

Applied Mathematics (XII)

(Code-241)

Session- 2022-23

Secondary School Education prepares students to explore future career options after graduating from schools. Mathematics is an important subject that helps students to choose various fields of their choices. Mathematics is widely used in higher studies as an allied subject in the field of Economics, Commerce, Social Sciences and many others. It has been observed that the syllabus of Mathematics in senior secondary grades meant for Science subjects may not be appropriate for the students who wish to pursue Commerce or Social Science-based subjects in university education. By keeping this in mind, one more elective course in the Mathematics syllabus is developed for Senior Secondary classes with an aim to provide students relevant experience in Mathematics that can be used in fields other than Physical Sciences.

This course is designed to develop substantial mathematical skills and methods needed in other subject areas. Topics covered in two years aim to enable students to use mathematical knowledge in the field of business, economic and social sciences. It aims to promote appreciation of mathematical power and simplicity for its countless applications in diverse fields. The course continues to develop mathematical language and symbolism to communicate and relate everyday experiences mathematically. In addition, it reinforces the logical reasoning skills of formulating and validating mathematical arguments, framing examples, finding counterexamples. It encourages students to engage in mathematical investigations and to build connections within mathematical topics and with other disciplines. The course prepares students to use algebraic methods as a means of representation and as a problem-solving tool. It also enables students to interpret two-dimensional geometrical figures using algebra and to further deduce properties of geometrical figures in a coordinate system. The course content will help students to develop a sound understanding of descriptive and inferential statistics which they can use to describe and analyze a given set of data and to further make meaningful inferences out of it. Data based case studies from the field of business, economics, psychology, education, biology and census data will be used to appreciate the power of data in contemporary society.

It is expected that the subject is taught connecting concepts to the applications in various fields. The objectives of the course areas are as follows:

Objectives:

- a) To develop an understanding of basic mathematical and statistical tools and their applications in the field of commerce (business/ finance/economics) and social sciences.
- b) To model real-world experiences/problems into mathematical expressions using numerical/algebraic/graphical representation.
- c) To make sense of the data by organizing, representing, interpreting, analysing, and making meaningful inferences from real-world situations.
- d) To develop logical reasoning skills and apply the same in simple problem-solving.
- e) To reinforce mathematical communication by formulating conjectures, validating logical arguments and testing hypothesis.
- f) To make connections between Mathematics and other disciplines.

Grade XII (2022-23)

Number of Paper: 1

Total number of Periods: 240 (35 Minutes Each)

Time: 3 Hours

Max Marks: 80

No.	Units	No. of Periods	Marks
I	Numbers, Quantification and Numerical Applications	30	11
II	Algebra	20	10
III	Calculus	50	15
IV	Probability Distributions	35	10
V	Inferential Statistics	10	05
VI	Index Numbers and Time-based data	30	06
VII	Financial Mathematics	50	15
VIII	Linear Programming	15	08
Total		240	80
Internal Assessment		20	

