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EXERCISE 7.9

Q No. 1 $\int_{-1}^{1} (x+1)\,dx$

Sol. $\int_{-1}^{1} (x+1)\,dx = \left[\frac{x^2}{2}+x\right]_{-1}^{1} = \left(\frac{1^2}{2}+1\right) - \left(\frac{(-1)^2}{2}+(-1)\right)$

$= \left(\frac{1}{2}+1\right) - \left(\frac{1}{2}-1\right) = 2$

$\left[\because \int_a^b f(x)\,dx = \left[F(x)\right]_a^b = F(b)-F(a)\right.$ where $\int f(x)\,dx = F(x)$

Q No. 2 $\int_2^3 \frac{1}{x}\,dx$

Sol. $\int_2^3 \frac{1}{x}\,dx = \left[\log|x|\right]_2^3 = \left[\log x\right]_2^3 = \log 3 - \log 2 = \log(3/2)$

Q No. 3 $\int_1^2 (4x^3 - 5x^2 + 6x + 9)\,dx$

Sol. $\int_1^2 (4x^3-5x^2+6x+9)\,dx = \left[4\cdot\frac{x^4}{4} - 5\cdot\frac{x^3}{3} + 6\frac{x^2}{2} + 9x\right]_1^2$

$= \left(2^4 - \frac{5\times 2^3}{3} + \frac{6\times 2^2}{2} + 9\times 2\right) - \left(1 - \frac{5}{3} + 3 + 9\right)$

$= \left(16 - \frac{40}{3} + 12 + 18\right) - \left(13 - \frac{5}{3}\right) = \left(46 - \frac{40}{3}\right) - \left(\frac{34}{3}\right) = \frac{138-74}{3} = \frac{64}{3}$

Q No 4: $\int_0^{\pi/4} \sin 2x \, dx$.

Sol. $\int_0^{\pi/4} \sin 2x \, dx = \left[-\frac{\cos 2x}{2}\right]_0^{\pi/4} = -\frac{1}{2}\left[\cos 2x\right]_0^{\pi/4} = -\frac{1}{2}\left[\cos \pi/2 - \cos 0\right]$

$= -\frac{1}{2}\left[0-1\right] = \frac{1}{2}$.

Q No 5. $\int_0^{5} \cos 2x \, dx$

Sol. $\therefore \int_0^{5} \cos 2x \, dx = \left[\frac{\sin 2x}{2}\right]_0^{\pi/2} = \frac{1}{2}\left[\sin \pi - \sin 0\right] = \frac{1}{2}(0-0) = 0$

Q No 6: $\int_4^5 e^x dx = \left[e^x\right]_4^5 = e^5 - e^4$.

Q No 7: $\int_0^{\pi/4} \tan x \, dx = \left[-\log|\cos x|\right]_0^{\pi/4} = -\log\left|\cos\frac{\pi}{4}\right| - \left(-\log|\cos 0|\right)$

$= -\log\left(\frac{1}{\sqrt{2}}\right) + \log 1 = -\log\left(\frac{1}{\sqrt{2}}\right) + 0 \quad \left[\because \log 1 = 0\right]$

$= -\log 1 - \left(-\log\sqrt{2}\right) = \log(\sqrt{2}) = \log(2^{1/2}) = \frac{1}{2}\log 2$

Q No 8: $\int_{\pi/6}^{\pi/4} \operatorname{cosec} x \, dx = \left[\log\left|\tan\frac{x}{2}\right|\right]_{\pi/6}^{\pi/4}$

$= \log\left|\tan\frac{\pi}{8}\right| - \log\left|\tan\frac{\pi}{12}\right| = \log\left(\tan\frac{\pi}{8}\right) - \log\left(\tan\frac{\pi}{12}\right)$

$= \log\left(\tan\frac{\pi}{8} / \tan\pi/12\right) \log\left(\frac{\sqrt{2}-1}{2-\sqrt{3}}\right)$

OR

$\int_{\pi/6}^{\pi/4} \operatorname{cosec} x \, dx = \left[\log\left|\operatorname{cosec} x - \cot x\right|\right]_{\pi/6}^{\pi/4}$

$= \log\left|\operatorname{cosec}\frac{\pi}{4} - \cot\frac{\pi}{6}\right| - \log\left|\operatorname{cosec}\frac{\pi}{6} - \cot\frac{\pi}{6}\right|$

$= \log\left|\sqrt{2}-1\right| - \log\left|2-\sqrt{3}\right|$

$= \log\left(\sqrt{2}-1\right) - \log\left(2-\sqrt{3}\right) = \log\left(\frac{\sqrt{2}-1}{2-\sqrt{3}}\right)$

Q No 9: $\int_0^1 \frac{dx}{\sqrt{1-x^2}} = \left[\sin^{-1}x\right]_0^1 = \sin^{-1}(1) - \sin^{-1}(0) = \frac{\pi}{2} - 0 = \frac{\pi}{2}$

