Ratio and Proportion

Conversion of Fractions, Ratios, Decimals and Whole Numbers to Equivalent Percentages and Vice-versa

The following table shows the numbers of matches played and won in a season by two cricket clubs − Balmain Star and New Star.

	BALMAIN STAR	**NEW STAR**
Number of matches played	75	90
Number of matches won	48	54

Now, how can we tell which one is the better team? Though New Star had won more matches than Balmain Star, but New Star had also played more matches than Balmain Star. Therefore, we cannot compare the better performing team just by looking at the number of matches won by the two teams.

We could have easily compared this data if the two teams had played the same number of matches. To solve such type of problems, we have to know the concept of **percentages.**

First of all, let us know what a percentage is and after that we will solve this problem.

Per cent is derived from Latin word 'per centum' meaning 'per hundred' and it is defined as

Percentages are numerators of fractions with denominator 100.

Now that we know how to convert whole numbers, fractions, decimals into percentages and vice-versa, let us use these concepts to find the better team between Balmain Star and New Star.

Before finding the percentage of match won by each team, we have to find the fraction of matches won by them out of the total number of matches played by the team.

Now, the fractions of matches won by Balmain Star and New Star are $\frac{48}{75}$ and $\frac{54}{90}$ respectively.

Therefore, percentage of matches won by Balmain Star $= \left(\frac{48}{75} \times 100\right)\% = 64\%$

Similarly, percentage of matches won by New Star $= \left(\frac{54}{90} \times 100\right)\% = 60\%$

Here, we can see that though New Star had won more matches than Balmain Star, the percentage of matches won by Balmain Star is more than that of New Star. Therefore, Balmain Star is a better team than New Star.

We know that a fraction represents a part of a whole and 1 represents the whole. Since 1 = 100%, 100% represents the whole of an object and the percentages below 100% represent parts of a whole.

Percentages can be added and subtracted, but the total percentage of a whole will be 100%.

For example, Sonu has a pizza and he ate 45% of it.

Now, percentage of remaining pizza = 100% − 45% = (100 − 45)% = 55%

Let us discuss some more examples based on the conversion of fractions, ratios, decimals, and whole numbers to equivalent percentage, and vice-versa.

Example 1:

Convert the percentages 325%, 9.5%, and 800% into fraction, whole number, and decimal.

Solution:

$325\% = \frac{325}{100} = \frac{13}{4}$

$\frac{13}{4}$ is a fraction. It cannot be a whole number.

Also, $325\% = \frac{325}{100} = 3.25$

$\therefore$ The fraction and decimal forms of 325% are $\frac{13}{4}$ and 3.25 respectively.

Now, $9.5\% = \frac{9.5}{100} = \frac{95}{1000} = \frac{19}{200}$

$\frac{19}{200}$ is a fraction. It cannot be a whole number.

Also, $9.5\% = \frac{9.5}{100} = 0.095$

$\therefore$ The fraction and decimal form of 9.5% are $\frac{19}{200}$ and 0.095 respectively.

Now, $800\% = \frac{800}{100} = 8$

Also, $800\% = \frac{800}{100} = 8.00$

$\therefore$ The whole number and the decimal form of 800% are 8 and 8.00 respectively.

Example 2:

Convert the numbers $\mathbf{0.0075, \frac{9}{25}, 2, \frac{31}{16}}$ into percentage form.

Solution:

$0.0075 = (0.0075 \times 100)\% = 0.75\%$

$\frac{9}{25} = \left(\frac{9}{25} \times 100\right)\% = 36\%$

$2 = (2 \times 100)\% = 200\%$

$\frac{31}{16} = \left(\frac{31}{16} \times 100\right)\% = 193.75\%$

Example 3:

In a hostel, 30% students like to watch BCB, 25% students like to watch NDVT, 27% students like to watch VT18 channel, and the remaining students like to watch NCN. If a student likes to watch only one channel, then what percentage of students like to watch NCN?

Solution:

Percentage of students who like to watch BCB = 30

Percentage of students who like to watch NDVT = 25

Percentage of students who like to watch VT18 = 27

Now, the percentage of students who like to watch these three channels = 30 + 25 + 27 = 82

We know that all the parts that form the whole when added together give the whole or 100%.

Therefore, percentage of students who like to watch NCN = 100 − 82 = 18

Thus, 18% students like to watch NCN.

Example 4:

A certain amount of money is divided among Pallavi, Manjri, and Payal in the ratio 3:9:8. What percentage of the amount distributed does each girl get?

Solution:

Ratio in which the amount is divided = 3:9:8

Sum of the parts of the ratio = 3 + 9 + 8 = 20

Therefore, Pallavi, Manjri, and Payal respectively got $\frac{3}{20}, \frac{9}{20}$ and $\frac{8}{20}$ of the total amount that was distributed among them.

Thus, Pallavi got $\left(\frac{3}{20}\times 100\right)\% = 15\%$ of the total amount that was distributed.

Similarly, Manjri got $\left(\frac{9}{20}\times 100\right)\% = 45\%$ of the total amount that was distributed.

Payal got $\left(\frac{8}{20}\times 100\right)\% = 40\%$ of the total amount that was distributed.

Example 5:

A child won 12 games out of a number of games. If the win percentage was 30, how many games were there in total?

Solution:

It is given that win percentage is 30. So,
30 games were won out of 100 games.

∴ 12 games were won out of $\frac{100}{30}\times 12 = 40$ games.

Hence, there were 40 games in total.

Example 6:

The salary of Rohan is increased by 20%. If his new salary is Rs 18,000, then what was his salary before increment?

Solution:

Let the salary before increment be Rs 100. Since his increment is 20%, so his salary after increment is Rs 100 + Rs 20 = Rs 120.

If the new salary is Rs 120, then the salary before increment is Rs 100.

So, if the new salary is Rs 18,000, then the salary before increment is
Rs $\frac{100}{120}\times 18000 = \text{Rs}\,15,000$

Thus, the salary of Rohan before the increment was Rs 15, 000.

Example 7:
Shyam's income is 40% more than that of Ram. What percent is Ram's income less than that of Shyam?
Solution:

Let Ram’s income be Rs 100. Then Shyam’s income will be Rs 140.

So, if Shyam’s income is Rs 140, then Ram’s income will be Rs 100.

If Shyam’s income is Rs 1, then Ram’s income will be Rs 100/140**.**

If Shyam’s income is Rs 100, then Ram’s income will be

Rs $\frac{100}{140}\times 100 = \text{Rs } \frac{5}{7}\times 100 = \text{Rs } 71\frac{3}{7}$

Hence, Ram’s income is $\left(100 - 71\frac{3}{7}\right)\% = 28\frac{4}{7}\%$ less than that of Shyam’s income.

Word Problems on Percentages

The following tables list the marks obtai ned by Bhaskar in his first unit test and the final term examinations.

1st Unit Test Mark Sheet

Subject	Marks Obtained	Total Marks
English	16	20
Mathematics	19.5	20
Science	16	20
French	18.5	20
Social Studies	16	20

Final Term Mark Sheet

Subject	Marks Obtained	Total Marks
English	75	100
Mathematics	95	100
Science	83	100
French	81	100
Social Studies	66	100

Now, how can we determine whether Bhaskar's overall performance was better in the 1^{st} unit test or in the final term examination?

Total marks in the 1^{st} unit test = 20 × 5 = 100

Total marks obtained by Bhaskar in the 1^{st} unit test = 16 + 19.5 + 16 + 18.5 + 16

= 86

Total marks in the final term examination = 100 × 5 = 500

Total marks obtained by Bhaskar in the final term examination = 75 + 95 + 83 + 81 + 66 = 400

Here, the total marks in the unit test and the final term examination are not equal. Therefore, we cannot compare Bhaskar's performance just by looking at the marks obtained by him in the test and in the examination. We can solve this problem by calculating his percentage score in the unit test and in the final term examination.

'Percent' means 'per hundred' or 'out of hundred'. To express *x* as a percentage of *y*, we use the following formula:

To express *x* as a percentage of *y*, percentage = $\left(\frac{x}{y}\times 100\right)\%$

Thus, Bhaskar's percentage score in the 1^{st} unit test

$$=\frac{\text{Marks obtained}}{\text{Total marks}}\times 100=\frac{86}{100}\times 100=86$$

Bhaskar's percentage score in the final term examination $=\frac{400}{500}\times 100=80$

Thus, Bhaskar scored 80% in the final term examination and 86% in the 1^{st} unit test. Looking at this information, we can safely say that his performance was better in the 1^{st} unit test as compared to that in the final term examination.

Let us consider another situation where Ramesh, Deepika, and Devyani are cousins. One day, their uncle gave them Rs 800 and told Deepika and Devyani to take 35% and 40% of this amount respectively and Ramesh to take the remaining amount. However, the three cousins were confused about how they would share this money. Let us try to help them out.

For this, first of all we have to know the method to find certain percentage of a quantity. For this, we use the following formula:

***x* % of a given quantity**
= (x100 × given quantity) x100 × given quantity

Deepika's share = 35%

Devyani's share = 40%

∴ Ramesh's share = [100 − (35 + 40)]% = (100 − 75)% = 25%

Thus, Deepika's share out of Rs 800 $= \text{Rs}\left(\frac{35}{100}\times 800\right) = \text{Rs } 280$

Similarly, Devyani's share out of Rs 800 $= \text{Rs}\left(\frac{40}{100}\times 800\right) = \text{Rs } 320$

Similarly, Ramesh's share out of Rs 800 $= \text{Rs}\left(\frac{25}{100}\times 800\right) = \text{Rs } 200$

Therefore, the shares of Ramesh, Deepika, and Devyani are Rs 200, Rs 280, and Rs 320 respectively.

Now, let us consider another situation where Rohit scored 68% mark in an exam. If he scored 340 marks, then can we find the total marks for which the exam was held?

For this, we have to know the formula to calculate the total quantity, when its certain quantities are given in terms of percentage.

If *x* % of a given quantity is *y*, then quantity
= $\frac{y}{x} \times \mathbf{100}$

Hence, the total marks for which the exam was held = $\frac{340}{68}\times 100 = 500$ marks

In this way, we can find the whole quantity or a part of the given quantity when the percentage is given. This concept is known as the “How Many” concept.

Let us discuss some more examples based on the discussed concepts.

Example 1:

Find the following values.

(i) 16% of 100

(ii) 25% of 325

(iii) 65% of 1.2 kg

Solution:

(i) 16% of 100 $= \frac{16}{100} \times 100 = 16$

(ii) 25% of 325 $= \frac{25}{100} \times 325 = 81.25$

(iii) 65% of 1.2 kg = 65% of 1200g = $\left(\frac{65}{100} \times 1200\right)$ g = 780 g = 0.78 kg

Example 2:

There were 25 chocolates in a bag, out of which Rohit ate 7 chocolates. What percentage of chocolates was eaten by Rohit?

Solution:

Number of chocolates eaten by Rohit = 7

Total number of chocolates in the bag = 25

∴ Fraction of chocolates eaten by Rohit = $\frac{7}{25}$

Therefore, percentage of chocolates eaten by Rohit = $\frac{7}{25}\times 100 = 28$

Thus, Rohit ate 28% chocolates in the bag.

Example 3:

A shopkeeper has a stock of 200 cricket bats of three brands − A, B, and C. Out of these, 65 bats are of brand A, 90 bats are of brand B, and the remaining bats are of brand C. Find the percentage of bats of each brand with the shopkeeper.

Solution:

Total number of cricket bats = 200

Number of bats of brand A = 65

Number of bats of brand B = 90

∴ Number of bats of brand C = 200 − (65 + 90) = 200 − 155 = 45

Thus, fraction of bats of brand A $=\frac{65}{200}$

Similarly, fraction of bats of brand B $=\frac{90}{200}$

Similarly, fraction of bats of brand C $=\frac{45}{200}$

∴ Percentage of bats of brand A $=\left(\frac{65}{200}\times 100\right)\% = 32.5\%$

∴ Percentage of bats of brand B $=\left(\frac{90}{200}\times 100\right)\% = 45\%$

∴ Percentage of bats of brand C $=\left(\frac{45}{200}\times 100\right)\% = 22.5\%$

Example 4:

There are 500 students in a school. Out of these, 325 students are day scholars while the remaining students are hostlers. Find the percentage of the students who are hostlers?

Solution:

Number of day scholars in the school = 325

Total number of students in the school = 500

Thus, number of hostlers in the school = 500 − 325 = 175

Thus, percentage of students who are hostlers $= \left(\frac{175}{500} \times 100\right)\% = 35\%$

This question can also be solved by another method.

