CBSE Test Paper 05

Chapter 4 Quadratic Equation

1. $2x^2 - 3x + 2 = 0$ have **(1)**
   a. Real and Distinct roots
   b. Real and Equal roots
   c. Real roots
   d. No Real roots
2. $2x^2 + 5\sqrt{3}\ x + 6 = 0$ have **(1)**
   a. Real and equal root
   b. Real roots
   c. No Real roots
   d. Real and Distinct roots
3. If one root of the equation $4x^2 - 2x + (\lambda - 4) = 0$ be the reciprocal of the other, then the value of λ is **(1)**
   a. 8
   b. 4
   c. - 8
   d. - 4
4. A quadratic equation $ax^2 + bx + c = 0$ has non – real roots, if **(1)**
   a. $b^2 - 4ac > 0$
   b. $b^2 - 4ac = 0$
   c. $b^2 - 4ac < 0$
   d. $b^2 - ac = 0$
5. The quadratic equation whose roots are $\frac{-1}{3}$ and $\frac{5}{2}$ is **(1)**
   a. $6x^2 + 13x - 5 = 0$
   b. $6x^2 + 13x + 5 = 0$
   c. $6x^2 - 13x + 5 = 0$
   d. $6x^2 - 13x - 5 = 0$
6. What is the nature of roots of the quadratic equation $5x^2 - 2x - 3 = 0$? **(1)**
7. If $x = \frac{-1}{2}$, is a solution of the quadratic equation $3x^2 + 2kx - 3 = 0$, find the value of k.

(1)

8. Check whether(x - 7)x = 3x^2 - 5 is a quadratic equation: **(1)**
9. If 2x^2 - (2 + k)x + k = 0 where k is a real number, find the roots of the equation. **(1)**
10. Solve the following problem: x^2−55x +750=0 **(1)**
11. Solve the quadratic equation $x^2 + (\frac{a}{a+b} + \frac{a+b}{a})x + 1 = 0$ by factorization. **(2)**
12. A water tank is connected with three pipes of uniform flow. The second pipe fills the tank 5 hours faster than the first pipe and 4 hours slower than the third pipe. If the fist two pipes operating simultaneously, fill the tank in the same time as the time taken by the third pipe alone to fill the tank. Find the time taken by each pipe individually to fill the tank. **(2)**
13. Determine whether the given values are solution of the given equation or not:

    6x^2 - x - 2 = 0, x = -1/2, x = 2/3 **(2)**
14. A farmer wishes to grow a 100 m^2 rectangular vegetable garden. Since he has with him only 30 m barbed wire, he fences three sides of the rectangular garden letting compound wall of his house act as the fourth side-fence. Find the dimensions of his garden. **(3)**
15. Solve for x : $\frac{2x}{x-3} + \frac{1}{2x+3} + \frac{3x+9}{(x-3)(2x+3)} = 0$, $x \neq 3, -\frac{3}{2}$ **(3)**
16. A motor boat, whose speed is 15 km/hr in still water, goes 30 km downstream and comes back in a total time of 4 hr 30 minutes, find the speed of the stream. **(3)**
17. Solve for x by quadratic formula p^{2}x^2 + (p^2 - q^2)x - q^2 = 0 **(3)**
18. A train travels a distance of 480 km's at a uniform speed. If the speed had been 8 km/hr less, then it would have taken 3 hours more to cover the same distance. Find the quadratic equation in terms of the speed of the train. **(4)**
19. A journey of 192 km from a town A to town B takes 2 hours more by an ordinary passenger train than a super fast train. If the speed of the faster train is 16 km/h more, find the speed of the faster and the passenger train. **(4)**
20. Solve for x: $\left(\frac{2x}{x-5}\right)^2 + 5\left(\frac{2x}{x-5}\right) - 24 = 0, x \neq 5$**(4)**

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Solution

1. d. No Real roots

   Explanation: D = b^2 - 4ac

   D = $(-3)^2$ - $4 \times 2 \times 2$

   D = 9 - 16

   D = - 7

   D < 0. Hence No Real roots.

