Short notes for Heat transfer

Fourier's Law of Heat Conduction

$$Q = -kA\frac{dT}{dx}$$

- Q = Heat transfer in given direction.
- A = Cross-sectional area perpendicular to heat flow direction.
- dT = Temperature difference between two ends of a block of thickness dx
- dx = Thickness of solid body
- $\frac{dT}{dx}$ = Temperature gradient in direction of heat flow.

General Heat Conduction Equation

- **Carterisan Coordinates (side parallel to *x, y* and *z*-directions)**

$$\frac{\partial}{\partial x}\left(k_x\frac{\partial t}{\partial x}\right)+\frac{\partial}{\partial y}\left(k_y\frac{\partial t}{\partial y}\right)+\frac{\partial}{\partial z}\left(k_z\frac{\partial t}{\partial z}\right)+q_g=\rho c\frac{\partial t}{\partial \tau}$$

q_g = Internal heat generation per unit volume per unit time

t = Temperature at left face of differential control volume

k_x, k_y, k_z = Thermal conductivities of the material in *x, y* and *z*-directions respectively

c = Specific heat of the material

ρ = Density of the material

α = Thermal diffusivity $\left(\frac{k}{\rho c}\right)$

$d\tau$ = Instantaneous time.

- **For homogeneous and isotropic material**

$$k_x = k_y = k_z = k,\ \alpha = \frac{k}{\rho c}$$

$$\frac{\partial^2 t}{\partial x^2}+\frac{\partial^2 t}{\partial y^2}+\frac{\partial^2 t}{\partial z^2}+\frac{q_g}{k}=\frac{1}{\alpha}\frac{\partial t}{\partial \tau}$$

- **For steady state condition (Poisson's equation)**

$$\frac{\partial^2 t}{\partial x^2}+\frac{\partial^2 t}{\partial y^2}+\frac{\partial^2 t}{\partial z^2}+\frac{q_g}{k}=0$$

- **For steady state and absence of internal heat generation (Laplace equation)**

$$\frac{\partial^2 t}{\partial x^2}+\frac{\partial^2 t}{\partial y^2}+\frac{\partial^2 t}{\partial z^2}=0$$

- **For unsteady heat flow with no internal heat generation**

$$\frac{\partial^2 t}{\partial x^2}+\frac{\partial^2 t}{\partial y^2}+\frac{\partial^2 t}{\partial z^2}=\frac{1}{\alpha}\frac{\partial t}{\partial \tau}$$

- **Cylindrical Coordinates**
  - **For homogeneous and isotropic material,**

$$\left[\frac{\partial^2 t}{\partial r^2}+\frac{1}{r}\frac{\partial t}{\partial r}+\frac{1}{r^2}\frac{\partial^2 t}{\partial \phi^2}\frac{\partial^2 t}{\partial \tau^2}\right]+\frac{q_g}{k}=\frac{1}{\alpha}\frac{\partial t}{\partial \tau}$$

  - **For steady state unidirectional heat flow in radial direction with no internal heat generation,**

$$\frac{\partial^2 t}{\partial x^2}+\frac{1}{r}\frac{\partial t}{\partial r}=0$$

$$\frac{1}{r}\frac{d}{dr}\left(r\frac{dt}{dr}\right)=0 \Rightarrow r\frac{dt}{dr}=$$

- **Spherical Coordinates**
  - **For homogeneous and isotropic material**

$$\frac{1}{r^2\sin^2\theta}\frac{\partial^2 t}{\partial \phi^2}+\frac{1}{r^2\sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta\frac{\partial t}{\partial \theta}\right)+\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial t}{\partial r}\right)+\frac{q_g}{k}=\frac{1}{\alpha}\frac{\partial t}{\partial \tau}$$

  - **For steady state uni-direction heat flow in radial direction with no internal heat generation,**

$$\frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{dt}{dr}\right)=0$$

- **Thermal resistance of hollow cylinders**

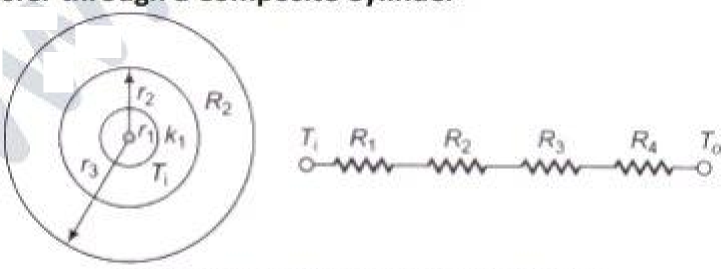
$$R=\frac{\log_e \frac{r_2}{r_1}}{2\pi k L}$$