Number of day scholars in the school = 325

Total number of students in the school = 500

Percentage of students who are day scholars $= \left(\frac{325}{500} \times 100\right)\% = 65\%$

Thus, percentage of students who are hostlers = 100% − 65% = 35%

Example 5:

Vicky spends Rs 4000 and saves Rs 1000 every month. What percentage of his monthly income constitutes his monthly savings?

Solution:

Vicky’s monthly expenditure = Rs 4000

Vicky’s monthly savings = Rs 1000

∴ Vicky’s monthly income = Rs (4000 + 1000) = Rs 5000

∴ Percentage of money saved by Vicky every month $= \left(\frac{\text{Rs } 1000}{\text{Rs } 5000} \times 100\right)\% = 20\%$

Thus, 20% of Vicky’s monthly salary constitutes his monthly savings.

Example 6:

In badminton, Sonia played 25 matches and lost 5 matches whereas Sunita played 32 matches and lost 6 matches. Who is a better player among them?

Solution:

Number of matches played by Sonia = 25

Number of matches lost by Sonia = 5

Therefore, number of matches won by Sonia = 25 − 5 = 20

∴ Percentage of matches won by Sonia $= \left(\frac{20}{25} \times 100\right)\% = 80\%$

Number of matches played by Sunita = 32

Number of matches lost by Sunita = 6

Therefore, number of matches won by Sunita = 32 − 6 = 26

∴ Percentage of matches won by Sunita $= \left(\frac{26}{32} \times 100\right)\% = 81.25\%$

Since the winning percentage of Sunita is more than that of Sonia, we can say that Sunita is the better player among the two.

Example 7:

What percentage of 1 minute is 21 seconds?

Solution:

1 minute = 60 seconds

Let *x*% of 60 seconds be 21 seconds.

∴ *x*% of 60 = 21

We know that, *x*% $= \frac{x}{100}$

$\Rightarrow \frac{x}{100}$ of $60 = 21$

$\Rightarrow \frac{x}{100} \times 60 = 21$

$\Rightarrow x \times \frac{3}{5} = 21$

Multiplying both sides by 5, we obtain

$x \times \frac{3}{5} \times 5 = 21 \times 5$

$\Rightarrow x \times 3 = 105$

Dividing both sides by 3, we obtain

$\frac{x \times 3}{3} = \frac{105}{3}$

$\Rightarrow x = 35$

Thus, 35% of 1 minute is 21 seconds.

Example 8:

In a class, 60% of all the students are boys. It was observed that 90 girls were present in the class on a specific day. If these girls were 75% of the entire girls, then how many boy students are there in the class?

Solution:

Let the total number of students in the class be *x*.

It is given that 60% of all the students are boys.

Therefore, number of boy students in the class = $\frac{60}{100} \times x = \frac{3x}{5}$

And, number of girl students in the class = $x - \frac{3x}{5} = \frac{2x}{5}$

It is also given that 75% of all the girls were present on the day.

Therefore, number of girls present in the class = $\frac{75}{100} \times \frac{2x}{5} = \frac{3x}{10}$

It is also given that 90 girls were present in the class.

$\therefore \frac{3x}{10} = 90$

$\Rightarrow 3x = 900$

$\Rightarrow x = \frac{900}{3} = 300$

Therefore, number of boy students in the class $= \frac{3x}{5} = \frac{3 \times 300}{5} = 180$

Example 9:

Sujit saved 33% of his salary last month. If he saved Rs 4950 that month, then what is his monthly salary?

Solution:

Let Rs *n* be his monthly salary.

Now, 33% of *n* = 4950

⇒ 33% × *n* = 4950

$\Rightarrow \frac{33}{100} n = 4950$

Multiplying both sides by 100, we obtain

33*n* = 495000

Now, dividing both sides by 33, we obtain

$\frac{33n}{33} = \frac{495000}{33}$

⇒ *n* = 15000

Therefore, his monthly salary is Rs 15000.

Example 10:

In a carton of 250 bulbs, 4.8% bulbs are found to be defective. How many bulbs in that carton are in working condition?

Solution:

Total number of bulbs in the carton = 250

Percentage of defective bulbs = 4.8%

$\therefore$ Number of defective bulbs in the carton $= 4.8\% \text{ of } 250 = \frac{4.8}{100} \times 250 = \frac{1200}{100} = 12$

Therefore, number of bulbs that are in working condition = 250 − 12 = 238

Concept of Profit and Loss

What is every shopkeeper's motto? "To buy an item cheap and to sell it at a higher price". The extra money that he makes in the process is known as profit. However, a shopkeeper is not always lucky and sometimes has to sell an item at a lesser price than the one he bought it at. This is a case of loss for the shopkeeper. Let us try and understand the concept of profit and loss with the help of some examples.

Generally when an item is purchased, some additional expenses such a labour charges, transportation charges, maintenance charges etc are made before the selling of the item. These expenses are known as overhead charges. These expenses have to be added in the cost price of the item.

∴ Real cost price = Price for purchasing the goods + Overhead charges

Let us discuss some examples based on the above concept.

Example 1:

Find out the profit or loss in the following transactions.

(i) Rahul bought a bicycle for Rs 1200 and sold it for Rs 1150.

(ii) Tanmay bought a pair of trousers for Rs 700 and sold them for Rs 725.

Solution:

(i) Here, C.P. = Rs 1200

S.P. = Rs 1150

∴ Loss incurred = C.P. − S.P. = Rs 1200 − Rs 1150 = Rs 50

Hence, a loss of Rs 50 was incurred in this transaction.

(ii) Here, C.P. = Rs 700

S.P. = Rs 725

∴ Profit made = S.P. − C.P. = Rs 725 − Rs 700 = Rs 25

Hence, a profit of Rs 25 was made in this transaction.

Example 2:

Javed bought 10 pens for Rs 120 and sold them for Rs 80. Find out the loss incurred on five pens.

Solution:

Cost price (C.P.) of 1 pen = $\text{Rs}\left(\frac{120}{10}\right)$ = Rs 12

Selling price (S.P.) of 1 pen = $\text{Rs}\left(\frac{80}{10}\right) = \text{Rs } 8$

∴ Loss incurred on 1 pen = Rs 12 − Rs 8 = Rs 4

Hence, loss incurred on 5 pens = 5 × Rs 4 = Rs 20

Example 3:

Kanika sold three bottles for Rs 135 and incurred a loss of Rs 15. What is the cost price of one bottle?

Solution:

SP of 3 bottles = Rs 135

∴ SP of 1 bottle = $\text{Rs } \frac{135}{3} = \text{Rs } 45$

Loss incurred on 3 bottles = Rs 15

∴ Loss incurred on 1 bottle = $\text{Rs } \frac{15}{3} = \text{Rs } 5$

CP = Loss + SP

= Rs 5 + Rs 45

= Rs 50

Thus, the cost price of one bottle is Rs 50.

Concept of Profit Percent and Loss Percent

In buying and selling articles, sometimes there is loss and sometimes profit. We can also write profit and loss as a percentage. Profit percent or loss percent is always calculated on the cost price of the article.

Let us consider an example in the given video.

In the same way, we can find loss percent.

Remember the following formulae.

Profit % $= \frac{\text{Profit}}{\text{C.P}} \times 100$

Loss % $= \frac{\text{Loss}}{\text{C.P.}} \times 100$

$$\text{S.P.} = \frac{(100 + \text{Profit}\,\%)}{100} \times \text{C.P.}$$

$$\text{S.P.} = \frac{(100 - \text{Loss}\,\%)}{100} \times \text{C.P.}$$

$$\text{C.P.} = \frac{100}{(100 + \text{Profit}\,\%)} \times \text{S.P.}$$

$$\text{C.P.} = \frac{100}{(100 - \text{Loss}\,\%)} \times \text{S.P.}$$

Now, let us solve some examples.

Example 1:

Apurva sold his bike for Rs 30000 at a loss of 40%. At what price did Apurva buy the bike?

Solution:

Let the price at which Apurva bought the bike be x.

C.P. = x

S.P. = Rs 30000

Loss = C.P. − S.P. = Rs (x − 30000)

We know that, Loss % $= \frac{\text{Loss}}{\text{C.P.}} \times 100$

$$40 = \frac{(x - 30000)}{x} \times 100$$

$40x = 100x - 3000000$

$100x - 40x = 3000000$

$60x = 3000000$

$x = \frac{3000000}{60}$

x = Rs 50000

Thus, Apurva bought the bike for Rs 50000.

Example 2:

Javed sold a refrigerator and a washing machine for Rs 15000 and Rs 10000 respectively. He made a gain of 25% on the refrigerator and a loss of 20% on the washing machine. Find his overall gain or loss.

Solution:

Let the cost price of refrigerator be x and that of washing machine be y.

S.P. of refrigerator = Rs 15000

Gain on refrigerator = S.P. − C.P. = Rs (15000 − x)

We know that, gain % $= \frac{\text{Gain}}{\text{C.P.}} \times 100$

$$25 = \frac{(15000 - x)}{x} \times 100$$

$25x = (1500000 - 100x)$

$125x = 1500000$

$$x = \frac{1500000}{125}$$

x = Rs 12000

C.P. of refrigerator = Rs 12000

Now, selling price of washing machine = Rs 10000

Loss on washing machine = Rs ($y - 10000$)

We know that, Loss% $= \frac{\text{Loss}}{\text{C.P.}} \times 100$

$$20 = \frac{(y - 10000)}{y} \times 100$$

$20y = 100y - 1000000$

$100y - 20y = 1000000$

$80y = 1000000$

$$y = \frac{1000000}{80}$$

y = Rs 12500

C.P. of washing machine = Rs 12500

Now, total cost price of refrigerator and washing machine is, (Rs 12000 + Rs 12500)

= Rs 24500

Total S.P. = Rs 15000 + Rs 10000 = Rs 25000

Overall gain = Rs 25000 − Rs 24500 = Rs 500

Thus, there is an overall gain of Rs 500 on the selling of the refrigerator and the washing machine.

Example 3:

Rahul purchased a television for Rs 20000 and its transportation cost was Rs 100. For how much should the television be sold so that he makes a profit of 7%?

Solution:

C.P. of the television = Rs 20000

Overhead charges = Rs 100

∴ Total cost of TV = Rs 20000 + Rs 100 = Rs 20100

Profit % = 7%

We know that, Profit % $= \frac{\text{Profit}}{\text{C.P.}} \times 100$

$$7 = \frac{\text{Profit}}{20100} \times 100$$

7 × 201 = Profit

∴ Profit = Rs 1407

Now,

S.P. = C.P. + Profit = Rs 20100 + Rs 1407 = Rs 21507

Thus, the television should be sold for Rs 21507.

Example 4:

Arun bought an umbrella for Rs 125 and sold it for a profit of Rs 20. What was the selling price of the umbrella and the profit percent of the transaction?

Solution:

Cost price (C.P.) of the umbrella = Rs 125

Profit made in the transaction = Rs 20

∴ Selling price (S.P.) of the umbrella = C.P. + Profit = Rs 125 + Rs 20 = Rs 145

Hence, profit percent of the transaction $= \frac{\text{Profit}}{\text{C.P.}} \times 100 = \frac{20}{125} \times 100 = 16$ %

Example 5:

Julie bought a washing machine for Rs 20000. She then sold it at 10% profit. At what price did she sell the washing machine?

Solution:

Profit per cent = 10%

Cost price (C.P.) of the washing machine = Rs 20000

Profit per cent of the transaction $= \frac{\text{Profit}}{\text{C.P.}} \times 100 = \frac{\text{S.P.} - \text{C.P.}}{\text{C.P.}} \times 100$

$$\therefore 10 = \frac{\text{S.P.} - 20000}{20000} \times 100$$

$$\text{S.P.} - 20000 = \frac{10 \times 20000}{100}$$

$$\text{S.P.} - 20000 = 2000$$

$$\text{S.P.} = 20000 + 2000 = 22000$$

Hence, Julie sold the washing machine for Rs 22000.

Example 6:

If the cost price of 16 chocolates is equal to selling price of 12 chocolates, then find the profit percent.

Solution:

Let the C.P. of each chocolate be Re 1. Then the C.P. of 16 chocolates will be Rs 16.

By the given data, S.P. of 12 chocolates = C.P. of 16 chocolates = Rs 16

∴ S.P. of 1 chocolate = Rs $\frac{16}{12}$

Profit = S.P. – C.P. = $\text{Rs}\frac{16}{12}-\text{Re}1=\text{Re}\frac{4}{12}=\text{Re}\frac{1}{3}$

Thus, there is a profit of Rs $\frac{1}{3}$ on each chocolate.

