## 5.4 Kinetic theory

### Monatomic gas

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|---|---|---|---|
| Pressure | $p=\frac{1}{3}nm\langle c^2\rangle$ | (5.77) | $p$ pressure<br>$n$ number density $=N/V$<br>$m$ particle mass<br>$\langle c^2\rangle$ mean squared particle velocity |
| Equation of state of an ideal gas | $pV=NkT$ | (5.78) | $V$ volume<br>$k$ Boltzmann constant<br>$N$ number of particles<br>$T$ temperature |
| Internal energy | $U=\frac{3}{2}NkT=\frac{N}{2}m\langle c^2\rangle$ | (5.79) | $U$ internal energy |
| Heat capacities | $C_V=\frac{3}{2}Nk$ | (5.80) | $C_V$ heat capacity, constant $V$<br>$C_p$ heat capacity, constant $p$<br>$\gamma$ ratio of heat capacities |
| | $C_p=C_V+Nk=\frac{5}{2}Nk$ | (5.81) | |
| | $\gamma=\frac{C_p}{C_V}=\frac{5}{3}$ | (5.82) | |
| Entropy (Sackur–Tetrode equation)[a] | $S=Nk\ln\left[\left(\frac{mkT}{2\pi\hbar^2}\right)^{3/2}\mathrm{e}^{5/2}\frac{V}{N}\right]$ | (5.83) | $S$ entropy<br>$\hbar$ $=$(Planck constant)$/(2\pi)$<br>e $=2.71828\ldots$ |

[a]For the uncondensed gas. The factor $\left(\frac{mkT}{2\pi\hbar^2}\right)^{3/2}$ is the quantum concentration of the particles, $n_Q$. Their thermal de Broglie wavelength, $\lambda_T$, approximately equals $n_Q^{-1/3}$.

### Maxwell–Boltzmann distribution[a]

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|---|---|---|---|
| Particle speed distribution | $\mathrm{pr}(c)\,\mathrm{d}c=\left(\frac{m}{2\pi kT}\right)^{3/2}\exp\left(\frac{-mc^2}{2kT}\right)4\pi c^2\,\mathrm{d}c$ | (5.84) | pr probability density<br>$m$ particle mass<br>$k$ Boltzmann constant<br>$T$ temperature<br>$c$ particle speed |
| Particle energy distribution | $\mathrm{pr}(E)\,\mathrm{d}E=\frac{2E^{1/2}}{\pi^{1/2}(kT)^{3/2}}\exp\left(\frac{-E}{kT}\right)\mathrm{d}E$ | (5.85) | $E$ particle kinetic energy $(=mc^2/2)$ |
| Mean speed | $\langle c\rangle=\left(\frac{8kT}{\pi m}\right)^{1/2}$ | (5.86) | $\langle c\rangle$ mean speed |
| rms speed | $c_{\mathrm{rms}}=\left(\frac{3kT}{m}\right)^{1/2}=\left(\frac{3\pi}{8}\right)^{1/2}\langle c\rangle$ | (5.87) | $c_{\mathrm{rms}}$ root mean squared speed |
| Most probable speed | $\hat{c}=\left(\frac{2kT}{m}\right)^{1/2}=\left(\frac{\pi}{4}\right)^{1/2}\langle c\rangle$ | (5.88) | $\hat{c}$ most probable speed |

[a]Probability density functions normalised so that $\int_0^\infty \mathrm{pr}(x)\,\mathrm{d}x=1$.

## Transport properties

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|---|---|---|---|
| Mean free path[a] | $l = \frac{1}{\sqrt{2}\pi d^2 n}$ | (5.89) | $l$ mean free path<br>$d$ molecular diameter<br>$n$ particle number density |
| Survival equation[b] | $\mathrm{pr}(x) = \exp(-x/l)$ | (5.90) | pr probability<br>$x$ linear distance |
| Flux through a plane[c] | $J = \frac{1}{4} n\langle c\rangle$ | (5.91) | $J$ molecular flux<br>$\langle c\rangle$ mean molecular speed |
| Self-diffusion (Fick's law of diffusion)[d] | $\boldsymbol{J} = -D\nabla n$<br>where $D \simeq \frac{2}{3} l\langle c\rangle$ | (5.92)<br>(5.93) | $D$ diffusion coefficient |
| Thermal conductivity[d] | $\boldsymbol{H} = -\lambda\nabla T$<br>$\nabla^2 T = \frac{1}{D}\frac{\partial T}{\partial t}$<br>for monatomic gas $\lambda \simeq \frac{5}{4}\rho l\langle c\rangle c_V$ | (5.94)<br>(5.95)<br>(5.96) | $\boldsymbol{H}$ heat flux per unit area<br>$\lambda$ thermal conductivity<br>$T$ temperature<br>$\rho$ density<br>$c_V$ specific heat capacity, $V$ constant |
| Viscosity[d] | $\eta \simeq \frac{1}{2}\rho l\langle c\rangle$ | (5.97) | $\eta$ dynamic viscosity |
| Brownian motion (of a sphere) | $\langle x^2\rangle = \frac{kTt}{3\pi\eta a}$ | (5.98) | $x$ displacement of sphere in $x$ direction after time $t$<br>$k$ Boltzmann constant<br>$t$ time interval<br>$a$ sphere radius |
| Free molecular flow (Knudsen flow)[e] | $\frac{\mathrm{d}M}{\mathrm{d}t} = \frac{4R_\mathrm{p}^3}{3L}\left(\frac{2\pi m}{k}\right)^{1/2}\left(\frac{p_1}{T_1^{1/2}} - \frac{p_2}{T_2^{1/2}}\right)$ | (5.99) | $\frac{\mathrm{d}M}{\mathrm{d}t}$ mass flow rate<br>$R_\mathrm{p}$ pipe radius<br>$L$ pipe length<br>$m$ particle mass<br>$p$ pressure |

[a]For a perfect gas of hard, spherical particles with a Maxwell–Boltzmann speed distribution.
[b]Probability of travelling distance $x$ without a collision.
[c]From the side where the number density is $n$, assuming an isotropic velocity distribution. Also known as "collision number."
[d]Simplistic kinetic theory yields numerical coefficients of $1/3$ for $D$, $\lambda$ and $\eta$.
[e]Through a pipe from end 1 to end 2, assuming $R_\mathrm{p} \ll l$ (i.e., at very low pressure).

## Gas equipartition

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|---|---|---|---|
| Classical equipartition[a] | $E_\mathrm{q} = \frac{1}{2}kT$ | (5.100) | $E_\mathrm{q}$ energy per quadratic degree of freedom<br>$k$ Boltzmann constant<br>$T$ temperature |
| Ideal gas heat capacities | $C_V = \frac{1}{2}fNk = \frac{1}{2}fnR$<br>$C_p = Nk\left(1 + \frac{f}{2}\right)$<br>$\gamma = \frac{C_p}{C_V} = 1 + \frac{2}{f}$ | (5.101)<br>(5.102)<br>(5.103) | $C_V$ heat capacity, $V$ constant<br>$C_p$ heat capacity, $p$ constant<br>$N$ number of molecules<br>$f$ number of degrees of freedom<br>$n$ number of moles<br>$R$ molar gas constant<br>$\gamma$ ratio of heat capacities |

[a]System in thermal equilibrium at temperature $T$.