Long Answer Type Questions

Q.1. Explain periodic function with illustrations state Fouriers' theorem and deduce the conditions for S.H.M.?

Ans. Periodic functions are those functions which are used to represent periodic motion.

A function $f(t)$ is said to be periodic, if

$$f(t) = f(t + \mathrm{T})$$

$$= f(t + 2\mathrm{T}) \quad \text{...(i)}$$

Where T is called the period of periodic function, sin θ and cos θ are the exponents of periodic functions with period equal to 2π radians because

$$\sin\theta = \sin(\theta + 2\pi)$$

$$= \sin(\theta + 4\pi) \quad \text{...(ii)}$$

And $$\cos\theta = \cos(\theta + 2\pi)$$

$$= \cos(\theta + 4\pi) \quad \text{...(iii)}$$

The periodic function T (t) can also be represented as

$$f(t) = \sin\frac{2\pi t}{\mathrm{T}} \quad \text{...(iv)}$$

Fourier theorem: According to this theorem, any periodic function $f(t)$ of the period T, however complex it may, can be represented by a unique combination of the functions $fc(t)$ and $gn(t)$.

Here, $$fn(t) = \sin\frac{2\pi nt}{\mathrm{T}}$$

And $$gn(t) = \cos\frac{2\pi nt}{\mathrm{T}}$$

Where n = 0, 1, 2, 3.

Mathematically, Fourier theorem can be written as:

$$\mathrm{F}(t) = \left[a_n sin\frac{2\pi nt}{\mathrm{T}} + b_n \cos\frac{2\pi nt}{\mathrm{T}}\right]$$

Or $\mathrm{F}(t) = a_1 \sin t + a_2 \sin(t + \mathrm{T}) + a_3 \sin(t + 2\mathrm{T}) \ldots. + b_0 + b_1 \cos t + b_2 \cos(t + \mathrm{T}) + b_3 \cos(t + 2\mathrm{T}) + \ldots ...$ (i)

The relation (i) is called Fourier series where $a_1, a_2, a_3, \ldots, b_0, b_1, b_2, b_3, \ldots$ ... are amplitudes of Fourier series and cosines terms are called Fourier constants.

A periodic motion for which only the Fourier coefficients a_1 and b_1 are non-zero, is called simple harmonic motion.

Similarly other periodic function of t can be

$$g(t) = \cos\frac{2\pi t}{\mathrm{T}} \qquad \ldots\text{(iv)}$$

$$f(t) = \sin\frac{2\pi t}{\mathrm{T}}$$

In order to check that these two functions 4 and 5 has a period T_1 can be tested by substituting (t + T) in place of t in those relations.

$$f(t+\mathrm{T}) = \sin\frac{2\pi}{\mathrm{T}}(\mathrm{T}+t)$$

$$= \sin\left(\frac{2\pi t}{\mathrm{T}} + 2\pi\right)$$

$$= \sin\frac{2\pi t}{\mathrm{T}} = \mathrm{F}(t)$$

$$g(t+\mathrm{T}) = \cos\frac{2\pi}{\mathrm{T}}(t+\mathrm{T})$$

$$= \cos\left(\frac{2\pi t}{\mathrm{T}} + 2\pi\right)$$

$$= \cos\frac{2\pi t}{\mathrm{T}} = g(t)$$

It can easily verified that:

$$f(t+nt) = f(t)$$

And $\quad g(t+nt) = g(t)$

Therefore, the infinite sets of periodic functions of period T may be represented as

$$fn(t) = \frac{\sin 2\pi nt}{\mathrm{T}}$$

$$gn(t) = \frac{\cos 2\pi nt}{\mathrm{T}}$$

Q.2. Explain displacement in S.H.M. Find the relations between them.

Ans. Displacement: Consider a reference particle moving on a circle of reference of radius a with uniform angular velocity ω. Let the particle start from the point X and trace angular distance θ radian in time t as it reaches the point P.

Therefore, $\quad \omega = \frac{\theta}{t}$ or $\theta = \omega t$.

Let the projection of the particle P on diameter YOY’ be at M. Then OM = y is the displacement in S.H.M. at time t.

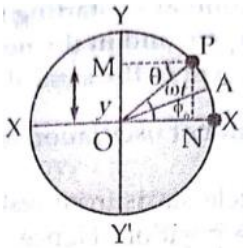
In ΔOPM,

$$\sin\theta = \frac{\mathrm{OM}}{\mathrm{OP}} = \frac{y}{a}$$

or $\quad y = a\sin\theta = a\sin\omega t \qquad \ldots\text{(i)}$

Important notes.

(i) If projection of P is taken on diameter XOX’, then point N will be executing S.H.M. Here, ON = x = Displacement in S.H.M. at time t.