Profit % = $\frac{\text{Profit}}{\text{C.P.}}\times 100\%=\frac{\text{Re}\frac{1}{3}}{\text{Re}1}\times 100\%=\frac{100}{3}\%=33\frac{1}{3}\%$

Thus, the profit percent is $33\frac{1}{3}\%.$

Example 7:
On selling a bicycle for Rs 5,600, a dealer loses 20%. For how much should he sell it to gain 15%?

Solution:

Selling price of the bicycle is Rs 5,600 and the loss is 20%. Therefore,

$$\begin{aligned}\text{C.P.}&=\frac{100}{(100-\text{loss}\%)}\times\text{S.P.}\\&=\frac{100}{(100-20)}\times\text{Rs } 5{,}600\\&=\frac{100}{80}\times\text{Rs } 5{,}600\\&=\text{Rs } 7{,}000\end{aligned}$$

Expected profit = 15%

$$\begin{aligned} S.P. &= \frac{(100+\text{profit}\%)}{100} \times C.P. \\ &= \frac{(100+15)}{100} \times \text{Rs } 7{,}000 \\ &= \frac{115}{100} \times \text{Rs } 7{,}000 \\ &= \text{Rs } 8{,}050 \end{aligned}$$

Thus, the selling price of the bicycle to gain 15% is Rs 8,050.

Example 8:

The cost price of a laptop is Rs 24,000. An additional Rs 1000 was spent on installing a software. If it is sold at 15% profit, then find the selling price of the laptop.

Solution:

Cost price of the laptop = Rs 24,000 + Rs 1,000 (overhead charges) = Rs 25,000
The laptop is sold at a profit of 15%. Therefore,

$$\begin{aligned} S.P. &= \frac{(100+\text{Profit}\%)}{100} \times C.P. \\ &= \frac{(100+15)}{100} \times \text{Rs } 25{,}000 \\ &= \text{Rs } \frac{115}{100} \times 25{,}000 \\ &= \text{Rs } 28{,}750 \end{aligned}$$

Thus, the selling price of the laptop is Rs 28, 750.

Concept of Discount

We have come across situations where some shops give **discount (rebate)** and we can obtain the article for less than the **marked price** or **catalogue price** of the article. Let us consider such a situation.

A book shop offers a discount of 20% on the purchase of any book. If the marked price of a book is Rs 350, then what will be the cash discount and the sale price of the book?

Go through the given video to find out the answer.

In the above example, we calculated the sale price of the book when its marked price was Rs 350 and the discount was 20%. What will be the sale price of the book, if the above bookshop offers yet another discount of 25% after giving the discount of 20%?

In this case, the bookshop offers two successive discounts (one is 20% and the other is 25% after giving the discount of 20%) on Rs 350. So, first of all, we calculate the sale price of the book when M.P. = Rs 350 and discount % = 20%. After that, we take this sale price as the M.P., on which the discount of 25% is offered and then, we calculate the sale price as follows:

For the 1st part of the successive discounts:

M.P. (marked price) = Rs 350, discount (*d*)% = 20%

So, the selling price (*S.P.*) of the book after 1st discounts is:

$$\text{S.P.} = \frac{100-d}{100} \times \text{M.P.}$$
$$\Rightarrow \text{S.P.} = \frac{100-20}{100} \times \text{ Rs } 350 = \text{Rs } 280$$

For the 2nd part of the successive discounts:

M.P. (marked price) = Rs 280, discount (*d*)% = 25%

So, the selling price (S.P.) of the book after 2nd successive discounts is:

$$\text{S.P.} = \frac{100-d}{100} \times \text{M.P.}$$
$$\Rightarrow \text{S.P.} = \frac{100-25}{100} \times \text{ Rs } 280 = \text{Rs } 210$$

So, the bookshop sells the given book for Rs 210.

For successive discounts on an item, we can generalise a formula as given below:

If the successive discount %, d_1%, d_2%, d_3% … are given, then

S.P. = $\text{M.P.} \times \left(\frac{100 - d_1}{100}\right) \times \left(\frac{100 - d_2}{100}\right) \times \left(\frac{100 - d_3}{100}\right) \times \ldots$

We can also find the marked price, if the discount and the sale price are known.

Now, let us solve some more examples to understand the concept better.

Example 1:

The price of a movie ticket is Rs 200, but there is a discount of 15% on the tickets of the morning show. Calculate the discount and cost of tickets of the morning show.

Solution:

Actual cost of a movie ticket = Rs 200

Discount = 15% of Rs 200

$= \text{Rs} \frac{15}{100} \times 200$

= Rs 30

Cost of a ticket of the morning show

= Rs 200 − Rs 30

= Rs 170

Thus, the cost of a ticket of morning show is Rs 170 and cash discount is Rs 30.

Example 2:

A shop offers a discount of 25% on each purchase of item made of leather. A customer purchases a leather belt for Rs 375. What is the marked price of the belt?

Solution:

Let the marked price of the leather belt be *x*.

Discount = 25% of x

$$= \frac{25}{100} \times x$$
$$= \frac{x}{4}$$

We know that,

Sale price = M.P. − Discount

Rs 375 $= x - \frac{x}{4}$

Rs 375 $= \frac{4x - x}{4}$

$$\frac{3x}{4} = \text{ Rs } 375$$
$$x = \text{Rs} \frac{375 \times 4}{3}$$

x = Rs 500

Thus, the marked price of the belt is Rs 500.

Example 3:

For the stock sale at the end of a season, a garment shop offers 50% and then 40% on the garments. What is the marked price of a shirt if the shop offers a total discount of Rs 840 after giving two successive discounts?

Solution:

Let the marked price of the shirt be Rs x.

In two successive discounts, d_1% = 50% and d_2% = 40%.

We know that

$$\text{S.P.} = \left(\frac{100-d_1}{100}\right)\times\left(\frac{100-d_2}{100}\right)\times \text{M.P.}$$
$$= \left(\frac{100-50}{100}\right)\times\left(\frac{100-40}{100}\right)\times x$$
$$= \frac{50}{100}\times\frac{60}{100}\times x$$
$$= \frac{3x}{10}$$

We know that

Discount = M.P. − S.P.

$$\Rightarrow 840 = x - \frac{3x}{10}$$
$$\Rightarrow \frac{7x}{10} = 840$$
$$\Rightarrow x = \frac{840\times 10}{7} = 1200$$

Hence, the marked price of the shirt is Rs 1200.

Example 4:

A shop keeper buys an article for Rs 750. He marks it at 10% above the cost price. If he sells it at 8% discount, then find the selling price.

Solution:
Cost price of the article = Rs 750

Profit = 10% of Rs 750 $= \frac{10}{100}\times \text{Rs } 750 = \text{Rs } 75$

Marked price = Cost price + Profit = Rs 750 + Rs 75 = Rs 825

Discount = 8%

For Rs 825, discount $= \frac{8}{100}\times \text{Rs } 825 = \text{Rs } 66$

∴ Selling price = Marked price – Discount = Rs 825 – Rs 66 = Rs 759
Thus, the selling price of article is Rs 759.
Example 5:

A cloth seller marks a dress at 35% above the cost price and allows a discount of 10%. What profit does he make in selling the dress?

Solution:
Suppose the cost price of the dress material is Rs 100. Since the seller marks it at 35% above the C.P., the marked price would be Rs 100 + Rs 35 = Rs 135.

Discount of 10% on this marked price = 10% of Rs 135 $= \frac{10}{100} \times \text{Rs}135 = \text{Rs}13.50$

∴ Selling price = Rs 135 – Rs 13.50 = Rs 121.50. So,
Profit = S.P. – C.P. = Rs 121.50 – Rs 100 = Rs 21.50

Hence, profit percent = $\frac{\text{Profit}}{\text{C.P.}} \times 100 = \frac{21.50}{100} \times 100 = \text{Rs } 21.50$

Thus, the cloth seller makes a profit of 21.50% on selling the dress.

Example 6:
The catalogue price of a book is Rs 450. If a shopkeeper offers some discount on the book and sells it at Rs 405. What is the rate of discount?

Solution:
Marked price of the book = Rs 450
Price after discount = Rs 405

Discount = Marked price – Price after discount
= Rs 450 – Rs 405
= Rs 45

Rate of discount = $\frac{\text{Discount } \times 100}{\text{Marked price}}$

$= \frac{45 \times 100}{450}$
= 10 %

Sales Tax (ST) And Value Added Tax (VAT)

Sales tax is the amount that is added to the price of an item. It is charged by the government from the customers.

The tax which customers pay these days is known as Value Added Tax (VAT).

The sales tax or value added tax is generally given as a percentage. Therefore, we can calculate our bill amount using the concept of percentages.

Let us consider an example in the given video.

Let us solve some more examples to understand the concept better.

Example 1:

The cost of a shirt including 12% VAT is Rs 896. Find the cost of the shirt without including VAT.

Solution:

Let x be the cost of the shirt without including VAT.

We know that,

Cost of shirt including VAT = Selling price + VAT

Rs 896 = x + 12% of x

Rs 896 = $x+\left(\frac{12}{100}\times x\right)$

Rs 896 = $\frac{112x}{100}$

$$x = \text{Rs}\,\frac{896\times 100}{112}$$
$$x = \text{Rs } 800$$

Thus, the selling price of the shirt without including VAT is Rs 800.

Example 2:

The original cost of a food item is Rs 175. What is the bill amount, if the sales tax is charged as 5%?

Solution:

Original cost of the food item = Rs 175

Sales tax = 5% of Rs 175 = $\frac{5}{100}\times \text{Rs } 175$ = Rs 8.75

Bill amount = Rs 175 + Rs 8.75 = Rs 183.75

Thus, the bill amount is Rs 183.75.

Value Added Tax (VAT)

VAT is the new method of raising tax on the sale/purchase of goods within a state. It is the replacement for sales tax.

Most state governments have accepted the VAT system.

Let us understand VAT with the help of an example.

A retailer purchases an article from a manufacturer for Rs 1000. The rate of sales tax is 10%.

The sales tax paid by the retailer = 10% of 1000 = Rs 100

If he sells the article for Rs 1200, then

Tax recovered on the sale = 10% of Rs 1200 = Rs 120

“**The difference between the tax paid on the sale value and that paid on the purchase value is deposited with the government as VAT.”**

Hence, in the above example,

VAT = Tax recovered on sale − Tax paid on purchase

= Rs 120 − Rs 100 = Rs 20

Here, the value added by the retailer is Rs 1200 − Rs 1000 = Rs 200

Tax on the added value = 10% of Rs 200 = Rs 20

This is the reason why it is called value added tax (VAT).

The difference between sales tax and value added tax is that sales tax is realised at single point only, while value added tax is realised at different stages.

Let us look at the following video to better understand the concept of VAT.

Let us now solve some problems to understand the concept of VAT more clearly.

Example 1

A shopkeeper sells an article to a consumer at the list price of Rs 2200. The rate of sales tax is 6%. The shopkeeper deposits a VAT of Rs 24 to the government. If the shopkeeper purchases the article from the wholesaler, then find the tax paid by the shopkeeper to the wholesaler.

Solution:

List price of the article = Rs 2200

Tax paid by the consumer = 6% of Rs 2200 = Rs 132

VAT paid by the shopkeeper = Rs 24

We know that

VAT = Tax recovered on the sale − Tax paid on the purchase

Rs 24 = Rs 132 − Tax paid on the purchase

Tax paid on the purchase = Rs 132 − Rs 24 = Rs 108

Thus, the tax paid by the shopkeeper to the wholesaler is Rs 108.

Example 2

A manufacturer sells an almirah to a wholesaler for Rs 6000. The wholesaler sells it to a retailer and earns Rs 1200 as profit. The retailer sells it to a customer at a profit of Rs 800. If the rate of VAT is 7%, then find the amount of tax deposited with the state government as VAT.

Solution:

Tax collected by the manufacturer = 7% of Rs 6000 = Rs 420

The wholesaler earns a profit of Rs 1200.

Hence, the value added by the wholesaler is Rs 1200.

VAT paid by the wholesaler = 7% of Rs 1200 = Rs 84

The retailer earns a profit of Rs 800.

Hence, the value added by the retailer is Rs 800.