$$Q=\frac{T_i=T_0}{R}=\frac{\Delta T}{R}$$

- **Thermal Resistance of a Hollow Sphere**

$$R=\frac{r_2-r_1}{4\pi k r_1 r_2}$$

- **Heat Transfer through a Composite Cylinder**



Heat transfer through a composite cylinder

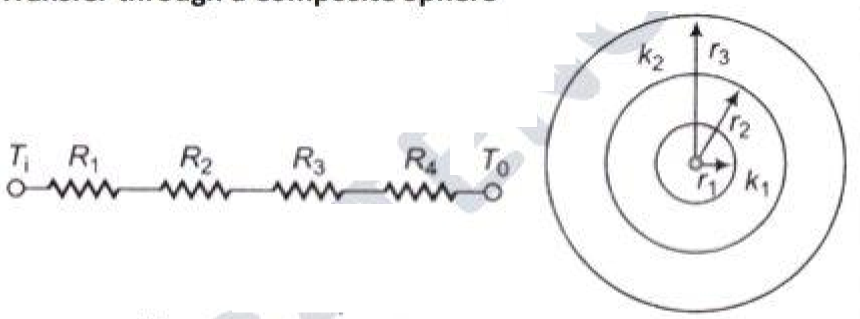
$$Q = \frac{T_i - T_0}{R_{eq}}$$

$$R_{eq} = R_1 + R_2 + R_3 + R_4$$

$$R_1 = \frac{1}{h_i 2\pi r_1 l}, \; R_2 = \frac{In\, r_2 / r_1}{2\pi k_1 l}$$

$$R_3 = \frac{In\, r_3 / r_2}{2\pi k_2 l}, \; R_4 = \frac{1}{h_0 2\pi r_2 l}$$

- **Heat Transfer through a Composite Sphere**



Heat transfer through a composite sphere

$$R_1 = \frac{1}{h_j A_i} = \frac{1}{h_j 4\pi r_1^2}$$

$$R_3 = \frac{r_2 - r_1}{4\pi k r_1 r_2}, \; R_3 = \frac{r_3 - r_2}{4\pi k r_2 r_3}$$

$$R_4 = \frac{1}{h_0 4\pi r_2^{\,2}}, \; Q \, \frac{T_1 - T_0}{R_1 + R_2 + R_3 + R_4}$$

- **Critical Thickness of Insulation:**
  - In case of cylinder,

$$r_c = \frac{k_0}{h}$$

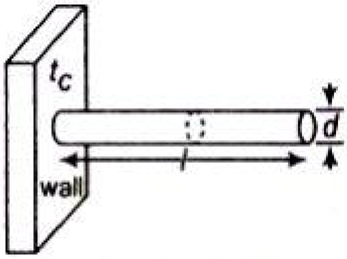
where, k_0 = Thermal conductivity, and *h* = Heat transfer coefficient

  - The drop in temperature across the wall and the air film will be proportional to their resistances, = **hL/k.**

- **Steady Flow of Heat along a Rod Circular fin**

$$\rho = \pi d$$

$$A_c = \frac{\pi}{4} d^2$$



Circular fin diagram

- **Generalized Equation for Fin Rectangular fin**

$$\frac{d^2\theta}{dx^2}+\frac{1}{A_c}\frac{dA_c}{dx}\frac{d\theta}{dx}-\frac{h}{kA_c}\frac{dA_s}{dx}0-0$$

- **Heat balance equation if A_c constant and $A_s \propto P(x)$ linear**

$$\frac{d^2t}{dx^2}-\frac{hp}{kA_c}(t-t_a)=0$$

$$\frac{d^2\theta}{dx^2}-m^2\theta=0$$

$$m=\sqrt{\frac{hp}{kA_c}}$$

- **General equation of 2nd order**

$$\vartheta = c_1e^{mx} + c_2e^{-mx}$$

  - **Heat Dissipation from an Infinitely Long Fin ($l \rightarrow \infty$)**

$$\frac{t-t_a}{t_0-t_a}=e^{-mx}$$

    - **Heat transfer by conduction at base**

$$Q_{fin}= \sqrt{kA_cPh}(t_0-t_a)$$

  - **Heat Dissipation from a Fin Insulated at the End Tip**

$$\frac{\theta}{\theta_0}=\frac{t-t_0}{t_0-t_a}=\frac{\cosh\ m(l-x)}{\cosh\ m\ l}$$

$$Q_{fin}=\sqrt{Phk\ A_c}\ (t_0-t_a)\tanh\ ml$$

  - **Heat Dissipation from a Fin loosing Heat at the End Tip**

$$-kA\left(\frac{dt}{dx}\right)_{x=l} = h\ A_S(t-t_a)$$

$$\theta = \theta_0 \left(\frac{\cosh m(l-x) + \frac{h}{mk}\sinh m(l-x)}{\cosh ml + \frac{h}{mk}\sinh ml} \right)$$

$$Q_{fin} = \sqrt{hPkA_c}\ \theta_0 \frac{\tanh ml + \frac{h}{mk}}{1 + \frac{h}{mk}\tanh ml}$$

