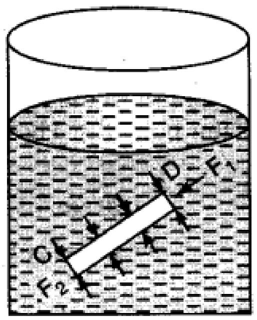
Long Answer Type Questions

Q. 1. State and prove Pascal's law and give its experimental proof also.

Ans. It states that if gravity effect is neglected, the pressure at every point of liquid in equilibrium of rest is same.

Proof: Consider two points C and D inside the liquid in a container which is in equilibrium of rest. Imagine a right circular cylinder with axis CD of uniform cross-sectional area A such that points C and D lie on float faces of the cylinder in figure.



The liquid inside the cylinder is in equilibrium under the action of forces exerted by the liquid outside the cylinder. These forces are acting everywhere perpendicular to the surface of the cylinder. Thus force on the flat faces of the cylinder at C and D will perpendicular to the forces on the curved surface of the cylinder. Since the liquid is in equilibrium, therefore, the sum of forces acting on the curved surface of the cylinder must be zero. If P1 and P2 are the pressure at points C and D and F1 and F2 are the forces acting on the flat faces of the cylinder due to liquid, then

$$F_1 = P_1A$$

and $$F_2 = P_2A$$

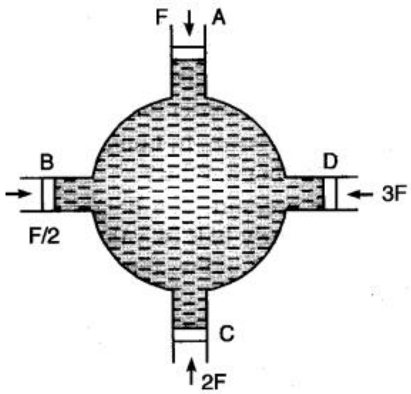
Since the liquid is in equilibrium, therefore

$$F_1 = F_2$$

or $$P_1A = P_2A$$

or $$P_1 = P_2$$

It means the pressure at C and D are the same.



Experimental Proof. Consider a spherical vessel having for cylindrical tubes A, B, C and D each fitted with air tight frictionless piston of area of cross-section a, $a/2$, $2a$ and $3a$ respectively.

Fill the vessel with an incompressible liquid so that no air gap is left inside the vessel and piston fitted in the various cylindrical tubes. Push the piston A with force F. The pressure developed on the liquid = F/a = P(say).

It is seen that all other pistons will be pushed outwards. To keep the pistons at their respective original positions, the force F/2, 2F and 3F respectively required to be applied on pistons of tubes B, C and D respectively to hold them. Now pressure developed on liquid in tubes, B, C and D are $\frac{F/2}{a/2}$, 2F/2a, 3F/3a i.e., such,equal to F/a. This indicates that the pressure applied is transmitted equally to all parts of liquid. This proves Pascal law.

Q.2. Find the excess pressure:

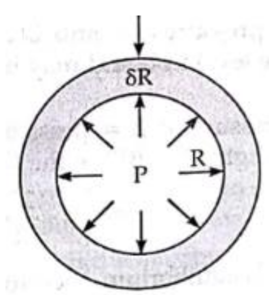
(i) Inside a liquid drop,
(ii) Inside liquid bubble,
(iii) Inside a bubble in liquid,

Ans. (i) Inside a liquid drop. Consider a liquid drop of radius R. the molecules lying on the surface of liquid drop, due to surface tension will experience resultant force acting inward ⊥ to the surface.

Let S = surface tension of liquid drop

P = excess pressure inside the drop

Due to excess of pressure, let there be increase in the radius of the drop by a small quantity δR as shown in figure.



Then work done by the excess pressure.

W = Force × Displacement

= (Excess pressure × Area × Increase in radius)

= P × $4\pi R^2$ × δR …(i)

Increase in surface area of the drop

= Final surface area – Initial surface area

= $4\pi(R+\delta R)^2 - 4\pi R^2$

= $4\pi[R^2 + 2R(\delta R) + (\delta R)^2 - R^2]$

= $8\pi R\delta R$ [Neglecting, $(\delta R)^2$ being very very small]

∴ Increase in surface energy

= increase in surface area × surface tension

= $8\pi R(\delta R) \times S$ …(ii)

As the increase in the surface energy is at the cost of work done by the excess pressure, therefore from (i) and (ii) P × $4\pi R^2$ × δR = $8\pi R\delta R \times S$

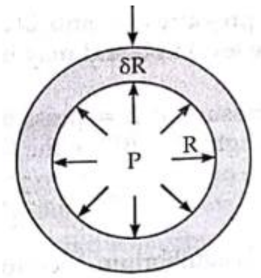
$$P = \frac{2S}{R}$$

(ii) Inside a liquid bubble. Consider a soap bubble of radius R the molecules lying on the surface of liquid bubble will experience a resultant force acing on water ⊥ to the surface due to the surface tension.