VAT paid by the retailer = 7% of Rs 800 = Rs 56

VAT deposited with the state government

= Rs 420 + Rs 84 + Rs 56 = Rs 560

Example 3

A wholesaler sells an article to a retailer at 25% discount on the list price. The retailer sells the article to a customer at 15% discount. The rate of sales tax is 10% and the list price of the article is Rs 12000. Find the profit earned and VAT paid by the retailer.

Solution:

List price of the article = Rs 12000

Discount given by wholesaler = 25%

Purchase price for the retailer

= Rs 12000 − 25% of Rs 12000

= Rs 12000 − Rs 3000

= Rs 9000

Discount given by the retailer = 15%

Sale price for the retailer

= Rs 12000 − 15% of Rs 12000

= Rs 12000 − Rs 1800

= Rs 10200

Profit earned by the retailer

= Rs 10200 − Rs 9000

= Rs 1200

Tax paid by the retailer to the wholesaler = 10% of purchase price = 10% of Rs 9000

= Rs 900

Tax recovered by the retailer from the customer

= 10% of sale price

= 10% of Rs 10200

= Rs 1020

VAT = Tax recovered on sale price − Tax paid on purchase price

= Rs 1020 − Rs 900 = Rs 120

Thus, the profit earned by the retailer is Rs 1200, while the VAT paid by the retailer is Rs 120.

Concept of Simple Interest

Sometimes, we need a large amount of money for bigger purposes such as buying a house, buying a car, paying fee for higher education etc. If we do not readily have that much of money then we have to borrow it from money lenders, banks or co-operative credit societies for a particular period of time along with a condition of paying some extra money at a particular rate. This borrowed money is called **loan**.

For example, Jatin borrowed Rs 10000 from Shashank. He promised to give him back Rs 11000 after one year.

Here, Rs 10000 is loan taken by Jatin from Shashank.

Now, the question is that why did Jatin promise to pay more money than he borrowed?

Have you heard about interest? Let us suppose that someone borrows some money for a specific time period. Then, the borrower has to pay some extra money along with the original amount after the passage of that fixed time period. This extra money that is paid by the borrower is called **interest**.

In the given example, extra money paid = Rs (11000 − 10000) = Rs 1000

Thus, here, he paid Rs 1000 as the interest for one year.

The original amount of money borrowed is called **principal**. In this case, Rs 10000 is the principal.

The total amount that has to be paid back after the specific time period is called **amount**. In the above example, Rs 11000 is the amount.

Thus, we can conclude that

Amount = Principal + Interest **Interest = Amount − Principal** **Principal = Amount − Interest**
The interest is always calculated on the principal and is given in the form of percentage for a certain period of time. This percentage is known as the rate of interest.

Let us now calculate the rate of interest for the above example.

Principal = Money borrowed by Jatin for one year = Rs 10000

Amount = Money returned by Jatin after one year = Rs 11000

Interest = Extra money given by Jatin = Rs (11000 − 10000) = Rs 1000

Now, to express the rate of interest, we need to calculate the interest per Rs 100 as principal.

Interest on Rs 10000 in one year = Rs 1000

Interest on Re 1 in one year = $\frac{\text{Rs } 1000}{\text{Rs } 10000}$

Therefore, interest per Rs 100 in one year = $\frac{\text{Rs } 1000}{\text{Rs } 10000} \times \text{Rs } 100$ = Rs 10

Thus, the rate of interest is 10% per year, which can also be expressed as 10% p.a. (per annum).

Now, how will you calculate the interest applicable on Rs 7000, at the rate of 12% p.a., at the end of one year?

Rate of interest = 12% p.a.

This means, if Rs 100 is borrowed, then interest after one year = Rs 12

Thus, if Re 1 is borrowed, then interest after 1 year = $\frac{\text{Rs } 12}{\text{Rs } 100} = \frac{12}{100}$

And, if Rs 7000 is borrowed, then interest after 1 year = $\frac{12}{100} \times$ (Rs 7000) = Rs 840

Thus, the interest on Rs 7000 at the end of one year = Rs 840

Now, what is the amount?

Amount = Principal + Interest = Rs 7000 + Rs 840 = Rs 7840

Therefore, from the above example, it is clear that

Interest = Rate of interest × Principal

If we express interest as *I*, principal as *P*, and rate of interest as *R*%, then the above statement can be expressed as follows:

$$I = P\% \text{ of } R = \frac{P}{100} \times R = \frac{PR}{100}$$

Now, if the amount is expressed as *A*, then *A* = *P* + *I*

$$\therefore A = P + \frac{PR}{100} = P\left(1 + \frac{R}{100}\right)$$

Remember:

1. Interest = *I* $= \frac{\boldsymbol{PR}}{\mathbf{100}}$

2. Amount = $\boldsymbol{A = P\left(1+\frac{R}{100}\right)}$

Note that the above formulae are applicable only in cases where **the money has to be returned after 1 year, i.e., the time period is 1 year.**

These formulae are not applicable if the time period is, say, 1 month, 2 months, 3 months, etc, or it is 2 years, 3 years, etc.

Let us apply the above formulae in an example.

What amount is to be paid at the end of one year for Rs 10000 at the rate of 10% p.a.?

Principal = P = Rs 10000

Rate of interest = R = 10%

Thus, amount payable at the end of one year = $A = P\left(1+\frac{R}{100}\right)$

$$
\begin{aligned}
&= 10000\left(1+\frac{10}{100}\right) \\
&= 10000\left(\frac{100+10}{100}\right) \\
&= 10000\times\frac{110}{100} \\
&= 11000
\end{aligned}
$$

Therefore, amount payable at the end of one year = Rs 11000

Now, how should we proceed if we have to find the interest when the time period is more than one year?

To understand the method used for solving such types of problems, go through the following video.

Let us go through some examples to understand the concepts of simple interest better.

Example 1:

Rahul borrowed some money from Parul. In return, Rahul had to pay Rs 350 as interest, along with the actual sum. If he paid a total of Rs 4850 to Parul, then find the amount that he borrowed.

Solution:

Interest = Rs 350

Amount = Rs 4850

∴ Principal = Amount − Interest = Rs 4850 − Rs 350 = Rs 4500

Thus, Rahul borrowed Rs 4500 from Parul.

Example 2:

What will be the interest on Rs 8700 at the end of one year at the rate of 20% per year? Also find the amount payable at the end of the year.

Solution:

Principal (P) = Rs 8700

Rate of interest (R) = 20%

Time period (T) = 1 year

$$\therefore I = \frac{P \times R \times T}{100} = \text{Rs } \frac{8700 \times 20 \times 1}{100} = \text{Rs } \frac{174000}{100} = \text{Rs } 1740$$

Therefore, interest (I) payable at the end of the year = Rs 1740

Now, amount (A) payable at the end of the year = $P + I$

= Rs 8700 + Rs 1740

= Rs 10440

Example 3:

To buy a car, Jitendra borrowed a sum of Rs 200000 for 12 years at the rate of 4% p.a. What is the total amount that he has to pay to repay the loan?

Solution:

Principal (*P*) = Rs 200000

Rate of interest (*R*) = 4%

Time period (*T*) = 12 years

Now, amount (*A*) payable at the end of the time period $= P\left(1+\frac{R\times T}{100}\right)$

$$= 200000\left(1+\frac{4\times 12}{100}\right)$$
$$= 200000\left(\frac{100+48}{100}\right)$$
$$= 200000\times\frac{148}{100}$$
$$= 296000$$

Therefore, Jitendra has to pay Rs 296000 in order to repay his loan.

Example 4:

Kiran invested a sum of Rs 15000 for 4 years and received a total amount of Rs 18000 at the end of this time period. Find the applicable rate of interest.

Solution:

Principal (*P*) = Rs 15000

Time period (*T*) = 4 years

Amount (*A*) = Rs 18000

∴ Interest = *A* − *P* = Rs 18000 − Rs 15000 = Rs 3000

Let the rate of interest be *R*% p.a.

We know that $A = P + \frac{P\times R\times T}{100}$.

$$\Rightarrow 18000 = 15000 + \frac{15000 \times R \times 4}{100}$$
$$\Rightarrow 15000 + \frac{15000 \times R \times 4}{100} = 18000$$
$$\Rightarrow \frac{15000 \times R \times 4}{100} = 18000 - 15000$$
$$\Rightarrow \frac{15000 \times R \times 4}{100} = 3000$$
$$\Rightarrow 15000 \times R \times 4 = 3000 \times 100$$
$$\Rightarrow R = \frac{300000}{15000 \times 4} = 5$$

Therefore, the applicable rate of interest is 5% p.a.

Example 5:

Mr. Sharma borrowed some money from Mr. Gupta for $\frac{3}{2}$ years at the rate of 11% per annum. Also, he borrowed the same amount of money from Mr. Verma for same time period at the rate of 12% per annum. If Mr. Sharma had to pay Rs 6210 as total interest to repay the whole debt then what was the total money that he borrowed?

Solution:

Case I: Money borrowed from Mr. Gupta

Time period (T) = $\frac{3}{2}$ years

Rate (R) = 11% per annum

Let principal be P and interest be I_1.

$$I_1 = \frac{P \times R \times T}{100}$$
$$\Rightarrow I_1 = \frac{P \times 11 \times 3}{100 \times 2}$$
$$\Rightarrow I_1 = \frac{33P}{200}$$

Case II: Money borrowed from Mr. Verma

Time period (T) = $\frac{3}{2}$ years

Rate (R) = 12% per annum

Principal = P

Let interest be I_2.

$$I_2 = \frac{P \times R \times T}{100}$$
$$\Rightarrow I_2 = \frac{P \times 12 \times 3}{100 \times 2}$$
$$\Rightarrow I_2 = \frac{36P}{200}$$

Total interest = Rs 6210

$\therefore I_1 + I_2 =$ Rs 6210

$$\frac{33P}{200} + \frac{36P}{200} = \text{Rs } 6210$$
$$\Rightarrow \frac{69P}{200} = \text{Rs } 6210$$
$$\Rightarrow P = \text{Rs} \frac{6210 \times 200}{69}$$
$$\Rightarrow P = \text{Rs } 18000$$

Thus, Mr. Sharma borrowed Rs 18000 from Mr. Gupta and the same amount from Mr. Verma.

Therefore, the total money borrowed by Mr. Sharma = Rs 18000 + Rs 18000 = Rs 36000

Example 6:

Surya deposited Rs 25000 in the bank at the rate of 8% per annum. After how much time the money will get doubled?

Solution:

Principal (P) = Rs 25000

Amount (A) = 2P = Rs (2 × 25000) = Rs 50000

Interest = $A - P$ = Rs 50000 – Rs 25000 = Rs 25000

Rate (R) = 8%

$$I = \frac{P \times R \times T}{100}$$
$$\Rightarrow 25000 = \frac{25000 \times 8 \times T}{100}$$
$$\Rightarrow T = \frac{100}{8}$$
$$\Rightarrow T = 12.5$$

Thus, after 12 years and 6 months, the money will get doubled.

Compound Interest Using the Concept of Simple Interest

When we deposit some money in a bank, the extra money paid by the bank is called interest.

We know how to calculate simple interest. However, the interest paid by the banks or post offices and charged by the money lenders is not of this kind. The interest paid or charged in such cases is compound interest.

Let us know the method of finding compound interest by taking an example.

Deepak borrowed Rs 16000 from Manish at a rate of 5% per annum for two years. Can you calculate the amount paid by Deepak as compound interest?

Look at the given video to find out the answer.

Now, let us solve one more example to understand the concept better.

Example:

Javed invested Rs 15000 for three years in a bank at the rate of 10% per annum compounded annually. On the other hand, Sujata invested the same sum in some other bank at the same rate for the same duration but on simple interest. Who will earn more interest?

Solution:

It is given that,

Principal = Rs 15000

Rate of interest = 10% per annum

Time = 3 years

Let us first calculate compound interest.

Interest for the 1st year $= \text{Rs } \frac{15000 \times 10}{100}$

= Rs 1500

Amount at the end of 1st year

= Rs 15000 + Rs 1500

= Rs 16500

Interest for 2nd year $= \text{Rs } \frac{16500 \times 10}{100}$

= Rs 1650

Amount at the end of 2nd year

= Rs 16500 + Rs 1650

= Rs 18150

Interest for 3rd year $= \text{Rs } \frac{18150 \times 10}{100}$

= Rs 1815

∴ Total interest received by Javed after 3 years

= Rs 1500 + Rs 1650 + Rs 1815

= Rs 4965

Simple interest received by Sujata after 3 years

$$= \text{Rs } \frac{15000 \times 10 \times 3}{100}$$

= Rs 4500

Thus, Javed received more interest than Sujata.