- **Fin Efficiency**
- Fin efficiency is given by

$$\eta = \frac{\text{Actual heat rate from fin Q}}{\text{Maximum heat transfer rate } Q_{max}}$$

- If $l \to \infty$ (infinite length of fin),

$$\eta = \frac{\sqrt{hPkA_c}\ \theta_0}{h(Pl + b\delta)\theta_0} = \frac{1}{l}\sqrt{\frac{kA_c}{hP}}$$

$$\eta = \frac{\theta_0\sqrt{hPkA_c}\ \tan h\ ml}{hPl\theta_0}$$

- If finite length of fin,

$$\eta = \frac{\theta_0\sqrt{hPkA_c}\left[\frac{\tan h\ ml + \frac{h}{mk}}{1 + \frac{h}{mk}\tan h\ ml}\right]}{h(Pl + b\delta)\theta_0}$$

- **Fin Effectiveness**

$$\varepsilon = \frac{\text{Actual heat transfer from fin surface (Q)}}{\text{Rate of heat transfer without fin}}$$

$$\varepsilon = \frac{Q}{hA\theta_0} = \frac{\theta_0\sqrt{hPkA_c}\ \tanh\ ml}{hA\theta_0} = \frac{\tan hml}{\sqrt{\frac{hA_c}{Pk}}} \quad (\text{if } l \to \infty)$$

- **Lumped Parameter System**

$$Q = -\rho VCp\frac{dT}{dt} = hA(T - Ta)$$

$$\int \frac{dT}{(T - Ta)} = -\frac{hA}{\rho VCp}\int dt$$

$$\ln(T - Ta) = -\frac{hA}{\rho VCp}t + C_1$$

$$\ln(T - Ta) = -\frac{hA}{\rho VCp}t + \ln(Ti - Ta)$$

$$\frac{T - Ta}{Ti - Ta} = \exp\left[-\frac{hA}{\rho VCp}t\right]$$

- **Nusselt Number (Nu)**
    - It is a dimensionless quantity defined as= hL/ k,

h = convective heat transfer coefficient,

L is the characteristic length

k is the thermal conductivity of the fluid.

- The Nusselt number could be interpreted physically as the ratio of the temperature gradient in the fluid immediately in contact with the surface to a reference temperature gradient (T_s – T_∞) /L.
- Newton's Law of Cooling says that the rate of heat transfer per unit area by convection is given by

$$\dot{Q}/A = h(T_w - T_\infty)$$

$$\frac{\dot{Q}}{A} = h(T_w - T_\infty) = -k\frac{T_w - T_\infty}{\delta_t}$$

$$h = \frac{k}{\delta_t}Nu = \frac{hL}{k} = \frac{L}{\delta_t}$$

$$Nu = \frac{\text{Rate of heat transfer by convection}}{\text{Rate of heat transfer by conduction}} = \frac{h.A.\Delta T}{kA\frac{\Delta T}{1}}$$

- **Temperature distribution in a boundary layer: Nusselt modulus**
- The heat transfer by convection involves conduction and mixing motion of fluid particles. At the solid fluid interface (y = 0), the heat flows by conduction only, and is given by

$$\frac{\dot{Q}}{A} = -k\left(\frac{dT}{dy}\right)_{Y=0}$$

$$h = \frac{\left(-k^{dT}/_{dy}\right)_{y=0}}{(T_w - T_\infty)}$$

$$\frac{hL}{k} = \frac{\left(\frac{dT}{dy}\right)_{y=0}}{(T_w - T_\infty)/L}, \text{and}$$

In dimensionless form,

$$= \left(\frac{d(T_w - T)/(T_w - T_\infty)}{d(y/L)}\right)_{y=0}$$

- **Reynold Number (Re):**

$$Re = \frac{\text{Inertia force}}{\text{Viscous force}}$$

$$Re = \frac{\rho v l}{\mu}$$

$$Re = \frac{v l}{\nu}$$

- **Critical Reynold Number:** It represents the number where the boundary layer changes from laminar to turbine flow.
  - **For flat plate,**
    - $Re < 5 \times 10^5$ (laminar)
    - $Re > 5 \times 10^5$ (turbulent)
  - **For circular pipes,**
    - $Re < 2300$ (laminar flow)
    - $2300 < Re < 4000$ (transition to turbulent flow)
    - $Re > 4000$ (turbulent flow)
- **Stanton Number (St)**

$$St = \frac{\text{Heat transfer coefficient}}{\text{Heat flow per unit temperature rise}}$$

$$St = \frac{Nu}{Re \times Pr}$$

- **Grashof Number (Gr)**

If a body with a constant wall temperature T_w is exposed to a qui scent ambient fluid at T_∞, the force per unit volume can be written as:

$$\rho g \beta (t_w - t_\infty)$$

where ρ = mass density of the fluid, β = volume coefficient of expansion and g is the acceleration due to gravity.

