Chapter-16

Fun with numbers

F9Y1F6

In earlier classes, you have learnt about different numbers like natural numbers, whole numbers, even numbers, odd numbers prime numbers, composite numbers, integers, rational numbers etc. with their interesting properties.

Greek Philosopher Pythagoras and his followers tried to explain everything in the universe by numbers. Their doctrine was '*All is number.*'

Indian mathematician Ramanujan had extraordinary command on numbers. So he is known as "Friend of Numbers". During his short span of life, his only aim was to study on numbers. His theorems and remarks on numbers are still challenging to the world community of mathematics. Here we will discuss divisibility of numbers, some structural beauty, attractive models, different funny games in addition to some additional and necessary topics.

16.1 Tests of Divisibility

(i) Divisibility by 2, 4 and 8

(a) Consider the number 816.

$816 = 810 + 6$
$= 10 \times 81 + 6$
$= 2 \times 5 \times 81 + 6$

First term in the right hand side is a multiple of 2. So it is divisible by 2. If the number is to be divisible by 2, the second term in Right Hand Side must be either zero or divisible by 2. Here, the number in unit's place is 6 which is divisible by 2. Therefore, 816 is divisible by 2. On the other hand, 713 is not divisible by 2. Because, the digit in unit's place is 3 which is not divisible by 2.

Thus, **any number whose unit's digit is either 0 or even, is divisible by 2.**

(b) Consider the number 712.

$712 = 700 + 12$
$= 100 \times 7 + 12$
$= 4 \times 25 \times 7 + 12$

Here the first term in RHS is a multiple of 4. So, it is divisible by 4. If the given number is to be divisible by 4, the second two digit number must 00 or divisible by 4. The two digit number is 12 and 12 is divisible by 4. So 712 is divisible by 4. Same reasons are applicable to 2572, 324, 576, 157792 for the divisibility by 4. On the other hand, 411, 5227, 6431 are not divisible by 4.

A number will be divisible by 4 if the number formed by the digit in unit's place and ten's place is either 00 or divisible by 4.

(c) Consider a number more than three digit, say 57275

$$\begin{aligned} 57275 &= 57000 + 275 \\ &= 1000 \times 57 + 275 \\ &= 8 \times 125 \times 57 + 275 \end{aligned}$$

Here, the first term in RHS is a multiple of 8. So it is divisible by 8. Now, the number 57275 will be divisible by 8, if the second term in RHS (i.e 275) i.e the three digit number formed by the digit in unit's place, ten's place and hundred's place must be 000 or divisible by 8.

Here 275 is not divisible by 8, so 57275 is not divisible by 8.

By samilar reasons, 2001, 45022, 743125 are not divisible by 8. But 1008, 5120, 395000, 12328 are divisible by 8.

If the number formed by the digits in unit's place, ten's place and hundred's place of a number is either 000 or divisible by 8, then the number is divisible by 8.

(ii) Divisibility by 5, 25 and 125

(a) Consider any number 25435

$$\begin{aligned} 25435 &= 25430 + 5 \\ &= 10 \times 2543 + 5 \\ &= 5 \times 2 \times 2543 + 5 \end{aligned}$$

Here, the first term in RHS is a multiple of 5. So it is divisible by 5. For the given number to be divisible by 5, the second term in RHS must be either 0 or divisible by 5. Here the digit in unit's place is 5. So the given number is divisible by 5. Similarly, the numbers 525, 320, 4795 etc. are divisible by 5. On the other hand, 302, 441, 10283 are not divisible by 5.

If the digit in unit's place of a number is either 0 or 5, then the number is divisible by 5.

(b) Consider a number of two or more digits say 3175

Here, 3175 = 3100 + 75

$= 100 \times 31 + 75$
$= 25 \times 4 \times 31 + 75$

Here, the first term is a multiple of 25. So, it is divisible by 25. For the given number to be divisible by 25, the second term in RHS i.e the number formed by the digit in unit's place and ten's place must be divisible by 25. Here, this number is 75 which is divisible by 25. So, the given number is divisible by 25.

For example 350, 7100, 854325 etc. are divisible by 25. On the other hand, 4752, 3710, 111507 etc. are not divisible by 25.

If the number formed by the digits in unit's place and ten's place of a number is either 00 or divisible by 25. then the given number is divisible by 25.

(c) Consider a three or more than three digit number like 12350

Here, $12350 = 12000 + 350$
$= 1000 \times 12 + 350$
$= 125 \times 8 \times 12 + 350$

The first term in RHS is a multiple of 125, so it is divisible by 125. The given number will be divisible by 125 if the second term in RHS consisting of the digits in unit's place, ten's place and hundred's place i.e 350 is divisible by 125. But 350 is not divisible by 125. So, the given number 12350 is not divisible by 125.

Similarly, the number 11251, 32503, 52310 etc. are not divisible by 125. On the other hand, 23000, 54250, 5750, 413250 etc. are divisible by 125.

If the number formed by the digits in unit's place, ten's place and hundred's place of a given number is either 000 or divisible by 125, then the given number is divisible by 125.