Formula for Calculating Compound Interest

Supriya invested Rs 75000 in a bank at the rate of 10% per annum compounded annually. What is the amount received by her after 2 years?

We know the method of calculating compound interest using the concept of simple interest. However, this method is very lengthy as we have to calculate the interest and then amount for each year one by one.

We can also find the amount directly using a formula. Before solving the given problem, let us know about the formula.

When the interest is compounded annually, the amount after *n* years is given by

$$A = P\left(1+\frac{R}{100}\right)^{n}$$

where P is the principal and R is the rate of interest per annum.

Let's now look at the following video to understand the proof of this formula.

Now, using this formula, let us calculate the amount received by Supriya after 2 years.

In the given problem,

Principal, P = Rs 75000

Rate of interest, R = 10% per annum

Number of years, n = 2 years

By the formula, we obtain

$$A = 75000\left(1+\frac{10}{100}\right)^{2}$$

$$A = 75000\left(\frac{11}{10}\right)^{2}$$

$$A = 75000\times\frac{11}{10}\times\frac{11}{10}$$

A = 750 × 121

A = Rs 90750

Thus, Supriya received Rs 90750 after 2 years.

Using this formula, we can find any one of the values - amount, time, rate of interest or principal, if the other three values are known to us.

For example, the compound interest on a certain sum invested for two years at the rate of 8% p.a. compounded annually is Rs 3328. The sum can be calculated as:

Let *x* be the sum.

Amount = Principal + Interest

Amount = Rs (*x* + 3328)

Now using the formula, we obtain

$$A = P\left(1 + \frac{R}{100}\right)^n$$

⇒ $(x + 3328) = x\left(1 + \frac{8}{100}\right)^2$

⇒ $(x + 3328) = x\left(\frac{27}{25}\right)^2$

⇒ $(x + 3328) = x \times \frac{27}{25} \times \frac{27}{25}$

⇒ $x + 3328 = \frac{729x}{625}$

⇒ 625*x* + 2080000 = 729*x*

⇒ 729*x* − 625*x* = 2080000

⇒ 104 *x* = 2080000

⇒ $x = \frac{2080000}{104}$

⇒ *x* = 20000

Thus, the sum is Rs 20000.

Now, let us solve some more examples.

Example 1:

Anurag borrowed Rs 50000 from his friend at the rate of 4% p.a. compounded annually. After some time, he repaid Rs 56243.20 to his friend. For how many years did Anurag borrow the money?

Solution:

Principal, P = Rs 50000

Rate, R = 4% p.a.

Amount, A = Rs 56243.2

Let *n* be the number of years for which Anurag borrowed the money.

Using the formula, $A = P\left(1+\frac{R}{100}\right)^n$,

⇒ Rs 56243.2 = Rs 50000 $\left(1+\frac{4}{100}\right)^n$

$$\Rightarrow \frac{56243.2}{50000} = \left(1+\frac{4}{100}\right)^n$$

$$\Rightarrow \frac{562432}{500000} = \left(1+\frac{1}{25}\right)^n$$

$$\Rightarrow \frac{17576}{15625} = \left(\frac{26}{25}\right)^n$$

$$\Rightarrow \left(\frac{26}{25}\right)^3 = \left(\frac{26}{25}\right)^n$$

$$\Rightarrow n = 3$$

Thus, Anurag borrowed the money for three years.

Example 2:

Find the rate at which Rs 700 becomes Rs 847 in two years, when the interest is compounded annually.

Solution:

Let R be the rate of interest.

Principal, P = Rs 700

Amount, A = Rs 847

Number of years, *n* = 2

Now, using the formula, $A = P\left(1+\frac{R}{100}\right)^n$, we obtain

$$\Rightarrow 847 = 700\left(1+\frac{R}{100}\right)^2$$
$$\Rightarrow \frac{847}{700} = \left(1+\frac{R}{100}\right)^2$$
$$\Rightarrow \frac{121}{100} = \left(1+\frac{R}{100}\right)^2$$

$$\Rightarrow \left(\frac{11}{10}\right)^2 = \left(1+\frac{R}{100}\right)^2$$
$$\Rightarrow \frac{11}{10} = 1+\frac{R}{100}$$
$$\Rightarrow \frac{11}{10} - 1 = \frac{R}{100}$$
$$\Rightarrow \frac{1}{10} = \frac{R}{100}$$
$$\Rightarrow R = 10\%$$

Thus, the rate of interest is 10%.

Problems Where the Interest is not Compounded Annually

We know how to find the compound interest using the following formula:

Amount, $A = P\left(1+\frac{R}{100}\right)^n$

Here, *P* is the principal, *R* is the rate of interest per annum, and *n* is the number of years.

Let us now try to solve an example.

What is the amount received after $1\frac{1}{2}$ years when Rs 20000 are invested at the rate of 6% p.a. compounded half yearly?

Do you find any difference between this question and the questions you have solved before?

Yes, in this example, the interest is compounded half yearly. In the questions we have solved before, the interest was compounded annually.

The time interval after which the interest is compounded is known as the conversion period.

In the given example, the conversion period is six months as the interest is compounded half yearly.

To solve such questions where the interest is not compounded annually, we have to change the rate and time accordingly.

In the given example, the time period is given as $1\frac{1}{2}$ years.

Here, the conversion period is six months.

$\therefore$ Number of conversion periods = 3

Rate of interest = 6% per annum

= 3% per half yearly

Now, we can solve this question using the same formula.

$$A = P\left(1+\frac{R}{100}\right)^n$$

Where,

n is the number of conversion periods and *R* is the rate per conversion period.

$$A = 20000\left(1 + \frac{3}{100}\right)^3$$
$$A = 20000\left(\frac{103}{100}\right)^3$$
$$A = 20000 \times \frac{103}{100} \times \frac{103}{100} \times \frac{103}{100}$$
$$A = \text{Rs } 21854.54$$

Hence, the amount received after $1\frac{1}{2}$ years is Rs 21854.54.

Before solving more examples, let us see the following table that will help us find the number of conversion periods and the rate of interest per conversion period.

Let *R* be the rate of interest per annum and *t* be the number of years.

Then,

When the interest is compounded	Number of conversion periods (n)	Rate of interest per conversion period (r)
Half yearly	t × 2	$\frac{R}{2}$
Quarterly	t × 4	$\frac{R}{4}$
Monthly	t × 12	$\frac{R}{12}$

Now, let us solve some more examples.

Example 1:

The compound interest on a certain sum invested for one year at the rate of 10% per annum compounded half yearly is Rs 5125. Find the sum.

Solution:

Let P be the sum.

Compound interest = Rs 5125

Amount, A = P + 5125

Number of conversion periods, n = 1 × 2 = 2

Rate of interest per conversion period $= \frac{10}{2}\% = 5\%$

We know that,

$$A = P\left(1+\frac{r}{100}\right)^{n}$$
$$\Rightarrow (P+5125) = P\left(1+\frac{5}{100}\right)^{2}$$
$$\Rightarrow (P+5125) = P\left(\frac{21}{20}\right)^{2}$$
$$\Rightarrow (P+5125) = \frac{441}{400}P$$
$$\Rightarrow 400P + 2050000 = 441P$$
$$\Rightarrow 441P - 400P = 2050000$$
$$\Rightarrow 41P = 2050000$$
$$\Rightarrow P = \frac{2050000}{41}$$
$$\Rightarrow P = 50000$$

Hence, the sum is Rs 50000.

Example 2:

Lalit borrowed Rs 125000 from his friend at the rate of 8% per annum compounded quarterly. After some time, he re-paid Rs 132651 to his friend. For how long did Lalit borrow the money?

Solution:

Principal, P = Rs 125000

Amount, A = Rs 132651

Let *n* be the number of conversion periods.

Rate of interest per conversion period, $r = \frac{8}{4}\% = 2\%$

We know that,

$$A = P\left(1 + \frac{r}{100}\right)^n$$
$$132651 = 125000\left(1 + \frac{2}{100}\right)^n$$
$$\frac{132651}{125000} = \left(1 + \frac{1}{50}\right)^n$$
$$\left(\frac{51}{50}\right)^3 = \left(\frac{51}{50}\right)^n$$
$$\Rightarrow n = 3$$

∴ Number of conversion periods = 3

Here, the interest is compounded quarterly and we know that, one quarter = 3 months

∴ 1 conversion period = one quarter = 3 months

∴3 conversion periods = 3 × 3 months = 9 months

Hence, Lalit borrowed the money for a period of 9 months.

Direct Proportion

Suppose you go to a grocery shop to buy eggs. If the shopkeeper is selling 2 eggs for Rs 4, then what amount is required to buy 5 eggs?

We can solve this problem using the unitary method.

First, let us determine the cost of 1 egg.

It is given that cost of 2 eggs = Rs 4

∴ Cost of 1 egg = $\frac{\text{Rs } 4}{2} = \text{Rs } 2$

Hence, cost of 5 eggs = 5 × Rs 2 = Rs 10

What do we observe in the above example?

One simple thing that we observe is that as the number of eggs increases, its cost also increases. Such situations are examples of **direct variation** or **direct proportion**. In our day-to-day lives, we come across various such situations. For example, if a car moves with constant speed, then the distance covered by it is in direct proportion with the time taken to cover the distance.

Direct variation or **Direct proportion** can be defined as follows:

When two variable quantities increase or decrease simultaneously such that their ratio remains unchanged, then it is an example of direct variation and the quantities are said to be in direct proportion. It is said that one variable "varies directly" with the other.

Writing direct variation or direct proportion in the form of symbols:

If there is direct variation between two variables *x* and *y,* then it is represented as:

x* α *y

It is read as "***x* varies directly as *y***" or "***x* is directly proportional to *y***".

Also, in direct proportion or direct variation, the ratio of variables is always a constant value.

Thus, $\frac{x}{y} = k$ or $x = ky$, where *k* is known as the **constant of proportionality**.

So, it can be concluded that:

If *x* α *y,* then $\frac{x}{y} = k$ or $x = ky$, where *k* is a constant.

or

If $\frac{x}{y} = k$ or $x = ky$, then *x* α *y*, where *k* is a constant.

$\frac{x}{y} = k$ **is the equation of direct proportion.**

Now, suppose variables *x* and *y* are in direct proportion. If y_1 and y_2 are the values of *y* corresponding to the respective values x_1 and x_2 of *x*, then $\frac{x_1}{y_1} = k$ and $\frac{x_2}{y_2} = k$.

Therefore, we can write the equation as follows:

$$\boxed{\frac{x_1}{y_1} = \frac{x_2}{y_2}}$$

Now, let us consider the following situation to check whether the variables involved in it are in direct proportion or not.

Arnab goes to a stationery shop to buy some pens. If each pen costs Rs 5, then what amount is required to buy 4 such pens? Also, determine the amount that Arnab need to pay to buy 10 such pens?

It is given that cost of 1 pen = Rs 5

Hence, cost of 4 pens = 4 × Rs 5 = Rs 20

Similarly, cost of 10 pens = 10 × Rs 5 = Rs 50

This information can be represented in the tabular form, where the number of pens is denoted by variable *x* and their corresponding cost is denoted by variable *y*, as

Number of pens: *x*	1	4	10
Cost (Rs): *y*	5	20	50

If we observe the ratio of the corresponding values of *x* and *y*, then we see that

$$\frac{1}{5} = \frac{1}{5}$$
$$\frac{4}{20} = \frac{1}{5}$$
$$\frac{10}{50} = \frac{1}{5}$$

We, thus, observe that as the number of pens (x) increases, their cost (y) also increases in such a manner that their ratio $\left(\frac{x}{y}\right)$ remains constant, say k. Thus, in this case, the value of k is $\frac{1}{5}$.

So, this is an example of **direct variation.**

Hence, we say that ***x* and *y* are in direct proportion**, **if** $\boldsymbol{\frac{x}{y} = k \text{ or } x = ky}$

Thus, to check whether the variables x and y are in direct proportion, we need to find the ratio $\frac{x}{y}$ for their corresponding values. If this ratio remains constant, then the variables are in direct proportion, otherwise they are not.

Let us now discuss some examples based on this concept.

Example 1:

The scale of a map is given as 1:10000. The distance between two buildings in a city on the map is 5 cm. What is the actual distance between the two buildings?

Solution:

Given situation is an example of direct variation.

So, it can be said that the distance on the map and the actual distance between the buildings are in direct proportion.

