Chapter - 7
Congruence of Triangles

S5Q4T8

7.1 Congruent objects are exact copies of one another. The relation of two objects being congment is called congruance. Here we shall discuss about **Congruance of Triangles.**

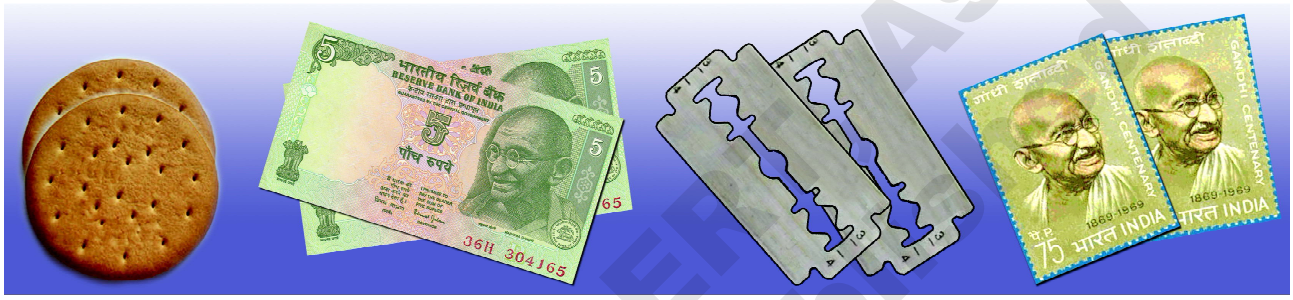


Figure - 7.1

If we place one rajor blade over another, one postal stamp over another of same kind or a five rupee note over another five rupee note, what will we observe? We shall notice them as one being superimposed over another completely. These give us the idea of congruence. We can find many such examples of congruence around us.

To see the congruence of two figures, we have to place one figure over another and observe if they fit with each other parts by parts or not.

The areas of congruent objeects are equal.

The corresponding angles and sides are equal.

The measure of congruent line segments are equal.

Congruence is expressed by symbol ≅.

7.2 Congruence of Line Segments

When one line segment coincides with another completely then the two segments are called congruent

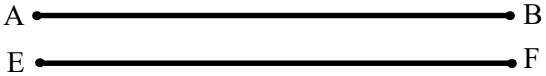


Figure - 7.2

Two line segments are congruent, if they have the same length. In Fig. 7.2, AB and EF are two line segments. Both are equal in length. Hence these are congruent.

We write, $\overline{AB} \cong \overline{EF}$ and read as $\overline{AB}$ is congruent to $\overline{EF}$.

7.3 Congruence of Angles:

Look at the Fig. 7.3. Place ∠ABC upon ∠PQR such that $\overline{AB}$ coincides with $\overline{PQ}$. Since the two angles are of the same measure $\overline{BC}$ will coincide with $\overline{QR}$.

Here, ∠ABC ≅ ∠PQR ie, the angles of equal measure are congruent.

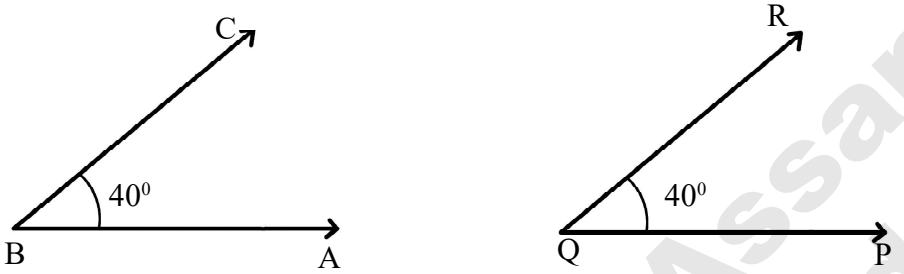


Figure -7.3

7.4 Congruence of Triangles

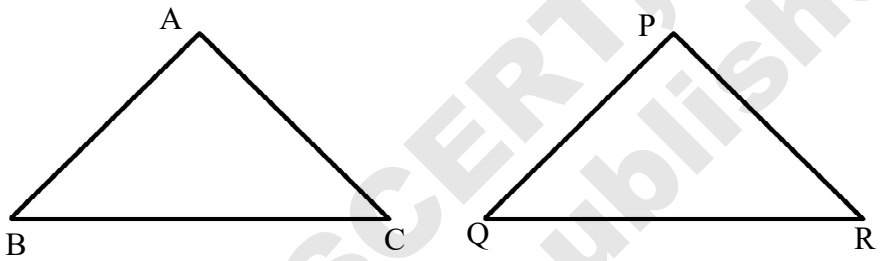


Figure - 7.4

Look at triangles of Fig. 7.4. When the first figure is placed over the second they fit with each other exactly. We can say that the triangles are congruent. We will express it as

ΔABC ≅ ΔPQR and read as :"triangle ABC is congruent to triangle PQR".