Let S = surface tension of the sop solution

P = excess of pressure inside the bubble.

Due to it, let there be increase in the radius of the bubble by a small amount Δr in fig.



Then work done,

W = Force × Distance

= (Excess pressure × Area) × Increase in radius

= P × $4\pi R^2$ × δR …(iii)

The soap bubble has two free surface, one outside the bubble when soap solution and air are in contact.

∴ The effective increase in surface area of the bubble

= 2[Final S.A. – Initial S.A.]

= $2[4\pi(R+\delta R)^2 - 4\pi R^2]$

= $2 \times 4\pi[R^2 + 2R(\delta R) - R^2]$

= $8\pi \times 2R(\delta R)$ [Neglecting $(\delta R)^2$, being very small]

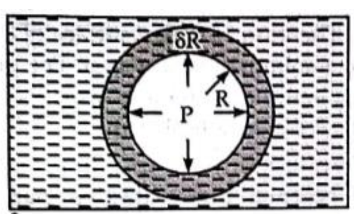
= $16\pi R\delta R \times S$ …(iv)

∵ Increase in surface energy is as the cost of work done by the excess pressure therefore from (iii) and (iv),

$P \times 4\pi R^2 \times (\delta R) = 16\pi R(\delta R) \times S$

or $P = \frac{4S}{R}$

(iii) Inside a bubble in liquid. Consider an air bubble of radius R. Just inside a liquid of surface tension S.



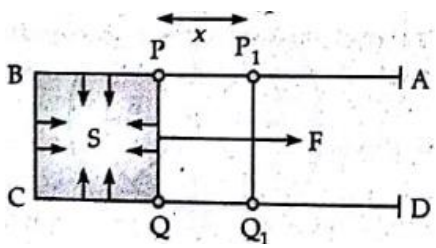
The air bubble will have only one free surface as shown in figure. It can be shown that pressure inside the air bubble is given by

$P = \frac{2S}{R}$

Q.3. Explain surface energy. Establish its relation with surface tension.

Ans. Surface energy is defined as the amount of the work done against the force of surface tension, in forming the liquid surface of a given area at a constant temperature.

To obtain an expression for surface energy, take a rectangular frame ABCD having a wire PQ which can slide along the sides AB and CD. Dip the frame in soap solution and form a soap film BCQP on the rectangular frame. There will be two free surfaces of film where air and soap are in contact.



Let S = Surface of the soap solution.

$l =$ Length of the wire PQ.

Since there are two free surface of the film and surface tension acts on both of them, hence total inward force on the wire PQ is

$$F = S \times 2l$$

To increase the area of the soap film we have to pull the sliding wire PQ outwards with a force F. Let the film be stretched by displacing wire PQ through a small distance x to the position P_1Q_1.

The increase in area of film PQQ_1P_1

$$= a$$

$$= 2(l + x)$$

∴ Work done in stretching film is

$$E = \text{Force applied} \times \text{Distance moved}$$

$$= (S \times 2l) \times x$$

$$= S \times (2ix)$$

$$= S \times a$$

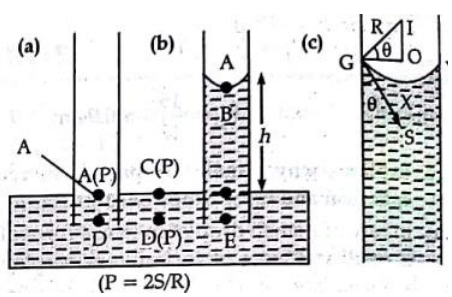
Where $2lx = a =$ increase in area of the film in both sides

∴ $l \times x$ = increase in area of film on both sides

If temperature of the film remains constant in this process, this work done is stored in the film as its surface energy.

Q.4. Derive the formula for rise of liquid in a capillary tube (Ascent formula).

