{2}\right)\mathbf{i} + \frac{\lambda}{2}\,\mathbf{j} + \left(\frac{6 + \lambda}{2}\right)\mathbf{k}$$

$$= \left(\frac{\lambda}{2} + 1\right)\mathbf{i} + \frac{\lambda}{2}\,\mathbf{j} + \left(3 + \frac{\lambda}{2}\right)\mathbf{k}$$

Since, R lies on the plane

$\mathbf{r} \cdot (\mathbf{i} + \mathbf{j} + \mathbf{k}) = 1$

$\therefore \quad \left[\left(\frac{\lambda}{2} + 1\right)\mathbf{i} + \frac{\lambda}{2}\,\mathbf{j} + \left(3 + \frac{\lambda}{2}\right)\mathbf{k}\right] \cdot (\mathbf{i} + \mathbf{j} + \mathbf{k}) = 1$

$\Rightarrow \quad \left[\frac{\lambda}{2} + 1 + \frac{\lambda}{2} + 3 + \frac{\lambda}{2}\right] = 1$

$\Rightarrow \quad \lambda = -2$

Hence, the position vector of Q is $-\mathbf{i} - 2\,\mathbf{j} + \mathbf{k}$.

22. Since, $\mathbf{a}$, $\mathbf{b}$, $\mathbf{c}$ form a right handed system

$\therefore \quad \mathbf{c} = \mathbf{b} \times \mathbf{a}$

$= \mathbf{j} \times (x\,\mathbf{i} + y\,\mathbf{j} + z\,\mathbf{k})$

$= x\,(\mathbf{j} \times \mathbf{i}) + z(\mathbf{j} \times \mathbf{k})$

$= -x\,\mathbf{k} + z\,\mathbf{i} = z\,\mathbf{i} - x\,\mathbf{k}$

23. Since, plane is parallel to a given vector, it implies that normal of plane must perpendicular to the given vectors. Given point to which plane passes through is $(2, -1, 3)$. Let A, B and C are direction ratios of its normal.

$\therefore$ Equation of plane is

$$A(x-2)+B(y+1)+C(z-3)=0$$

Now, normal to plane $A\mathbf{i}+B\mathbf{j}+C\mathbf{k}$ is perpendicular to the given vectors

$$\mathbf{a}=3\mathbf{i}+0\mathbf{j}-\mathbf{k}$$

and $\mathbf{b}=-3\mathbf{i}+2\mathbf{j}+2\mathbf{k}$

$\therefore \quad 3A+0B-C=0 \quad ...(i)$

and $-3A+2B+2C=0 \quad ...(ii)$

From Eqs. (i) and (ii), we get

$$\frac{A}{2}=\frac{B}{-3}=\frac{C}{6}$$

$\therefore$ Equation of plane be

$$2(x-2)-3(y+1)+6(z-3)=0$$

i. e., $2x-3y+6z-25=0$

24. $\lim\limits_{x\to 1}\dfrac{(2x-3)(\sqrt{x}-1)}{2x^2+x-3}$

$$=\lim_{x\to 1}\frac{(2x-3)(\sqrt{x}-1)}{(2x+3)(x-1)}$$

$$=\lim_{x\to 1}\frac{(2x-3)}{(2x+3)(\sqrt{x}+1)}$$

$$=\frac{2-3}{(2+3)(\sqrt{1}+1)}=-\frac{1}{10}$$

25. Since, 10, 3, e, 2 are in decreasing order.

$\therefore$ $\log_{10}\alpha, \log_3\alpha, \log_e\alpha, \log_2\alpha$ are in increasing order.

26. Given that, $y=\dfrac{\sqrt{5}-\sqrt{2}}{\sqrt{5}+\sqrt{2}}$

and $x=\dfrac{\sqrt{5}+\sqrt{2}}{\sqrt{5}-\sqrt{2}}$

$\Rightarrow \quad y=\dfrac{1}{x}$

$\Rightarrow \quad xy=1$

$\therefore \quad 3x^2+4xy-3y^2=3(x-y)(x+y)+4xy$

$$=3\left(\frac{\sqrt{5}+\sqrt{2}}{\sqrt{5}-\sqrt{2}}-\frac{\sqrt{5}-\sqrt{2}}{\sqrt{5}+\sqrt{2}}\right)\times\left(\frac{\sqrt{5}+\sqrt{2}}{\sqrt{5}-\sqrt{2}}+\frac{\sqrt{5}-\sqrt{2}}{\sqrt{5}+\sqrt{2}}\right)+4$$

$$=3\left[\frac{(\sqrt{5}+\sqrt{2})^2-(\sqrt{5}-\sqrt{2})^2}{(5-2)(5-2)}\right][(\sqrt{5}+\sqrt{2})^2+(\sqrt{5}-\sqrt{2})^2]+4$$

$$=\frac{1}{3}\cdot 4\sqrt{10}\cdot 2(5+2)+4$$

$$=\frac{56}{3}\sqrt{10}+4$$

$$=\frac{1}{3}(56\sqrt{10}+12)$$

27. $\sum\limits_{r=0}^{n} r^2\,{}^nC_r\,x^r y^{n-r}$

$$=\sum_{r=0}^{n}[r(r-1)+r]\,{}^nC_r\,x^r y^{n-r}$$

$$=\sum_{r=0}^{n}r(r-1)\,{}^nC_r\,x^r y^{n-r}+\sum_{r=0}^{n}r\,{}^nC_r\,x^r y^{n-r}$$