We also observe, ∠A = ∠P, ∠B = ∠Q and ∠C = ∠R. Further, $\overline{AB} = \overline{PQ}$, $\overline{BC} = \overline{QR}$, $\overline{AC} = \overline{PR}$.

7.5 Criteria for Congruence of Triangles

7.5.1 Side-Side-Side criterion (S-S-S Criteria).

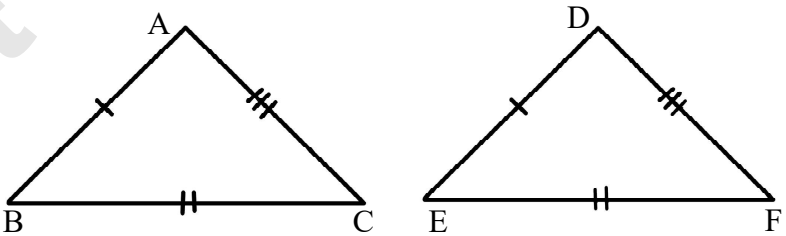


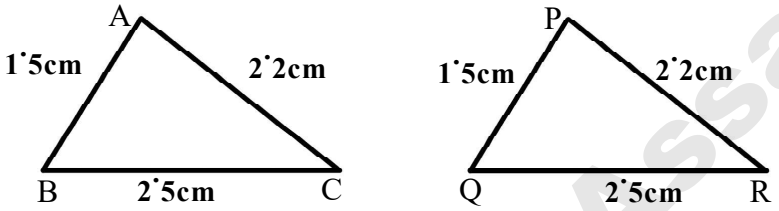
Figure - 7.5

If in ΔABC and ΔDEF

$\overline{AB} = \overline{DE}$, $\overline{BC} = \overline{EF}$, $\overline{AC} = \overline{DF}$. Then ΔABC ≅ Δ DEF

If the three sides of one triangle are equal to the three corresponding sides of another triangle, then by the side-side-side criterion of congruence, the triangles will be congruent to each other.

Example 1 : In ΔABC and ΔPQR, AB = 1·5 cm, BC = 2·5 cm, AC = 2·2 cm, PQ = 1·5 cm, QR = 2·5 cm, PR = 2·2 cm. Check whether the triangles are congruent. If they are congruent, then express the congruence relation in symbol.



Solution :

AB = PQ = 1·5 cm

BC = QR = 2·5 cm

AC = PR = 2·2 cm

We see that three sides of one triangle is equal to the three sides of the other triangle. So, by S-S-S congruence criterion ΔABC and ΔPQR are congruent. From the above, we have three relations of equality.

Corresponding vertices coincide : $A \leftrightarrow P, B \leftrightarrow Q, C \leftrightarrow R$

Corresponding angles coincide : $\angle A \leftrightarrow \angle P, \angle B \leftrightarrow \angle Q, \angle C \leftrightarrow \angle R$

Corresponding sides coincide : $\overline{AB} \leftrightarrow \overline{PQ}$, $\overline{BC} \leftrightarrow \overline{QR}$, $\overline{AC} \leftrightarrow \overline{PR}$

7.5.2 Side-Angle-Side (S-A-S) criterion :

If two sides and the angle included between them of a triangle are equal to two corresponding sides and the angle included between them of another triangles then the triangles are congruent by **'Side-Angle-Side Criterion'.**