Ans. When one end of capillary tube of radius r is immersed into a liquid of density ρ which wets the sides of capillary tube (say water and capillary tube of glass), the shape of the liquid is in the tube becomes concave upwards in figure.



Let R = Radius of curvature of liquid meniscus

P = Atmospheric pressure

S = Surface tension of the liquid,

The pressure at point A just above the liquid meniscus in the capillary tube is atmospheric pressure = P

The pressure at point B, just below the liquid meniscus (on convex side).

= P -2S/R.

Pressure at point C and D, just above and below the place surface of liquid in the vessel is also P (i.e. atmospheric pressure). The point B and D are in the same horizontal plane in liquid but the pressure at these point is different. Hence there will be an equilibrium.

In order to maintain an equilibrium the liquid level rises in the capillary tube upto a height h, so that the pressure at point D and E which are in the same level in liquid may become equal, from figure.

Now, pressure at E = pressure at B + pressure due to height h (= BE) of the liquid column

$$= \left(\mathrm{P} - \frac{2\mathrm{S}}{\mathrm{R}}\right) + h\rho g$$

As there is equilibrium, therefore

Pressure at E = Pressure at D

i.e., $\mathrm{P} - \frac{2\mathrm{S}}{\mathrm{R}} + h\rho g = \mathrm{P}$

or $h\rho g = \frac{2\mathrm{S}}{\mathrm{R}}$

or $h = \frac{2\mathrm{S}}{\mathrm{R}\rho g}$ …(1)

Calculation of R. Let I be the centre of curvature of liquid meniscus GXY in the tube and GS be the tangent to the liquid surface at point G.

GI = R, GO = r

∠IGO = θ = angle of contact

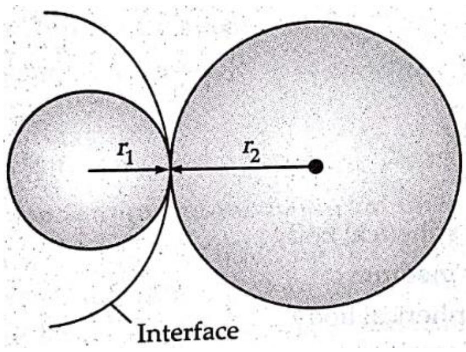
In ΔIGO, $\cos\theta = \frac{\mathrm{GO}}{\mathrm{GI}} = r/\mathrm{R}$

$$\mathrm{R} = \frac{r}{\cos\theta}$$

Putting this value in (1) eqn.

$$h = \frac{2\mathrm{S}\cos\theta}{r\rho g}$$

Q.5. Derive an expression for the radius of interference when two soap bubble of different radii are in contact.



Ans. Consider two soap bubbles of radii r_1 and r_2 in contact with each other as shown in figure.

Let r be the radius of the common boundary. If P_1 and P_2 are the excess pressure on the two sides of one interface then the resultant excess pressure

P = $P_1 - P_2$

$$\frac{4S}{r} = \frac{4S}{r_1} - \frac{4S}{r_2}$$

Or $$\frac{1}{2} = \frac{1}{r_1} - \frac{1}{r_2}$$

$$r = \frac{r_1 r_2}{r_2 - r_1}$$

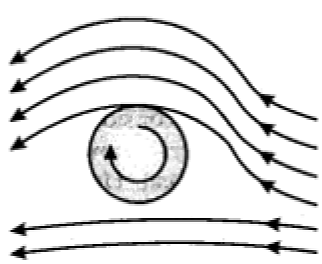
Q.6. (a) Explain Magnus effect in case of spinning ball.

(b) Name a device used to measure the fluid flow. On which principle it works. Derive its expression with diagram.

OR

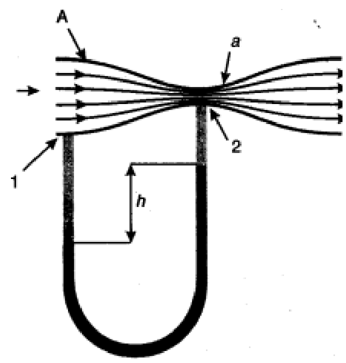
Liquid drops are spherical in shape. Why?
Derive the expression of excess pressure inside the liquid drop.
Give two similarities and dissimilarities between friction and viscosity.