(iii) Divisibility by 3 and 9

Consider a three or more digit number say 23594

$23594 = 10000 \times 2 + 1000 \times 3 + 100 \times 5 + 10 \times 9 + 4$
$= (9999 + 1) \times 2 + (999 + 1) \times 3 + (99 + 1) \times 5 + (9 + 1) \times 9 + 4$
$= 9999 \times 2 + 999 \times 3 + 99 \times 5 + 9 \times 9 + (2 + 3 + 5 + 9 + 4)$

Each of the first four terms in RHS is a multiple of 9. So, all the four terms are divisible by both 3 and 9. Therefore, if the sum within the bracket is divisible by 3 or 9, then the given number is divisible by 3 or 9.

Here $2 + 3 + 5 + 9 + 4 = 23$, which is not divisible by any of 3 and 9.

Thus, we can say that **a number will be divisible by 3 or 9 if the sum of all the digits of the number is divisible by 3 or 9.**

Example : For the number 123, sum of the digits of the number is $1 + 2 + 3 = 6$. which is divisible by 3. therefore, 123 is divisible by 3.

Again, for the number 563211 sum of the digits of the number is $5 + 6 + 3 + 2 + 1 + 1 = 18$, which is divisible by 3 and 9 both. Therefore 563211 is divisible by 3 and 9 both. On the other hand, sum of the digits of the number 51134 is $5 + 1 + 1 + 3 + 4 = 14$. But 14 is not divisible by either 3 or 9. Therefore 51134 is not divisible by 3 or 9.

(iv) Divisibility by 6

We know that $6 = 2 \times 3$

From this it follows that if a number is divisible by 6, it is divisible by 2 and 3 both. Conversely, if a number is divisible by 2 and 3 both, then, it is divisible by 6.

Example : 12348 is divisible by 2, for the digit in unit's place i.e 8 is divisible by 2. Also, sum of the digits of the number is $1 + 2 + 3 + 4 + 8 = 18$ and 18 is divisible by 3. Therefore, 12348 is divisible by $2 \times 3 = 6$.

Again, sum of the digits of the number 111111 is $1 + 1 + 1 + 1 + 1 + 1 = 6$ and 6 is divisible by 3. But the number is not divisible by 2, as unit place digit is 1. So, 111111 is not divisible by 6.

A number is divisible by 6 if it is divisible by 2 and 3 both.

Carefully note here that if a number is divisible by two different numbers, the number may not be divisible by the product of two numbers always.

For example, 12 is divisible by 2 and 4 both. But it is not divisible by $2 \times 4 = 8$. Actually, **a number is divisible by two different numbers separately as well as their product if the HCF of two different numbers is 1. i.e. they are Co prime numbers or relatively prime numbers.** In the case of 2 and 3, HCF of 2 and 3 is 1. So a number is divisible by $2 \times 3 = 6$ if it is divisible by 2 and 3 both. Again HCF of 2 and 4 is 2. So a number may not be divisible by $2 \times 4 = 8$, although it may be divisible by 2 and 4 both.

(v) Divisible by 11

Consider a three or more than three digit number say 45371

Now, $45371 = 10000 \times 4 + 1000 \times 5 + 100 \times 3 + 10 \times 7 + 1$

$= (9999 + 1) \times 4 + (1001 - 1) \times 5 + (99 + 1) \times 3 + (11 - 1) \times 7 + 1$

$= 9999 \times 4 + 1001 \times 5 + 99 \times 3 + 11 \times 7 + 1 \times 4 - 1 \times 5 + 1 \times 3 - 1 \times 7 + 1$

$= 11 \times 909 \times 4 + 11 \times 91 \times 5 + 11 \times 9 \times 3 + 11 \times 7 + (4 - 5 + 3 - 7 + 1)$

Here , each of the first four terms is a multiple of 11.

So, for the number to be divisible by 11, the sum of the numbers within bracket must be either 0 or divisible by 11. Here, the sum within the bracket i.e $4 - 5 + 3 - 7 + 1 = -4$, which is not divisible by 11. So, the number 45371 is not divisible by 11. Mind that, $4 - 5 + 3 - 7 + 1 = (4 + 3 + 1) - (5 + 7)$ is the difference of the sums of digits taken form odd and even places of the given number. So, the condition of a number to be divisible by 11 can be stated as–

A number will be divisible by 11 if the difference of the sums of digits taken separately from odd and even places of the number is either 0 or divisible by 11.

Example : Difference of the sums of digits taken separately from odd and even places of the number 753214

$= (7 + 3 + 1) - (5 + 2 + 4)$
$= 11 - 11$
$= 0$

By condition 753214 is divisible by 11

Similarly, 9190909090 is divisible by 11, since

$(9 + 9 + 9 + 9 + 9) - (1 + 0 + 0 + 0 + 0) = 45 - 1 = 44$ is divisible by 11.

Note that we may forward the condition of divisibility of a number by 11 in two different ways as follows.

(A) A number *abcdefg* (7 digit number) will be divisible by 11 if $(g+e+c+a) - (f + d + b)$ is divisible by 11

(B) A number *abcdefg* will be divisible by 11 if the difference of $efg + a$ and bcd is 0 or divisible by 11.

