EXERCISE 7.1 (NCERT)

Find the anti derivative (or Integral) of the following functions by method of inspection.

QNo 1 $\sin 2x$.

Sol. $\because \frac{d}{dx}\left(-\frac{1}{2}\cos 2x\right) = -\frac{1}{2}(-\sin 2x)\cdot 2 = \sin 2x$.

$\therefore \int \sin 2x\,dx = -\frac{1}{2}\cos 2x$.

QNo 2: $\cos 3x$

Sol: $\because \frac{d}{dx}\left(\frac{1}{3}\sin 3x\right) = \frac{1}{3}\cdot\cos 3x\cdot 3 = \cos 3x$.

$\therefore \int \cos 3x\,dx = \frac{1}{3}\sin 3x$.

QNo 3. e^{2x}.

Sol. $\because \frac{d}{dx}\left(\frac{1}{2}e^{2x}\right) = \frac{1}{2}\cdot e^{2x}\cdot 2 = e^{2x}$.

$\therefore \int e^{x}dx = \frac{1}{2}e^{2x}$

QNo 4 $(ax+b)^2$

Sol: $\because \frac{d}{dx}\left(\frac{1}{3a}(ax+b)^3\right) = \frac{1}{3a}\times 3(ax+b)^2\cdot a = (ax+b)^2$.

$\therefore \int (ax+b)^2 dx = \frac{1}{3a}(ax+b)^3$

QNo 5. $\sin 2x - 4e^{3x}$

Sol. $\int(\sin 2x - 4e^{3x})\,dx = \int \sin 2x\,dx - 4\int e^{3x}dx$

Now $\because \frac{d}{dx}\left(-\frac{1}{2}\cos 2x\right) = -\frac{1}{2}(-\sin 2x)\cdot 2 = \sin 2x$.

$\therefore \int \sin 2x\,dx = -\frac{1}{2}\cos 2x$.

and $\because \frac{d}{dx}\left(\frac{1}{3}e^{3x}\right) = \frac{1}{3}e^{3x}\cdot 3 = e^{3x}$.

$\therefore \int e^{3x}dx = \frac{1}{3}e^{3x}$

$\therefore \int(\sin 2x - 4e^{3x})\,dx = -\frac{1}{2}\cos 2x - \frac{4}{3}e^{3x}$.

Find the integrals from Q 6 to Q.No 20.

QNo. 6. $\int(4e^{3x}+1)\,dx$

Sol.. $\int(4e^{3x}+1)\,dx = 4\int e^{3x}dx + \int 1\cdot dx = 4\cdot\frac{e^{3x}}{3} + x + C$

Q No. 7 $\int x^2\left(1-\frac{1}{x^2}\right)dx$

Sol. $\int x^2\left(1-\frac{1}{x^2}\right)dx = \int (x^2-1)\,dx = \int x^2 dx - \int 1\,dx$

$= \frac{x^3}{3} - x + C$

Q No. 8 $\int (ax^2+bx+c)\,dx = a\int x^2 dx + b\int x\,dx + c\int dx$

$= a\frac{x^3}{3} + b\frac{x^2}{2} + cx + k.$ $\left[\because \int x^n dx = \frac{x^{n+1}}{n+1} + c\right]$

Q No. 9 $\int (2x^2+e^x)\,dx = 2\int x^2 dx + \int e^x dx.$

$= 2\cdot\frac{x^3}{3} + e^x + C$

Q No 10 $\int\left(\sqrt{x}-\frac{1}{\sqrt{x}}\right)^2 dx = \int\left(x+\frac{1}{x}-2\right)dx = \int x\,dx + \int\frac{1}{x}\,dx - \int 2\,dx$

$= \frac{x^2}{2} + \log|x| - 2x + C$

Q No 11. $\int \frac{x^3+5x^2-4}{x^2}\,dx = \int (x+5-4x^{-2})\,dx = \int x\,dx + 5\int dx - 4\int x^{-2}dx$

$= \frac{x^2}{2} + 5x - 4\cdot\frac{x^{-2+1}}{-2+1} + C = \frac{x^2}{2} + 5x + \frac{4}{x} + C$

Q No 12 $\int \frac{x^3+3x+4}{\sqrt{x}}\,dx$

Sol. $\int\left(\frac{x^3}{\sqrt{x}} + 3\frac{x}{\sqrt{x}} + \frac{4}{\sqrt{x}}\right)dx = \int x^{\frac{5}{2}}dx + 3\int x^{\frac{1}{2}}dx + 4\int x^{-1/2}dx$