$$Gr = \frac{\text{Inertia force} \times \text{buoyancy foce}}{\text{viscous force}}$$

β = Coefficient of volumetric expansion = 1/*T*

$$Gr = \frac{(\rho V^2 L^2) \times \rho g \beta (T_w - T_\infty) L^3}{(\mu V L)^2}$$

$$= \frac{\rho^2 g \beta (T_w - T_\infty) L^3}{\mu^2} = g\beta L^3 (T_w - T_\infty)/\nu^2$$

- The magnitude of Grashof number indicates whether the flow is **laminar or turbulent**.
- If the Grashof number is greater than 10^9, the flow is turbulent and
- For Grashof number less than 10^8, the flow is laminar.
- For $10^8 < Gr < 10^9$, It is the transition range.

- **Prandtl Number (Pr):**

$$Pr = \frac{\text{Momentum diffusivity through the fluid}}{\text{Thermal diffusivity through the fluid}}$$

$$Pr = \mu C_p / k = \nu / \alpha$$

$$Pr = \frac{\nu}{\alpha} = \frac{\mu/\rho}{k/\rho c_p} \Rightarrow Pr = \frac{\mu c_p}{k}$$

- For liquid metal, $Pr < 0.01$
- For air and gases, $Pr \approx 1$
- For water, $Pr \approx 10$
- For heavy oil and grease, $Pr > 10^5$
- For $Pr << 1$ (in case of liquid metals), the thickness of the thermal boundary layer will be much more than the thickness of the momentum boundary layer and vice versa.
- The product of Grashof and Prandtl number is called Rayleigh number. Or,

Ra = Gr × Pr

- **Rayleigh Number (Ra)**

$$Ra = Gr.Pr, Ra = \frac{g\, \beta\, l^3 \Delta t}{\nu . \alpha}$$

- Free or natural convection
  - $10^4 < Ra < 10^9$ (laminar flow)
  - $Ra > 10^9$ (turbulent flow)

- **Turbulent flow over flat plate**

$$Nu_x = 0.0292\,(Re)^{0.8}\,(Pr)^{0.33}$$

$$\overline{Nu} = 0.036\,(Re)^{0.8}\,(Pr)^{0.33}$$

$$\delta = \frac{0.37\,x}{(Re_x)^{\frac{1}{5}}}$$

- **Turbulent flow in tubes**

$$Nu = 0.023\,(Re)^{0.8}\,(Pr)^{n}$$

where, n = 0.4 if fluid is being heated,

= 0.3 if fluid is being cooled.

$$\overline{Nu} = 0.0036\,(Re)^{0.8}\,(Pr)^{n}$$

Empirical Correlation for Free Convection

- **Heated surface up or cooled surface down**
- **Laminar flow**

$$2 \times 10^5 < Gr.Pr < 2 \times 10^7$$

$$Nu = 0.54\,(Gr\,Pr)^{0.25}$$

- **Turbulent flow**

$$2 \times 10^7 < Gr.Pr < 3 \times 10^{10}$$

$$Nu = 0.14\,(Gr\,Pr)^{0.33}$$

- **Heated surface down or cooled surface up**
- **Laminar flow**

$$3 \times 10^5 < Gr.Pr < 7 \times 10^8$$

$$Nu = 0.27\,(Gr\,Pr)^{0.25}$$

- **Turbulent flow**

$$7 \times 10^8 < Gr.Pr < 11 \times 10^{10}$$

$$Nu = 0.107\,(Gr\,Pr)^{0.33}$$

- **Vertical plates and Large cylinder**
- **Laminar flow**

$$10^4 < GrPr < 10^9$$

$$Nu = 0.59\ (GrPr)^{0.25}$$

- **Turbulent flow**

$$10^9 < GrPr < 10^{12}$$

$$Nu = 0.13\ (GrPr)$$

Empirical Correlation for Forced Convection

- **Laminar Flow over Flat Plate**

$$Nu_x = \frac{hx}{k} = 0.332(\mathrm{Re}_x)^{\frac{1}{2}}\mathrm{Pr}^{\frac{1}{3}}$$

$$Nu_x = \frac{\bar{h}l}{k} = 0.64(\mathrm{Re}_x)^{\frac{1}{2}}\mathrm{Pr}^{\frac{1}{3}}$$