Example 1 : Investigate the divisibility of 2359874 by 2, 3, 4, 5, 6, 7, 8,9,10,11,12 and 13.

Solution : Divisibility by 2, 4, 8

Digit in unit's place of the number is 4. It is an even number and is divisible by 2. Therefore, the number is divisible by 2. The number consisting of the digits in unit's place and ten's place is 74. It is not divisible by 4. So, the given number is not divisible by 4. For similar reasons, the number is not divisible by 8.

Divisibility by 3 and 9 :

Sum of the digits of the number is $2 + 3 + 5 + 9 + 8 + 7 + 4 = 38$ which is not divisible by 3 or 9. Therefore, the given number is neither divisible by 3 nor by 9.

Divisibility by 5, 25 and 125 :

A number will be divisible by 5 if the digit in unit's place is either 0 or 5. Here the digit in unit's place is 4. and it is not divisible by 5. Thus the given number is not divisible by 5.

If a number is not divisible by 5, then it is neither divisible by 25 nor 125 i.e the number is divisible by none of 5, 25 and 125.

Divisibility by 6 :

For a number to be divisible by 6, it must be divisible by both 2 and 3. The given number is not divisible by 3. So, it is not divisible by 6.

Divisibility by 7, 11, 13 :

Taking three digits at a time from right hand side we get groups of numbers as 874, 359 and 2.

Now adding two alternate groups we get 874 + 2 and 359 i.e, 876 and 359

Difference of these groups is 876 – 359 = 517

Here, 517 is divisible by 11. So the given number is divisible by 11. But 517 is divisible by none of 7 and 13. Therefore, the given number is not divisible by 7 and 13.

Example 2 : Show that 151452 is divisible by 21

Solution : We know that $21 = 3 \times 7$

Let us investigate the cases of divisibility by 3 and 7. Here sum of the digits of the number is 2 + 5 + 4 + 1 + 5 + 1 = 18 and 18 is divisible by 3. So the given number is divisible by 3.

Again, grouping the numbers taking three digits at a time from right hand side, we get 452 and 151.

Now, $452 - 151 = 301$ and $301 = 7 \times 43$. So, 301 is divisible by 7.

Therefore, the number 151452 is divisible by both 3 and 7. Also 3 and 7 are co-prime numbers. So, the given number is divisible by 21.

Example 3 : Find the value of x where the number $13x\,2741$ is divisible by 11.

Solution : For the number to be divisible by 11, we must have the difference of the sums of digits in even and odd places must be 0 or divisible by 11.

i.e $(1 + 7 + x + 1) - (4 + 2 + 3)$

$= 9 + x - 9$

$= x$

x must be 0 or divisible by 11.

i.e $x = 0$ or multiple of 11. But the non-zero multiple of 11 must have at least two digits.

So, $x = 0$

$\therefore$ The number will be 1302741

Example 4 : Find the least number which when added to 2311 is divisible by (i) 3 (ii) 4

Solution : (i) The sum of the digits of 2311 is $2 + 3 + 1 + 1 = 7$. 7 is not divisible 3. If 2 is added to 7.

it becomes $7 + 2 = 9$ which is divisible by 3. So, 2 is the least number when added to the given number, will be divisible by 3.

(ii) For the number 2311 to be divisible by 4, the number formed by the digits in unit's place and ten's place i.e 11 must be divisible by 4. But 11 is not divisible by 4. But $11 + 1 = 12$ is divisible by 4. Hence the least number to be added to the given number is 1. i.e $2311 + 1 = 2312$ is divisible by 4.

Example 5 : Put a digit in the right hand side of 2785 so that the new number becomes divisible by (i) 9 (ii) 11.

Solution : (i) Let the new number be 2785x. Now $2 + 7 + 8 + 5 + x = 22 + x$. Since x is one digit, it must be a digit between 0 and 9. i.e $22 + x$ must be in between $22 + 0$ and $22 + 9$ i.e 22 and 31. 27 is the only number between 22 and 31 which is divisible by 9.

So, $22 + x = 27$

$\therefore \quad x = 5$

(ii) To check the divisibility by 11, we must have $x - 5 + 8 - 7 + 2 = x - 2$ and $x - 2$ must be divisible by 11

So, $x - 2 = 0, 11, 22...$ etc. Since x is one digit we must have $x - 2 = 0$ i.e $x = 2$

Exercise 16.1

1. Examine the divisibility of the following numbers by 2, 3, 5, 9

   (i) 4253 (ii) 18935 (iii) 12123232

   (iv) 8753973 (v) 333666 (vi) 785634

2. Examine the divisibility of the following numbers by 4, 6, 8, 11

   (i) 532740 (ii) 347435 (iii) 123456

   (iv) 693011 (v) 1238932

3. Examine the divisibility of the following numbers by 7 and 13

   (i) 2561876 (ii) 864192 (iii) 1604928

4. Find the value of x in 25372x so that the number becomes divisible by (i) 3 (ii) 9

5. Find the value of x of the number 25x043 so that it becomes divisible by 11.

16.2 Simple games of numbers

You can play games of numbers inside or outside of the class room or at leisure with your friends or some other members of your family. There are some interesting numbers. If you learn to play with those numbers, you will definitely find pleasure and mathematics will become friendly. Let us discuss few games of numbers.