$$=\sum_{r=2}^{n-2}r(r-1)\frac{n}{r}\cdot\frac{n-1}{r-1}\,{}^{n-2}C_{r-2}\,x^2\cdot x^{r-2}y^{n-r}+\sum_{r=1}^{n}r\cdot\frac{n}{r}\,{}^{n-1}C_{r-1}\,x\cdot x^{r-1}y^{n-r}$$

$$=n(n-1)x^2\sum_{r=2}^{n-2}{}^{n-2}C_{r-2}\,x^{r-2}y^{(n-2)-(r-2)}+nx\sum_{r=1}^{n}{}^{n-1}C_{r-1}\,x^{r-1}y^{(n-1)-(r-1)}$$

$$=n(n-1)x^2(x+y)^{n-2}+nx(x+y)^{n-1}$$

$$=n(n-1)x^2+nx \qquad (\because x+y=1)$$

$$=nx(nx-x+1)$$

$$=nx(nx+y) \qquad (\because x+y=1)$$

28. Given that,

$$Z=f(x+ay)+\phi(x-ay)$$

$$Z_x=f'(x+ay)+\phi(x-ay)$$

$\Rightarrow \quad Z_{xx}=f''(x+ay)+\phi''(x-ay)$

and $Z_y=af'(x+ay)-a\phi'(x-ay)$

$\Rightarrow \quad Z_{yy}=a^2f''(x+ay)-a^2\phi'(x-ay)$

Hence, $a^2Z_{xx}=Z_{yy}$

29. For unique solution of the given system, $\Delta \neq 0$

$$i.e., \quad \begin{vmatrix} 1 & 1 & 1 \\ 5 & -1 & \mu \\ 2 & 3 & -1 \end{vmatrix} \neq 0$$

Hence, it depends only on μ.

30. Given, $\begin{bmatrix} 4 & 2 & 2 \\ -5 & 0 & \alpha \\ 1 & -2 & 3 \end{bmatrix} = 10A^{-1}$

$$\therefore \quad \begin{bmatrix} 4 & 2 & 2 \\ -5 & 0 & \alpha \\ 1 & -2 & 3 \end{bmatrix} \begin{bmatrix} 1 & -1 & 1 \\ 2 & 1 & -3 \\ 1 & 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 10 & 0 & 0 \\ 0 & 10 & 0 \\ 0 & 0 & 10 \end{bmatrix}$$

$$\Rightarrow \quad \begin{bmatrix} 10 & 0 & 0 \\ -5+\alpha & 5+\alpha & -5+\alpha \\ 0 & 0 & 10 \end{bmatrix}$$

$$= \begin{bmatrix} 10 & 0 & 0 \\ 0 & 10 & 0 \\ 0 & 0 & 10 \end{bmatrix}$$

$$\Rightarrow \quad -5+\alpha = 0$$

$$\Rightarrow \quad \alpha = 5$$

31. Given equation of curves are

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \text{ and } x^2 + y^2 = ab$$

$$\therefore \quad \frac{ab - y^2}{a^2} + \frac{y^2}{b^2} = 1$$

$$\Rightarrow \quad y^2\left(\frac{a^2 - b^2}{a^2 b^2}\right) = \frac{a-b}{a}$$

$$\Rightarrow \quad y^2 = \frac{ab^2}{a+b}$$

and $\quad x^2 = \frac{a^2 b}{a+b}$

$$\Rightarrow \quad x = a\sqrt{\frac{b}{a+b}}$$

and $\quad y = b\sqrt{\frac{a}{a+b}}$

Slope of tangent at ellipse $= \frac{-b^2 x}{a^2 y}$

$$= -\frac{b^2}{a^2}\sqrt{\frac{a}{b}}$$

Slope of tangent at circle $= -\frac{x}{y} = -\sqrt{\frac{a}{b}}$

$$\therefore \quad \theta = \tan^{-1}\left[\frac{\sqrt{\frac{a}{b}} - \frac{b^2}{a^2}\sqrt{\frac{a}{b}}}{1 + \frac{b^2}{a^2}\cdot\frac{a}{b}}\right]$$

$$\Rightarrow \quad \theta = \tan^{-1}\left(\frac{a-b}{\sqrt{ab}}\right)$$

32. Given that,

$$S = \sum_{n=0}^{\infty} \frac{(\log x)^{2n}}{(2n)!}$$

$$= \left(\frac{e^{\log x} + e^{-\log x}}{2}\right) = \frac{x + x^{-1}}{2}$$

33. $\log 2 + 2\left(\frac{1}{5} + \frac{1}{3}\cdot\frac{1}{5^3} + \frac{1}{5}\cdot\frac{1}{5^5} + \ldots \infty\right)$