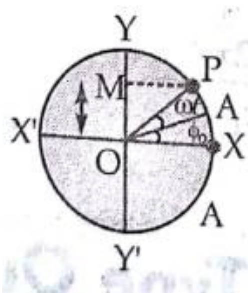


In ΔONP, $\cos\theta = \frac{\text{ON}}{\text{OP}}$

or $x = a\cos\theta = a\cos\omega t$ ...(ii)

(ii) If A is the starting position of the particle of reference such that ∠AOX = ϕ_0 and ∠XOP = ωt.
Θ = ∠XOA = ω$t - \phi_0$

Here (–) ϕ_0 is called the initial phase or epoch of S.H.M.



(ii) If B is the starting position of the particle of reference such that

∠BOX = ϕ_0

∠BOP = ωt.

Θ = ∠XOP = ω$t + \phi_0$

From equation (i)

$$y = a\sin(\omega t + \varphi_0)$$

From equation (ii)

$$x = a\cos(\omega t + \phi_0)$$

Here (+) ϕ_0 is called initial phase or epoch of S.H.M.

Q.3. Discuss some of the imported characteristics of wave motion.

Ans. The important characteristics of wave motion:

(i) Wave motion is a sort of disturbance which travels through a medium.
(ii) A material medium is essential for the propagation of mechanical waves. The medium must possess three properties, viz., elasticity, inertia and minimum friction amongst the particles of

the medium.
(iii) The velocity of the particles during their vibration is different at different position.
(iv) Energy is propagated along with the wave motion without any net transported of the medium.
(v) There is a continuous phase difference amongst successive particles of the medium.

Q.4. What are longitudinal waves? Give examples. How do such waves propagate?

Ans. A longitudinal wave motion is that wave motion in which individual particles of the medium execute simple harmonic motion about their mean position along the same direction of propagation of wave.

Example: Vibration of air column of organ pipes are longitudinal.

Propagation of longitudinal waves.

Let us consider a series of 5 equally spaced particles 1, 2, 3, 4, 5 in one horizontal level. When these particles execute simple harmonic motion of equal amplitudes and equal time period about their mean position along the straight line in which they are placed, we can show that a longitudinal wave travels to the right.

In given figure, solid lines represent the mean positions dotted lines around the solid lines represent the extreme positions of the particles while vibrating XY = XY' is the amplitude of vibration.

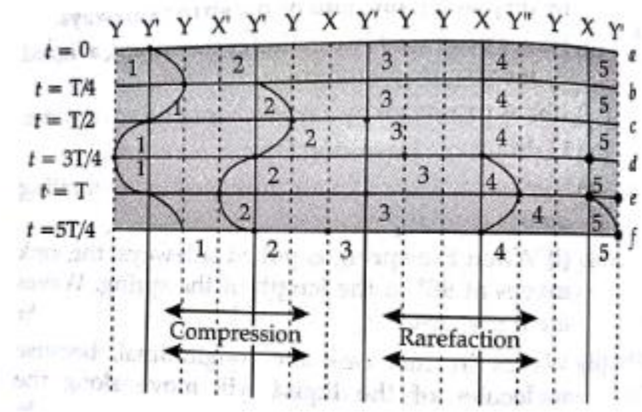
Let т is the time period of vibration of each particle, the disturbance is handed on from one particle to the adjoining particle in T/4 sec.

(i) At t = 0, all the particles 1, 2, 3, 4, 5 are at rest at their mean positions and the disturbance just reaches particles 1.

(ii) At t = T/4 particle 1 completes 1/4 of its vibration and reaches the right extreme position. The disturbance just reaches particle 2, which starts vibrating. The particles 3, 4, 5 are at rest.

(iii) At t = T/2, particle 1 completes 1/2 its vibration returning to the mean position and particle 2 reaches its right extreme position. The disturbance just reaches particle 3, which starts vibrating. The particles 4 and 5 are at rest.

(iv) At t = 3T/4, particle 1 goes to its left extreme position, particle 2 completes 1/2 vibration reaching its mean position, particles 3 has executed 1/4th of its vibration, reaching its right extreme position. Disturbance has just reached particles 4 and 5 continues to be at rest.



(v) At t = T, particle 1 has just completed one vibration, particle 2 has reached its left extreme position particle 3 has completed half its vibration reaching the mean position, particle 4 has reached its right extreme position and disturbance has just reached particle 5.

(vi) At t = 5T/4, particle 1 reaches again its right extreme position, particle 2 has completed on vibration returning to its mean position, particle 3 reaches its extreme left position, particle 4 comes to the mean position after completing 1/2 its vibration and particle 5 goes to its right extreme position.