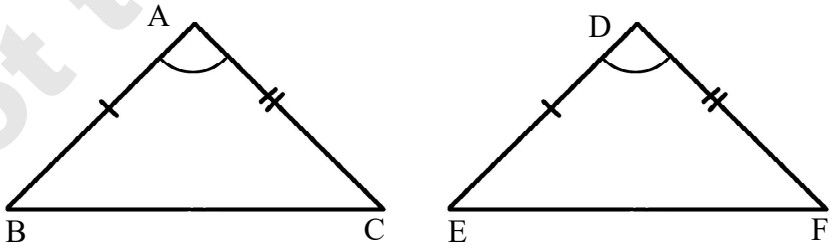


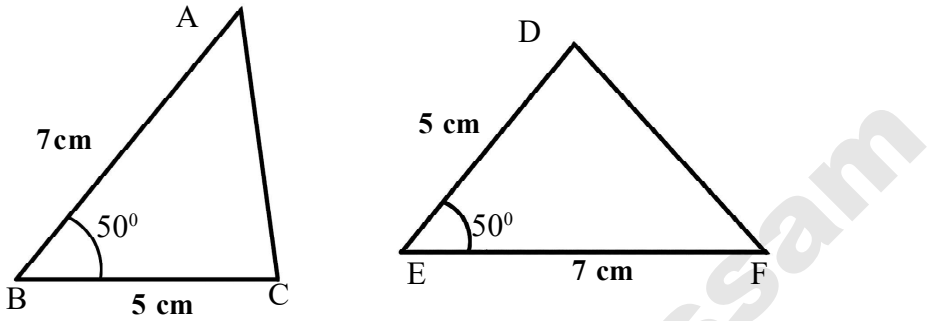
Figure - 7.6

If in ΔABC and ΔDEF

$\overline{AB} = \overline{DE}$, $\angle A = \angle D$ and $\overline{AC} = \overline{DF}$

then ΔABC ≅ ΔDEF

Example 2 : In the figure given below ABC and DEF are two triangles with AB = 7cm, BC = 5cm, $\angle B = 50^0$, DE = 5 cm, EF = 7 cm, $\angle E = 50^0$. Examine if the two triangles are congruent or not.



Solution :

From the given conditions AB = FE = 7 cm

BC = ED = 5 cm

$\angle B = 50^0 = \angle E = 50^0$

Therefore by 'Side-Angle-Side' criterion of congruence $\Delta ABC \cong \Delta FED$

7.5.3. Angle -Side- Angle (A-S-A) criterion :

If two angles and the included side of a triangle are equal to two corresponding angles and the included side of another triangle, then the triangles are congruent.

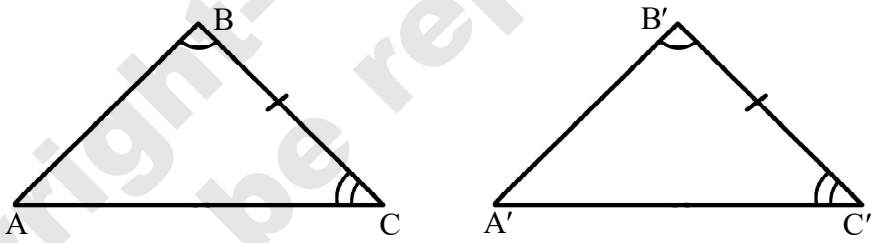


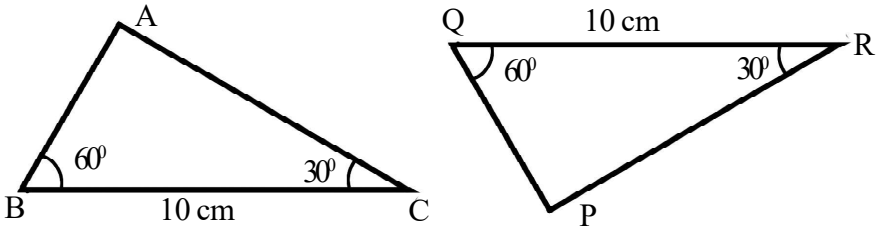
Figure -7.7

In 7.7, ΔABC and $\Delta A'B'C'$

$\angle B = \angle B'$, BC = B′C′ and $\angle C = \angle C'$

In this case $\Delta ABC \cong \Delta A'B'C'$

Example 3 : In ΔABC and ΔPQR, BC = 10 cm, $\angle B = 60^0$, $\angle C = 30^0$, QR = 10 cm, $\angle Q = 60^0$, $\angle R = 30^0$. Check if the triangles are congruent by **A-S-A Criterion.**



Solution :

Here, $\overline{BC} = \overline{QR}$ = 10 cm

$\angle B = \angle Q = 60^0$

$\angle C = \angle R = 30^0$

Therefore, by the **A-S-A Criterion** $\Delta ABC \cong \Delta PQR$.