- **Hydrodynamic boundary layer thickness**

$$\rho = \frac{5x}{\sqrt{\mathrm{Re}_x}}$$

- **Laminar Flow over Inside Tube**

$$Nu = \frac{hd}{k},\ T_\infty = \frac{T_o + T_i}{2}$$

  - Constant heat flux, Nu = 4.36

- **Fouling Factor (R_f)**

$$\mathrm{R}_f = \frac{1}{\mathrm{v}_{\text{dirty}}} - \frac{1}{\mathrm{v}_{\text{clear}}}$$

Fin Efficiency and Fin Effectiveness

- **η_{fin} = (actual heat transferred) / (heat which would be transferred if the entire fin area were at the root temperature)**

- **For a very long fin, effectiveness**:

$$E = \dot{Q}_{\text{with fin}} / \dot{Q}_{\text{without fin}}$$

$$= (hpkA)^{1/2}\theta_0 / hA\theta_0 = (kp/hA)^{1/2}$$

And

$$\eta_{fin} = (hpkA)^{1/2}\theta_0 (hpL\theta_0) = (hpkA)^{1/2} / (hpL)$$

$$\frac{E}{\eta_{fin}} = \frac{(kp/hA)^{1/2}}{(hpkA)^{1/2}} \times hpL = \frac{pL}{A} = \frac{\text{Surface area of fin}}{\text{Cross-sectional area of the fin}}$$

i.e., effectiveness increases by increasing the length of the fin but it will decrease the fin efficiency.

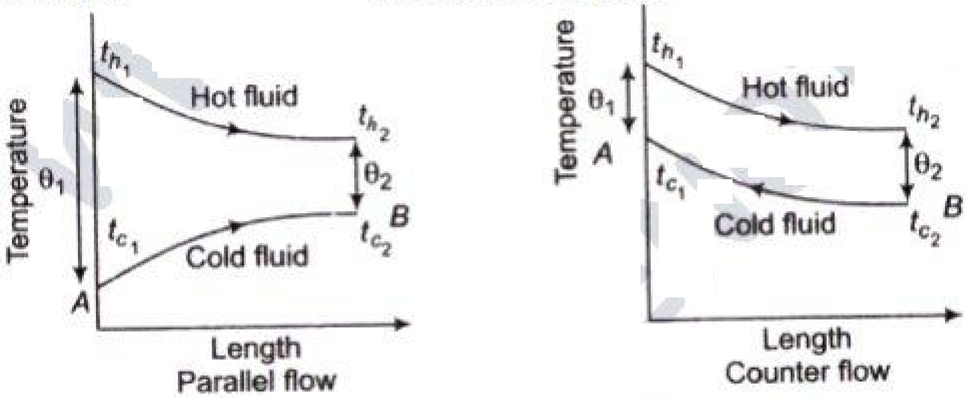
- **Expressions for Fin Efficiency for Fins of Uniform Cross-section:**
    - Very long fins:

$$(hpkA)^{1/2}(T_0 - T_\infty)/[hpL(T_0 - T_\infty)] = 1/mL$$

- For fins having insulated tips

$$\frac{(hpkA)^{1/2}(T_0 - T_\infty)\tanh(mL)}{hpL(T_0 - T_\infty)} = \frac{\tanh(mL)}{mL}$$

Logarithmic Mean Temperature Difference (LMTD)



Temperature distribution for parallel and counter flow heat exchanger

$$Q = UA\frac{\theta_1 - \theta_2}{\log_e\left(\frac{\theta_1}{\theta_2}\right)} = UA\theta_m$$

$$\theta_m = \frac{\theta_1 - \theta_2}{\log_e \frac{\theta_1}{\theta_2}}$$

LMTD

- **Capacity Ratio**

- **Capacity ratio *c = mc*, where *c* = Specific heat**

$$If\ m_h c_h > m_c c_c,\ R = \frac{m_c c_c}{m_h c_h}$$

$$If\ m_h c_h < m_c c_c,\ R = \frac{m_h c_h}{m_c c_c}$$

- **Effectiveness of Heat Exchanger:**

$$\varepsilon = \frac{(Q_{act})\text{actual heat transfer}}{(Q_{max})\text{maximum possible heat transfer}}$$

$$Q_{act} = m_h c_h (t_{h_1} - t_{h_2})$$

$$= m_c c_c (t_{c_2} - t_{c_1})$$

$$Q_{max} = c_{min} (t_{h_1} - t_{c_1})$$

  - If $m_c c_n < m_h c_h \Rightarrow c_{min} = m_c c_c$

$$\Rightarrow Q_{max} = m_c c_c (t_{h_1} - t_{c_1})$$

$$\varepsilon = \frac{m_c c_c (t_{c_2} - t_{c_1})}{m_c c_c (t_{h_1} - t_{c_1})} = \frac{t_{c_2} - t_{c_1}}{t_{h_1} - t_{c_1}}$$