Q.5. What are transverse waves? Give examples, how such wave propagate?

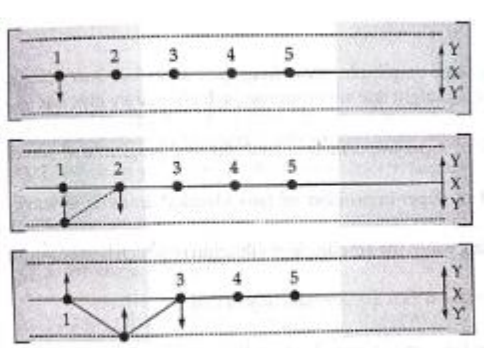
Ans. A transverse wave motion is that motion in which individual particle of the medium execute simple harmonic motion about their mean position is a direction perpendicular to the direction propagation of wave.

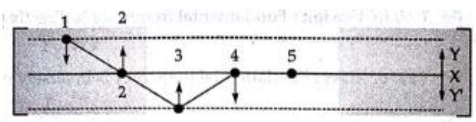
Examples: (i) Movement of string of sitar or a violin.

(ii) Movement of a kink on a rope.

A transverse wave travel through a medium in the form of crests and troughs.

Propagation of transverse waves: Let us consider a series of 5 equally spaces particles 1, 2, 3, 4, 5 in a straight line. When these particles execute S.H.M. of equal amplitudes and time period about their mean position in the upward and downward directions. Solid line represent the mean position and dotted lines up and down represents the extreme positions, while vibrating XY = XY' is the amplitude of vibration. Let T is the time period of vibration of each particle, and the disturbance is handed on from one particle to the adjoining particle in T/4 sec.





(i) At t = 0, all the particles 1, 2, 3, 4, 5 are at rest at their mean positions and the disturbance just reaches particle 1 which start moving downwards. The other particles 2, 3, 4 and 5 are at rest.

(ii) At t = T/4, particle 1 completes 1/4th of tis vibration and reaches the lower extreme positions, the disturbance just reaches particles 2, which starts vibrating downwards. The particles 3, 4, 5 are at rest.

(iii) At t = T/2, particle 1 completes 1/2 of tis vibration reaching the mean position, particle 2 reaches its lower extreme position and disturbance just reaches particle 3, which starts vibrating. The particles 4 and 5 are at rest.

(iv) At t = 3T/4 particle 1 goes to its upper extreme position, particle 2 has completed half its vibration reaching its mean position, particle 3 has executed, 1/4th of its vibration, reaching its lower extreme position. Disturbance has just reached particle 4 and 5 continues to be at rest.

(v) At t = T, particle 1 has completed one vibration, particle 2 has reached its upper position, particle 3 has completed half of its vibration has just reached particle 5.

Q.6. Define Doppler effect. Deduce the apparent frequency of sound in case of:

(i) Source is in rest and observer is moving toward source.
(ii) Observer is in rest and source is moving away from observer.

Ans. It is everyday experience that the pitch for frequency of the whistle of a fast moving train decreases as, it recedes away. When we approach a stationary source of sound with high speed, the pitch of the sound heard appears to be higher than that of the source. As the observer recedes away from the source, the observed pitch or frequency becomes lower than that of the source, the motion related frequencies change, is called Doppler effect.

(i) Observer moving, source stationary: Now, to derive the doppler shift when the observer is moving with velocity v_0 towards the source and the source is at the rest, we have to proceed in a different manner. We work in the reference frame of the moving observer in this reference from the source and medium are approaching at speed v_0 and the speed with which the observer approaches is $v_0 + v$, following a similar procedure as in the previous case. We find that the time interval between the arrival of the first and the $(n+1)^{\text{th}}$ crest is

$$t_{n+1} - t_1 = nt_0 - \frac{nv_0 \mathrm{T}_0}{v_0 + v}$$

The observer thus measure the period of the wave to be

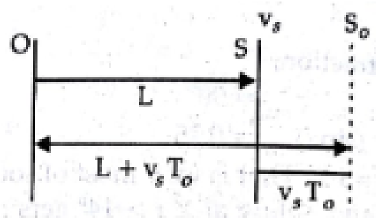
$$= \mathrm{T}_0 \left(1 - \frac{v_0}{v_0 + v}\right)$$

$$= \mathrm{T}_0 \left(1 + \frac{v_0}{v}\right)$$

Given: $$v = v_0 \left(1 + \frac{v_0}{v}\right)$$

if $\frac{v_0}{v}$ is small, the doppler shift is almost same.