7.5.4. Right angle, Hypotenuse- Side Criterion (R-H-S) of congruence :

If the hypotenuse and one side of one right-angled triangle are equal to the hypotenuse and the corresponding side of another right-angled triangle respectivly, then the triangles are congruent.

In ΔABC and ΔDEF

$\angle B = \angle E = 90^0$

$\overline{BC} = \overline{EF}$, $\overline{AC} = \overline{DF}$

In this case, $\Delta ABC \cong \Delta DEF$

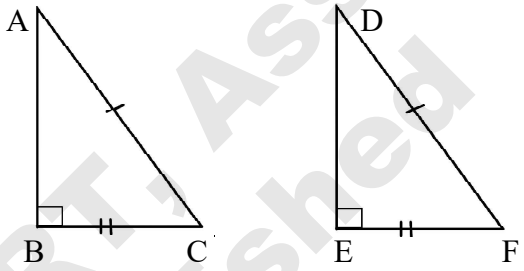


Figure -7.8

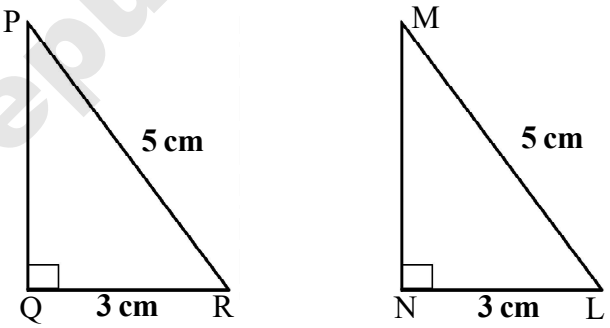
Example 4 : In ΔPQR and ΔMNL, $\angle Q = 90^0$, $\overline{QR}$ = 3 cm, PR = 5cm, $\angle N = 90^0$, $\overline{NL}$ = 3cm, $\overline{ML}$ = 5 cm. Check if the triangles are congruent by R-H-S criterion.

Solution :

Given that $\angle Q = \angle N = 90^0$

$\overline{QR} = \overline{NL}$ = 3 cm

$\overline{PR} = \overline{ML}$ = 5 cm



The RHS criterion for congruence of triangles is staisfied here.

Hence $\Delta PQR \cong \Delta MNL$

Corollary : Angle-Angle-Side criterion :

If two angles and any one side of a triangle are correspondingly equal totwo angles and side of another triangle, then the two triangles are said to be congruent by A-A-S criterion.

In ΔABC and ΔDEF

$\angle A = \angle D$, $\angle B = \angle E$, BC = EF

In this case, $\Delta ABC \cong \Delta DEF$

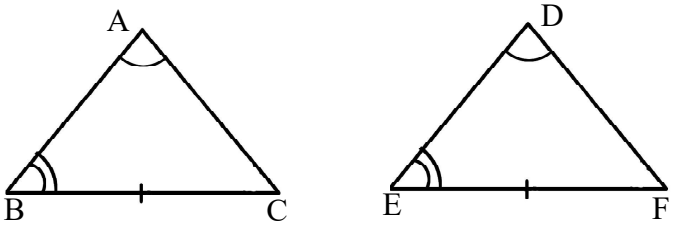


Figure -7.9

Note :

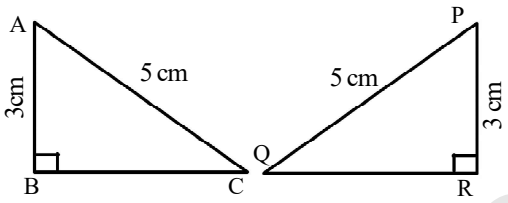
The criterionAAS is a special case of the criterion A-S-A.

Note carefully, A-A-A is not a congruence criterion for two triangles. Because,the measures of three angles of a triangle may be equal to the measures of three coressponding angles of another triangle though their corresponding sides are not equal.

Exarcise -7.1

1. Measures of sides of pairs of triangles are given in figure below. Show that the pairs are congruent. Mention the congruence criteria.