(i) Four numbers with amazing behaviour

Four numbers like 153, 370, 371 and 407 possess quiet amazing behaviour. Each number is equal to the sum of the cubes of its digits.

$$153 = 1^3 + 5^3 + 3^3 = 1 + 125 + 27 = 153$$
$$370 = 3^3 + 7^3 + 0^3 = 27 + 343 + 0 = 370$$
$$371 = 3^3 + 7^3 + 1^3 = 27 + 343 + 1 = 371$$
$$407 = 4^3 + 0^3 + 7^3 = 64 + 0 + 343 = 407$$

Is it not really interesting?

(ii) Fun with 37 : The number 37 possesses a magical property. If you multiply 37 by 3 or multiples of 3, you will find the products arranged in a special order.

$37 \times 3 = 111$	$37 \times 18 =$ ....
$37 \times 6 = 222$	$37 \times 21 =$ ....
$37 \times 9 = 333$	$37 \times 24 =$ ....
$37 \times 12 = 444$	$37 \times 27 = 999$
$37 \times 15 = 555$	

After that a new order begins.

$37 \times 30 = 1110$	$37 \times 39 =$ ....
$37 \times 33 = 1221$	$37 \times 42 =$ ....
$37 \times 36 = 1332$	$37 \times 45 =$ ....
	.... $\times$ .... = ...

Moreover $37037 \times 3 = 111111$

$37037037 \times 3 = 111111111$

[These were invented by Mahaviracharya in 850 AD. He wrote a book namely **Ganitasarasangraha**.]

(iii) Games of three consecutive numbers

Mili : write any three consecutive numbers

Rangmili : I write 9, 10, 11

Mili : Add these three numbers

Rangmili : Yes, it is $9 + 10 + 11 = 30$

Mili : What is the sum? Tell me.

Rangmili : 30
Mili : Then, your numbers are 9, 10, 11
Rangmili : That's right! But how did you know? Tell me.
Let us learn the technique used by Mili.
Let the middle of three consecutive numbers be = x

$\therefore$ Previous number = $x - 1$ and next number = $x + 1$

$\therefore$ Sum of these = $x - 1 + x + x + 1 = 3x$

$$\therefore \text{ Middle number} = \frac{\text{Sum of the numbers}}{3}$$

$$= \frac{30}{3} = 10$$

$\therefore$ Previous number = 9
and next number = 11

Similarly, $1 + 2 + 3 = 2 \times 3 = 6$

$3 + 4 + 5 = 4 \times 3 = 12$

$9 + 10 + 11 = 10 \times 3 = 30$

Do it with your friends

(i) 24 + 25 + 26 (ii) 28 + 29 + 30 (iii) 69 + 70 + 71

(iv) Game with four consecutive numbers :

This time Rangmili starts the game

Rangmili: Mili, you write four consecutive numbers so that I cannot see

Mili: Okay, I have written 2, 3, 4, 5

Rangmili: Add these four numbers

Mili: Done 2 + 3 + 4 + 5

Rangmili: Tell me the sum

Mili: Sum is 14

Rangmili : Then your numbers are 2, 3, 4 and 5

Mili : How could you do it so immediately?

Let us learn the technique of used by Rangmili.

Let the four consecutive numbers be x, $x + 1$, $x + 2$ and $x + 3$

$$\text{Sum of these numbers} = x + x + 1 + x + 2 + x + 3$$

$$\text{or,}\quad \text{Sum} = 4x + 6$$

$$\text{or,}\quad 4x = \text{Sum} - 6$$

$$\text{or,}\quad x = \frac{\text{Sum - 6}}{4}$$

Mili got sum as 14

$$\therefore \quad \text{1st number} = \frac{14 - 6}{4} = 2$$

$\therefore$ Consecutive numbers are 2, 3, 4 and 5

Similarly, $3 + 4 + 5 + 6 = 3 \times 4 + 6 = 18$

$5 + 6 + 7 + 8 = 5 \times 4 + 6 = 26$

Alternately, multiplying the sum of the middle numbers by 2 also you will get the sum of four consecutive numbers.

$$9 + 10 + 11 + 12 = (10 + 11) \times 2 = 42$$

$$15 + 16 + 17 + 18 = (16 + 17) \times 2 = 66$$

Do it in group

(i) 7 + 8 + 9 + 10 (ii) 12 + 13 + 14 + 15 (iii) 20 + 21 + 22 + 33

(v) Game of 9 :

Now, Pankaj, Parvez, Manash and Daniel started another interesting game of numbers. The game of numbers with two digits.

Pankaj: Think a two digit number

Parvez: Okay, it is 24

Manash: Okay, it is 20

Daniel: Okay, it is 91

Pankaj: Now reverse the digits

Parvez: Okay, it becomes 42

Manash: Okay, it becomes 02

Daniel: Okay, it becomes19

Now, subtract the smaller number from the bigger

Okay, $42 - 24 = 18$

Okay, $20 - 02 = 18$

Okay, $91 - 19 = 72$

Divide the difference by 9. You will find no remainder, it is multiple of 9

$18 \div 9 = 2$ Yes, how do you know?