(ii) Let us choose the convention to take the direction from the observer to the source as the positive direction of velocity. Consider a source S moving with velocity v_s and an observer who is stationary in a reference in which the medium is also at rest. Let the speed of a wave of angular frequency (i) and period of both measured by an observer at rest with respect to the medium be v. we assume that the observer has a detector that counts every time a wave crest reaches it as shown in fig. At time t = 0, the source is at point S, located at a distance L from the observer and this reaches the observer at time t_1 = L/v at time T = T_0. The source has moved a distance $\mathrm{v}_s\mathrm{T}_0$ and is at point S_2, located at a distance (L + u_s T_0) from the observer at S_2, the source emits a second crest and this reaches the observer at



$\mathrm{T}_2 = \mathrm{T}_0 + \frac{(\mathrm{L}+\mathrm{v}_0\mathrm{T}_0)}{\mathrm{v}}$

At time $n\mathrm{T}_0$, the source emits its $(n+1)$th crest and this reaches the observer at time

$\mathrm{T}_n + 1 = n\mathrm{T}_0 + \frac{(\mathrm{L}+n\mathrm{v}_0\mathrm{T}_0)}{\mathrm{v}}$

Hence, in a time interval

$\left[n\mathrm{T}_0 + \frac{(\mathrm{L}+n\mathrm{v}_0\mathrm{T}_0)}{\mathrm{v}} - \frac{\mathrm{L}}{\mathrm{v}}\right] - n$

The observer's detector counts n crests and the observer records the period of the waves at T given by

T = $\left[n\mathrm{T}_0 + \frac{(\mathrm{L}+n\mathrm{v}_0\mathrm{T}_0)}{\mathrm{v}} - \frac{\mathrm{L}}{\mathrm{v}}\right] - n$

= $\mathrm{T}_0 + \frac{\mathrm{v}_s\mathrm{T}_0}{\mathrm{v}}$

= $\mathrm{T}_0\left[1 + \frac{\mathrm{u}_s}{\mathrm{v}}\right]$

Equation may be rewritten in terms of the frequency v_0 that would be measured if the source and observer were stationary and the frequency observed, when the source is moving as:

v = $\mathrm{v}_0\left(1 + \frac{\mathrm{v}_s}{\mathrm{v}}\right)^{-1}$

If v_s is small compared with the wave speed V. Taking binomial expansion to terms in first order in v_s/v and neglecting higher power, above equation may be approximated as :

v = $\mathrm{v}_0\left(1 - \frac{\mathrm{v}_0}{\mathrm{v}}\right)$

For a source approaching the observer we replace V_0 by V_s to get v = $v_0\left(1+\frac{v_s}{v}\right)$

Q.7. State newton's formula for velocity of sound in air. Point out the error and hence, discussion Laplace's correction.

Ans. Newton's formula for velocity of sound in air:

$$v = \sqrt{\frac{E}{\rho}} \qquad \ldots(1)$$

Where v = velocity of sound in the medium.

E = coefficient of elasticity of medium.

ρ = density of medium.

Newton's formula for velocity of sound in gas:

$$v = \sqrt{\frac{K_i}{\rho}}$$

Since, a gas has only one type of elasticity, i.e., bulk modulus (K),

Sound travel through a gas in the form of compression and rarefactions.

Newton assumed that changes in pressure and volume of a gas, when sound waves are propagated through it, are isothermal. Using coefficient of isothermal elasticity, i.e., K_i in (eqn. 1)

$$v = \sqrt{\frac{K_i}{\rho}} \qquad \ldots (2)$$

Error in Newton's formula:

Let us consider the velocity of sound in air at N.T.P.

$$v = \sqrt{\frac{P}{\rho}} \qquad \ldots (3)$$

As, P = hdg

h = 0.76 m of Hg column

d = 13.6×10^{3} kgm^{-3}

∴ P = $0.76 \times 13.6 \times 10^{-3} \times 9.8$ Nm^{-2}

Density of air, P = 1.293 kg/m^{3}

Here from equ (3) = $\sqrt{\frac{0.76\times13.6\times10^{3}\times9.8}{1.293}}$

v = 280 ms^{-1}

The experimental value of the velocity of sound in air at N.T.P. is 332 ms^{-1}

Difference between the experimental and theoretical value of velocity or sound in air

= (332 – 280) ms^{-1}

= 52 ms^{-1}

Percentage Error = $\frac{52}{332} \times 100$

= 15.7 %

Or ≈ 16%

Laplace's correction:

According to Laplace, the changes in pressure and volume of a gas, when sound waves are propagated through it, are not isothermal, but adiabatic. This is because.

(i) Velocity of sound in a gas is quite large.

(ii) A gas is a bad conductor of heat.