2. d. Real and Distinct roots

   Explanation: Comparing the given equation to the below equation

   ax^2 + bx + c = 0

   a = 2, b = 5√3, c = 6

   D = b^2 - 4ac

   D = $(5\sqrt{3})^2$ - $4 \times 2 \times 6$

   D = 75 - 48

   D = 27

   D > 0.

   If b^2−4ac > 0, then equation have real and distinct roots.

3. a. 8

   Explanation: Let one root be α then other root will be $\frac{1}{\alpha}$
   Product of the root $\Rightarrow \alpha \times \frac{1}{\alpha} = \frac{c}{a}$
   $\Rightarrow 1 = \frac{\lambda - 4}{4}$
   $\Rightarrow \lambda - 4 = 4 \Rightarrow \lambda = 8$

4. c. $b^2 - 4ac < 0$

   Explanation: The roots of the quadratic equation ax^2 + bx + c = 0, In this formula the term b^2 - 4ac is called the discriminant. If b^2 - 4ac = 0, so the equation has a single repeated root. If b^2 - 4ac > 0, the equation has two real roots. If b^2- 4ac < 0, the equation has two complex roots.

5. d. $6x^2 - 13x - 5 = 0$

Explanation: Given: $\alpha = \frac{-1}{3}$ and $\beta = \frac{5}{2}$

$x^2 - (\alpha + \beta)x + \alpha\beta = 0$

$\Rightarrow x^2 - \left(\frac{-1}{3} + \frac{5}{2}\right)x + \left(\frac{-1}{3} \times \frac{5}{2}\right) = 0$

$\Rightarrow x^2 - \left(\frac{-2+15}{6}\right)x + \left(\frac{-5}{6}\right) = 0$

$\Rightarrow x^2 - \left(\frac{13}{6}\right)x + \left(\frac{-5}{6}\right) = 0$

$\Rightarrow 6x^2 - 13x - 5 = 0$

6. On comparing equation with standard form of equation i.e, $ax^2 + bx + c = 0$, we get

a = 5, b = -2, c = -3

Now , $D = b^2 - 4ac = (-2)^2 - 4 \times 5 \times (-3)$

= 4 + 60 = 64

Therefore, D = 64

We know , For D>0 , the roots of equation are real and distinct.

Therefore , $5x^2 - 2x - 3 = 0$ has real and distinct roots.

7. We have,

$3x^2 + 2kx - 3 = 0$

since x = $\frac{-1}{2}$

$\therefore 3(\frac{-1}{2})^2 + 2k(\frac{-1}{2}) - 3 = 0$

$\Rightarrow 3(\frac{1}{4}) - k - 3 = 0$

$\Rightarrow \frac{3}{4} - k - 3 = 0$

$\Rightarrow k = \frac{3}{4} - 3 = \frac{3-12}{4} = -\frac{9}{4}$

8. Given equation is $(x - 7)x = 3x^2 - 5$

$\Rightarrow x^2 - 7x = 3x^2 - 5$

$\Rightarrow 2x^2 + 7x - 5 = 0$

which of the form $ax^2 + bx + c = 0$

Hence, given equation is a quadratic equation.

9. Given quadratic equation is $2x^2 - (2 + k)x + k = 0$.

Here, $a = 2, b = -(2 + k), c = k$

Now, $a + b + c = 2 + [-(2 + k)] + k = 0$

$\therefore$ roots are 1 and $\frac{k}{2}$

(If a + b + c = 0, then roots of the quadratic equation are 1 and $\frac{c}{a}$)

10. $x^2-55x+750=0$

 $\Rightarrow x^2-25x-30x+750=0 \Rightarrow x(x-25)-30(x-25)=0$

 $\Rightarrow (x-30)(x-25) \Rightarrow x=30, 25$

11. We have, $x^2+(\frac{a}{a+b}+\frac{a+b}{a})x+1=0$

 $\implies x^2+\frac{a}{a+b}x+\frac{a+b}{a}x+1=0$

 $\implies x(x+\frac{a}{a+b})+\frac{a+b}{a}(x+\frac{a}{a+b})=0$

 $\implies (x+\frac{a}{a+b})(x+\frac{a+b}{a})=0$

 $Either\, x+\frac{a}{a+b}=0\, or\, x+\frac{a+b}{a}=0$

 $\implies x=-\frac{a}{a+b},-\frac{a+b}{a}$

 $\therefore x=-\frac{a}{a+b},-\frac{a+b}{a}$ are the required roots.