Ans. (a) Magnus effect: A ball which spinning, drags air along with it. If the surface is rough, more air will dragged. Figure shows the streamlines of air for a ball which is moving and spinning at the same time. The ball is moving forward and relative to it the air is moving backwards. Therefore, the velocity of air below the ball relative to it is large and above it is smaller. The streamlines get crowded below and rarified above:



This difference in the velocity of air results in the pressure difference between the lower and upper faces and there is a net downward force on the ball. The difference in lateral pressure, which causes a spinning ball to take a curved path which is convex towards greater pressure side is called magnus effect.

(b) Venturi – Meter : The venture – meter is a device to measure the flow of speed of incompressible fluid. It consists of a tube with a broad diameter and small construction at the middle as shown in figure. A manometer is in the form of a U tube is also attached to it, with one arm at the board neck point of the tube and the other constriction as show in figure. The manometer contains a liquid of density P_m. The speed v_1 of the liquid flowing through the tube at the broad neck area. A is to be measured from equation of continuity e.g., the speed at the constriction becomes,



A schematic diagram of Venturi – meter

$$v_2 = \frac{A}{a} v_1.$$

Then using Bernoulli's is equation, we get

$$P_1 + \frac{1}{2}\rho v_1^2 = P_2 + \frac{1}{2}\rho v_1^2 (A/a)^2$$

so that

$$P_1 - P_1 = \frac{1}{2}\rho v_1^2 \left[\left(\frac{A}{a}\right)^2 - 1\right]$$

This pressure difference causes the fluid in the U tube connected at the narrow neck to rise in comparison of the other arm. The difference in height h measure the pressure difference

$$P_1 - P_2 = \rho_m g h$$

$$= \frac{1}{2}\rho \mathrm{v}_1^2 \left[\left(\frac{A}{a}\right)^2 - 1\right]$$

So, that the speed of fluid at wide neck is

$$\mathrm{V}_1 = \sqrt{\left(\frac{2\rho_m g h}{\rho}\right)}\left[\left(\frac{A}{a}\right)^2 - 1\right]$$

The principle behind this meter has many applications. The carburetor of an automobile has a venture channel (nozzle) through which air flows with a large speed. The pressure is then lowered at the narrow neck and the petrol (gasoline) is sucked up in the chamber to provide the correct mixture of air to fuel necessary for combustion. Filter pumps or aspirators, Bunsen burner atomisers and sprayers used for perfumes or to spray insecticide work on the same principle.

OR

(a) Liquid drops: One consequence of surface tension is that free liquid drops and bubbles are spherical if effect of gravity can be neglected.

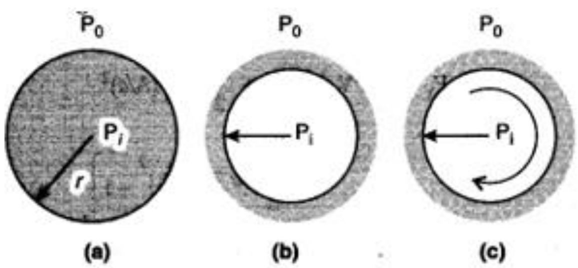
A liquid air interface has energy, so for a given value the surface with minimum energy is the one with the least area. The sphere has this property. So, if gravity and outer forces were ineffective, liquid drops would be spherical and the pressure inside a spherical drop is more than that the pressure outside.

(b) If the drop is in equilibrium, this energy cost is balanced by the energy gain due to expansion under the pressure difference $(P - P_0)$ between the inside of the bubble and the outside, the work done is

$$W = [P_1 - P_0]4\pi r^2 \Delta r$$

i.e., $[P_1 - P_0] = \left(\frac{2S}{r}\right)$

in general, for a liquid gas interface, the concave side has a higher pressure than convex side. For example, an air bubble in a liquid, would have higher pressure inside it.



Drop, cavity and bubble of radius r.

A bubble in fig. (c) differs from a drop. Applying the above argument, we have

$$[P_1 - P_0] = (4S/r)$$

This probably why you have to blow hard, but not too hard, to from a soap bubble. A little extra air pressure is needed inside.

(c) Similarities between friction & viscosity:

(i) Viscosity is due to the friction between neighbouring particles in a fluid that are moving at different velocities. Fluid friction also called as viscosity.

(ii) Viscosity and friction both depends on the size and shape of its particles and the attractions between the particles.

Dissimilarities:

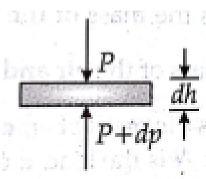
(i) Viscosity is the measure of the resistance of a fluid, which is deformed by either shear stress or tensile stress. While friction is caused by the contact of two rough surfaces.