Using the coefficient of adiabatic elasticity, i.e., K_i instead of K_a :

$$v = \sqrt{\frac{\mathrm{K}_i}{\rho}} = \sqrt{\frac{\mathrm{K}_a}{\rho}}$$

Calculation of 'K_a'

Let P be the initial pressure and V be the initial volume of the certain mass of the gas. Under adiabatic condition

$$\mathrm{PV}^r = \text{constant} \quad \ldots(1)$$

Where γ = $\frac{\mathrm{C}_p}{\mathrm{C}_v}$ = ratio of two principal specific heats of the gas.

Differentiating both sides of eqn. 1.

$$\mathrm{P}(\gamma \mathrm{V}^{\gamma-1} d\mathrm{V}) + \mathrm{V}'(d\mathrm{P}) = 0$$

or $$\gamma \mathrm{P} \mathrm{V}^{\gamma-1} d\mathrm{V} = -\mathrm{V}^{\gamma}(d\mathrm{P})$$

or $$\gamma \mathrm{P} = -\frac{\mathrm{V}^{\gamma}}{\mathrm{V}^{\gamma-1}}\left(\frac{d\mathrm{P}}{d\mathrm{V}}\right)$$

$$= \frac{d\mathrm{P}}{\frac{d\mathrm{V}}{\mathrm{V}}}$$

$$= \mathrm{K}_a$$

(By definition)

∴ $$\mathrm{K}_a = \gamma \mathrm{P}$$

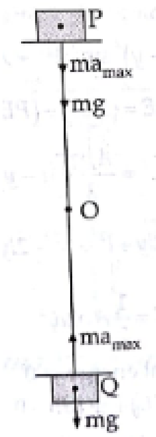
Corrected formula: Substituting this value of K_a in

$$v = \sqrt{\frac{\mathrm{K}_a}{\rho}} = \sqrt{\frac{\gamma \mathrm{P}}{\rho}}$$

The value of γ depends on nature of the gas.

Q.8. A person normally weighing 50 kg stands on a massless platform which oscillated up and down harmonically at a frequency of 2.0 s^{-1} and an amplitude 5.0 cm. A weighting machine on the platform gives the persons weight against time.

(a) Will there be any change in weight of the body, during the oscillation?
(b) If answer to part (a) is yes, what will be the maximum and minimum reading in the machine and at which position?



Ans. (a) Yes, change of weight will be there,

(b) Given mass = m = 50 kg, v = $2s^{-1}$,

A = 5 cm = 0.05 m

Let the platform vibrate between two extreme positions O

P and Q about mean position O.

a_{max} = Maximum acceleration towards mean position O

$a_{max} = w^2 A = (2\pi \text{v})^2 A$

$= 7.9 ms^{-1}$

At P, restoring force (ma_{max}) and weight (mg) are directed towards mean position. so net weight at P given by

$\text{W}_1 = mg + ma_{max} = m(g + a_{\text{max}})$

$= 50(9.8 + 7.9) = 885\ N$

Similarly, net weight at Q

$\text{W}_2 = mg - ma_{max} = m(g - a_{max})$

$= 50(9.8 - 7.9) = 95N$

Weight is maximum at top position and minimum at lowermost position.

Q.9. A body of mass m is attached to one end of a mass less spring which is suspended vertically from a fixed pint. The mass is held in hand so that the spring is neither stretched nor compressed. Suddenly the support of the hand is removed. The lowest position attained by the mass during oscillation is 4 cm below the point, where it was held in hand.

(a) What is the amplitude of oscillation?
(b) Find the frequency of oscillation?

Ans. (a) Amplitude = $\frac{\text{distance between 2 extreme positions}}{2}$

$= \frac{4\ cm}{2} = 2\ cm$

(b) Freq, $v = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$

At mean position, $mg = k(2)$ or $\frac{k}{m} = \frac{g}{2}$

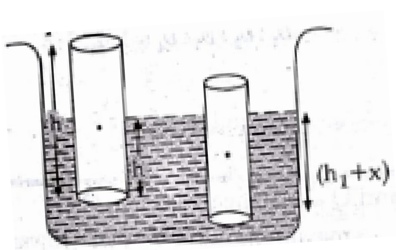
$\therefore$ v = $\frac{1}{2\pi}\sqrt{\frac{g}{2}} = \frac{1}{2\times3.14}\sqrt{\frac{980}{2}}$

$= \frac{1}{6.28}\sqrt{490} = 3.5s^{-1}$

Q.10. A cylindrical log of wood of height H and area of cross-section A floats in water. It is pressed and then released. Show that the log would execute S.H.M. with a time period.

T = $2\pi\sqrt{\frac{m}{A\rho g}}$

Where m is mass of the body and is density of the liquid.