Let the actual distance between the buildings be x.

$$\therefore \frac{1}{10000} = \frac{5 \text{ cm}}{x}$$
$$\Rightarrow x = (10000 \times 5) \text{ cm}$$
$$x = 50000 \text{ cm}$$
$$x = \frac{50000}{100} \text{ m}$$
$$x = 500 \text{ m}$$

Thus, the actual distance between the buildings is 500 m.

Example 2:

If 1 kg 600 g of rice is sufficient for 20 people, then what quantity of rice will be sufficient for 27 people? Also calculate for how many people 2 kg 400 g of rice will be sufficient.

Solution:

Let *x* kg of rice be sufficient for 27 people and 2 kg 400 g of rice be sufficient for *y* number of people.

1 kg 600 g = 1.6 kg

2 kg 400 g = 2.4 kg

The given information can be represented by the following table:

Quantity of rice (kg): ***x***	1.6	*x*	2.4
Number of people: ***y***	20	27	*y*

In the given case, quantity of rice increases and decreases as the number of people increases and decreases respectively. So, this is an example of direct variation.

So, it can be said that the quantity of rice and the number of people are in direct proportion.

$$\therefore \frac{1.6}{20} = \frac{x}{27} \text{ and } \frac{1.6}{20} = \frac{2.4}{y}$$

$$x = \frac{27 \times 1.6}{20} \text{ and } y = \frac{2.4 \times 20}{1.6}$$

$$x = 2.16 \text{ kg and } y = 30 \text{ people}$$

$$x = 2 \text{ kg } 160 \text{ g and } y = 30 \text{ people}$$

Hence, 2 kg 160 g of rice will be sufficient for 27 people and 2 kg 400 g of rice will be sufficient for 30 people.

Example 3:

A car travels at a constant speed of 35 km/h. How far can it travel in 15 minutes?

Solution:

Speed of car = 35 km/h

This means that the car travels 35 km in 60 minutes.

Let the car travel *x* km in 15 minutes.

The given information can be represented by the following table:

Distance covered by the car (km): ***x***	35	*x*
Time taken by the car (min): ***y***	60	15

In the given case, the time taken by the car increases and decreases as the distance covered by the car increases and decreases respectively. So, this is an example of direct variation.

So, it can be said that the distance covered by the car and the time taken by it are in direct proportion.

$$\therefore \frac{35\text{ km}}{60\text{ min}} = \frac{x}{15\text{ min}}$$

$$\Rightarrow x = \left(\frac{35}{60} \times 15\right)\text{ km}$$

$$\Rightarrow x = \left(\frac{35}{4}\right)\text{ km} = 8.75\text{ km}$$

Thus, the car travels 8.75 km in 15 minutes.

Example 4:

If 12 machines can be assembled in 4 hours, then how many machines can be assembled in 8 hours?

Solution:

Let *x* machines be assembled in 8 hours.

The given information can be represented by the following table:

Number of machines: *x*	12	*x*
Time taken (hours): *y*	4	8

In the given case, the time taken increases and decreases as the number of machines increases and decreases respectively. So, this is an example of direct variation.

So, it can be said that the number of machines and the time taken to assemble them are in direct proportion.

$$\therefore \frac{12}{4} = \frac{x}{8}$$

$$x = \frac{12 \times 8}{4}$$

$$x = 24$$

Thus, 24 machines can be assembled in 8 hours.

Example 5:

At a particular time of the day, the length of the shadow of a 28 feet high tree is 1.4 feet. Determine the height of a tree that has a shadow of length 2.3 feet at the same time of the day?

Solution:

Let the height of the tree having shadow of length 2.3 feet at the given time be *x* feet.

The given information can be represented by the following table:

Height of the tree (ft): *x*	28	*x*

Length of the shadow (ft): *y*	1.4	2.3

In the given case, length of the shadow increases and decreases as height of the tree increases and decreases respectively. So, this is an example of direct variation.

So, it can be said that at a particular time of the day, the height of the tree and length of its shadow are in direct proportion.

$$\therefore \frac{28\text{ ft}}{1.4\text{ ft}} = \frac{x}{2.3\text{ ft}}$$
$$x = \left(\frac{28}{1.4} \times 2.3\right)\text{ ft}$$
$$x = 46\text{ ft}$$

Hence, the height of the second tree is 46 feet.

Example 6:

State whether the given situations involve two variables in direct proportion.

1. **Distance covered by a car and the quantity of petrol consumed by it**
2. **Number of workers and the time taken by them to complete a work**
3. **Speed of a person and the time taken by him/her to cover a fixed distance**
4. **Speed of a person and the distance covered by him/her in a fixed time**
5. **Time period and simple interest if the rate of interest is fixed**
6. **Time period and rate of interest if the simple interest is fixed**
7. **Area of cultivated land and the crop harvested**

Solution:

(a) The quantity of petrol consumed by a car increases if the distance covered by it also increases. Thus, the distance covered by a car is in direct proportion with the quantity of petrol consumed by it.

(b) More number of workers will take less time to complete a work. Thus, the number of workers and the time taken by them to complete the work are not in direct proportion.

(c) If a person travels at a higher speed, then he/she takes less time to cover a fixed distance. Thus, the speed of a person and the time taken by him/her to cover a fixed distance are not in direct proportion.

(d) If a person increases his/her speed, then he/she will cover more distance in a fixed time. Hence, the speed of a person and the distance covered by him/her in a fixed time are in direct proportion.

(e) For a fixed rate of interest, if the time period is more, then the simple interest will also be more. Hence, the time period and the simple interest are in direct proportion if the rate of interest is fixed.

(f) For a fixed simple interest, if the time period is more, then the rate of interest will be less. Hence, the time period and the rate of interest are not in direct proportion if the rate of interest is fixed.

(g) The more the area of land cultivated, the more will be the amount of crop harvested. Hence, the area of cultivated land and the crop harvested are in direct proportion.

Example 7:

The following table lists the distance covered by a person and the corresponding time taken by him to cover this distance. Check whether the distance covered changes in direct proportion with the time taken.

Distance covered	**Time taken**
4 km	**16 minutes**
6 km	**24 minutes**
12 km	**48 minutes**
20 km	**72 minutes**

Solution:

The above information can be represented in a table by taking the distance covered (km) as variable ***x*** and the corresponding time taken (minutes) as ***y***.

Distance covered (km): ***x***	4	6	12	20
Time taken (minutes): ***y***	16	24	48	72

Thus, we can find the ratio $\frac{x}{y}$ for the corresponding values of *x* and *y* as

$$\frac{4}{16}=\frac{1}{4}$$
$$\frac{6}{24}=\frac{1}{4}$$
$$\frac{12}{48}=\frac{1}{4}$$
$$\frac{20}{72}=\frac{5}{18}$$

Since the ratio $\frac{x}{y}$ does not remain constant, the distance covered by the person does not change in direct proportion with the time taken.

Example 8:

Observe the following tables and find whether *x* and *y* are directly proportional.

(a)

x	**2**	**9**	**14**	**15**	**17**	**19**
y	**6**	**27**	**42**	**45**	**51**	**57**

(b)

x	**50**	**45**	**37.5**	**30**	**25**	**5**
y	**40**	**36**	**30**	**24**	**20**	**4**

(c)

x	**50**	**48**	**42**	**39**	**30**	**17**
y	**40**	**38**	**32**	**29**	**20**	**7**

Solution:

(a)

x	2	9	14	15	17	19
y	6	27	42	45	51	57

Thus, $\frac{2}{6} = \frac{1}{3}$

$\frac{9}{27} = \frac{1}{3}$

$\frac{14}{42} = \frac{1}{3}$

$\frac{15}{45} = \frac{1}{3}$

$\frac{17}{51} = \frac{1}{3}$

$\frac{19}{57} = \frac{1}{3}$

Since the ratio $\frac{x}{y}$ remains constant for the corresponding values of *x* and *y*, the variables *x* and *y* are directly proportional.

(b)

x	50	45	37.5	30	25	5
y	40	36	30	24	20	4

Thus, $\frac{50}{40} = \frac{5}{4}$

$\frac{45}{36} = \frac{5}{4}$

$\frac{37.5}{30} = \frac{5}{4}$

$\frac{30}{24} = \frac{5}{4}$

$\frac{25}{20} = \frac{5}{4}$

$\frac{5}{4} = \frac{5}{4}$

Since the ratio x/y remains constant for the corresponding values of *x* and *y*, the variables *x* and *y* are directly proportional.

(c)

x	**50**	**48**	**42**	**39**	**30**	**17**
y	**40**	**38**	**32**	**29**	**20**	**7**

Thus, $\frac{50}{40} = \frac{5}{4}$

$\frac{48}{38} = \frac{24}{19}$

$\frac{42}{32} = \frac{21}{16}$

$\frac{39}{29} = \frac{39}{29}$

$\frac{30}{20} = \frac{3}{2}$

$\frac{17}{7} = \frac{17}{7}$

Since the ratio x/y does not remain constant for the corresponding values of *x* and *y*, the variables *x* and *y* are **not** directly proportional.

Example 9:

If *a* varies directly as *b* and *b* = 56 when *a* = 49 then find the value of *a* when *b* = 64.

Solution:

We have

$a \propto b$

$\Rightarrow a = kb$ (Where k is the constant of proportionality)

$\Rightarrow 49 = k \times 56$ ($b = 56$ when $a = 49$)

$\Rightarrow k = \frac{7}{8}$

On substituting the value of k, the equation of variation becomes

$a = \frac{7}{8} \times b$

$\Rightarrow a = \frac{7}{8} \times 64$ (When $b = 64$)

$\Rightarrow a = 56$

Example 10:

The area of a circle varies directly as the square of its radius. The area of a circle having diameter of 21 cm is 346.5 cm^2. Find the constant of proportionality. Also, find the radius of the circle having an area of 154 cm^2.

Solution:

Let the area, radius and diameter of the circle be A, r and d respectively.

It is given that

$A \propto r^2$

$\Rightarrow A = kr^2$ (Where k is the constant of proportionality)

$$\Rightarrow A = k \times \left(\frac{d}{2}\right)^2 \qquad (r = \frac{d}{2})$$

$$\Rightarrow 346.5 = k \times \left(\frac{21}{2}\right)^2 \qquad (A = 346.5\,\text{cm}^2 \text{ when } d = 21\text{cm})$$

$$\Rightarrow 346.5 = k \times \frac{441}{4}$$

$$\Rightarrow 346.5 = k \times 110.25$$

$$\Rightarrow k = \frac{22}{7}$$

Substituting the value of *k*, the equation of variation becomes

$$A = \frac{22}{7} r^2$$

When $A = 154 \text{ cm}^2$, we have

$$154 = \frac{22}{7} r^2$$

$$\Rightarrow r^2 = \frac{154 \times 7}{22}$$

$$\Rightarrow r^2 = 7 \times 7$$

$$\Rightarrow r = 7 \qquad \text{(Radius cannot be negative)}$$

Thus, the value of constant of proportionality is $\frac{22}{7}$ and radius of the required circle 7 cm.

Example 11:

If the wages of 9 workers is Rs 1350, find the wages of 14 workers.

Answer:

Wages of 9 workers = Rs 1350

Let the wages of 14 workers be *x*.

When the number of workers increases, total wage will also increases. So, this is the case of direct proportion.

Therefore,

9 : 14 :: 1350 : *x*

9*x* = 14 × 1350

$$x = \frac{14 \times 1350}{9}$$

$x = 2100$

Thus, the wages of 14 workers is Rs 2100.

Inverse Proportion

We have learnt to solve the problems where variables vary directly, but there are situations where variables can vary inversely.

When the first quantity increases with the decrease in the second quantity and when the first quantity decreases with the increase in the second quantity, such that their product remains unchanged, then the quantities are said to be in inverse proportion. It is said that one variable "varies inversely" with the other.

Let us consider such a problem.

6 men can whitewash a house in 20 days. Can we calculate the number of men required to whitewash that house in (i) 10 days (ii) 30 days?

Go through the following video to answer these questions.

Writing inverse variation or inverse proportion in the form of symbols:

If there is inverse variation between variables *x* and *y,* then it is represented as:

$x \propto \frac{1}{y}$

It is read as "***x* varies inversely as *y***" or "***x* is inversely proportional to *y***".

Also, in inverse variation or inverse proportion, the product of variables is always a constant value.
Thus**, *xy = k*,** where *k* is known as the **constant of proportionality.**

So, it can be concluded that:

If *xy = k,* then $x \propto \frac{1}{y}$, where *k* is the constant of proportionality.

or

If $x \propto \frac{1}{y}$, then $xy = k$, where k is the constant of proportionality.