CLASS XII

Sl. No.	Contents	Learning Outcomes: Students will be able to	Notes / Explanation
UNIT-1 NUMBERS, QUANTIFICATION AND NUMERICAL APPLICATIONS			
1.1	Modulo Arithmetic	<ul style="list-style-type: none"> Define modulus of an integer Apply arithmetic operations using modular arithmetic rules 	<ul style="list-style-type: none"> Definition and meaning Introduction to modulo operator Modular addition and subtraction
1.2	Congruence Modulo	<ul style="list-style-type: none"> Define congruence modulo Apply the definition in various problems 	<ul style="list-style-type: none"> Definition and meaning Solution using congruence modulo Equivalence class
1.4	Alligation and Mixture	<ul style="list-style-type: none"> Understand the rule of alligation to produce a mixture at a given price Determine the mean price of a mixture Apply rule of alligation 	<ul style="list-style-type: none"> Meaning and Application of rule of alligation Mean price of a mixture
1.5	Numerical Problems	Solve real life problems mathematically	
	Boats and Streams (upstream and downstream)	<ul style="list-style-type: none"> Distinguish between upstream and downstream Express the problem in the form of an equation 	<ul style="list-style-type: none"> Problems based on speed of stream and the speed of boat in still water
	Pipes and Cisterns	<ul style="list-style-type: none"> Determine the time taken by two or more pipes to fill or empty the tank 	<ul style="list-style-type: none"> Calculation of the portion of the tank filled or drained by the pipe(s) in unit time
	Races and Games	<ul style="list-style-type: none"> Compare the performance of two players w.r.t. time, distance 	<ul style="list-style-type: none"> Calculation of the time taken/ distance covered / speed of each player
1.6	Numerical Inequalities	<ul style="list-style-type: none"> Describe the basic concepts of numerical inequalities Understand and write numerical inequalities 	<ul style="list-style-type: none"> Comparison between two statements/situations which can be compared numerically Application of the techniques of numerical solution of algebraic inequations
UNIT-2 ALGEBRA			
2.1	Matrices and types of matrices	<ul style="list-style-type: none"> Define matrix Identify different kinds of matrices Find the size / order of matrices 	<ul style="list-style-type: none"> The entries, rows and columns of matrices Present a set of data in a matrix form
2.2	Equality of matrices, Transpose of a matrix, Symmetric and Skew symmetric matrix	<ul style="list-style-type: none"> Determine equality of two matrices Write transpose of given matrix Define symmetric and skew symmetric matrix 	<ul style="list-style-type: none"> Examples of transpose of matrix A square matrix as a sum of symmetric and skew symmetric matrix Observe that diagonal elements of skew symmetric matrices are always zero

2.3	Algebra of Matrices	<ul style="list-style-type: none"> Perform operations like addition & subtraction on matrices of same order Perform multiplication of two matrices of appropriate order Perform multiplication of a scalar with matrix 	<ul style="list-style-type: none"> Addition and Subtraction of matrices Multiplication of matrices (It can be shown to the students that Matrix multiplication is similar to multiplication of two polynomials) Multiplication of a matrix with a real number
2.4	Determinants	<ul style="list-style-type: none"> Find determinant of a square matrix Use elementary properties of determinants 	<ul style="list-style-type: none"> Singular matrix, Non-singular matrix $AB = A B$ Simple problems to find determinant value
2.5	Inverse of a matrix	<ul style="list-style-type: none"> Define the inverse of a square matrix Apply properties of inverse of matrices 	<ul style="list-style-type: none"> Inverse of a matrix using: <ol style="list-style-type: none"> cofactors If A and B are invertible square matrices of same size, <ol style="list-style-type: none"> $(AB)^{-1} = B^{-1}A^{-1}$ $(A^{-1})^{-1} = A$ $(A^T)^{-1} = (A^{-1})^T$
2.6	Solving system of simultaneous equations using matrix method, Cramer's rule and	<ul style="list-style-type: none"> Solve the system of simultaneous equations using <ol style="list-style-type: none"> Cramer's Rule Inverse of coefficient matrix Formulate real life problems into a system of simultaneous linear equations and solve it using these methods 	<ul style="list-style-type: none"> Solution of system of simultaneous equations upto three variables only (non-homogeneous equations)

UNIT- 3 CALCULUS

Differentiation and its Applications

3.1	Higher Order Derivatives	<ul style="list-style-type: none"> Determine second and higher order derivatives Understand differentiation of parametric functions and implicit functions 	<ul style="list-style-type: none"> Simple problems based on higher order derivatives Differentiation of parametric functions and implicit functions (upto 2nd order)
3.2	Application of Derivatives	<ul style="list-style-type: none"> Determine the rate of change of various quantities Understand the gradient of tangent and normal to a curve at a given point Write the equation of tangents and normal to a curve at a given point 	<ul style="list-style-type: none"> To find the rate of change of quantities such as area and volume with respect to time or its dimension Gradient / Slope of tangent and normal to the curve The equation of the tangent and normal to the curve (simple problems only)
3.3	Marginal Cost and Marginal Revenue using derivatives	<ul style="list-style-type: none"> Define marginal cost and marginal revenue Find marginal cost and marginal revenue 	<ul style="list-style-type: none"> Examples related to marginal cost, marginal revenue, etc.