$$= \log 2 + \log\left(\frac{1 + \frac{1}{5}}{1 - \frac{1}{5}}\right)$$

$$= \log 2 + \log\left(\frac{3}{2}\right) = \log 3$$

34. Since, vertex is (0, 4) and focus is (0, 2).

$$\therefore \quad a = 2$$

$\therefore$ Equation of parabola is

$$(x-0)^2 = -4\cdot 2(y-4)$$

$$\Rightarrow \quad x^2 + 8y = 32$$

35. The equation of a line parallel to $x + 2y = 4$ is $x + 2y = k$. Since, the distance between these two lines is 3, therefore

$$\frac{k}{\sqrt{1+4}} - \frac{4}{\sqrt{1+4}} = 3$$

$$\Rightarrow \quad k = 4 + 3\sqrt{5}$$

This shifted line cuts x-axis at $(k, 0)$. After rotation the slope of the line is $\tan(\theta - 30^\circ)$, where $\tan\theta$ = (slope of

$$(x + 2y = 4) = -\frac{1}{2}$$

$\therefore$ The equation of the line in the new position is

$$y - 0 = \tan(\theta - 30^\circ)(x - k),$$

$\Rightarrow$ $y = \tan(\theta - 30^\circ)(x - k)$

where $k = 4 + 3\sqrt{5}$

36. According to given condition, $2ae = 2 \cdot 2a$

$\Rightarrow$ $e = 2$

and $2b = 6 \Rightarrow b = 3$

Hence, $a = \frac{3}{\sqrt{3}} = \sqrt{3}$

$\therefore$ Required equation is

$$\frac{x^2}{3} - \frac{y^2}{9} = 1$$

$\Rightarrow$ $3x^2 - y^2 = 9$

37. The second degree equation will represent a pair of perpendicular straight lines, if

$$\begin{vmatrix} a & h & g \\ h & b & f \\ g & f & c \end{vmatrix} = 0 \text{ and } a + b = 0$$

Given pair of line is

$$12x^2 + 7xy - py^2 - 18x + qy + 6 = 0$$

$\therefore$ $\begin{vmatrix} 12 & 7/2 & -9 \\ 7/2 & -p & q/2 \\ -9 & q/2 & 6 \end{vmatrix} = 0$

and $12 - p = 0$

$\Rightarrow$ $p = 12$

$\therefore$ $\begin{vmatrix} 12 & 7/2 & -9 \\ 7/2 & -12 & q/2 \\ -9 & q/2 & 6 \end{vmatrix} = 0$

$\Rightarrow$ $12\left(-72 + \frac{q^2}{4}\right) - \frac{7}{2}\left(-21 + \frac{9q}{2}\right) - 9\left(\frac{7q}{4} - 108\right) = 0$

$\Rightarrow$ $-864 - 3q^2 - \frac{147}{2} - \frac{63q}{4} - \frac{63q}{4} + 972 = 0 \Rightarrow q = 1$

38. Given equation can be rewritten as

$$x^2 - 2(\alpha + \beta)x + 3\alpha\beta = 0$$

Let roots be α' and $-\alpha'$.

$\therefore$ $\alpha' + (-\alpha') = 2(\alpha + \beta)$

$\Rightarrow$ $\alpha + \beta = 0$

39. Let $f(x) = ax^2 + bx + c$

Again, let $f(x) = \int f(x)\,dx$

$$= \frac{a}{3}x^3 + \frac{bx^2}{2} + cx$$

At $x = 0$,

$f(0) = 0$

Agin, at $x = 1$

$$f(1) = \frac{a}{3} + \frac{b}{2} + c$$

$$= \frac{2a + 3b + 6c}{6} = 0$$

Hence, one root lies between (0, 1).

40. Since, the lines $3x + 4y + 2 = 0$ and $3x + 4y + 5 = 0$ are on the same side of the origin. The distance between these lines is $5 - 2 = 3$.

And the line $3x + 4y + 2 = 0$ and $3x + 4y - 5 = 0$ are opposite side of the origin, The distance between these lies is 7.

Hence, required ratio is 3 : 7.

41. Given equation of line $y = x$ is written in polar form is

$$\frac{x}{\cos\theta} = \frac{y}{\sin\theta} = r, \text{ where } \theta = \frac{\pi}{4}$$

For point P, $r = 6\sqrt{2}$. Therefore, coordinates of P are given by

$$\frac{x}{\cos\frac{\pi}{4}} = \frac{y}{\sin\frac{\pi}{4}} = 6\sqrt{2}$$

$\Rightarrow$ $x = 6,\ y = 6$

Since, point (6, 6) lies on

$$x^2 + y^2 + 2gx + 2fy + c = 0$$

$\therefore$ $72 + 12(g + f) + c = 0$ ...(i)

Since, $y = x$ touches the circle, therefore the equation $2x^2 + 2x(g + f) + c = 0$ has equal roots.