$xy = k$ is the equation of inverse variation.

Now, if y_1 and y_2 are two values of y, and x_1 and x_2 are two values of x corresponding to y_1 and y_2 respectively, then we have

$x_1y_1 = k$ … (1)

$x_2y_2 = k$ … (2)

From the equations (1) and (2), we obtain

$$\boldsymbol{x_1y_1 = x_2y_2}$$

We use this expression to solve problems when two variables are in inverse proportion.

Let us solve a problem where the variables are in inverse proportion using the above relation.

Converting the proportion into inverse proportion:
While solving the problems related to inverse proportion, we should be careful about equating the ratios. This can be understood with the help of an example.

The water in the tank is sufficient for 5 members for 21 days. The same amount of water is sufficient to 15 people for 7 days.

In this case, if the number of people increases, the number of days decrease.
Thus, this is the case of inverse proportion.

Ratio of people = 5 : 15
Ratio of days = 21 : 7
We have, 5 : 15 ≠ 21: 7

But when the proportion of the days is written in inverse proportion, we get

5 : 15 = 7 : 21
⇒ 5 × 21 = 15 × 7

So, it can be concluded that when $p : q$ and $r : s$ are in inverse proportion, then $p : q = s : r$.

Now, let us learn to verify whether the given variables are in inverse proportion or not.

The following table shows the speed of a train and time taken to cover the same distance.

Speed (in km/hr)	**Time taken (in hr)**
20	6
30	4
40	3
60	2

Is the speed of the train and the time taken to cover the distance in inverse proportion?

We know that two variables x and y will be in inverse proportion if $xy = k$, where k is constant of proportionality.

Therefore, to check whether the two variables x and y of a given situation are in inverse proportion or not, we have to calculate the product of the value of variable x with its corresponding value of the variable y. If all these products are equal, then we can say that the variables x and y are in inverse proportion, otherwise not.

Using this concept, let us check whether the speed of train and time taken to cover the distance is in inverse proportion or not.

In the given table, there are two variables − speed of the train and time taken to cover the distance. For the first observation, the product of these variables is $20 \times 6 = 120$.

Similarly, for the other observations, the products of corresponding values of the variables are

$30 \times 4 = 120$

$40 \times 3 = 120$

$60 \times 2 = 120$

Here, the products of the values of the variables for all observations are same.

Thus, the speed and time taken to cover the same distance are in inverse proportion.

Using this concept, we can check whether the variables of a given situation, given in a tabular form, are in inverse proportion or not.

Let us discuss one more example based on verification of inverse proportion.

Example 1:

Observe the following tables and check whether *x* and *y* are related inversely or not.

(i)

x	**1**	**2**	**5**	**10**
y	**20**	**10**	**4**	**2**

(ii)

x	**7**	**8**	**9**	**10**
y	**27**	**20**	**12**	**7**

Solution:

We know that two quantities x and y are in inverse proportion, if $x \times y$ = constant.

(i) Here, $1 \times 20 = 20$

$2 \times 10 = 20$

$5 \times 4 = 20$

$10 \times 2 = 20$

It can be seen that $x \times y = 20$, which is constant for each observation.

Therefore, x and y are in inverse proportion.

(ii) Here, $7 \times 27 = 189$

$8 \times 20 = 160$

$9 \times 12 = 108$

$10 \times 7 = 70$

It can be seen that $x \times y$ is not constant for each observation.

Therefore, x and y are not in inverse proportion.

Example 2:

In each of the following statements, find the situation where two variables are in inverse proportion.

1. **Distance covered by a car and the amount of petrol required**
2. **Number of workers and time taken by them to complete the work**
3. **Speed of a person and distance covered by him in a fixed time**
4. **Length and breadth of a rectangle to keep its area constant**

Solution:

(a) If we want to cover more distance, then we will require more amount of petrol. Hence, distance covered by a car and amount of petrol are in direct proportion, not in inverse proportion.

(b) If number of workers will be more, then they will take less time to complete the work. Hence, the number of workers and time taken by them to complete the work are in inverse proportion.

(c) If a person will increase his speed, then he will cover more distance in a fixed time. Hence, speed of a person and distance covered by him in a fixed time are not in inverse proportion.

(d) If we will increase the length of rectangle, then we will have to decrease its breadth in order to keep its area constant. Hence, to keep the area of a rectangle constant, its length and breadth should be in inverse proportion.

Example 3:

A packet of chocolates is to be distributed among 25 children such that each of them will get 4 chocolates. How many more chocolates would each of them get if the number of children is reduced by 5?

Solution:

Let each child get x more chocolates.

Hence, each child will get $(4 + x)$ chocolates.

There were 25 children.

If number of children is reduced by 5, then there are 25 − 5 = 20 children

We can represent the given situation by constructing a table between the number of children and number of chocolates as shown below.

Number of children	25	20
Number of chocolates	4	$4 + x$

In the given case, the number of chocolates increases as the number of children decreases and the number of chocolates decreases as the number of children increases. So, this is an example of inverse variation.

Thus, it can be said that the number of children and the number of chocolates that each child got are in inverse proportion.

$\therefore 25 \times 4 = 20\,(4 + x)$

$$4 + x = \frac{25 \times 4}{20}$$

$4 + x = 5$

$x = 5 - 4 = 1$

Therefore, each child will get 1 more chocolate if 5 children are reduced.

Example 4:

In one full day in a school, there are 8 periods of 40 min each. The school management decided to increase the number of periods in a day so that more number of classes can take place in a single day. However, they want to keep the school duration the same. What will be the duration of new periods if the number of periods is increased to 10?

Solution:

Let the duration of new periods be x.

The given situation can be represented with the help of the following table.

Number of periods	8	10
Time duration (minutes)	40	x

In the given case, the time duration increases as the number of periods decreases and the time duration decreases as the number of periods increases. So, this is an example of inverse variation.

Here, the number of periods and time duration of each period are in inverse proportion.

$\therefore 40 \times 8 = x \times 10$

$$x = \frac{40 \times 8}{10} = 32$$

Therefore, the duration of new periods will be 32 minutes.

Example 5:

If *a* varies inversely as *b* and *b* = 15 when *a* = 4, then find the value of *a* when *b* = 20.

Solution:

Since a varies inversely as b, we have

$ab = k$ (Where k is constant of proportionality)

$\Rightarrow k = 4 \times 15$ ($b = 15$ when $a = 4$)

$\Rightarrow k = 60$

Substituting the value of k, the equation of variation becomes

$ab = 60$

$\Rightarrow a \times 20 = 60$ (When $b = 20$)

$\Rightarrow a = 3$

Example 6:

The time taken by a train to travel a particular distance is inversely proportional to its average speed. When the average speed of train is 60 km/hr, the journey can be completed in 8.5 hours, find the time taken by the train to finish the journey when its average speed is 85 km/hr.

Solution:

Let us denote the average speed and time taken by *s* and *t* respectively.

Then

$t \propto \frac{1}{s}$

$\Rightarrow st = k$ (Where *k* is constant of proportionality)

$\Rightarrow k = 60 \times 8.5$ ($t = 8.5$ when $s = 60$)

$\Rightarrow k = 510$

Substituting the value of *k*, the equation of variation becomes

$st = 510$

$\Rightarrow 85 \times t = 510$ (When $s = 85$)

$\Rightarrow t = 6$

Thus, the required time is 6 hours.

Example 7:

Six workers can do a job in 60 days. How many days will 10 workers take to do the same work?

Solution:

Number of days taken by 6 workers to complete the work = 60

Number of days taken by 10 workers to complete the work = *x*

Ratio of workers = 6 : 10

Ratio of days = 60 : x

Inverse of ratio of days = x : 60

Therefore,

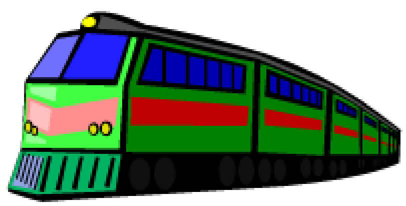
6 : 10 = x : 60

$10x = 360$

$x = 36$

Thus, 10 workers will take 36 days to complete the work.

Unitary Method



Suppose train A travels 210 km in 3 hours and train B travels 300 km in 4 hours. **Can we find how much distance train A travels in 5 hours? Can we say which train travels at more speed?**

To answer such types of questions, we will have to learn a method called **unitary method**.

This method requires two steps.

- The value of one unit is evaluated first.
- Then, the value of the required number of units is evaluated.

Let us now view this video and solve the above problem.

Now, while solving the problems based on unitary method, we come across two types of variations:

(a) Direct Variation
(b) Indirect Variation

Let us study about these one by one.

1. **Direct Variation:** Two quantities are said to vary directly, if the increase (or decrease) in one quantity causes increase (or decrease) in the other quantity. For example, more

the quantity of sugar, more will be its cost.

2. **Indirect Variation:** Two quantities are said to vary inversely, if the increase (or decrease) in one quantity causes decrease (or increase) in the other quantity. For example, more the duration of a period in a school, less will be the number of periods in the school.

   Let us take an example to understand these variations better.

   Six men can whitewash a house in 20 days. Calculate the number of men required to whitewash the house in 10 days.

   The number of days is decreasing here, so we need to increase the number of men to finish the work on time. When number of days decreases, number of men increases, so this is a case of indirect variation.

   Let us find the required number of men now.

   From the given condition, we find that the number of men required to whitewash the house in one day is 20 × 6.

   Number of men required to whitewash the house in 10 days = $\frac{20\times 6}{10}$ = 12

   Thus, we find that 12 men are required to whitewash the house in 10 days.

   Let us take one more example.

   Mohan is a taxi driver. He charges Rs 36 as fare for 6 kilometres. Rajesh hired Mohan's taxi for 10 kilometres. How much money will Rajesh pay as fare?

   If the distance increases the fare of taxi will also increase, so this is a case of direct variation.

   Let us find the money to be paid as fare by Rajesh.
   From the given condition, it is clear that fare for 1 kilometre is Rs $\frac{36}{6}$.

   Thus, fare for 10 kilometres = Rs $\frac{36}{6}\times 10$ = Rs 60

   Similarly, we can identify the variation in the given conditions and solve the problem.

   Let us look at some more examples for a better understanding of this concept.

   Example 1:
   The cost of 3 shirts is Rs 675. How many shirts can be purchased with Rs 2250?

Solution:

Number of shirts purchased in Rs 675 = 3

Number of shirts purchased in Re 1 = $\frac{3}{675}$

∴ Number of shirts purchased in Rs 2250 = $\frac{3}{675}\times 2250$

$$=\frac{6750}{675}$$
$$=10$$

Thus, 10 shirts can be purchased with Rs 2250.

Example 2:
Ranveer purchases 5.5 kg apples for Rs 44 and 7 kg mangoes for Rs 52.5. Which of the two fruits costs more per kg?

Solution:
Cost of 5.5 kg apples = Rs 44

Therefore, cost of 1 kg apples = Rs $\frac{44}{5.5}$
= Rs 8

Cost of 7 kg mangoes = Rs 52.5

Therefore, cost of 1 kg mangoes = Rs $\frac{52.5}{7}$
= Rs 7.5

Therefore, the cost of 1 kg apples is more than the cost of 1 kg mangoes.
Hence, apples are costlier than mangoes.

Example 3:
A bus travels 180 km in 6 hours. How much time is required by the bus to travel 225 km?

Solution:
Time required in travelling 180 km = 6 hours

Time required in travelling 1 km $=\frac{6}{180}$ hours

∴ Time required in travelling 225 km $= \frac{6}{180} \times 225$ hours

$= \frac{1350}{180}$ hours

= 7.5 hours

Thus, the time required in travelling 225 km is 7.5 hours i.e., 7 hours 30 minutes.

Example 4:

On a map, the distance between two places A and B is 2.5 cm. However, the actual distance between them is 45 km. If the distance between the other two places X and Y on the same map is 6.5 cm, then find the actual distance between these two places?

Solution:

If the distance on map is 2.5 cm, then actual distance = 45 km

If the distance on map is 1 cm, then actual distance $= \frac{45}{2.5}$ km

Therefore, if the distance on map is 6.5 cm, then actual distance $= \frac{45}{2.5} \times 6.5$ = 117 km

Thus, the actual distance between the places X and Y is 117 km.

Example 5:

A businessman makes a profit of Rs 15400 in 7 days. How much profit will the businessman make in 19 days?

Solution:

In 7 days, businessman makes a profit of Rs 15400.