3.4	Increasing /Decreasing Functions	<ul style="list-style-type: none"> Determine whether a function is increasing or decreasing Determine the conditions for a function to be increasing or decreasing 	<ul style="list-style-type: none"> Simple problems related to increasing and decreasing behaviour of a function in the given interval
3.5	Maxima and Minima	<ul style="list-style-type: none"> Determine critical points of the function Find the point(s) of local maxima and local minima and corresponding local maximum and local minimum values Find the absolute maximum and absolute minimum value of a function Solve applied problems 	<ul style="list-style-type: none"> A point $x = c$ is called the critical point of f if f is defined at c and $f'(c) = 0$ or f is not differentiable at c To find local maxima and local minima by: <ol style="list-style-type: none"> First Derivative Test Second Derivative Test Contextualized real life problems

Integration and its Applications

3.6	Integration	<ul style="list-style-type: none"> Understand and determine indefinite integrals of simple functions as anti-derivative 	<ul style="list-style-type: none"> Integration as a reverse process of differentiation Vocabulary and Notations related to Integration
3.7	Indefinite Integrals as family of curves	<ul style="list-style-type: none"> Evaluate indefinite integrals of simple algebraic functions by method of: <ol style="list-style-type: none"> substitution partial fraction by parts 	<ul style="list-style-type: none"> Simple integrals based on each method (non-trigonometric function)
3.8	Definite Integrals as area under the curve	<ul style="list-style-type: none"> Define definite integral as area under the curve Understand fundamental theorem of Integral calculus and apply it to evaluate the definite integral Apply properties of definite integrals to solve the problems 	<ul style="list-style-type: none"> Evaluation of definite integrals using properties
3.9	Application of Integration	<ul style="list-style-type: none"> Identify the region representing C.S. and P.S. graphically Apply the definite integral to find consumer surplus-producer surplus 	Problems based on finding <ul style="list-style-type: none"> Total cost when Marginal Cost is given Total Revenue when Marginal Revenue is given Equilibrium price and equilibrium quantity and hence consumer and producer surplus

Differential Equations and Modeling

3.10	Differential Equations	<ul style="list-style-type: none"> Recognize a differential equation Find the order and degree of a differential equation 	<ul style="list-style-type: none"> Definition, order, degree and examples
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3.11	Formulating and Solving Differential Equations	<ul style="list-style-type: none"> Formulate differential equation Verify the solution of differential equation Solve simple differential equation 	<ul style="list-style-type: none"> Formation of differential equation by eliminating arbitrary constants Solution of simple differential equations (direct integration only)
3.12	Application of Differential Equations	<ul style="list-style-type: none"> Define Growth and Decay Model Apply the differential equations to solve Growth and Decay Models 	<ul style="list-style-type: none"> Growth and Decay Model in Biological sciences, Economics and business, etc.
UNIT- 4 PROBABILITY DISTRIBUTIONS			
4.1	Probability Distribution	<ul style="list-style-type: none"> Understand the concept of Random Variables and its Probability Distributions Find probability distribution of discrete random variable 	<ul style="list-style-type: none"> Definition and example of discrete and continuous random variable and their distribution
4.2	Mathematical Expectation	<ul style="list-style-type: none"> Apply arithmetic mean of frequency distribution to find the expected value of a random variable 	<ul style="list-style-type: none"> The expected value of discrete random variable as summation of product of discrete random variable by the probability of its occurrence.
4.3	Variance	<ul style="list-style-type: none"> Calculate the Variance and S.D. of a random variable 	<ul style="list-style-type: none"> Questions based on variance and standard deviation
4.4	Binomial Distribution	<ul style="list-style-type: none"> Identify the Bernoulli Trials and apply Binomial Distribution Evaluate Mean, Variance and S.D of a binomial distribution 	<ul style="list-style-type: none"> Characteristics of the binomial distribution Binomial formula: $P(r) = {}^nC_r p^r q^{n-r}$ Where n = number of trials P = probability of success q = probability of failure Mean = np Variance = npq Standard Deviation = \sqrt{npq}
4.5	Poisson Distribution	<ul style="list-style-type: none"> Understand the Conditions of Poisson Distribution Evaluate the Mean and Variance of Poisson distribution 	<ul style="list-style-type: none"> Characteristics of Poisson Probability distribution Poisson formula: $P(x) = \frac{\lambda^x \cdot e^{-\lambda}}{x!}$ • Mean = Variance = λ
4.6	Normal Distribution	<ul style="list-style-type: none"> Understand normal distribution is a Continuous distribution Evaluate value of Standard normal variate Area relationship between Mean and Standard Deviation 	<ul style="list-style-type: none"> Characteristics of a normal probability distribution Total area under the curve = total probability = 1 Standard Normal Variate: $Z = \frac{x - \mu}{\sigma}$ where x = value of the random variable μ = mean σ = S.D.