$\Rightarrow \quad 4(g + f)^2 = 8c$

$\Rightarrow \quad (g + f)^2 = 2c \qquad \ldots(ii)$

From Eq. (i)

$[(12(g + f)]^2 = [-(c + 72)]^2$

$\Rightarrow \quad 144(g + f)^2 = (c + 72)^2$

$\Rightarrow \quad 144(2c) = (c + 72)^2$ [from Eq. (ii)]

$\Rightarrow \quad (c - 72)^2 = 0$

$\Rightarrow \quad c = 72$

42. $(1 + x + x^2 + x^3)^n = (1 + x)^n (1 + x^2)^n$

$= (1 + {}^nC_1\, x + {}^nC_2 x^2 + \ldots\ldots + {}^nC_n\, x^n)$
$\times (1 + {}^nC_1\, x^2 + {}^nC_2 x^4 + \ldots\ldots + {}^nC_n x^{2n})$

$\therefore$ The coefficient of x^4

$= {}^nC_2 + {}^nC_2 \cdot {}^nC_1 + {}^nC_4$

$= {}^nC_4 + {}^nC_2 + {}^nC_1 \cdot {}^nC_2$

43. $T_n = \dfrac{1 + x + x^2 + \ldots + x^{n-1}}{n!}$

$= \dfrac{x^n - 1}{x - 1} \cdot \dfrac{1}{n!}$

$= \dfrac{1}{1 - x}\left(\dfrac{x^n}{n!} - \dfrac{1}{n!}\right)$

$\Rightarrow \quad \sum_{n=1}^{\infty} \dfrac{1}{1 - x}\left(\dfrac{x^n}{n!} - \dfrac{1}{n!}\right)$

$= \dfrac{1}{(x - 1)}\left[\sum_{n=1}^{\infty} \dfrac{x^n}{n!} - \sum_{n=1}^{\infty} \dfrac{1}{n!}\right]$

$= \dfrac{1}{(x - 1)}(e^x - e)$

44. Given inequation is

$n! > 2^{n-1}$

For $n = 1, 2$

$1! \not> 1$

$\Rightarrow \quad 1 \not> 1$

and $\quad 2! \not> 2^1 \Rightarrow 2 \not> 2$

For $n = 3$

$3! > 2^2$

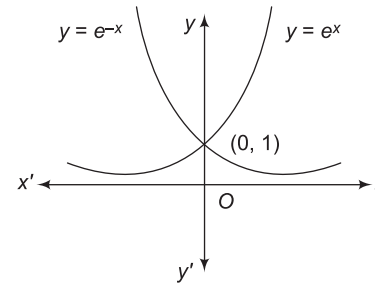
$\Rightarrow \quad 6 > 4$

Hence, it is true for $n > 2$.

45. Given sets are

$A = \{(x, y) : y = e^x,\ x \in R\}$ and

$B = \{(x, y) : y = e^{-x},\ x \in R\}$



It is clear from the graph that two curves intersect at one point.

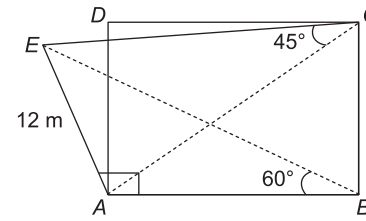
$\therefore \quad A \cap B \neq \phi$

46. Now, $\dfrac{b - c}{r_1} + \dfrac{c - a}{r_2} + \dfrac{a - b}{r_3}$

$= \dfrac{[(b - c)(s - a) + (c - a)(s - b) + (a - b)(s - c)]}{\Delta}$

$= 0$

47. Let AE is a vertical lamp post



In ΔAEC,

$\tan 45° = \dfrac{AE}{AC}$

$\Rightarrow \quad AC = AE = 12\text{ m}$

and in ΔABE,

$\tan 60° = \dfrac{AE}{AB}$

$\Rightarrow \quad AB = \dfrac{AE}{\sqrt{3}} = 4\sqrt{3}$

Now, $BC = \sqrt{AC^2 - AB^2}$

$= \sqrt{144 - 48} = \sqrt{96}$

$= 4\sqrt{6}$

$\therefore$ Area of rectangle $ABCD$

$= AB \times BC$

$= 4\sqrt{3} \times 4\sqrt{6}$

$= 48\sqrt{2}\ \text{m}^2$

48. Given that,

$f(x) = \max\{(1 - x), 2, (1 + x)\}$

For $x \le -1$, we find that $1 - x \ge 2$

and $1 - x \ge 1 + x$

$\therefore \max\{(1 - x), 2, (1 + x)\} = 1 - x$

For $-1 < x < 1$, we find that

$0 < 1 - x < 2$ and $0 < 1 + x < 2$

$\therefore \max\{(1 - x), 2, (1 + x)\} = 2$

For $x \ge 1$, we find that

$1 + x \ge 2, 1 + x > 1 - x$

$\therefore \max\{(1 - x), 2, (1 + x)\} = 1 + x$

Hence, $f(x) = \begin{cases} 1 - x & , \quad x \le -1 \\ 2 & , \quad -1 < x < 1 \\ 1 + x & , \quad x \ge 1 \end{cases}$

49. Given, $3\sin^2 x - 7 \sin x + 2 = 0$

$\Rightarrow \quad (2 \sin x - 1)(\sin x - 2) = 0$

$\Rightarrow \quad \sin x = \frac{1}{3} \qquad (\because \sin x \ne 2)$

Let $\sin^{-1}\frac{1}{3} = \alpha$, $0 < \alpha < \frac{\pi}{2}$ are the solutions in $[0, 5\pi]$. Then,

$\alpha, \pi - \alpha, 2\pi + \alpha, 3\pi - \alpha, 4\pi + \alpha, 5\pi - \alpha$ are the solutions.