$18 \div 9 = 2$ Yes, how do you know?

$72 \div 9 = 8$ Yes, how do you know?

Now, let us discuss the technique used here.

Let ab be the two digit number choosen by Daniel. Its general form is $10a + b$.

Reversing the digits, the new number will be $ba = 10b + a$,

If $a > b$, then $10a + b > 10b + a$. Now subtracting the smaller number from the bigger

$$\begin{aligned}(10a + b) - (10b + a) &= 10a + b - 10b - a \\ &= 9a - 9b \\ &= 9(a - b)\end{aligned}$$

Which is a multiple of 9 as said by Pankaj

Do in your group

(i) 16 (ii) 45 (iii) 62

(vi) Game of 1089

Take a three digit number so that the digits are not same

Okay

Now, make the greatest and smallest number that can be made by the three digits

Okay 831 and138

Subtract the smaller from the bigger

Okay $831 - 138 = 693$

Reverse the order of the digits of new number and add.

Okay
693 + 396 = 1089

Is not the sum = 1089

Yes, how do you know?

What is the game here? In fact, if we proceed with any three digit number with unequal digits and follow the procedure as given, we would get finally 1089.

Do with friends

(i) 327 (ii) 291 (iii) 456 (iv) 786

Note :

The number 1089 is known as Kaprekar Number. Kaprekar, a school teacher of Nachik in Maharastra invented this number.

Some interesting results of this number are :

$1089 \times 1 = 1089, \longleftrightarrow 9801 = 1089 \times 9$

$1089 \times 2 = 2178, \longleftrightarrow 8712 = 1089 \times 8$

$1089 \times 3 = 3267, \longleftrightarrow 7623 = 1089 \times 7$

$1089 \times 13 = 14,157 \longleftrightarrow 75,141 = 1089 \times 69$

Carefully observe the products in left hand side and right hand side. Do you find here any speciality?

Again

$33^2 = 1089$

$333^2 = 110889$

$3333^2 = 11108889$ ..... etc.

(vii) Game of 1001

Take a three digit number, say 125.

Write it twice to get 125125

Divide this number by 7 so that the quotient is 17875

Now divide this number by11 so that the quotient is 1625

Again divide this number by 13 so that the quotient is 125

Thus, original number is found

Observe, $7 \times 11 \times 13 = 1001$,

So, dividing by 1001, quotient will be obtained as 125

(i) Proceeding from opposite direction, $125125 = 125 \times 1001$,

(ii) considering the number $125125 = 125 \times a$, what is a?

Let us discuss the technique.

Actually, the six digit number obtained from a three digit number writing twice is 1001 times the original number.

$$\begin{aligned} \text{Here } 125125 &= 125{,}000 + 125 \\ &= 125(1000 + 1) \\ &= 125 \times 1001 \quad \therefore \quad a = 1001 \end{aligned}$$

Again, $7 \times 11 \times 13 = 1001$. Do you follow the reason?

Do it with your friends

(i) 234 (ii) 175 (iii) 432

16.3 Some numerical patterns

. (i) Look at the following pattern

$9 \times 9 = 81$, $9 + 9 = 18$

$24 \times 3 = 72$ $24 + 3 = 27$

$47 \times 2 = 94$ $47 + 2 = 49$

$497 \times 2 = 994$ $497 + 2 = 499$

(ii) Another formation

$9 \times 9 + 7 = 88$

$98 \times 9 + 8 = 888$

$987 \times 9 + 5 = 8888$

$98765432 \times 9 + 0 = 88888888$

$987654321 \times 9 + (-1) =$

$9876543210 \times 9 + (-2) =$

Fill up the blanks by guessing. Compare the results with those found by actual multiplicaton . Is not it amusing?

(iii) $6^2 - 5^2 = 11 \times 1$

$(56)^2 - (45)^2 = 101 \times 11$

$(556)^2 - (445)^2 = 1001 \times 111$ etc.

(iv) Look at the number 3816547290

The significance of this number formed by all the digits from 0 to 9 is that if any group of numbers taken from left hand side to right hand side in order, then it is divisible by the number of the digits in the group. For example 1 | 3, 2 | 38, 3 | 381, 4 | 3816 ... etc. Here, the sign ' | ' denotes the divisibility of the number in right hand side by the number in left hand side.

(v) $\frac{148}{296}+\frac{35}{70}=1$ What is its significance?

(x) Some Patterns of Squares of Numbers

We know that we can find the square of any number. But each number may not be the square of a number. Here, we discuss some beautiful patterns of squares of numbers.

(a) One pattern is like this–

$$4^2 = 16$$
$$34^2 = 1156$$
$$334^2 = 111556$$
$$3334^2 = 11115556$$
$$33334^2 = 1111155556$$
$$333334^2 = 111111555556$$

You write some other patterns yourself below.