  - If $m_c c_n < m_h c_h \Rightarrow c_{min} = m_h c_h$

$$\Rightarrow Q_{max} = m_h c_h (t_{h_1} - t_{c_1})$$

$$\varepsilon = \frac{m_h c_h (t_{h_1} - t_{h_2})}{m_h c_h (t_{h_1} - t_{c_1})} = \frac{t_{h_1} - t_{h_2}}{t_{h_1} - t_{c_1}}$$

- **Number of Transfer Units (NTU):**

$$NTU = \frac{UA}{c_{min}}$$

U = Overall heat transfer coefficient

A = Surface area

C min = Minimum capacity rate

If $m_h c_h < m_c c_c \Rightarrow c_{min} = m_c c_c$

$$\Rightarrow NTU = \frac{UA}{m_h c_c}$$

If $m_h c_h < m_c c_c \Rightarrow c_{min} = m_h c_h$

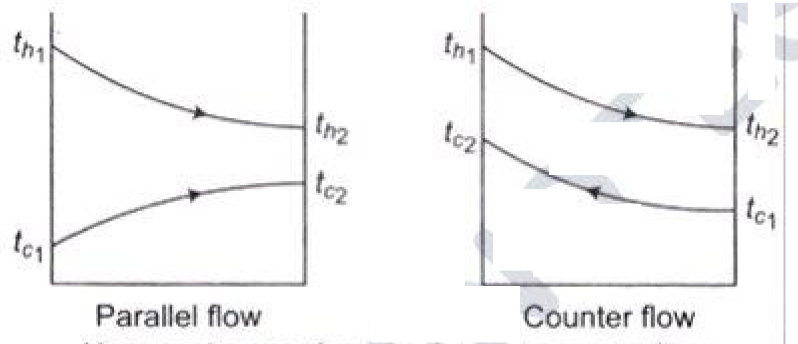
$$\Rightarrow NTU = \frac{UA}{m_h c_h}$$

Effectiveness for Parallel Flow Heat Exchanger

$$\varepsilon = \frac{1 - \exp[-NTU(1+R)]}{1+R}$$

$$R = \frac{C_{min}}{C_{max}}$$

$$NTU = \frac{UA}{C_{min}}$$



Heat exchanger for parallel and counter flow

- **Effectiveness for the Counter Flow Heat Capacity:**

$$\varepsilon = \frac{1-\exp[-NTU(1-R)]}{1-R\exp[-NTU\ (1-R)]}$$

- **Heat Exchanger Effectiveness Relation:**
- **Concentric tube:**
  - Parallel flow:

$$\epsilon = \frac{1-\exp[-N(1+R)]}{(1+R)}; R = C_{min}/C_{max}$$

  - Counter flow:

$$\epsilon = \frac{1-\exp[-N(1-R)]}{1-R\ \exp[-N(1-R)]}; R<1$$

$$\epsilon = N/(1+N)\ for R = 1$$

- **Cross flow (single pass):**
  - Both fluids unmixed:

$$\epsilon = 1-\exp\left[(1/R)(N)^{0.22}\left\{\exp\left(-R(N)^{0.78}\right)-1\right\}\right]$$

  - C_{max} mixed , C_{min} unmixed:

$$\epsilon = (1/R)\left[1-\exp\left\{-R\left(1-\exp(-N)\right)\right\}\right]$$

  - C_{min} mixed, C_{max} unmixed:

$$\epsilon = 1-\exp\left[-R^{-1}\left\{1-\exp(-RN)\right\}\right]$$

Total Emissive Power (E)

- It is defined as the total amount of radiation emitted by a body per unit time and area.

$$E = \sigma T^4 \ W/m^2$$

σ = Stefan Boltzmann constant

$\sigma = 5.67 \times 10^{-8}$ W/m² K⁴

Monochromatic (Spectral) Emissive Power (E_λ)

- It is defined as the rate of energy radiated per unit area of the surface per unit wavelength.

$$E = \int_0^\infty E_\lambda d\lambda \, w/m^2$$

Emission from Real Surface

- The emissive power from a real surface is given by

$$E = \varepsilon \sigma A T^4 \ W$$

ε = Emissivity of the surface,

T = Surface temperature

Emissivity (ε)

- It is defined as the ratio of the emissive power of any body to the emissive power of a black body of same temperature.

$$\varepsilon = \frac{E}{E_b}$$

  - For black body, ε = 1
  - For white body, ε = 0
  - For gray body, 0< ε<1

Reflectivity (ρ)

- It is defined as the fraction of total incident radiation that are reflected by material.

$$\text{Reflectivity}(\rho) = \frac{\text{Energy reflected } (Q_r)}{\text{Total incident radiation } (Q)}$$

Absorptivity

- It is defined as the fraction of total incident radiation that are absorbed by material.