12. Let the time taken by the second pipe to fill the tank individually be x hrs.

 Then, the time taken by the first and third pipes to fill the tank individually are (x +5) hrs and (x – 4) hrs respectively.

 We have, $\frac{1}{x+5}+\frac{1}{x}=\frac{1}{x-4}$

 $\implies \frac{x+x+5}{x(x+5)}=\frac{1}{x-4}$

 $\implies (2x+5)(x-4)=x^2+5x$

 $\implies 2x^2-8x+5x-20=x^2+5x$

 $\implies x^2-8x-20=0$

 $\implies x^2-10x+2x-20=0$

 $\implies x(x-10)+2(x-10)=0$

 $\implies (x-10)(x+2)=0$

 Either x = 10 or x = - 2

 But x cannot be negative. So, x = 10

 Hence, the time taken by first, second and third pipe to fill the tank individually are 15 hours, 10 hours and 6 hours respectively.

13. We have, $6x^2-x-2=0$

 Substituting $x=-\frac{1}{2}$, we get

 $\text{LHS}=6\times\left(-\frac{1}{2}\right)^2-\left(-\frac{1}{2}\right)-2=\frac{6}{4}+\frac{1}{2}-2=0=\text{RHS}$

Therefore, $x = \frac{1}{2}$ is a solution of the given equation.
Now, $x = \frac{2}{3}$,
$\text{LHS} = 6 \times \left(\frac{2}{3}\right)^2 - \frac{2}{3} - 2 = 0 = \text{RHS}$
Therefore, $x = \frac{2}{3}$ is also a solution of the given equation.

14. Let the length of one side be x metres and other side be y metres.
Then, x + y + x = 30 $\implies$ y = 30 - 2x
$\therefore$ Area of the garden = 100 m^2
$\implies$ xy = 100
$\implies$ x (30-2x) = 100
$\implies$ $2x^2$ - 30x +100 = 0
$\implies$ 2(x^2-15x+50) = 0 or x^2-15x+50 = 0
$\implies$ x^2-10x - 5x +50 = 0
$\implies$ x (x-10) - 5 (x-10) = 0
$\implies$ (x-10) (x-5) = 0
Either x-10 = 0 or x-5 = 0
$\implies$ x= 10, 5
$\therefore$ y = 30 - 20 = 10 or 30 -10 =20
Hence, the dimensions of the vegetable garden are 5m $\times$20m or 10m $\times$10m.

15. Given, $\frac{2x}{x-3} + \frac{1}{2x+3} + \frac{3x+9}{(x-3)(2x+3)} = 0$
$\Rightarrow \frac{2x(2x+3)+x-3+3x+9}{(x-3)(2x+3)} = 0$
$\Rightarrow \frac{4x^2+6x+x-3+3x+9}{2x^2+3x-6x-9} = 0$
$\Rightarrow \frac{4x^2+10x+6}{2x^2-3x-9} = 0$
Cross multiplying equation, we get,
$\Rightarrow 4x^2 + 10x + 6 = 0$
$\Rightarrow 4x^2 + 6x + 4x + 6 = 0$
$\Rightarrow x(4x+6) + 1(4x+6) = 0$
$\Rightarrow (x+1) = 0$ or $(4x+6) = 0$
$\Rightarrow x = -1$
$\Rightarrow x = -\frac{3}{2}$
Hence , the roots of the given quadratic equation are -1 and $-\frac{3}{2}$