50. $\left(1 + \cos\frac{\pi}{8}\right)\left(1 + \cos\frac{3\pi}{8}\right)\left(1 - \cos\frac{3\pi}{8}\right)\left(1 - \cos\frac{\pi}{8}\right)$

$= \left(1 - \cos^2\frac{\pi}{8}\right)\left(1 - \cos^2\frac{3\pi}{8}\right)$

$= \left(\sin\frac{\pi}{8} \sin\frac{3\pi}{8}\right)^2$

$= \frac{1}{4}\left(2\sin\frac{3\pi}{8} \sin\frac{\pi}{8}\right)^2$

$= \frac{1}{4}\left(\cos\frac{\pi}{4} - \cos\frac{\pi}{2}\right)^2 = \frac{1}{8}$

51. Since, $\sin^{-1}\sqrt{x}$, $\cos^{-1}\sqrt{x}$ are defined for $x \le 1$ and $x \ge 0$.

$\therefore \quad 0 \le \sqrt{x^2 - x + 1} \le 1$

and $\quad 0 \le \sqrt{x^2 - x} \le 1$

$\Rightarrow \quad -1 \le x^2 - x \le 0$

and $\quad 0 \le x^2 - x \le 0$

$\Rightarrow \quad x^2 - x = 0$

$\Rightarrow \quad x = 0, 1$

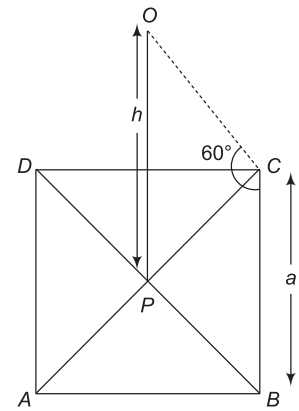
Hence, number of solutions are two.

52. Let $ABCD$ be a square of each side of length a. It is given that $\angle OCP = 60°$.

Length of diagonal

$AC = \sqrt{a^2 + a^2} = a\sqrt{2}$

$\therefore \quad PC = \frac{AC}{2} = \frac{a}{\sqrt{2}}$



In $\quad \Delta OCP$, $\tan 60° = \frac{OP}{PC}$

$\Rightarrow \quad \sqrt{3} = \frac{h}{a/\sqrt{2}}$

$\Rightarrow \quad \sqrt{3}a = \sqrt{2}h$

$\Rightarrow \quad 3a^2 = 2h^2$

53. $\cos^{-1}\left(\cos\frac{7\pi}{6}\right) = \cos^{-1}\left(\cos\left(2\pi - \frac{5\pi}{6}\right)\right)$

$= \cos^{-1}\left(\cos\left(\frac{5\pi}{6}\right)\right) = \frac{5\pi}{6}$

54. Let $T = |\sin nx|$

Put $n = 2$, then

$T = |\sin 2x|$

$= |2 \sin x \cos x| \leq 2|\sin x|$

55. Given sides of a triangle are

$a = 30,\ b = 24,\ c = 18$

$\therefore \quad s = \frac{a+b+c}{2}$

$= \frac{30+24+18}{2} = 36$

Now, $\Delta = \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{36\,(36-30)\,(36-24)\,(36-18)}$

$= \sqrt{36 \times 6 \times 12 \times 18}$

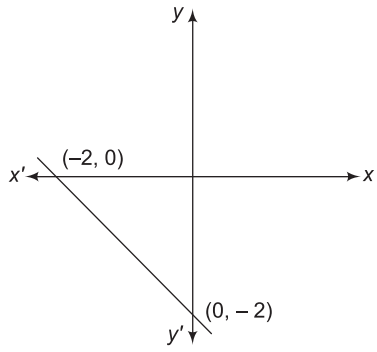
$= \sqrt{36 \times 36 \times 36} = 216$

$\therefore \quad r_3 = \frac{\Delta}{s-c} = \frac{216}{18} = 12$

56. Given lines are

$xy + 2x + 2y + 4 = 0$ ...(i)

and $x + y + 2 = 0$ ...(ii)



From Eqs. (i) and (ii), we get

$xy = 0$

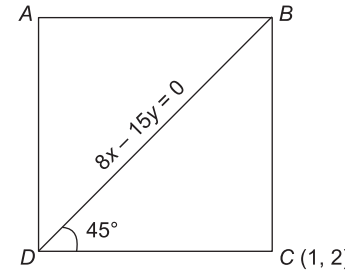
$\Rightarrow \quad x = y = 0$

$\therefore$ Vertices of triangle are
$(-2, 0), (0, 0), (0, -2)$.

In a right angled triangle circumcentre is the mid-point of hypotenuse.

$\therefore$ Point $(-1, -1)$ is the required coordinate of the circumcentre.