$$\text{Absorptivity}(\alpha) = \frac{\text{Energy absorbed } (Q_a)}{\text{Total incident radiation } (Q)}$$

Transmissivity

- It is defined as the fraction of total incident radiation that are transmitted through the material.

$$\text{Transmissivty}(\tau) = \frac{\text{Energy transmitted } (Q_t)}{\text{Total incident radiation } (Q_w)}$$

$$Q_0 = Q_a + Q_r + Q_t$$

$$\frac{Q_0}{Q_0} = \frac{Q_a}{Q_0} + \frac{Q_r}{Q_0} + \frac{Q_t}{Q_0}$$

$$\alpha + \rho + \tau = 1$$

- **For black body** $\alpha = 1, \rho = 0, \tau = 0$
- **For opaque body** $\tau = 0, \alpha + \rho = 1$
- **For white body** $\rho = 1, \alpha = 1$ and $\tau = 0$

Kirchoff's Law

- The emissivity ε and absorptivity α of a real surface are equal for radiation with identical temperature and wavelength.

$$\alpha = \varepsilon = \frac{E}{E_b}$$

- Emissive power of a black body is directly proportional to the fourth power of its absolute temperature.

$$E_b = \sigma T^4$$

E_b = Emissive power of a black body,

σ = Stefan Boltzmann constant ($5.67 \cdot 10^{-8} W/m^2 \cdot K^4$),

T = Absolute temperature of the emitting surface, K.

Wien's Displacement Law

- Wien's displacement law state that the product of λ_{max} and T is constant.

$$\lambda_{max} T = \text{constant}$$

λ_{max} = Wavelength at which the maximum value of monochromatic emissive power occurs.

Gray Surfaces

The gray surface is a medium whose monochromatic emissivity (ε_λ) does not vary with wavelength.

$$\varepsilon_\lambda = E_\lambda / E_{\lambda, b}$$

But, we know the following.

$$E = \int_0^\infty \varepsilon_\lambda E_{\lambda,b} d\lambda$$

$$E_b = \int_0^\infty E_{\lambda,b} d\lambda = \sigma T^4$$

Therefore,

$$\varepsilon = \frac{\int_0^\infty \varepsilon_\lambda E_{\lambda,b} d\lambda}{\sigma T^4}$$

View Factors:

- Define the view factor, F_{1-2}, as the fraction of energy emitted from surface 1, which directly strikes surface 2.

$$F_{1\to 2} = \frac{q_{1\to 2}}{q_{emitted}} = \frac{\int_{A_1}\int_{A_1} \frac{I \cdot \cos\theta_1 \cdot}{R^2} dA_1 \cdot \cos\theta_2 dA_2}{\pi \cdot I \cdot A_1}$$

$$F_{1\to 2} - \frac{1}{A_1} = \int_{A_1}\int_{A_1} \frac{\cos\theta_1 \cdot \cos\theta_2 \cdot dA_1 \cdot dA_2}{\pi \cdot R^2}$$

Reciprocity:

$$A_i \cdot F_{1\to j} = \int_{Aj}\int_{A_i} \frac{\cos\theta_i \cdot \cos\theta_j \cdot dA_i \cdot dA_j}{\pi \cdot R^2}$$

$$A_j \cdot F_{j\to i} = \int_{Aj}\int_{A_i} \frac{\cos\theta_j \cdot \cos\theta_i \cdot dA_j \cdot dA_i}{\pi \cdot R^2}$$

$$A_i \cdot F_{i\to j} = A_j \cdot F_{j\to i}$$

Planck's Law:

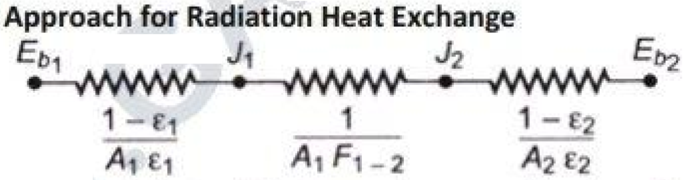
- Planck suggested following formula, monochromatic emissive power of a black body.

$$(E_\lambda)_b = \frac{2\pi c^2 h \lambda^{-5}}{\exp\left(\frac{ch}{\lambda k T}\right) - 1}$$

- Total emissive power

$$E_b = \int_0^\infty (E_\lambda)_b \, d\lambda$$

Electrical Network Approach for Radiation Heat Exchange



An electrical network between two non black surfaces

$$(Q_{1-2})_{net} = \frac{E_{b_1} - E_{b_2}}{\frac{1-\varepsilon_1}{A_1 \varepsilon_1} + \frac{1}{A_1 F_{1-2}} + \frac{1-\varepsilon_2}{A_2 \varepsilon_2}}$$

$$\text{or } (Q_{1-2})_{net} = (F_g)_{1-2} A_1 \sigma_b \left(T_1^4 - T_2^4\right)$$