(b)

$1^2 =$	1
$11^2 =$	1 2 1
$111^2 =$	1 2 3 2 1
$1111^2 =$	1 2 3 4 3 2 1
$11111^2 =$	1 2 3 4 5 4 3 2 1
$111111^2 =$	1 2 3 4 5 6 5 4 3 2 1
$1111111^2 =$	1 2 3 4 5 6 7 6 5 4 3 2 1

↑
(middle pillar)

What are the significance you can see in this pattern? Firstly, when you take the square of a number, the digits in the right hand side and left hand side of the digit in the middle pillar are same. Such pattern is known as *Palindrome.* Palidrome like patterns are found in different languages also. Nayan, noon, civic, *'Madam I'm Adam'* are examples in English language. Again, you see that when you make square of a number, the total number of 1 is the number in the middle pillar. For example, when you take the square of 1111111, the number of 1 is 7 and 7 occupies the middle position of its square. So, the square of 1111111 is 1 2 3 4 5 6 7 6 5 4 3 2 1

This triangular pattern of numbers is known as *Pascal's triangle*.

(c) Look at the following pattern

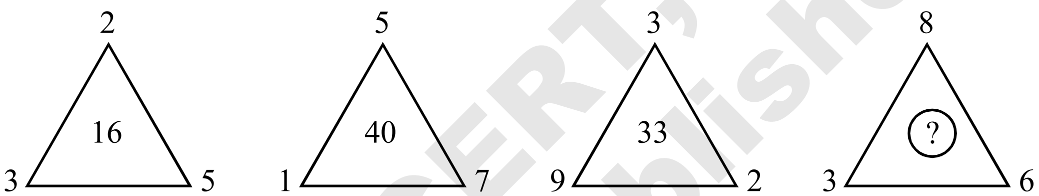
$$9^2 = 81$$
$$99^2 = 9801$$
$$999^2 = 998001$$
$$9999^2 = 99980001$$

You write some more numbers below of the same pattern and try to understand the special characteristics of the pattern.

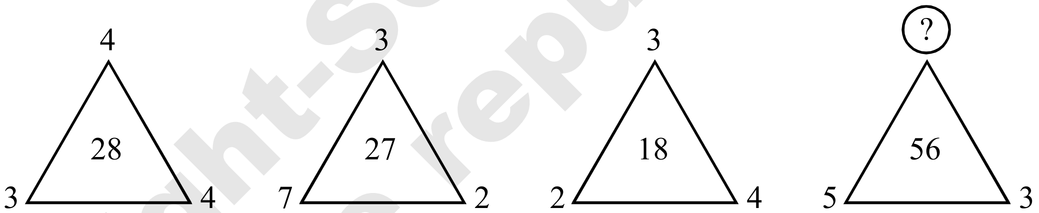
Exercise-16.2

Look, investigate and write the technique by filling up the blanks –

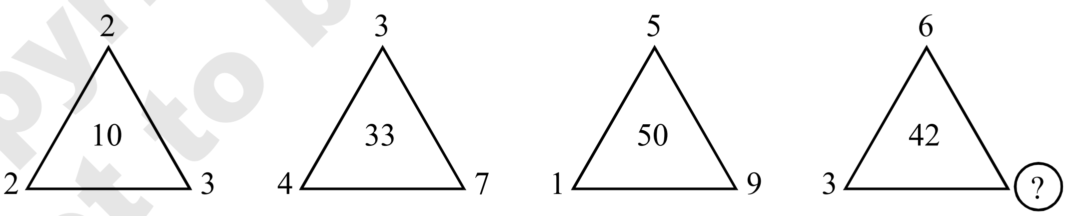
1. (a)



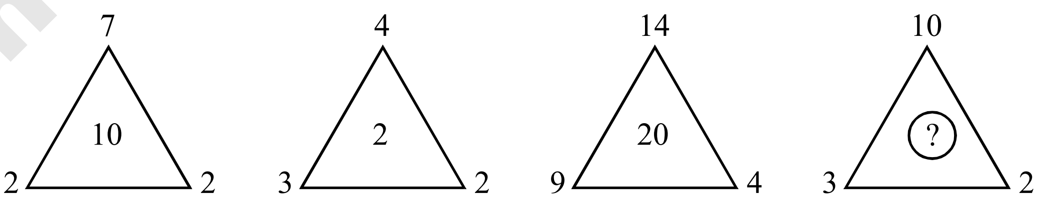
(b)



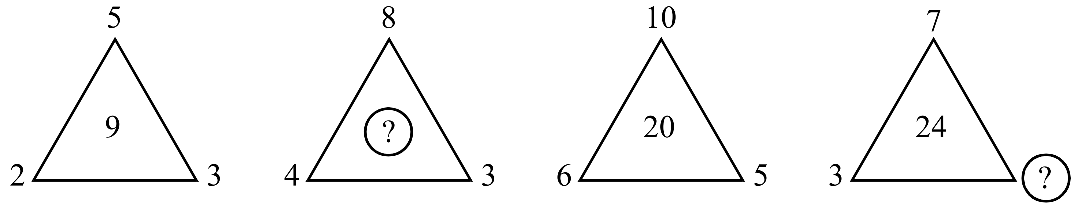
(c)