57. Slop of BD is $\frac{8}{15}$ and angle made by BD with AD and DC is 45°.



So, let slope of DC be m, then

$\tan 45° = \pm \frac{m - \frac{8}{15}}{1 + \frac{8}{15}m}$

$\Rightarrow \quad (15 + 8m) = \pm (15m - 8)$

$\Rightarrow \quad m = \frac{23}{7}$ and $-\frac{7}{23}$

Hence, the equation of DC and AD are

$y - 2 = \frac{23}{7}(x - 1)$

$\Rightarrow \quad 23x - 7y - 9 = 0$

and $y - 2 = -\frac{7}{23}(x - 1)$

$\Rightarrow \quad 7x + 23y - 53 = 0$

58. Given curves are

$x^3 - 3xy^2 + 2 = 0$ ...(i)

and $3x^2y - y^3 - 2 = 0$ ...(ii)

On differentiating Eqs. (i) and (ii), with respect to x, we get

$\left(\frac{dy}{dx}\right)_{C_1} = \frac{x^2 - y^2}{2xy}$

and $\left(\frac{dy}{dx}\right)_{C_2} = \frac{2xy}{x^2 - y^2}$

$\left(\frac{dy}{dx}\right)_{C_1} \times \left(\frac{dy}{dx}\right)_{C_2} = -1$

59. Given that,

$$\frac{2x+3}{(x+1)(x-3)} = \frac{a}{(x+1)} + \frac{b}{(x-3)}$$

$\Rightarrow \quad (2x+3) = a(x-3) + b(x+1)$

On comparing the coefficient of x and constant, we get

$$2 = a + b \text{ and } 3 = -3a + b$$

$\Rightarrow \quad a = -\frac{1}{4}$ and $b = \frac{9}{4}$

$\therefore \quad a + b = -\frac{1}{4} + \frac{9}{4} = 2$

60. $\lim\limits_{n\to\infty} \frac{1}{n}\left[\sec^2\frac{\pi}{4n} + \sec^2\frac{2\pi}{4n} + \ldots + \sec^2\frac{n\pi}{4n}\right]$

$$= \lim_{n\to\infty} \frac{1}{n} \sum_{r=1}^{n} \sec^2\frac{r\pi}{4n}$$

$$= \int_0^1 \sec^2\frac{\pi x}{4}\,dx$$

$$= \frac{4}{\pi}\left[\tan\frac{\pi x}{4}\right]_0^1 = \frac{4}{\pi}$$

61. Given that, $f(x) = |\cos x|$ and $g(x) = [x]$,

$\therefore \quad gof(x) = g(|\cos x|)$

$= [|\cos x|]$

62. Since, $\lim\limits_{x\to\infty}\left(1 + \frac{a}{x} + \frac{b}{x^2}\right)^{2x} = e^2$

$\Rightarrow \quad \lim\limits_{x\to\infty}\left(1 + \frac{ax+b}{x^2}\right)^{2x} - e^2 \qquad \ldots(i)$

$\Rightarrow \quad \lim\limits_{x\to\infty} \frac{ax+b}{x^2}$ must be equal to zero.

$\Rightarrow \quad \lim\limits_{x\to\infty} \frac{ax+b}{x^2} = 0$

$\Rightarrow \quad \lim\limits_{x\to\infty}\left(\frac{a}{x} + \frac{b}{x^2}\right) = 0$

$\Rightarrow \quad a$ and $b \neq 0$

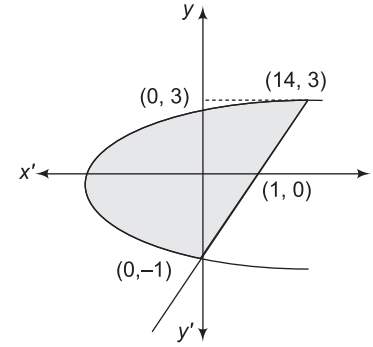
$\therefore$ From Eq. (i)

$$e^{\lim\limits_{x\to\infty}\frac{2(ax+b)}{x}} = e^2$$

$\Rightarrow \quad e^{2a} = e^2$

$\Rightarrow \quad a = 1$ and $b \in R$

63. The point of intersection of curve $y^2 = 2x + 1$ and line $x - y - 1 = 0$ is (14, 3) and (0, – 1)



$\therefore$ Required area $= \int_{-1}^{3} (x_2 - x_1)\,dy$

$$= \int_{-1}^{3} \left\{(y+1) - \frac{(y^2-1)}{2}\right\} dy$$

$$= \frac{1}{2}\int_{-1}^{3} (2y + 3 - y^2)\,dy$$

$$= \frac{1}{2}\left[y^2 + 3y - \frac{y^3}{3}\right]_{-1}^{3}$$

$$= \frac{1}{2}\left[(9 + 9 - 9) - \left(1 - 3 + \frac{1}{3}\right)\right]$$

$$= \frac{1}{2}\left(9 + \frac{5}{3}\right)$$

$$= \frac{16}{3} \text{ sq units}$$

64. For $f(x)$ to be continuous everywhere, we must have

$$f(0) = \lim_{x\to 0} f(x)$$

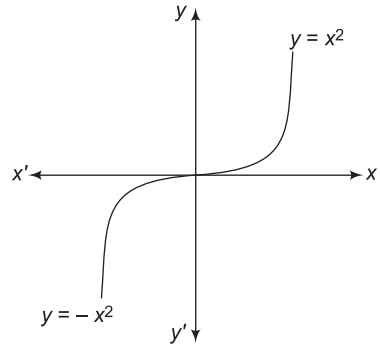
$$= \lim_{x\to 0} \frac{2 - (256 - 7x)^{1/8}}{(5x+32)^{1/5} - 2}$$

$$= \lim_{x\to 0} \frac{\frac{7}{8}(256 - 7x)^{-7/8}}{(5x+32)^{-4/5}}$$

$$= \frac{7}{8} \times \frac{2^{-7}}{2^{-4}} = \frac{7}{64}$$

65. Given that, $f(x) = x|x|$

$\therefore \quad f(x) = \begin{cases} x^2, & x \geq 0 \\ -x^2, & x < 0 \end{cases}$



It is clear from the graph that $f(x)$ is differentiable everywhere.