Ans. Given, m = mass of cylinder

h = height of cylinder

h_1 = length of cylinder dipping in liquid in equilibrium position

ρ = density of liquid

A = area of cross section of cylinder

mg = buoyant force

= weight of water displaced by body

= ρ(Ah_1)g …(i)

log is pressed gently through small distance x vertically and released.

$$F_B = \rho A(h_1 + x)g$$

∴ Net restoring force, F = Buoyant Force – weight

= ρA$(h_1 + x)g - mg$

= ρA$(h_1 + x)g$ – ρ(Ah_1)g [from (ii)]

= $(A\rho g)x$

∴ F and x are in opposite direction.

F = $-(A\rho g)x$

$a = \frac{-(A\rho g)}{m}x$ …(ii)

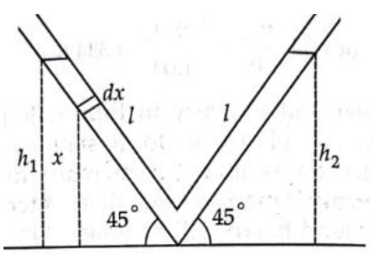
for standard SHM $a = w^2 x$ …(iii)

∴ by (ii) & (iii)

$w^2 = \frac{A\rho g}{m}$ or $w = \sqrt{\frac{A\rho g}{m}}$

∴ T = $2\pi\sqrt{\frac{m}{A\rho g}}$

Q.11. One end of a V-tube containing mercury is connected to a suction pump and the other end to atmosphere. The two arms of the tube are inclined to horizontal at an angle of 45^o each. A small pressure difference is created between two columns when the suction pump is removed. Will the column of mercury in V-tube execute simple harmonic motion? Neglect capillary and viscous forces. Find the time period of oscillation.



Ans. Let the liquid in the length dx is at a height x.

Its mass = Aρdx

PE = (Aρdx)dx

The PE of the left column = $\int_0^{h_1} A\rho g x dx$

=A$\rho g\left[\frac{x^2}{2}\right]_0^{h_1} = A\rho g\frac{l^2 \sin^2 45^o}{2}$

$h_1 = h_2 = l \sin 45^o$ where l = length in one arm of the tube.

Total PE = $A\rho g l^2 sin^2 45^o = \frac{A\rho g l^2}{2}$

If the change in liquid level in the tube in left side is y,

Then length of liquid on left side = $l - y$

And right side = $l + y$

Total

PE = $A\rho g(l-y)^2 \sin^2 45^o$ + $A\rho g(l+y)^2 \sin^2 45^o$

Change in PE = (PE)$_{\mathrm{F}} - (\mathrm{PE})_i = \frac{A\rho g}{2}[(l-y)^2 + (l+y)^2 - l^2]$

= $\frac{A\rho g}{2}[l^2 + y^2 - 2ly + l^2 + y^2 + 2ly - l^2] = \frac{A\rho g}{2}[y^2 + l^2]$

Change in KE = $\frac{1}{2}A\rho 2l\dot{y}^2$

Change in total energy = 0

Δ(PE) + Δ(KE) = 0

$\frac{A\rho g}{2}[2y^2 + l^2] + A\rho l\dot{y}^2 = 0$

Differentiating both wrt time

$A\rho g[0 + \dot{y}y] + 2A\rho l\dot{y}\ddot{y} = 0$

$2A\rho g + 2A\rho l\ddot{y} = 0$

$l\ddot{y} + gy = 0$

$\ddot{Y} + \frac{g}{l}y = 0$

Or $\ddot{y} = -\frac{g}{l}y$

$\therefore w^2 = \frac{g}{l}$ or $w = \sqrt{\frac{g}{l}}$ or T = $2\pi\sqrt{\frac{l}{g}}$

Q.12. A tunnel is dug through the centre of the Earth. Show that a body of mass 'm' when dropped from one end of the tunnel will execute simple harmonic motion.

Ans. Acceleration due to gravity inside Earth

$g' = \frac{GM}{R^3}x = \frac{g}{R}x$

Here x = distance of the point from centre of earth (x<R)

If, block of mass m is placed along the diameter inside the earth.

So, force on block

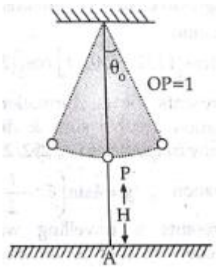
F = $-\frac{mg}{R}x = -kx$

$\therefore$ T = $2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{m}{\frac{mg}{R}}}$

T = $2\pi\sqrt{\frac{R}{g}}$

Q.13. A simple pendulum of time period 1 s and length l is hung from a fixed support at O, such that the bob is at a distance H vertically above A on the ground (Fig.)