UNIT - 5 INFERENCE STATISTICS			
5.1	Population and Sample	<ul style="list-style-type: none"> • Define Population and Sample • Differentiate between population and sample • Define a representative sample from a population • Differentiate between a representative and non-representative sample • Draw a representative sample using simple random sampling • Draw a representative sample using and systematic random sampling 	<ul style="list-style-type: none"> • Population data from census, economic surveys and other contexts from practical life • Examples of drawing more than one sample set from the same population • Examples of representative and non-representative sample • Unbiased and biased sampling • Problems based on random sampling using simple random sampling and systematic random sampling (sample size less than 100) •
5.2	Parameter and Statistics and Statistical Inferences	<ul style="list-style-type: none"> • Define Parameter with reference to Population • Define Statistics with reference to Sample • Explain the relation between Parameter and Statistic • Explain the limitation of Statistic to generalize the estimation for population • Interpret the concept of Statistical Significance and Statistical Inferences • State Central Limit Theorem • Explain the relation between Population-Sampling Distribution-Sample 	<ul style="list-style-type: none"> • Conceptual understanding of Parameter and Statistics • Examples of Parameter and Statistic limited to Mean and Standard deviation only • Examples to highlight limitations of generalizing results from sample to population • Only conceptual understanding of Statistical Significance/Statistical Inferences • Only conceptual understanding of Sampling Distribution through simulation and graphs
5.3	t-Test (one sample t-test and two independent groups t-test)	<ul style="list-style-type: none"> • Define a hypothesis • Differentiate between Null and Alternate hypothesis • Define and calculate degree of freedom • Test Null hypothesis and make inferences using t-test statistic for one group / two independent groups 	<ul style="list-style-type: none"> • Examples and non-examples of Null and Alternate hypothesis (only non-directional alternate hypothesis) • Framing of Null and Alternate hypothesis • Testing a Null Hypothesis to make Statistical Inferences for small sample size • (for small sample size: t- test for one group and two independent groups) • Use of t-table
UNIT – 6 INDEX NUMBERS AND TIME BASED DATA			
6.4	Time Series	<ul style="list-style-type: none"> • Identify time series as chronological data 	<ul style="list-style-type: none"> • Meaning and Definition