(ii) The viscosity of a fluid depends on the temperature. It decreases as the temperature is increased. While friction depends on the surface area.

Q.7. (a) Pressure decreases as one ascends the atmosphere. If the density of air is ρ, what is the change in pressure do over a differential height dh?

(b) Considering the pressure p to be proportional to the density, find the pressure p at a height h is the pressure on the surface of the earth is p_0.
(c) If p_0 = 1.03 × 10^5 Nm^{-2}, p_0 = 129 kgm^{-3} and g = 9.8 ms^{-2}, at which height will the pressure drop to (1/10) the value at the surface of the earth?
(d) This model of the atmosphere works for relatively small distances. Identify the underlying assumption that limits the model.

Ans. (a) Consider a horizontal parcel of air with cross section A and height dh. Le the pressure on the top surface and bottom surface be p and $p + dp$. If the parcel is in equilibrium, then the net upward force must be balanced by the weight.



i.e. $(p + dp) - pA = -\rho g A dh$ [such that, Weight = Density × Volume × g]

$or\ dp = -\rho g dh$ [here, ρ = density of air] (-v) sign indicated pressure decrease with height.

(b) Let the density of air on the earth's surface be ρ_0, then

$$\frac{p}{p_0} = \frac{\rho}{\rho_0}$$

Or $\rho = \frac{\rho_0}{p_0} p$

∴ $dp = -\frac{\rho_0 g}{p_0} p dh$ [such that, $dp = -pgdh$]

or $\frac{dp}{p} = \frac{\rho_0 g}{p_0} dh$

or $\int_{p_0}^{p} \frac{dp}{p} = -\frac{\rho_0 g}{p_0} \int_0^h dh$ $[at\ h = 0, r = p_0]$ $[at\ h = h,\ p = p]$

or $In \frac{p}{p_0} = -\frac{\rho_0 g}{p_0} h$

Taking antilog,

Or $p = p_0$ exp$\left(-\frac{\rho_0 g}{p_0} h\right)$

(c) $In \frac{p}{p_0} = -\frac{\rho_0 g h}{\rho_0}$

$In \frac{1}{10} = -\frac{\rho_0 g}{p_0} h_0$

$\therefore \quad h_0 = -\frac{p_0}{\rho_0 g} In\frac{1}{10} = \frac{-p_0}{\rho_0 g} In(10)^{-1} = \frac{p_0}{\rho_0 g} In\ 10$

$= \frac{p_0}{\rho_0 g} \times 2.303$

$= \frac{1.013\times 10^5}{1.29\times 9.8} \times 2.303 = 0.184 \times 10^5 m = 18.4$ km

(d) The assumption $p \propto \rho$ is valid only for the isothermal case which is only valid for small distances.

Temperature remains constant only near the surface of the earth, not at greater heights

Q.8. Surface tension is exhibited by liquids due to force of attraction between molecules of the liquid. The surface tension decrease with increase in temperature and vanishes at boiling point. Given that the latent heat of vaporization for water $L_0 = 540\ kcl\ kg^{-1}$, the mechanical equivalent of heat J = 4.2 J cal^{-1}, density of water $\rho_w = 10^3$ kgm^{-3}, Avagadro's No $N_A = 6.0 \times 10^{26}$ k $mole^{-1}$ and the molecular weight of water $M_A = 18$ kg for 1 mole.

(a) Estimate the energy required for one molecule of water to evaporate.

(b) Show that the inter-molecular distance for water is $d = \left[\frac{M_A}{N_A} \times \frac{1}{\rho_W}\right]^{1/3}$ and find its value.

(c) 1 g of water in the vapour state at 1 atm occupies 1601 cm^3. Estimate the intermolecular distance at boiling point, in the vapour state.

(d) During vaporization a molecule overcome a force F< assumed constant to go from an intermolecular distance d to d'. Estimate the value of F, where d = 3.1 × 10^{-10} m.

(e) Calculate F/d which is a measure of the surface tension.