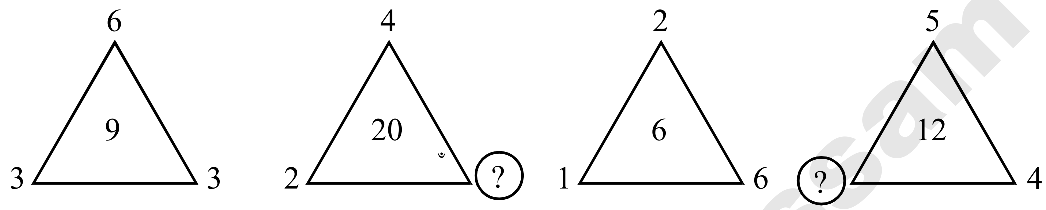
2. (a)



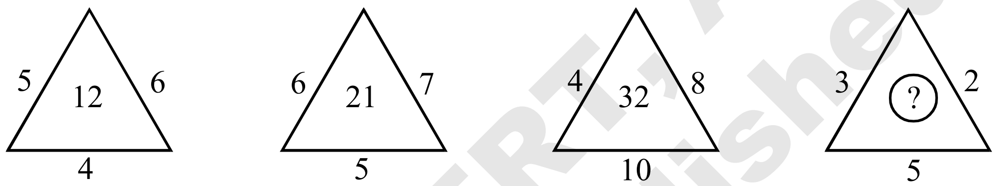
(b)



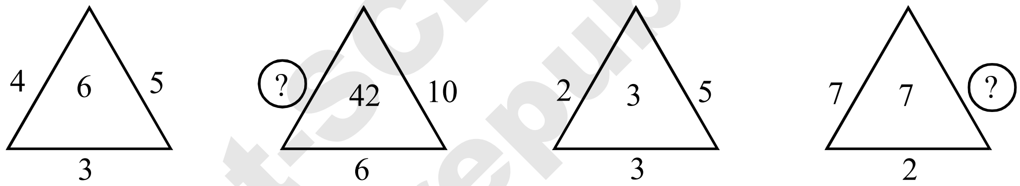
(c)



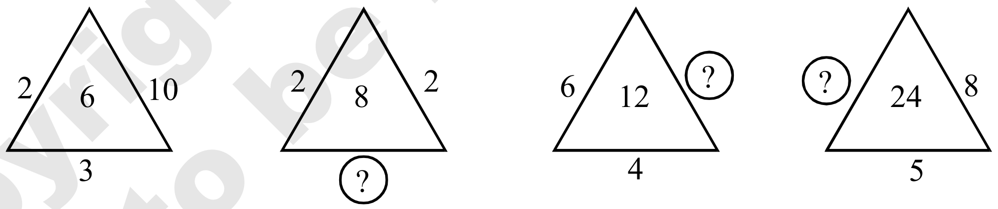
3. (a)



(b)



(c)



16.4 Game with letters for digits

In this section, letters are used in place of digits are used in some mathematical problems. We will try to find the techniques in solving those problems. Here, we will use addition and multiplication processes only.

To solve such problems, we will follow the following principles :

(a) One letter is used for one digit in each problem. One letter represents one digit only.

(b) The first digit of a number in a problem cannot be zero.

(c) A problem must have one solution.

Let us discuss the following examples –

Example 1 :

	A	4
+	1	B
	4	9

Find the values of A and B

Solution : The above is a problem of addition. Here, A and B are used for two digits. To find the numerical values of A and B. Where A and B represent different values.

Step 1 : See the addition in unit's place. Here, 4 + B = 9. Since 4 + 5 = 9, therefore value of B is 5.

Step 2 : See the addition in ten's place. Here A + 1 = 4. Since 3 + 1 = 4, therefore value of A is 3.

ten		unit	
A=	3		4
+	1	B =	5
	4		9

Answer : A = 3, B = 5

Example 2 :

	5	A	B
+	A	B	2
	B	7	8

Find the values of A and B.

Solution : It is a problem of addition of numbers of three digits.

Step 1 : In addition of unit's place B + 2 = 8. Since 6 + 2 = 8, so, B = 6

Step 2 : In addition of ten's place A + B = 7, Since B = 6 and 1 + 6 = 7, so A = 1.

Step 3 : In addition of hundred's place. 5 + A = B. Since A = 1 and B = 6 and 5 + 1 = 6

∴ Solution of the whole problem is–

	hundred		ten		unit	
		5	A=	1	B=	6
+	A=	1	B=	6		2
	B=	6		7		8

Answer : A = 1 and B = 6

Example 3 :

$$\begin{array}{cccc} & A & B & B \\ + & A & B & B \\ + & A & B & B \\ \hline 1 & 9 & A & B \end{array}$$

Find the values of A and B

Solution : This problem is somewhat different from the earliar two problems. We should be careful in solving it.

