CBSE Test Paper 03

CH-04 Principle of Mathematical Induction

1. If n is a +ve integer, then $2.4^{2n+1} + 3^{3n+1}$ is divisible by
    a. none of these
    b. 11
    c. 9
    d. 2
2. For each n $\in$ N , n (n + 1) (2n + 1) is divisible by :
    a. 8
    b. 15
    c. 2
    d. 6
3. For each n $\in$ N , $3(5^{2n+1}) + 2^{3n+1}$ is divisible by :
    a. 17
    b. 21
    c. 23
    d. 19
4. For each $n \in N, a^{2n-1} + b^{2n-1}$ is divisible by :
    a. none of these.
    b. $(a+b)^2$
    c. $(a+b)^3$

d. a + b

5. Let P (k) = 1 + 3 + 5 + ...+ (2k – 1) = $(3+k^2)$ .Then which of the following is true?

   a. P (1) is correct

   b. Principal of mathematical induction can be used to prove the formula

   c. P (k) $\Rightarrow$ P (k+1)

   d. P (k) $\Rightarrow$ P (k + 2)

6. Fill in the blanks:

   2^{3n} - 1 is divisible by ________, for all natural numbers n.

7. Fill in the blanks:

   If P(n) : $2.4^{2n+1} + 3^{3n+1}$ is divided by λ for all n $\in$ N is true, then the value of λ is ________.

8. Prove by the Principle of Mathematical Induction that $3^n > 2^n$, for all n$\in$N.

9. Prove the following by using the principle of mathematical induction for all $n \in N$: n (n + 1) (n + 5) is a multiple of 3.

10. Prove by the principle of mathematical induction that for all n $\in$ N,

    $$\sin\theta + \sin 2\theta + \sin 3\theta + \ldots + \sin n\theta = \frac{\sin\left(\frac{n+1}{2}\right)\theta \sin\frac{n\theta}{2}}{\sin\frac{\theta}{2}}$$

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Solution

1. (b) 11

   Explanation: When n is replaced by 1,2,3... the value obtained is a multiple of 11.
2. (d) 6

   Explanation: When n = 1 the value is 6 . The subsequent substitution will give the value as a multiple of 6.
3. (a) 17

   Explanation: When n = 1 we have 391 which is divisible by 17.
4. (d) a + b

   Explanation: When n = 1 we have a + b.And the subsequent substitution of n as 2,3,... will result in the expression whose factor is a + b.
5. (b) Principal of mathematical induction can be used to prove the formula

   Explanation: Since the statement is invalid for n = 1 , 2...

   (c) P (k) $\Rightarrow$P (k+1)

   Explanation: By the principle of mathematical induction the result follows

6. 7

7. 11

8. **Step I:** Let P(n) be the statement given by $P(n) : 3^n > 2^n$, n $\in$ N

   Step II: For $n = 1$, we have $3^1 > 2^1$

   $\Rightarrow 3 > 2$, which is true.

   Therefore, $P(1)$ is true.

   Step III: For $n = k$, assume that P(k) is true, i.e.,

   $P(k) : 3^k > 2^k$ ...(i)

   Step IV: For n = k + 1, we have to show that $P(k + 1)$ is true, whenever P(k) is true. i.e.,

   $P(k + 1) : 3^{k+1} > 2^{k+1}$

   Now, $3^{k+1} = 3^k \cdot 3 > 2^k \cdot 3$ [Eq. (i) multiply 3 on both sides]

   But $2^k .3 > 2^k .2$

$\therefore 2^k.3 > 2^{k+1}$

$\Rightarrow 3^k.3 > 2^{k+1}$

$\therefore 3^{k+1} > 2^{k+1}$

Thus, $P(k+1)$ is true, whenever P(k) is true.

Hence, by Principle of Mathematical Induction, P(n) is true for all n$\in$ N.