Ans. Given, $L_v = 540\frac{kcal}{Kg}$

$= 540 \times 10^3 \times 4.2$ J/Kg

(a) Such that, 1kg of water requires energy for evaporation = L_v k cal

$\therefore M_A$ kg of water requires $M_A L_v k$ cal

Since there are N_A molecules to evaporates is

$u = \frac{M_A L_v}{N_A} J$ [Here, $N_A = 6 \times 10^{26}$ = Avogadro number]

$= \frac{18\times 540\times 4.2\times 10^3}{6\times 10^{26}}$

$= 90 \times 18 \times 4.2 \times 10^{-23}$ J

$= 6.8 \times 10^{-20}$ J

(b) Let us consider the water molecules to be points at a distance d from each other.

Volume of N_A molecules = $\frac{M_A}{\rho_w} l$

Thus, the volume around one molecule is $\frac{M_A}{N_A \rho_w} l$

The volume around one molecule is $d^3 = (M_A / N_A \rho_w)$

$\therefore d = \left(\frac{M_A}{N_A \rho_w}\right)^{\frac{1}{3}} = \left(\frac{18}{6\times 10^{26} \times 10^3}\right)^{\frac{1}{3}}$

$= (3 \times 10^{-30})^{1/3}\ m \simeq 3.1\ \times 10^{-10}$ m

(c) 1 kg of vapour occupies = 1601×10^{-3} m^3

$\therefore$ 18 kg of vapour occupies = $18 \times 1601 \times 10^{-3}$ m^3

or 6×10^{26} molecules occupies = $18 \times 1601 \times 10^{-3}$ m^3

$\therefore$ 1 molecule occupies = $\frac{18 \times 1601 \times 10^{-3}}{6 \times 10^{26}} m^3$

If d_1 is the inter molecular distance. Then

$d_1^3 = (3 \times 1601 \times 10^{-29})$ m^3

$\therefore d_1 = (30 \times 1601)^{1/3} \times 10^{-10}$ m

$= 36.3 \times 10^{-10}$ m

(d) To change the distance from d to d_1,

Work done = F$(d_1 - d)$

Work done = energy required to evaporate 1 molecule

Or F = $\frac{6.8 \times 10^{-20}}{d_1 - d}$

$= \frac{6.8 \times 10^{-20}}{(36.3 - 3.1) \times 10^{-10}}$

= 0.2048×10^{-10} N

(e) Surface Tension = $\frac{F}{d}$

$= \frac{2.05 \times 10^{-20}}{3.1 \times 10^{-10}}$

= 6.6×10^{-2} Nm^{-1}

Q.9. A hot air balloon is a sphere of radius 8 m. the air inside is at a temperature of 60^{o}C. how large a mass can the balloon lift when the outside temperature is 20^{o}C? (Assume air is an ideal gas, R = 8.314 J mol^{-1} K^{-1}, 1 atm. = 1.013×10^3 Pa; the membrane tension is 5 Nm^{-1}.)

Ans. Let the pressure inside the balloon be P_i and the outside pressure be P_0

$P_i - P_0 = \frac{2T}{r}$

[T = Surface Tension, r = radius balloon]

Considering the air to be an ideal gas

$$P_i V = n_i R T_i$$

Where, V is the volume of the air inside the balloon, n_i is the number of moles inside and T_i is the temperature inside, and $P_0 V = n_0 R T_0$ where V is the volume of the air displaced and $n_0$is the number of moles displaced and T_0 is the temperature outside.

So, $n_i = \frac{P_i V}{R T_i} = \frac{M_i}{M_A}$

Where M_i is the mass of the air inside and M_A is the molar mass of the air and $n_0 = \frac{P_0 V}{R T_0} = \frac{M_0}{M_A}$ where M_0 is the mass of air outside that has been displaced. If W is the load it can raise, then

$$W + M_i g = M_0 g$$

Or $\quad W = M_0 g - M_i g$

In atmospheric air, 21% O_2 and 79% N_2 is present.

∴ Molar mass of air

$M_A = 0.21 \times 32 + 0.79 \times 28$ = 28.84 g

∴ Weight raised by balloon

or W = $\frac{M_A V}{R}\left(\frac{P_0.}{T_0} - \frac{P_i}{T_i}\right) g$ here, $P_0 = P_i$

$$= \frac{0.02884 \times \frac{4}{3}\pi \times 8^3}{8.314} \times 1.013 \times 10^5 \left(\frac{1}{293} - \frac{1}{333}\right) \times 9.8 \text{ N}$$

= 3044.2 N

Therefore, Mass lifted by balloon = $\frac{W}{g}$

$$= \frac{3044.2 \text{ N}}{9.8 \frac{m}{s^2}}$$

= 310.6 Kg