Step 1 : Sum of three B's is a number whose digit in unit's place is B. This possible only for B = 0 and B = 5. But B = 0 does not satisfy the condition of addition in ten's place. So, B = 5. Also 5 + 5 + 5 = 15 (we accept 5 = B in this problem) and 1 + 5 + 5 + 5 = 16 (we accept 6 = A in this problem)

Step 2 : In hundred's place, if we put 6 for A, then 6 + 6 + 6 = 18 with 1 from the ten's place becomes 19.

i.e Solution of the whole problem is

Thousand	Hundred	Ten	Unit
	1	1	B= 5
+	A= 6	B= 5	B= 5
	A= 6	B= 5	B= 5
	A= 6	B= 5	B= 5
1	9	A= 6	B= 5

Example 4 :

$$\begin{array}{ccc} & A & B \\ \times & & B \\ \hline C & A & B \end{array}$$

Find the values of A, B and C

Solution : It is a problem of multiplication of three letters.

Step 1 : In unit's place, the product of B and B is a number where digit in unit's place is again B. This is possible for B = 0, 5 and 6. If B = 0, then the product in ten's place must be zero. But it is not possible. So, B = 5 or 6.

Step 2 : Now $(10A + B) \times B = 100C + 10A + B$. If $B = 5$

then $(10A + 5) \times 5 = 100C + 10A + 5$

or, $50A + 25 = 100C + 10A + 5$

or, $50A - 10A + 25 - 5 = 100C$

or, $40A + 20 = 100C$

or, $2A + 1 = 5C$ [Dividing both sides by 20]

i.e $2A + 1$ is the multiple of 5 and it is odd.

So, $2A + 1 = 5$ or, $2A + 1 = 15$

or, $A = 2$ or, $A = 7$

If $A = 2$, then $2 \times 2 + 1 = 5C$

or, $5 = 5C$

$\therefore C = 1$

If $A = 7$, then $2 \times 7 + 1 = 5C$

or, $15 = 5C$

$\therefore C = 3$

So, $A = 2, B = 5$ and $C = 1$

The solution of the problem is

$$\begin{array}{rrr} & A=2 & B=5 \\ \times & & B=5 \\ \hline C=1 & A=2 & B=5 \end{array}$$

Answer : $A = 2,\ B = 5$ and $C = 1$

Exercise 16.3

1. Find the value of letters for digits (step wise) of the following.

(i)
$$\begin{array}{rrr} & 6 & A \\ + & 8 & 7 \\ \hline B & A & 2 \end{array}$$

(ii)
$$\begin{array}{rrr} 2 & 1 & A \\ +\ 1 & A & 3 \\ \hline 3 & 6 & 8 \end{array}$$

(iii)
$$\begin{array}{rrr} 1 & A & B \\ +\ A & B & 1 \\ \hline B & 0 & 7 \end{array}$$

(iv)
$$\begin{array}{rrr} B & 2 & A \\ +\ 3 & A & B \\ \hline A & 0 & 0 \end{array}$$

(v)
$$\begin{array}{rrrr} 3 & 0 & A & 6 \\ +\ 4 & 2 & 4 & B \\ +\ A & 3 & B & 6 \\ \hline C & 6 & A & 7 \end{array}$$

(vi)
$$\begin{array}{rrr} A & A & A \\ +\ A & A & A \\ +\ A & A & A \\ \hline C\ B & B & A \end{array}$$

(vii)
$$\begin{array}{rrr} & A & B \\ \times & & 3 \\ \hline C & A & B \end{array}$$

(viii)
$$\begin{array}{rrr} & B & A \\ \times & B & 3 \\ \hline 5 & 7 & A \end{array}$$

(ix)
$$\begin{array}{rrr} & A & B \\ \times & & 6 \\ \hline C & B & B \end{array}$$

(x)
$$\begin{array}{rrr} & A & B \\ \times & & 6 \\ \hline B & B & B \end{array}$$

(xi)
$$\begin{array}{rrr} & A & B \\ \times & & 5 \\ \hline C & A & B \end{array}$$

(xii)
$$\begin{array}{rrr} & A & B \\ \times & A & B \\ \hline C & A & B \end{array}$$

What we have learnt

1. **Divisibility by 2, 4 and 8**

   A number which has 0 or even digit in unit's place in divisible by 2

   A number is divisible by 4 if the number formed by the digits in unit's place and ten's place in either 00 or divisible by 4.

   A number is divisible by 8 if the number formed by the digits on unit's place, ten's place and hundred place in either 000 or divisible by 8.

2. **Divisible by 5, 25 and 125**

   If the digit in unit's place of a number is either 0 or 5, the number is divisible by 5.

   A number is divisible by 25 if the number formed by the digits in unit's place and ten's place is either 00 or divisible by 25.

   A number is divisible by 125 if the number formed by the digits in unit's place, ten's place and hundred's place is either 000 or divisible by 125.

3. **Divisibility 3 and 9**

   A number will be divisible by 3 or 9 if the sum of all digits of the number is divisible by 3 or 9 respectively.

4. **Divisibility 6**

   If a number is divisible by 6 if the number is divisible by both 2 and 3.

   A number is divisible by two numbers, then it will also be divisible by the product of the numbers if they are co-prime or relatively prime i.e if their HCF is 1.

5. **Divisibility by 11**

   A number is divisible by 11 if the difference of the sums of digits at even places and at odd places is either 0 or divisible by 11.

6. Some games that can be played with numbers.
7. Games of letters for digits.

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