(a)



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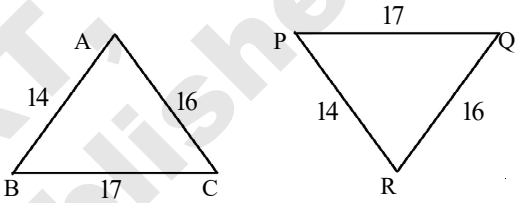

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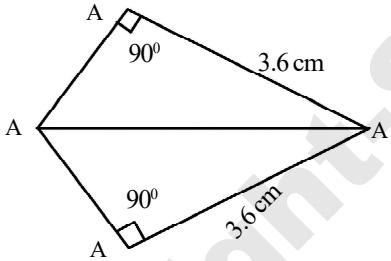

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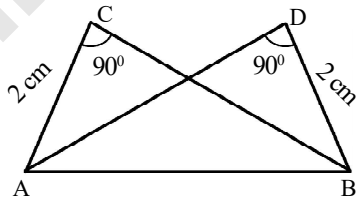

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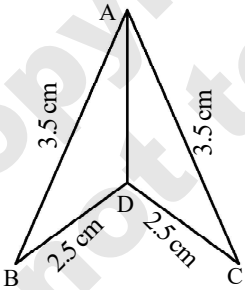

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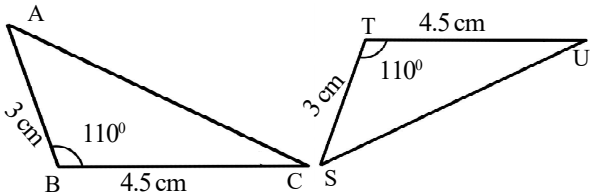
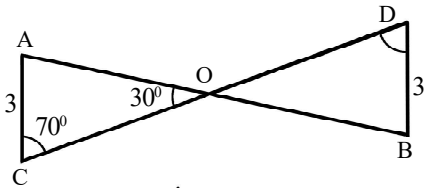
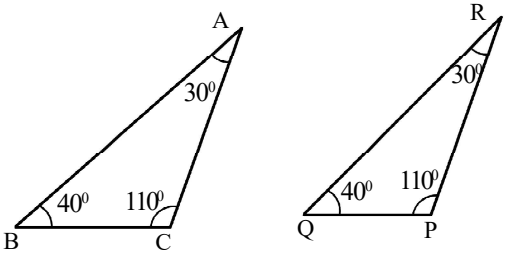


(g)



(h)



2. In Fig (i) $\overline{AB} = \overline{CD}$, $\overline{AD} = \overline{BC}$. Prove that $\Delta ADC \cong \Delta CBA$

3. In Fig (ii) ΔPQR and ΔPSQ, $\overline{PR} = \overline{PS}$ and $\overline{RQ} = \overline{SQ}$. Prove that $\Delta PRQ \cong \Delta PSQ$.

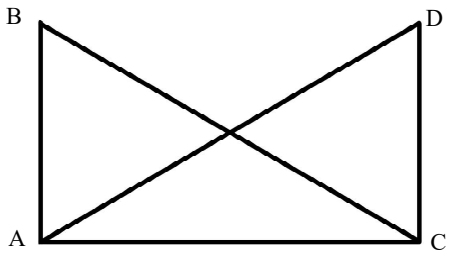


Figure (i)

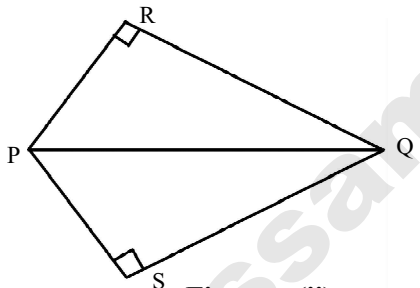
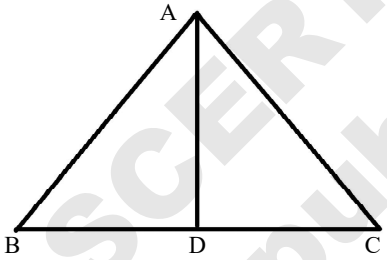


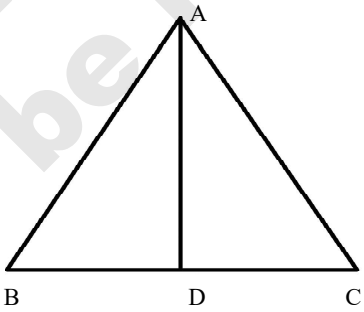
Figure - (ii)

4. In the figure below ΔABC is a Isoceles triangle, where $\overline{AB} = \overline{AC}$ and AD is its median. Prove that

$\Delta ABD \cong \Delta ACD$.