Q No 10. $\int_0^1 \frac{dx}{1+x^2} = \left[\tan^{-1}x\right]_0^1 = \tan^{-1}(1) - \tan^{-1}(0) = \frac{\pi}{4} - 0 = \frac{\pi}{4}$

Q No 11. $\int_2^3 \frac{dx}{x^2-1} = \left[\frac{1}{2}\log\left|\frac{x-1}{x+1}\right|\right]_2^3 = \frac{1}{2}\left[\log\left|\frac{3-1}{3+1}\right| - \log\left|\frac{2-1}{1+2}\right|\right]$

$$= \frac{1}{2}\left[\log\frac{1}{2} - \log\frac{1}{3}\right] = \frac{1}{2}\log\left[\frac{\frac{1}{2}}{1/3}\right] = \frac{1}{2}\log\frac{3}{2}.$$

Q No 12. $\int_0^{\pi/2} \cos^2 x\, dx = \int_0^{\pi/2}\left(\frac{1+\cos 2x}{2}\right)dx = \frac{1}{2}\int_0^{\pi/2}(1+\cos 2x)\,dx.$

$$= \frac{1}{2}\left[\left[x\right]_0^{\pi/2} + \left[\frac{\sin 2x}{2}\right]_0^{\pi/2}\right] = \frac{1}{2}\left[\left(\frac{\pi}{2}-0\right) + \left(\frac{\sin\pi}{2} - \frac{\sin 0}{2}\right)\right]$$

$$= \frac{\pi}{4} + 0 - 0 = \frac{\pi}{4}.$$

Q No 13 $\int_2^3 \frac{x\,dx}{x^2+1} = \frac{1}{2}\int_2^3 \frac{2x}{x^2+1}\,dx = \frac{1}{2}\int_2^3 \frac{\frac{d}{dx}(x^2+1)}{(x^2+1)}\cdot dx.$

$$= \frac{1}{2}\left[\log|x^2+1|\right]_2^3 = \frac{1}{2}\left[\log|3^2+1| - \log|2^2+1|\right]$$

$$= \frac{1}{2}\left[\log 10 - \log 5\right] = \frac{1}{2}\log\left(\frac{10}{5}\right) = \frac{1}{2}\log 2.$$

Q No 14 $\int_0^1 \frac{2x+3}{5x^2+1}\,dx = \int_0^1 \frac{2x}{5x^2+1}\,dx + \int_0^1 \frac{3}{5x^2+1}\,dx.$

$$= \frac{1}{5}\int_0^1 \frac{10x}{5x^2+1}\,dx + \frac{3}{5}\int \frac{1}{x^2+\frac{1}{5}}\,dx$$

$$= \frac{1}{5}\left[\log|5x^2+1|\right]_0^1 + \frac{3}{5}\times\frac{1}{1/\sqrt{5}}\left[\tan^{-1}\frac{x}{1/\sqrt{5}}\right]_0^1$$

$$= \frac{1}{5}\left[\log|6| - \log|1|\right] + \frac{3}{\sqrt{5}}\left[\tan^{-1}\sqrt{5} - 0\right]$$

$$= \frac{1}{5}\log 6 + \frac{3}{\sqrt{5}}\tan^{-1}(\sqrt{5})$$

Q No. 15 $\int_0^1 x \cdot e^{x^2} dx$

Sol.

Let $I = \int x e^{x^2} dx$

Put $x^2 = t \Rightarrow 2x\,dx = dt \Rightarrow x\,dx = \frac{dt}{2}$

$\therefore I = \frac{1}{2}\int e^t dt = \frac{1}{2}e^t = \frac{1}{2}e^{x^2}$

$\therefore \int_0^1 x \cdot e^{x^2} dx = \left[\frac{1}{2}e^{x^2}\right]_0^1 = \frac{1}{2}\left[e^1 - e^0\right] = \frac{e-1}{2}$

Q No 16 $\int_1^2 \frac{5x^2}{x^2+4x+3} dx$

Now $\frac{5x^2}{x^2+4x+3} = 5 + \frac{-20x-15}{x^2+4x+3}$ [By Long division method]

$= 5 - \frac{20x+15}{(x+1)(x+3)}$

Now $\frac{20x+15}{(x+1)(x+3)} = \frac{A}{x+1} + \frac{B}{x+3}$ [Partial fractions]

$\Rightarrow A = -\frac{5}{2}$; $B = \frac{45}{2}$

$\therefore \frac{5x^2}{x^2+4x+3} = 5 + \frac{5}{2} \times \frac{1}{x+1} - \frac{45}{2} \times \frac{1}{x+3}$

$\therefore \int_1^2 \frac{5x^2}{x^2+4x+3} = 5\int_1^2 dx + \frac{5}{2}\int_1^2 \frac{1}{x+1} - \frac{45}{2}\int_1^2 \frac{1}{x+3} dx$

$= 5\left[x\right]_1^2 + \frac{5}{2}\left[\log|x+1|\right]_1^2 - \frac{45}{2}\left[\log|x+3|\right]_1^2$

$= 5\left[2-1\right] + \frac{5}{2}\left[\log 3 - \log 2\right] - \frac{45}{2}\left[\log[5] - \log 4\right]$

$= 5 + \frac{5}{2}\log\frac{3}{2} - \frac{45}{2}\log\frac{5}{4}$.