16. Speed of motor boat in still water = 15 km/hr

Speed of stream = x km/hr

Speed in downward direction = 15 + x

Speed in upward direction = 15 - x

According to question,

$\frac{30}{15+x}+\frac{30}{15-x}=4\frac{1}{2}$

$\Rightarrow \frac{30(15-x)+30(15+x)}{(15+x)(15-x)}=\frac{9}{2}$

$\Rightarrow \frac{450-30x+450+30x}{225-x^2}=\frac{9}{2}$

$\Rightarrow$ 9(225 - x^2) = 1800

$\Rightarrow$ 225 - x^2 = 200

$\Rightarrow$ x = 5

Speed of stream = 5 km/hr

17. $p^2x^2 + (p^2 - q^2)x - q^2 = 0$

$a = p^2, b = p^2 - q^2, c = -q^2$

$D = b^2 - 4ac$

$= (p^2 - q^2)^2 - 4 \times p^2(-q^2)$

$= p^4 + q^2 - 2p^2q^2 + 4p^2q^2$

$= (p^2 + q^2)^2$

$x=\frac{-b\pm\sqrt{D}}{2a}$

$=\frac{-(p^2-q^2)\pm\sqrt{(p^2+q^2)}^2}{2\times p^2}=\frac{-p^2+q^2+p^2+q^2}{2p^2}$

or $x=\frac{-p^2+q^2-p^2-q^2}{2p^2}$

$x=\frac{2q^2}{2p^2}$ or $x=\frac{-2p^2}{2p^2}$

$x=\frac{q^2}{p^2}$ or $x=-1$

18. Distance travelled by the train = 480 km

Let the speed of the train be x kmph

Time taken for the journey = $\frac{480}{x}$

Given speed is decreased by 8 kmph

Hence the new speed of train = (x – 8) kmph

Time taken for the journey = $\frac{480}{x-8}$

$\frac{480}{x-8} = \frac{480}{x} + 3$

$\Rightarrow \frac{480}{x-8} - \frac{480}{x} = 3$

$\Rightarrow \frac{480(x-x+8)}{x(x-8)} = 3$

$\Rightarrow \frac{480\times 8}{x(x-8)} = 3$

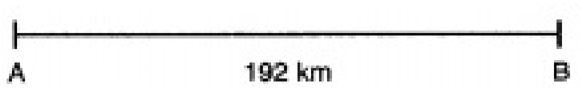
$\Rightarrow 3x(x-8) = 480 \times 8$

$\Rightarrow x(x-8) = 160 \times 8$

$\Rightarrow$ x^2 - 8x - 1280 = 0

On solving we get x = 40

Thus the speed of train is 40 kmph.

19. 

Let speed of passenger train be x km/h

$\therefore$ speed of superfast train = (x + 16) km/h

By question, $T_{passenger} = \frac{192}{x}$ and $\mathrm{T}_{\text{superfast}} = \frac{192}{(x+16)}$

or, $\frac{192}{x} - \frac{192}{x+16} = 2$

or, $192(x+16) - 192x = 2\left(x^2 + 16x\right)$

or, $192x + 192 \times 16 - 192x = 2\left(x^2 + 16x\right)$

192x + 3072 - 192x $= 2\left(x^2 + 16x\right)$ (divide throughout by 2, we get,

96x + 1536 - 96x $= \left(x^2 + 16x\right)$

or x(x + 48) - 32(x + 48) = 0

or, (x - 32) (x + 48) = 0

or, x = 32 or - 48

Since speed can't be negative, therefore - 48 is not possible.

$\therefore$ Speed of passenger train = 32 km/h and Speed of fast train = 48 km/h

20. We have given,

$\left(\frac{2x}{x-5}\right)^2 + 5\left(\frac{2x}{x-5}\right) - 24 = 0$

Let $\frac{2x}{(x-5)}$ be y

$\therefore \quad y^2 + 5y - 24 = 0$

Now factorise,

$y^2 + 8y - 3y - 24 = 0$

$$y(y+8)-3(y+8)=0$$
$$(y+8)(y-3)=0$$
$$y=3,-8$$
Putting y=3
$$\frac{2x}{x-5}=3$$
2x = 3x - 15

x = 15

Putting y = -8
$$\frac{2x}{x-5}=-8$$
2x = -8x + 40

10x = 40

x = 4Hence, x is 15 , 4