66. Since, $f(x) = |x|^{\sin x}$

when $x < 0$

$$f(x) = (-x)^{(-\sin x)}$$

$$= e^{-\sin x \log(-x)}$$

$$\Rightarrow \quad f'(x) = e^{-\sin x \log(-x)} \left(-\cos x \log(-x) - \frac{\sin x}{x}\right)$$

$$\Rightarrow \quad f'(x) = (-x)^{-\sin x} \left(-\cos x \cdot \log(-x) - \frac{\sin x}{x}\right)$$

$$\Rightarrow \quad f'\left(-\frac{\pi}{4}\right) = \left(\frac{\pi}{4}\right)^{1/\sqrt{2}} \left(-\frac{1}{\sqrt{2}} \log \frac{\pi}{4} + \frac{4}{\pi}\left(-\frac{1}{\sqrt{2}}\right)\right)$$

$$= \left(\frac{\pi}{4}\right)^{1/\sqrt{2}} \left(\frac{\sqrt{2}}{2} \log \frac{4}{\pi} - \frac{2\sqrt{2}}{\pi}\right)$$

67. Let the centre of a circle be $C_1(h, k)$

Since, $C_1C_2 = r_1 + r_2$ (given)

$$\Rightarrow \quad \sqrt{(h-0)^2 + (k-3)^2} = |k+2|$$

$$\Rightarrow \quad h^2 = 5(2k-1)$$

Hence, locus of a point is $x^2 = 5(2y-1)$ which represents a equation of parabola.

68. Let $S(a, 0)$ be the focus of the parabola $y^2 = 4ax$ and $P(at^2, 2at)$ be a point on it. Then, the equation of a circle on SP as diameter is $(x-a)(x-at^2) + (y-0)(y-2at) = 0$

It meets y-axis at $x = 0$

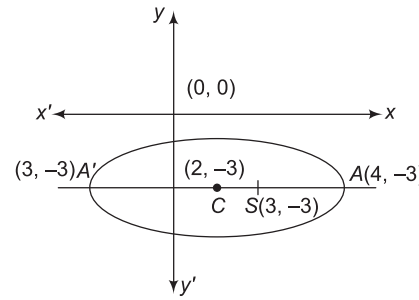
$$\therefore \quad y^2 - 2aty + a^2t^2 = 0$$

$$\Rightarrow \quad (y - at)^2 = 0$$

This shows that y-axis meets the circle in two coincident points. Hence, the circle touches the tangent at the vertex.

69. Let $2a$ and $2b$ be the major and minor axes of the ellipse. Then, its equation is

$$\frac{(x-2)^2}{a^2} + \frac{(y+3)^2}{b^2} = 1 \qquad \ldots(i)$$



Here, semi-major axis $CA = a$

$$\Rightarrow \quad \sqrt{(4-2)^2 + (-3+3)^2} = a$$

$$\Rightarrow \quad a = 2 \qquad \ldots(ii)$$

Here, $CS = ae$

$$\Rightarrow \quad \sqrt{(2-3)^2 + (-3+3)^2} = ae$$

$$\Rightarrow \quad ae = 1 \qquad \ldots(iii)$$

From Eqs. (ii) and (iii), we get

$$e = \frac{1}{2}$$

Now, $b^2 = a^2(1 - e^2)$

$$\Rightarrow \quad b^2 = 4\left(1 - \frac{1}{4}\right) = 3$$

On substituting the values of a and b in Eq. (i) we get

$$\frac{(x-2)^2}{4} + \frac{(y+3)^2}{3} = 1$$

70. The diameters $y = m_1 x$ and $y = m_2 x$ are conjugate diameters of the hyperbola

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1, \text{ if } m_1 m_2 = \frac{b^2}{a^2}$$

Given equation is

$$\frac{x^2}{9} - \frac{y^2}{16} = 1$$

Here, $a^2 = 9, b^2 = 16$

Also given, $x = 2y$

Then, $m_1 = \frac{1}{2}$

$\because \quad \frac{1}{2} \times m_2 = \frac{16}{9}$

$\Rightarrow \quad m_2 = \frac{32}{9}$

Hence, the required equation of diameter is

$$y = \frac{32x}{9}$$

71. $\therefore (2x + 3) = a(x - 3) + b(x + 1)$

Put $x = -1$ and $x = 3$ respectively, we get

$$1 = -4a \text{ and } 9 = 4b$$

$\Rightarrow \quad a = -\frac{1}{4}$ and $b = \frac{9}{4}$

$\therefore \quad a + b = -\frac{1}{4} + \frac{9}{4} = 2$

72. Here, $e^x + 2 = -3(2e^x - 3) + B(e^x - 1)$

On equating the coefficient of e^x and constant, we get

$$1 = -6 + B,$$

$$2 = 9 - B \Rightarrow B = 7$$

73. $\frac{3\sqrt{2}}{\sqrt{6}+\sqrt{3}} - \frac{4\sqrt{3}}{\sqrt{6}+\sqrt{2}} + \frac{\sqrt{6}}{\sqrt{3}+\sqrt{2}}$