In 1 day, businessman makes profit = Rs $\frac{15400}{7}$

∴ In 19 days, businessman will make profit = Rs $\frac{15400}{7} \times 19$ = Rs 41800

Thus, in 19 days, the businessman will make a profit of Rs 41800.

Example 6:

If 15 men can do a piece of work in 10 days, then in how many days can 6 men do the same work?

Solution:

This is the case of indirect variation since more the number of men, less will be the number of days required to finish the work.

It is given that 15 men can do the work in 10 days.

∴ One man can do the work in (10 × 15) days.

Hence, 6 men can do the work in $\frac{10 \times 15}{6}$ days = 25 days

Example 7:

A teacher got some number of toffees to distribute amongst the students of her class. If she gives 2 toffees to each student, then all the 45 students of her class get the toffees. Find the number of students who will get the toffees, if the teacher distributes 3 toffees to each student.

Solution:

This is the case of indirect variation since more the number of students, less will be the number of toffees each student will get.

It is given that 2 toffees were distributed to each of the 45 students.
∴ One toffee will be distributed to (45 × 2) students.

∴ Three toffees will be distributed to $\frac{45 \times 2}{3}$ students = 30 students

Thus, if the teacher distributes 3 toffees to each student, then the number of students who will get the toffees is 30.

Compound Variation

We are familiar with two types of variations, namely direct and indirect variations.

In **direct variation**, increase or decrease in one quantity leads to the corresponding increase or decrease in the other.

For example, if we increase the quantity of mobile phones purchased, then the amount spent on their purchase would also increase. Similarly, a decrease in the quantity of mobile phones leads to the decrease in the amount spent.

In **indirect variation**, increase or decrease in one quantity leads to the corresponding decrease or increase in the other.

For example, if the number of employees are increased, then the number of days required to complete a project would decrease. Similarly, a decrease in the number of employees leads to the increase in the number of days required to complete the project.

The situations discussed above involve either direct variation or indirect variation. However, we come across certain situations which involve a chain of two or more variations and such type of situation is called **compound variation**.

The table below shows the different possibilities of variations containing two variations.

Variation I	**Variation II**
Direct	Direct
Direct	Indirect
Indirect	Direct
Indirect	Indirect

Let us consider an example to understand this concept.

20 cars consume 3000 L of petrol in 15 days. We need to find the amount of petrol consumed by 35 cars in 20 days.

There are two methods to solve this problem. Let us discuss the **first method** to solve such type of problems.

Let 35 cars consume x L of petrol in 20 days.

The given problem can be tabulated as follows:

Number of cars	**Number of days**	**Quantity of petrol consumed (L)**
20	15	3000
35	20	x

When the number of cars increases, the consumption of petrol would also increase. Thus, there is a direct variation between the number of cars and the quantity of petrol consumed.

i.e., 20:35 = 3000:*x* ... (1)

Similarly, when the number of days increases, the consumption of petrol would also increase. Thus, there is a direct variation between the number of days and the quantity of petrol consumed.

i.e., 15:20 = 3000:*x* ... (2)

From (1) and (2), we get

$$\left.\begin{matrix} 20:35 \\ 15:20 \end{matrix}\right\} :: 3000:x$$

Here, 20, 15 and *x* are extremes and 35, 20 and 3000 are means.
We know that product of extremes = product of means

$$\Rightarrow 20 \times 15 \times x = 35 \times 20 \times 3000 \Rightarrow x = \frac{35 \times 20 \times 3000}{20 \times 15} = 7000$$

∴ In 20 days, 35 cars would consume 7000 L of petrol.

Now, let us try to solve the above problem by using the **second method**.
In this method, we first find the multiplying factors.

There is a direct variation between the number of cars and the quantity of petrol consumed. Also, the number of cars increase from 20 to 35.

∴ We take the multiplying factor as 35/20.

Similarly, there is a direct variation between the number of days and the quantity of petrol consumed. Also, the number of days increase from 15 to 20.

So, we take the multiplying factor as 2015.

$$\Rightarrow x = \frac{35}{20} \times \frac{20}{15} \times 3000 = 7000$$

∴ In 20 days, 35 cars would consume 7000 L of petrol.

The above example involved direct variations only. Now, let us take an example which has direct and indirect variation both.

10 men can dig 100 units of sand in 5 days. We need to find the number of men required to dig 240 units of sand in 4 days.

Method 1:
Let *x* be the number of men required.

The given problem can be tabulated as follows:

Number of men	Number of units	Number of days
10	100	5
x	240	4

When the number of men increases, the amount of sand dug also increases. Thus, there is a direct variation between the number of men and the amount of sand dug.
$\Rightarrow$ 100:240 = 10:*x* ... (1)

When the number of men increases, the number of days decreases. Thus, there is an indirect variation between the number of men and the number of days.
$\Rightarrow$ 4:5 = 10:*x* ... (2)

From (1) and (2), we get

$$\left.\begin{array}{r}100:240\\4:5\end{array}\right\}::\ 10:x$$

Here, 100, 4 and *x* are extremes and 240, 5 and 10 are means.

$$\Rightarrow 100 \times 4 \times x = 240 \times 5 \times 10 \Rightarrow x = \frac{240 \times 5 \times 10}{100 \times 4} = 30$$

∴ 30 men can dig 240 units of sand in 4 days.

Method 2:
There is a direct variation between the number of men and the number of units of sand dug. Also, the amount of sand dug increases from 100 to 240 units.

∴ We take the multiplying factor as 240100.

There is an indirect variation between the number of men and the number of days. Also, number of days decreases from 5 to 4.

∴ We take the multiplying factor as 5/4.

$$\Rightarrow x = \frac{240}{100} \times \frac{5}{4} \times 10 = 30$$

∴ 30 men can dig 240 units of sand in 4 days.

Note: If there is a direct variation between two quantities, then the multiplying factor is obtained by the fraction $\frac{\text{Final value}}{\text{Initial value}}$ and if there is an indirect variation between two quantities, then the multiplying factor is obtained by the fraction $\frac{\text{Initial value}}{\text{Final value}}$.

Let us solve some more problems based on this concept.

Example 1:
If 8 men can make 20 tables in 6 days, how many men would be required to make 60 tables in 9 days?

Solution:
Let *x* be the number of men required.
The given problem can be tabulated as follows:

Number of men	Number of tables	Number of days
8	20	6
x	60	9

When the number of men increases, the number of days decreases. Thus, there is an indirect variation between the number of men and the number of days. Also, the number of days decrease from 9 to 6.

∴ The multiplying factor is 6/9.

When the number of men increases, the number of tables made also increases. Thus, there is a direct variation between the number of men and the number of tables. Also, the number of tables increase from 20 to 60.

∴ The multiplying factor is 60/20.

$$\Rightarrow x = \frac{6}{9} \times \frac{60}{20} \times 8 = 16$$

∴ 16 men would be required to make 60 tables in 9 days.

Example 2:
A car travelling at a speed of 70 km/ h for 5 hours per day covered a certain distance in 9 weeks. How many weeks would the car take to cover the same distance if it travels at a speed of 50 km/ h for 3 hours per day?

Solution:
Let x be the number of weeks taken.
The given problem can be tabulated as follows:

Speed of the car (km/ h)	Number of weeks	Number of hours (per day)
70	9	5
50	x	3

When the speed of car increases, the number of weeks decreases. Thus, there is an indirect variation between them. Also, speed of the car decreases from 70 to 50 km/ h.
∴ The multiplying factor is 70/50.

Again, when the number of hours increases, the number of weeks decreases. Thus, there is an indirect variation between them. Also, the number of hours decreases from 5 to 3 h.

∴ The multiplying factor is 5/3.

$$\Rightarrow x = \frac{70}{50} \times \frac{5}{3} \times 9 = 21$$

∴ The car would take 21 weeks to cover the same distance if it travels at the speed of 50 km/ h for 3 hours per day.

Solving Problems Based on Time and Work

Consider a situation. Raj is a carpenter. He can make a computer table in 7 days.

Can we find out Raj's one day's work?

We can find it out by using a formula which is given below.

One day's work

$$= \frac{1}{\text{Number of days to complete the work}}$$

From the above formula we observe that:

Number of days to complete the work

$$= \frac{1}{\text{One day's work}}$$

So, we can now find Raj's one day's work by using the above formula.

We get that Raj's one day's work = 1/7

Let's now look at some more examples to understand this concept better.

Example 1:
A and B can separately complete a work in 48 days and 96 days respectively. They both worked for 16 days, then B took leave for 12 days. During this period A alone did the work. After B rejoined, A left the job. In how many days will B finish the work?

Solution:

A can complete the work in 48 days.

His/her one day's work = 1/48

B can complete the work in 96 days.

His/her one day's work = 1/96

Working together, their one day's work $= \frac{1}{48} + \frac{1}{96} = \frac{2+1}{96} = \frac{3}{96} = \frac{1}{32}$

Working together, their 16 days' work $= 16 \times \frac{1}{32} = \frac{1}{2}$

Work done by A (alone) when B was on leave for 12 days $= \frac{12}{48} = \frac{1}{4}$

So, the work remaining after deducting the joint effort of A and B for 16 days and the work done by A alone for 12 days

$$= 1 - \left(\frac{1}{2} + \frac{1}{4}\right) = 1 - \left(\frac{2+1}{4}\right) = \frac{4-3}{4} = \frac{1}{4}$$

So, time taken by B to complete the remaining work

$$= \frac{\text{Work to be done}}{\text{B's one day's work}}$$

$$= \left(\frac{\frac{1}{4}}{\frac{1}{96}} \right) \text{days}$$

$$= 24 \text{ days}$$

Example 2:

A, B and C together can complete a work in 16 days, where A and C can separately complete the work in 60 days and 80 days respectively. If A and B together did the work for 10 days and the remaining work was done by C, then in how many did C complete the work?

Solution:

A can complete the work in 60 days.

So, his/her one day’s work = 1/60

C can complete the work in 80 days.

So, his/her one day’s work = 1/80

So, one day’s work done by A and C $= \frac{1}{60} + \frac{1}{80} = \frac{4+3}{240} = \frac{7}{240}$

It is given that A, B and C can complete the work in 16 days.

So, one day’s work done by A, B and C = 1/16

∴ One day’s work of B $= \frac{1}{16} - \frac{7}{240} = \frac{15-7}{240} = \frac{8}{240} = \frac{1}{30}$

This means C can complete the work in 30 days.

Work done by A and B in 1 day $= \frac{1}{60} + \frac{1}{30} = \frac{1+2}{60} = \frac{3}{60} = \frac{1}{20}$

Work done by A and B in 10 days $= 10 \times \frac{1}{20} = \frac{1}{2}$

Remaining work to be done to complete the work $= 1 - \frac{1}{2} = \frac{1}{2}$

So, time taken by C to complete the remaining work $= \left(\frac{\frac{1}{2}}{\frac{1}{30}} \right)$ days $= \left(\frac{1}{2} \times 30 \right) \text{days} = 15\,\text{days}$

Example 3:

Nitika and Ruchika together can type 100 pages in 20 hours. Nitika alone can type 100 pages in 25 hours. How much time would Ruchika take to type 100 pages alone?

Solution:

Nitika and Ruchika together type 100 pages in 20 hours.

∴One hour's work done by Nitika and Ruchika together = 1/20hours

Now, Nitika alone can type 100 pages in 25 hours.

∴Nitika's one hour's work = 1/25hours

∴Ruchika one hour's work $= \frac{1}{20} - \frac{1}{25}$

$$= \frac{5-4}{100} = \frac{1}{100}$$

Thus, Ruchika alone would type 100 pages in 100 hours.

Example 4:

A can mend a wall in 14 days. B can mend the wall in 7 days. B started the work and worked for 4 days and then left the work. Find the number of days that A will take to complete the remaining work.

Solution:

It is given that A can mend a wall in 14 days.

∴A’s one day’s work = 1/14

B can mend the wall in 7 days.

∴B’s one day’s work = 1/7

∴B’s four day’s work $= \frac{1}{7} \times 4 = \frac{4}{7}$

Work left after B has worked for 4 days $= 1 - \frac{4}{7} = \frac{3}{7}$

Remaining work = 3/7

∴ Number of days taken by A to complete the work

$$= \frac{\text{Work to be done}}{\text{A's one day's work}}$$

$$= \frac{\frac{3}{7}}{\frac{1}{14}}$$

$$= \frac{3}{7} \times \frac{14}{1} = 6$$

Hence, A will take 6 days to complete the remaining work.