The amplitude is θ_0. The string snaps at θ = $\theta_0/2$. Find the time taken by the bob to hit the ground. Also find distance from A where bob hits the ground. Assume $\theta_0$to be small so that sin θ_o = θ_o and cos $\theta_o \simeq 1$.



Ans. Let t = 0 when θ = θ_o

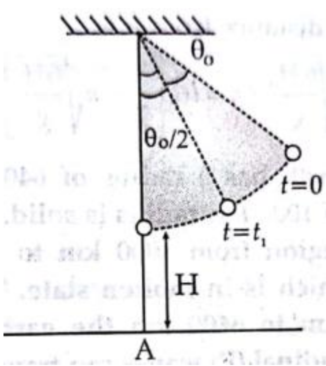
θ = θ_o

Given, T = 1 s $\therefore w = 2\pi rads^{-1}$

at time $t_1, let\ \theta = \frac{\theta_0}{2}$

$\therefore$ cos $2\pi t_1 = \frac{1}{2}$ or $t_1 = \frac{1}{6}s$

$\therefore$ θ = $-\theta_0 2\pi t$ $\left[where\ \theta = \frac{d\theta}{dt}\right]$



At $t_1 = \frac{1}{6s}$

θ = $-\theta_0 2\pi \sin\frac{2\pi}{6}$

$= -\sqrt{3}\pi\theta_0$

Thus, the linear velocity, $u = -\sqrt{3}\pi\theta_0 l$ perpendicular to the string

The vertical component is

$$u_y = -\sqrt{3}\pi\theta_0 l sin\,\theta_0$$

Horizontal component is

$$u_x = -\sqrt{3}\pi\theta_0 l cos\,\theta_0$$

The time when it snaps, vertical height,

$$H' = H + l\left(1 - \cos\frac{\theta_0}{2}\right)$$

Let the time required for fall be t, then

$$H' = u_y t + \left(\frac{1}{2}\right) g t^2$$

{g is also is –ve direction)

$$\frac{1}{2} g t^2 + \sqrt{3}\pi\theta_0 t - H' = 0$$

$$\therefore t = \frac{-\sqrt{3}\pi\theta_0 l sin\,\theta_0 \pm \sqrt{3\pi^2\theta_0^2 l^2 sin^2\theta_0 + 2gH'}}{g}$$

$$= \frac{-\sqrt{3}\pi l\theta_0^2 + \sqrt{3\pi^2\theta_0^4 l^2 + 2gH'}}{g}$$

$$t = \sqrt{\frac{2H'}{g}} \{Neglecting\ terms\ of\ order\ \theta_0^2\ and\ higher\}$$

Now, H' = H + l(1-1) = H $\therefore t = \sqrt{\frac{2H'}{g}}$

Distance travelled in x-direction is $u_x t$ to the left where snapped.

$$x = \sqrt{3}\pi\theta_0 l cos\,\theta_0 \sqrt{\frac{2H}{g}} = \sqrt{\frac{6H}{g}}\theta_0 l\pi$$

At the time of snapping ,the bob was at

$l sin\left(\frac{\theta_0}{2}\right) = l\left(\frac{\theta_0}{2}\right)$ distance from A

$\therefore$ the distance from A is

$$\frac{l\theta_0}{2} - \sqrt{\frac{6H}{g}} l\theta_0\pi = l\theta_0\left[\frac{1}{2} - \pi\sqrt{\frac{6H}{g}}\right]$$

Q.14. The earth has a radius of 6400 km. the inner core of 1000 km radius is solid. Outside it, there is a region from 1000 km to radius of 3500 km which is in molten state. Then again from 3500 km to 6400 km the earth is solid. Only longitudinal (P) waves can

travel inside a liquid. Assume that the P wave has a speed of 8 km s^{-1} in in solid parts and of 5 km s^{-1} in liquid parts of the earth. An earthquake occurs at some place close to the surface of the earth. Calculate the time after which it will be recorded in a seismometer at a diametrically opposite point on the earth if eave travels along diameter?

Ans. In liquid speed of wave = 5 km/s

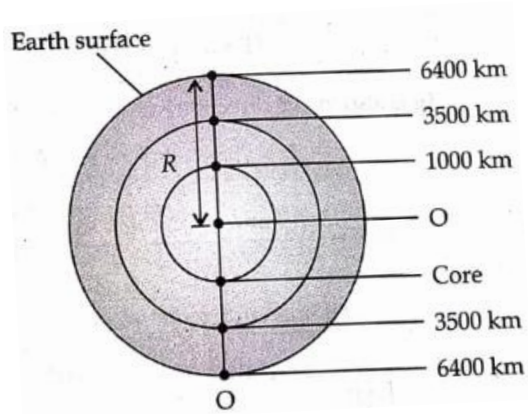
Time required = $\frac{\text{Distance}}{\text{speed}}$

Time = $\left[\frac{1000-0}{8}+\frac{3500-1000}{5}+\frac{6400-3500}{8}\right]\times 2$

Time = $\left[\frac{1000}{8}+\frac{2500}{5}+\frac{2900}{8}\right]\times 2$

Time required = 1975 sec

Time required = 32 min 55 sec.