New Gray Body Factor

$$(G_g)_{1-2} = \frac{1}{\frac{1-\varepsilon_1}{A_1\varepsilon_1} + \frac{1}{A_1F_{12}} + \frac{1-\varepsilon_2}{A_2\varepsilon_2}}$$

E_1 = Emissivity for body 1

$E2$ = Emissivity for body 2

- **In case of black surfaces, $\varepsilon_1 = \varepsilon_2 = 1$, $(F_g)_{12} = F_{1-2}$**

$$Q_{net} = F_{1-2} A_1 \sigma \left(T_1^4 - T_2^4 \right)$$

- **In case of parallel planes, $A_1 = A_2$ and $F_{1-2} = 1$**

$$(F_g)_{1-2} = \frac{1}{\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_2} - 1}$$

- **In case of concentric cylinder or sphere, $F_{1-2} = 1$**

$$(F_g)_{1-2} = \frac{1}{\frac{1-\varepsilon_1}{\varepsilon_1} + 1 + \frac{1+\varepsilon_2}{\varepsilon_2}\frac{A_1}{A_2}}$$

Where, $\frac{A_1}{A_2} = \frac{r_1}{r_2}$ (for concentric cylinder)

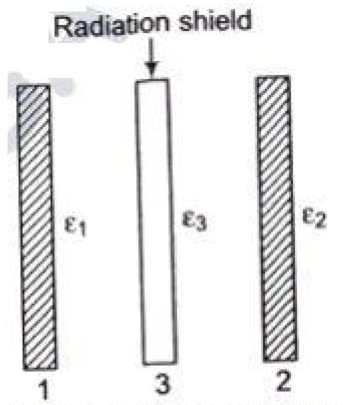
$\frac{A_1}{A_2} = \frac{r_1^2}{r_2^2}$ (for concentric sphere)

- **When a small body lies inside a large enclosure**

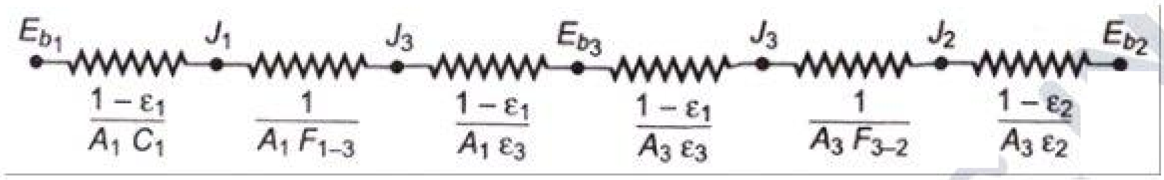
$$F_{1-2} = 1,\ A_1 << A_2 \Rightarrow \frac{A_1}{A_2} = 0$$

$$(F_g)_{1-2} = \frac{1}{\frac{1-\varepsilon_1}{\varepsilon_1} + 1}$$

Radiation Shield



Radiation shield diagram



- **Radiation network for 2 parallel infinite places separated by one shield**

$$(Q_{1-3})_{net} = (Q_{3-2})_{net} \quad (A_1 = A_3 = A_2)$$

$$\frac{A\sigma(T_1^4 - T_3^4)}{\dfrac{1}{\varepsilon_1} + \dfrac{1}{\varepsilon_3 - 1}} = \frac{A\sigma(T_3^4 - T_2^4)}{\dfrac{1}{\varepsilon_3} + \dfrac{1}{\varepsilon_2} - 1}$$

$$(Q_{1-2})_{net} = \frac{A\sigma(T_1^4 - T_2^4)}{\left(\dfrac{1}{\varepsilon_1} + \dfrac{1}{\varepsilon_3} - 1\right) + \left(\dfrac{1}{\varepsilon_3} + \dfrac{1}{\varepsilon_2} - 1\right)}$$

$$\frac{[(Q_{1-2})_{net}]\text{ with shield}}{[(Q_{1-2})_{net}]\text{ without shield}}$$

$$= \frac{\left(\dfrac{1}{\varepsilon_1} + \dfrac{1}{\varepsilon_2} - 1\right)}{\left(\dfrac{1}{\varepsilon_1} + \dfrac{1}{\varepsilon_3} - 1\right) + \left(\dfrac{1}{\varepsilon_3} + \dfrac{1}{\varepsilon_2} - 1\right)}$$

If $\varepsilon_1 = \varepsilon_2 = \varepsilon_3$

Then,

$$(Q_{1-3})_{net} = (Q_{3-2})_{net} = \frac{1}{2}(Q_{1-2})_{net}$$

and

$$T_3^4 = \frac{1}{2}(T_1^4 - T_2^4)$$