9. Let P(n) = n (n + 1) (n + 5) is a multiple of 3.

   For n = 1

   P(1) = 1 (1 + 1) (1 + 5) is a multiple of 3 $\Rightarrow$ $1 \times 2 \times 6$ is a multiple of 3

   $\Rightarrow$ 12 is a multiple of 3

   $\therefore$ P(1) is true

   Let P(n) be true for n = k

   $\therefore$ P(k) = k (k + 1) (k + 5) is a multiple of 3 $\Rightarrow$ k (k + 1) (k + 5) = 3λ

   $\Rightarrow k^3 = 3\lambda - 6k^2 - 5k$ .... (i)

   For n = k + 1

   P(k+1)= (k + 1) (k + 2) (k + 6) is a multiple of 3

   Now (k + 1) (k + 2) (k + 6) = (k + 1) (k^2 + 8k + 12) = k^3 + 9k^2 + 20k + 12

   $= 3\lambda - 6k^2 - 5k + 9k^2 + 20k + 12$ [using (i)]

   $= 3[\lambda + k^2 + 5k + 4]$

   $\Rightarrow$ (k + 1) (k + 2) (k + 6) is a multiple of 3

   $\therefore$ P(k + 1) is true

   Thus P(k) is true $\Rightarrow$ P(k + 1) is true

   Hence by principle of mathematical induction, P(n) is true for all $n \in N$

10. Let P(n) be the statement given by

    P(n): sin θ + sin 2θ + sin 3θ +... + sin nθ = $\frac{\sin\left(\frac{n+1}{2}\right)\theta \sin\frac{n\theta}{2}}{\sin\frac{\theta}{2}}$

    We have, P(1) : LHS = sin θ

    RHS = $\frac{\sin\left(\frac{1+1}{2}\right)\theta \sin\left(\frac{1\times\theta}{2}\right)}{\sin\frac{\theta}{2}}$ = sin θ

    $\therefore$ P(1) is true.

    Let p(m) be true. Then,

    sin θ + sin 2θ +... + sin mθ = $\frac{\sin\left(\frac{m+1}{2}\right)\theta \sin\frac{m\theta}{2}}{\sin\frac{\theta}{2}}$

We shall now show that P(m + 1) is true.

i.e. sin θ + sin 2θ + sin mθ + sin (m + 1)θ = $\dfrac{\sin\left(\frac{m+2}{2}\right)\theta \sin \frac{(m+1)\theta}{2}}{\sin \frac{\theta}{2}}$

Now,

sin θ + sin 2θ + ...+ sin mθ + sin (m + 1)θ

$= \dfrac{\sin\left(\frac{m+1}{2}\right)\theta \sin \frac{m\theta}{2}}{\sin \frac{\theta}{2}} + 2\sin\left(\frac{m+1}{2}\right)\theta \cos\left(\frac{m+1}{2}\right)\theta$

$= \sin\left(\frac{m+1}{2}\right)\theta \left\{ \dfrac{\sin\left(\frac{m\theta}{2}\right)}{\sin \frac{\theta}{2}} + 2\cos\left(\frac{m+1}{2}\right)\theta \right\}$

$= \sin\left(\frac{m+1}{2}\right)\theta \left\{ \dfrac{\sin\left(\frac{m\theta}{2}\right) + 2\sin \frac{\theta}{2} \cos\left(\frac{m+1}{2}\right)\theta}{\sin \frac{\theta}{2}} \right\}$

$= \sin\left(\frac{m+1}{2}\right)\theta \left\{ \dfrac{\sin\left(\frac{m\theta}{2}\right) + \sin\left(\frac{m+2}{2}\right)\theta - \sin \frac{m\theta}{2}}{\sin \frac{\theta}{2}} \right\}$

$= \dfrac{\sin\left(\frac{m+1}{2}\right)\theta \sin\left(\frac{m+2}{2}\right)\theta}{\sin \frac{\theta}{2}} = \dfrac{\sin\left\{\frac{(m+1)+1}{2}\right\}\theta \sin\left(\frac{m+1}{2}\right)\theta}{\sin \frac{\theta}{2}}$

$\therefore$ P(m + 1) is true

Thus, P(m) is true $\Rightarrow$ P(m + 1) is true.

Hence, by principle of mathematical induction P(n) is true for all n $\in$ N.