Q.15. If c is r.m.s. speed of molecules in a gas and v is the speed of sound waves in the gas, show that c/v is constant and independent of temperature for al diatomic gases.

Ans. r.m.s speed of molecules of a gas

$$c=\sqrt{\frac{3P}{\rho}}$$

$$c=\sqrt{\frac{3RT}{M}}$$ (I) [M = Molar mass]

∵ PV = nRT

$n=1$

Or P = $\frac{RT}{V}$

∴ $\frac{P}{\rho}=\frac{RT}{M}$ $\left[\therefore \frac{P}{\delta}=\frac{\frac{RT}{V}}{\frac{M}{V}}=\frac{RT}{M}\right]$

Speed of sound wave in gas, $v = \sqrt{\frac{rP}{\rho}}$

$$v = \sqrt{\frac{rRT}{M}} \text{ (II)}$$

Dividing eqn (II) by eq.n (I),

$\frac{c}{v} = \frac{\sqrt{\frac{3RT}{M}}}{\sqrt{\frac{rRT}{M}}}$

$\frac{c}{v} = \sqrt{\frac{3}{r}}$ [r = adiabatic constant for diatomic gas]

$r = \frac{7}{5}$

Thus, $\frac{c}{v}$ = constant

Q.16. Given below are some functions of x and t to represent the displacement of an elastic wave.

(a) y = 5cos (4x) sin (20t)

(b) $y = 4 \sin(5x - t/2) + 3 \cos(5x - t/2)$

(c) $y = 10 \cos[(252 - 250)\pi t] + \cos[(252 + 250)\pi t)]$

(d) $y = 100 \cos(100\pi t + 0.5x)$

State which of these represent
(a) A travelling wave along-x direction
(b) A stationary wave
(c) Beats
(d) A travelling wave along 4 + x direction.

Given reasons for your answers.

Ans. (a) Equation $y = 100\cos(100\pi t + 0.5x)$ represents a travelling wave along x-direction.

(b) Equation y = 5 cos (4x)sin (20t) represents stationary wave, as this is combination of progressive waves (contains sin, cos, terms)

(c) Equation $y = 10\cos[(252 - 250)\pi t] \cos[(252 + 250)\pi t]$ represent beats formation because this equation involve sum & difference of two nearby frequencies i.e., 252, 250.

(d) Equation $y = 4\sin\left(5x - \frac{t}{2}\right) + 3\cos\left(5x - \frac{t}{2}\right)$ represent a travelling wave along +x direction because it involves negative sign with x.

Q.17. In the given progressive wave $y = 5\sin(100\pi t - 0.4\pi x)$

Where y and x are in m, t is in s. What is the

(a) Amplitude
(b) Wavelength
(c) Frequency
(d) Wave velocity
(e) Particle velocity amplitude.

Ans. Standard progressive wave-

$$y = \text{a}\sin(wt - kx + \phi)$$

$$y = 5\sin(100\pi t - 0.4\pi x + 0)$$

(a) Amplitude, $a = 5$ m

(b) Wavelength, λ

$k = \frac{2\pi}{\lambda}\, or\, \lambda = \frac{2\pi}{k}$

$k = 0.4\pi$

$\lambda = \frac{2\pi}{0.4\pi} = 5$ m

(c) Frequency ν,

$w = 2\pi\nu$

Or $\nu = \frac{w}{2\pi}$

$\because w = 100\pi$

$\nu = \frac{100\pi}{2\pi} = 50$ Hz

(d) Wave velocity, $v = \nu\lambda = 50 \times 5$

$= 250\,\frac{\text{m}}{\text{s}}$

$$y = 5\sin(100\pi t - 0.4\pi x)$$

$\frac{dy}{dx} = 5 \times 100\pi\cos(100\pi t - 0.4\pi x)$

For maximum velocity at mean position –

$\cos(100\pi t - 0.4\pi x) = 1$

Or $100\pi t - 0.4\pi x = 0$

$\therefore \left(\frac{dy}{dx}\right)_{\text{maximum}} = 5 \times 100\pi \times 1$

$v_{max} = 500\pi$ m/s .