5. ABC is a Isosceles triangle with $\overline{AB} = \overline{BC}$ and AD is its altitude.



(i) Write down three equal parts of ΔADB and ΔADC.

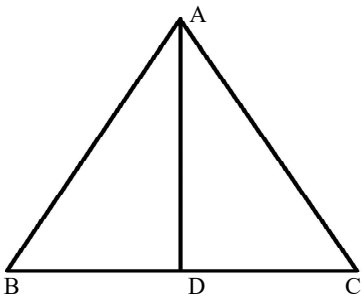
(ii) Is $\Delta ABD \cong \Delta ADC$? Give reason.

(iii) Is $\angle B = \angle C$?

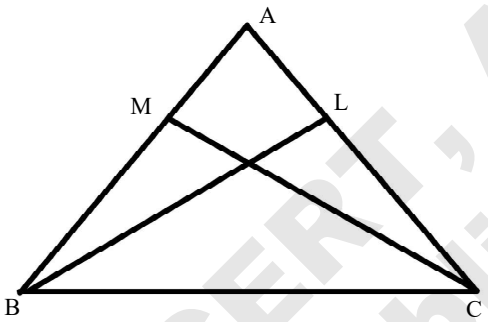
(iv) If $\overline{BD} = \overline{CD}$? Give reasons.

6. In ΔABC, $\angle A = 30^0$, $\angle C = 110^0$ and in ΔPQR, $\angle P = 30^0$, $\angle R = 110^0$.
Is $\Delta ABC \cong \Delta PQR$?

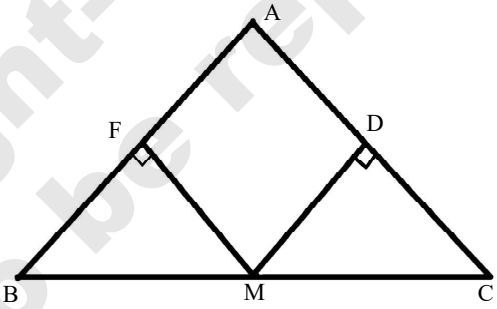
7. In the figure $\overline{AD}$ bisects $\angle A$ also, in ΔABC, $\overline{AB} = \overline{AC}$ show that the angles opposite to equal sides are equal.



8. In ΔABC, $\angle B = \angle C$, $\overline{BL}$ and $\overline{CM}$ are bisectors of $\angle B$ and $\angle C$ respectively. Prove that $\overline{BL} = \overline{CM}$

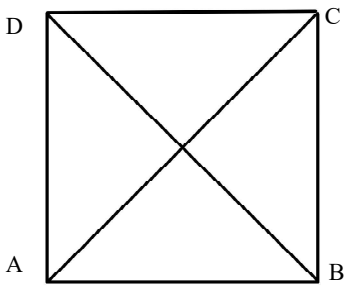


9. If the mid point M of the base $\overline{BC}$ is equidistant from the other two sides of a triangle ΔABC, then show that the triangle is isosceles.

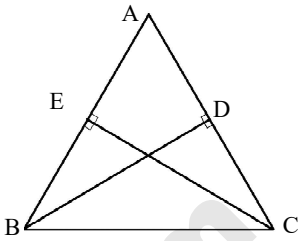


10. In Fig, $\overline{DA} = \overline{AB}$, $\overline{CB} = \overline{AB}$, $\overline{AC} = \overline{BD}$. Write down three equal parts of ΔABC and ΔDAB. Show that

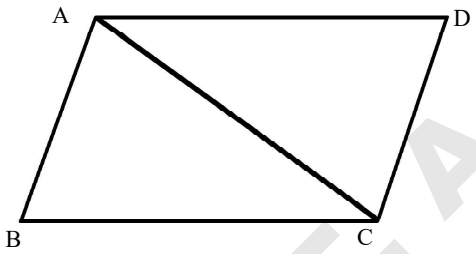
(i) $\Delta ABC \cong \Delta BAD$.



