

Topics : Statistics, Fundamentals of Mathematics, Three Dimensional Geometry, Indefinite Integration

Type of Questions		M.M., Min.
Single choice Objective (no negative marking) Q.1	(3 marks, 3 min.)	[3, 3]
Subjective Questions (no negative marking) Q.2 to Q.6	(4 marks, 5 min.)	[20, 25]
Match the Following (no negative marking) Q.7	(8 marks, 8 min.)	[8, 8]
Assertion and Reason (no negative marking) Q.8	(3 marks, 3 min.)	[3, 3]

1. If $P = \sum_{r=1}^n r^2$, $Q = \sum_{m=1}^n \sum_{r=1}^m r - \frac{1}{2} \sum_{r=1}^n r$, then $\frac{P}{Q}$ is equal to

- (A) 4 (B) $\frac{1}{2}$ (C) 2 (D) None of these

2. In a library a set of 96 books of mathematics, 240 books of physics and 336 books of chemistry. If stacks of same subject books have to be formed, find the minimum number of stacks required for the maths, physics, chemistry stacks (given each stacks as same number books).

3. Find the shortest distance and the equation of the line of shortest distance between the following pairs of lines.

$$\frac{x-2}{1} = \frac{y-3}{1}, z = 4 \text{ and } \frac{x-1}{1} = \frac{2-y}{1}, z = 3$$

4. Evaluate :

(i) $\int \frac{e^{3x} + e^{5x}}{e^x + e^{-x}} dx$ (ii) $\int \left(x^{3/2} + \frac{1}{\sqrt{x}} \right) dx$ (iii) $\int \frac{x^6 + 1}{x^2 + 1} dx$

5. Evaluate

(i) $\int (\sin^4 x - \cos^4 x) dx$ (ii) $\int e^{x \ln a} \cdot e^x dx$

6. Evaluate

(i) $\int \frac{e^{5 \ln x} - e^{4 \ln x}}{e^{3 \ln x} - e^{2 \ln x}} dx$ (ii) $\int \frac{1}{1 + \sqrt{x}} dx$

7. Match the column

Column – I

(A) The plane $x - 2y + 7z + 21 = 0$ contains the line

(B) An equation of the line passing through $3\hat{i} - 5\hat{j} + 7\hat{k}$ and perpendicular to the plane $3x - 4y + 5z = 8$ is

(C) Equation of the line of shortest distance between the lines

$$x = y = z \text{ and } \frac{x-1}{2} = \frac{y}{1} = \frac{z}{-1} \text{ is}$$

(D) The line of intersection of the planes $\vec{r} \cdot (3\hat{i} - \hat{j} + \hat{k}) = 1$ and $\vec{r} \cdot (\hat{i} + 4\hat{j} - 2\hat{k}) = 2$ is parallel to the line given by

Column – II

(p) $\frac{x-3}{3} = \frac{y+5}{-4} = \frac{z-7}{5}$

(q) $\frac{x+1}{-3} = \frac{y-3}{2} = \frac{z+2}{1}$

(r) $\frac{x-3}{-2} = \frac{y-1}{7} = \frac{z-4}{13}$

(s) $\frac{(7x-1)}{2} = \frac{(7y-1)}{-3} = \frac{(7z-1)}{1}$

8. **Statement-1** : The three planes $x + ay + (b + c)z + d = 0$, $x + by + (c + a)z + d = 0$ and $x + cy + (a + b)z + d = 0$ have a line in common.

Statement-2 : If $a_1x + b_1y + c_1z + d_1 = 0$, $a_2x + b_2y + c_2z + d_2 = 0$ and $a_3x + b_3y + c_3z + d_3 = 0$ passes

through a common line then
$$\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = 0.$$

- (A) Statement-1 is correct and statement-2 is correct and statement-2 is correct explanation of statement-1
- (B) Statement-1 and statement-2 both are correct but statement-2 is not correct explanation of statement-1
- (C) Statement-1 is false but statement-2 is true
- (D) Statement-1 is true but statement-2 is false

Answers Key

1. (C) 2. $x = 2, 5, 7$ 3. $PQ = 1, x = 1, y = 2$

4. (i) $\frac{e^{4x}}{4} + c$ (ii) $\frac{2}{5}x^{5/2} + 2\sqrt{x} + c$

(iii) $\frac{x^5}{5} - \frac{x^3}{3} + x + c$

5. (i) $\frac{-\sin 2x}{2} + c$ (ii) $\frac{(ae)^x}{\ln(ae)} + c$

6. (i) $\frac{x^3}{3} + c$ (ii) $2\sqrt{x} - 2 \log |1 + \sqrt{x}| + c$

7. (A) \rightarrow (q), (B) \rightarrow (p), (C) \rightarrow (s), (D) \rightarrow (r)

8. (B)