$= \frac{x^{5/2+1}}{5/2+1} + 3\cdot\frac{x^{\frac{1}{2}+1}}{1/2+1} + 4\,\frac{x^{-1/2+1}}{-1/2+1} + C$

$= \frac{2}{7}x^{7/2} + 2x^{3/2} + \frac{8}{\sqrt{x}} + C$

Q No. 13 $\int \frac{x^3-x^2+x-1}{x-1}\,dx = \int\left(\frac{x^2(x-1)+1(x-1)}{x-1}\right)dx$

$= \int \frac{(x^2+1)(x-1)}{(x-1)}\,dx = \int (x^2+1)\,dx = \int x^2 dx + \int 1\,dx = \frac{x^3}{3} + x + C$

Q No 14. $\int (1-x)\sqrt{x}\,dx = \int (\sqrt{x} - x\sqrt{x})\,dx = \int x^{1/2}dx - \int x^{3/2}dx$

$= \frac{x^{3/2}}{3/2} - \frac{x^{5/2}}{5/2} + C = \frac{2}{3}x^{3/2} - \frac{2}{5}x^{5/2} + C$

QNo15. $\int \sqrt{x}(3x^2+2x+3)\,dx = \int\left(3x^{\frac{5}{2}} + 2x^{\frac{3}{2}} + 3x^{1/2}\right)dx$

$= 3\int x^{5/2}dx + 2\int x^{3/2}dx + 3\int x^{1/2}dx = 3\frac{x^{7/2}}{7/2} + 2\frac{x^{5/2}}{5/2} + 3\frac{x^{3/2}}{3/2} + c$

$= \frac{6}{7}x^{7/2} + \frac{4}{5}x^{5/2} + 2x^{3/2} + C$

QNo 16. $\int(2x - 3\cos x + e^x)\,dx = 2\int x\,dx - 3\int\cos x\,dx + \int e^x dx$

$= 2\cdot\frac{x^2}{2} - 3\sin x + e^x + C = x^2 - 3\sin x + e^x + C$

QNo17 $\int(2x^2 - 3\sin x + 5\sqrt{x})\,dx = 2\int x^2dx - 3\int\sin x\,dx + 5\int x^{1/2}dx$

$= 2\cdot\frac{x^3}{3} - 3(-\cos x) + 5\frac{x^{3/2}}{3/2} + C = \frac{2}{3}x^3 + 3\cos x + \frac{10}{3}x^{3/2} + C$

Q.No. 18. $\int \sec x(\sec x + \tan x)\,dx = \int\sec^2 x\,dx + \int\sec x\tan x\,dx$

$= \tan x + \sec x + C$

QNo 19 $\int\frac{\sec^2 x}{\operatorname{cosec}^2 x}dx = \int\frac{\sin^2 x}{\cos^2 x}dx = \int\tan^2 x\,dx = \int(\sec^2 x - 1)\,dx$

$= \int\sec^2 x\,dx - \int 1\,dx = \tan x - x + C.$

QNo20. $\int\frac{2 - 3\sin x}{\cos^2 x}dx = \int\frac{2}{\cos^2 x}dx - 3\int\frac{\sin x}{\cos^2 x}dx$

$= 2\int\sec^2 x\,dx - 3\int\sec x\cdot\tan x\,dx$

$= 2\tan x - 3\sec x + C$

QNo21 $\int\left(\sqrt{x} + \frac{1}{\sqrt{x}}\right)dx = \int x^{1/2}dx + \int x^{-1/2}dx = \frac{x^{\frac{3}{2}}}{3/2} + \frac{x^{1/2}}{1/2} + C$

$= \frac{2}{3}x^{\frac{3}{2}} + 2x^{\frac{1}{2}} + C$

$\therefore$ C is correct option.

QNo22. If $\frac{d}{dx}(f(x)) = 4x^3 - \frac{3}{x^4}$ such that $f(2) = 0$ Then $f(x) = ?$

Sol. $\therefore \int\frac{d}{dx}(f(x)) = \int\left(4x^3 - \frac{3}{x^4}\right)dx = 4\frac{x^4}{4} - 3\frac{x^{-3}}{-3} + C = x^4 + \frac{1}{x^3} + C$

$\therefore f(x) = x^4 + \frac{1}{x^3} + C, \Rightarrow C = -(2)^4 - \frac{1}{(2)^3} = -16 - \frac{1}{8} = -\frac{129}{8}$

$\therefore f(x) = x^4 + \frac{1}{x^3} - \frac{129}{8}$