11. In the given figure BD and CE are two altitudes of ΔABC such that BD = CE.
    (i) Write three equal parts of ΔCBD and ΔBCE
    (ii) Is $\Delta CBD \cong \Delta BCE$?
    (iii) Is $\angle DCB = \angle EBC$? If not, Why?



12. In the given figure $\overline{AB} = \overline{DC}$ and $\overline{BC} = \overline{AD}$. Show that, $\Delta ABC \cong \Delta CDA$.



Exercise - 7.2

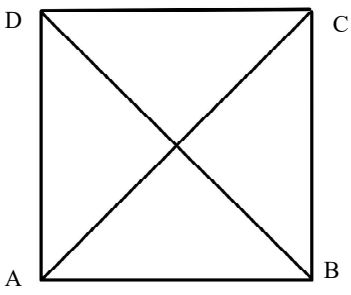
Find out the correct statements of the following question :

1. If in ΔABC and ΔPQR, AB = 4cm, BC = 5cm, AC = 6cm, PQ = 4cm, QR = 5 cm, PR = 6cm. then
   (a) $\Delta ABC \cong \Delta QRP$ (b) $\Delta ABC \cong \Delta PQR$
   (c) $\Delta ABC \cong \Delta PRQ$ (d) $\Delta ABC \cong \Delta QPR$

2. In ΔABC, $\angle A = 90^0$ and $\overline{AB} = \overline{AC}$. Then
   (a) $\angle B = \angle C = 60^0$ (b) $\angle B = \angle C = 30^0$
   (c) $\angle B = \angle C = 45^0$ (d) $\angle B = \angle B = 50^0$

3. The measure of each angle of an equilateral triangle is ....
   (a) 60^0 (b) 30^0 (c) 45^0 (d) 40^0

4. In the figure AB = CD , AD = CB, and $\angle DAB = \angle BCD$. Then
   (a) $\Delta ABC \cong \Delta ADC$
   (b) $\Delta ABC \cong \Delta ACD$
   (c) $\Delta BAD \cong \Delta DCB$
   (d) $\Delta ABC \cong \Delta CAD$



5. In ΔABC and ΔPQR, AB = 3·5 cm, BC = 7·1 cm, AC = 5 cm, PQ = 7·1 cm, QR= 5 cm and PR = 3·5cm. Which of the following statements is correct?

   (a) $\Delta ABC \cong \Delta QRP$ (b) $\Delta ABC \cong \Delta PQR$

   (c) $\Delta ABC \cong \Delta RPO$ (d) $\Delta ABC \cong \Delta OPR$

6. In ΔABC and ΔDEF, AB = 7 cm, BC = 5 cm, $\angle B = 50^0$, DE = 5 cm, EF = 7cm, $\angle E = 50^0$. Under which condition, are the triangles congruent?

   (a) SAS (b) RHS (c) ASA (d) SSS

7. In ΔABC and ΔPQR, $\angle B = \angle P = 90^0$ and AB = RP. Triangles are congruent if.

   (a) AC = RQ (b) $\angle A = \angle P$ (c) BC = QR (d) $\angle R = \angle C$ ।

8. If $\Delta ABC \cong \Delta DEF$ and $\angle A = 50^0$, $\angle E = 85^0$ then $\angle C$ = ?

   (a) 50^0 (b) 45^0 (c) 85^0 (d) 40^0

What we have learnt

1. When a triangle is placed on aother triangle such that the two triangles coincide with each other parts by parts, then the triangles are congruent.
2. Two line segments are congruent, if they have the same length.
3. Two triangles congruent if
   (i) The three sides of one triangle are equal to the three corresponding sides of another triangle then the triangles are congruent (SSS Criterion)
   (ii) If two sides and the intermediate angle between them of a triangle are equal to two corresponding sides and the intermediate Angle between them of another triangle, then the Triangles are congruent (SAS Criterion)
   (iii) If two angles and the common sides the angles of a triangle are equal to two corresponding angles and common side of another triangle then the triangles are congruent (ASA criterion).
   (iv) If the hypotenuse and one side of a right-angled triangle are respectively equal to the hypotenuse and coressponding side of another right-angled triangle, then the triangles are congruent (RHS criterion).
   (v) If two angles and any one side of a triangle are respectively equal to two angles and one side of another triangle, then the triangles are congruent (AAS criteria).