$$= \frac{3\sqrt{2}(\sqrt{6}-\sqrt{3})}{6-3} - \frac{4\sqrt{3}(\sqrt{6}-\sqrt{2})}{6-3} + \frac{\sqrt{6}(\sqrt{3}-\sqrt{2})}{3-2}$$

$$= \sqrt{2}(\sqrt{6}-\sqrt{3}) - \sqrt{3}(\sqrt{6}-\sqrt{2}) + \sqrt{6}(\sqrt{3}-\sqrt{2})$$

$$= 0$$

74. Since, it is an AP.

$\therefore \quad \log_3(2^x - 5) = \frac{\log_3 2 + \log_3\left(2^x - \frac{7}{2}\right)}{2}$

$\Rightarrow \quad 2\log_3(2^x - 5) = \log_3\left(2 \cdot \left(2^x - \frac{7}{2}\right)\right)$

$\Rightarrow \quad (2^x - 5)^2 = 2^{x+1} - 7$

$\Rightarrow \quad 2^{2x} - 12 \cdot 2^x - 32 = 0$

$\Rightarrow \quad x = 2, 3$

But $x = 2$, $\log_3(2^x - 5)$ is not defined.

75. Let $\frac{A}{R}, A, AR$ be the roots of the equation $ax^3 + bx^2 + cx + d = 0$, then

Product of roots

$$A^3 = -\frac{d}{a}$$

$\Rightarrow \quad = A = -\left(\frac{d}{a}\right)^{1/3}$

Since, A is a root of the equation.

$\therefore \quad aA^3 + bA^2 + cA + d = 0$

$\Rightarrow \quad a\left(-\frac{d}{a}\right) + b\left(-\frac{d}{a}\right)^{2/3} + c\left(-\frac{d}{a}\right)^{1/3} + d = 0$

$\Rightarrow \quad b\left(\frac{d}{a}\right)^{2/3} = c\left(\frac{d}{a}\right)^{1/3}$

$\Rightarrow \quad b^3 \cdot \frac{d^2}{a^2} = c^3 \cdot \frac{d}{a}$

$\Rightarrow \quad b^3 d = c^3 a$

76. The composition table is as given below

$\times_5$	1	2	3	4
1	1	2	3	4
2	2	4	1	3
3	3	1	4	2
4	4	3	2	1

Clearly, identity element = 1

$\therefore \quad 2^{-1} \times_5 (3 \times_5 x) = 4$

$\Rightarrow \quad 3 \times_5 (3 \times_5 x) = 4$

$\Rightarrow \quad (3 \times_5 3) \times_5 x = 4$

$\Rightarrow \quad 4 \times_5 x = 4$

$\Rightarrow \quad x = 4^{-1} \times_5 4 = 1$

77. Let p : Ram is in class x, q : Rashmi is in class XII

Given proposition is $p \vee q$.

Its negation is $\sim (p \vee q) = \sim p \wedge \sim q$

Ram is not in class X and Rashmi is not in class XII.

78. Inverse of $p \Rightarrow q$ is $\sim p \Rightarrow \sim q$

$\therefore$ Inverse of $(p \wedge \sim q) \Rightarrow r$ is

$\sim (p \wedge \sim q) \Rightarrow \sim r$

i. e., $(\sim p \vee q) \Rightarrow \sim r$

79. $(17)^2 = 289 \equiv (-1) \pmod 5$

$\Rightarrow \quad (17^2)^{15} \equiv (-1)^{15} \pmod 5$

$\Rightarrow \quad 17^{30} \equiv - \pmod 5$

i. e., $17^{30} \equiv 4 \pmod 5$

80. Given equation can be rewritten as

$\text{cosec } y \, dy = \sin x \, dx$

On integrating both sides, we get

$\log \tan \frac{y}{2} = - \cos x + \log C$

$\Rightarrow \quad \log \frac{\tan \frac{y}{2}}{C} = - \cos x$

$\Rightarrow \quad \frac{\tan \frac{y}{2}}{C} = e^{- \cos x}$

$\Rightarrow \quad e^{\cos x} \cdot \tan \frac{y}{2} = C$

General English and Aptitude

21. Knot, Watt and Fathom, all are units of measurement.

22. Majlis, Diet and Knesset, all are parliament name of countries.

23. Kanha, Periyar and Dachigam, all are animal sanctuaries.

24. Stirrup, Anvil and Drum, all are parts of ear.

25. Kennedy, Indira and Palme, all of them were assassinated.

26. Chair, Door and Stick all are made of the same raw material.

27. As, a performer plays music on a guitar. Similarly, an acrobat performs tricks on a rope.

28. First is used to make the second and the third.

29. Second and third are bigger and more sophisticated forms than the first and second respectively.

30. Second is more intense form of the first, while third is opposite of second.