Q.No 17 $\int_0^{\pi/4}(2\sec^2 x+x^3+2)\,dx = \left[2\tan x+\frac{x^4}{4}+2x\right]_0^{\pi/4}$

$= \left(2\tan\frac{\pi}{4}+\frac{1}{4}\left(\frac{\pi}{4}\right)^4+2\times\frac{\pi}{4}\right)-(0+0+0)$

$= 2+\frac{\pi^4}{4^5}+\frac{\pi}{2}.$

QNo 18: $\int_0^{\pi}\left(\sin^2\frac{x}{2}-\cos^2\frac{x}{2}\right)dx$

$= -\int_0^{\pi}\left(\cos^2\frac{x}{2}-\sin^2\frac{x}{2}\right)dx = -\int_0^{\pi}\cos x\,dx \quad \left[\because \cos 2x = \cos^2 x-\sin^2 x\right]$

$= -\left[\sin x\right]_0^{\pi} = -\left[\sin\pi-\sin 0\right] = -[0-0] = 0$

QNo 19: $\int_0^2\frac{6x+3}{x^2+4}dx = 3\int_0^2\frac{2x+1}{x^2+4}dx = 3\left[\int_0^2\left(\frac{2x}{x^2+4}+\frac{1}{x^2+4}\right)dx\right]$

$= 3\int_0^2\frac{\frac{d}{dx}(x^2+4)}{x^2+4}dx+3\int_0^2\frac{1}{x^2+(2)^2}dx.$

$= 3\left[\log|x^2+4|\right]_0^2+3\left[\frac{1}{2}\tan^{-1}\frac{x}{2}\right]_0^2$

$= 3\left[\log 8-\log 4\right]+\frac{3}{2}\left[\tan^{-1}(1)-\tan^{-1}(0)\right]$

$= 3\log\left(\frac{8}{4}\right)+\frac{3}{2}\left[\frac{\pi}{4}-0\right] = 3\log 2+\frac{3\pi}{8}$

QNo 20 $\int_0^1\left(xe^x+\sin\frac{\pi x}{4}\right)dx = \int_0^1 xe^x dx+\int_0^1\sin\frac{\pi x}{4}dx$

$= \left[xe^x-\int 1\,e^x dx\right]_0^1+\left[\frac{-\cos\frac{\pi x}{4}}{\pi/4}\right]_0^1$

$= \left[xe^x-e^x\right]_0^1-\frac{4}{\pi}\left[\cos\frac{\pi x}{4}\right]_0^1$

$= \left[(e-e)-(0-e^0)\right]-\frac{4}{\pi}\left[\cos\frac{\pi}{4}-\cos 0\right]$

$= 0-(-1)-\frac{4}{\pi}\cdot\frac{1}{\sqrt{2}}+\frac{4}{\pi}\times 1 = 1+\frac{4}{\pi}\left(1-\frac{1}{\sqrt{2}}\right)$ or $1+\frac{2}{\pi}(2-\sqrt{2})$

Q No. 21 $\int_1^{\sqrt{3}} \frac{dx}{1+x^2} = \int_1^{\sqrt{3}} \frac{dx}{(1)^2+(x)^2} dx = \left[\frac{1}{1} \tan^{-1}\frac{x}{1}\right]_1^{\sqrt{3}}$

$= \tan^{-1}(\sqrt{3}) - \tan^{-1}(1) = \frac{\pi}{3} - \frac{\pi}{4} = \frac{\pi}{12}$

So (D) is correct option.

Q No. 22. $\int_0^{2/3} \frac{dx}{4+9x^2} = \int_0^{2/3} \frac{dx}{9\left(\frac{4}{9}+x^2\right)} = \frac{1}{9}\int_0^{2/3} \frac{dx}{\left(\frac{2}{3}\right)^2+(x)^2}$

$= \frac{1}{9}\left[\frac{1}{2/3} \tan^{-1}\frac{x}{2/3}\right]_0^{2/3} = \frac{1}{6}\left[\tan^{-1}\left(\frac{3x}{2}\right)\right]_0^{2/3}$

$= \frac{1}{6}\left[\tan^{-1}(1) - \tan^{-1}(0)\right] = \frac{1}{6}\left[\frac{\pi}{4} - 0\right] = \frac{\pi}{24}$

So (C) is correct option.