6.5	Components of Time Series	<ul style="list-style-type: none"> • Distinguish between different components of time series 	<ul style="list-style-type: none"> • Secular trend • Seasonal variation • Cyclical variation • Irregular variation
6.6	Time Series analysis for univariate data	<ul style="list-style-type: none"> • Solve practical problems based on statistical data and Interpret the result 	<ul style="list-style-type: none"> • Fitting a straight line trend and estimating the value
6.7	Secular Trend	<ul style="list-style-type: none"> • Understand the long term tendency 	<ul style="list-style-type: none"> • The tendency of the variable to increase or decrease over a long period of time
6.8	Methods of Measuring trend	<ul style="list-style-type: none"> • Demonstrate the techniques of finding trend by different methods 	<ul style="list-style-type: none"> • Moving Average method • Method of Least Squares

UNIT - 7 FINANCIAL MATHEMATICS

7.1	Perpetuity, Sinking Funds	<ul style="list-style-type: none"> • Explain the concept of perpetuity and sinking fund • Calculate perpetuity • Differentiate between sinking fund and saving account 	<ul style="list-style-type: none"> • Meaning of Perpetuity and Sinking Fund • Real life examples of sinking fund • Advantages of Sinking Fund • Sinking Fund vs. Savings account
7.3	Calculation of EMI	<ul style="list-style-type: none"> • Explain the concept of EMI • Calculate EMI using various methods 	<ul style="list-style-type: none"> • Methods to calculate EMI: <ul style="list-style-type: none"> i) Flat-Rate Method ii) Reducing-Balance Method • Real life examples to calculate EMI of various types of loans, purchase of assets, etc.
7.4	Calculation of Returns, Nominal Rate of Return	<ul style="list-style-type: none"> • Explain the concept of rate of return and nominal rate of return • Calculate rate of return and nominal rate of return 	<ul style="list-style-type: none"> • Formula for calculation of Rate of Return, Nominal Rate of Return
7.5	Compound Annual Growth Rate	<ul style="list-style-type: none"> • Understand the concept of Compound Annual Growth Rate • Differentiate between Compound Annual Growth Rate and Annual Growth Rate • Calculate Compound Annual Growth Rate 	<ul style="list-style-type: none"> • Meaning and use of Compound Annual Growth Rate • Formula for Compound Annual Growth Rate
7.7	Linear method of Depreciation	<ul style="list-style-type: none"> • Define the concept of linear method of Depreciation • Interpret cost, residual value and useful life of an asset from the given information • Calculate depreciation 	<ul style="list-style-type: none"> • Meaning and formula for Linear Method of Depreciation • Advantages and disadvantages of Linear Method

UNIT - 8 LINEAR PROGRAMMING

8.1	Introduction and related terminology	<ul style="list-style-type: none"> • Familiarize with terms related to Linear Programming Problem 	<ul style="list-style-type: none"> • Need for framing linear programming problem • Definition of Decision Variable, Constraints, Objective function, Optimization and Non Negative conditions
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8.2	Mathematical formulation of Linear Programming Problem	<ul style="list-style-type: none"> Formulate Linear Programming Problem 	<ul style="list-style-type: none"> Set the problem in terms of decision variables, identify the objective function, identify the set of problem constraints, express the problem in terms of inequations
8.3	Different types of Linear Programming Problems	<ul style="list-style-type: none"> Identify and formulate different types of LPP 	<ul style="list-style-type: none"> Formulate various types of LPP's like Manufacturing Problem, Diet Problem, Transportation Problem, etc.
8.4	Graphical method of solution for problems in two variables	<ul style="list-style-type: none"> Draw the Graph for a system of linear inequalities involving two variables and to find its solution graphically 	<ul style="list-style-type: none"> Corner Point Method for the Optimal solution of LPP Iso-cost/ Iso-profit Method
8.5	Feasible and Infeasible Regions	<ul style="list-style-type: none"> Identify feasible, infeasible, bounded and unbounded regions 	<ul style="list-style-type: none"> Definition and Examples to explain the terms
8.6	Feasible and infeasible solutions, optimal feasible solution	<ul style="list-style-type: none"> Understand feasible and infeasible solutions Find optimal feasible solution 	<ul style="list-style-type: none"> Problems based on optimization Examples of finding the solutions by graphical method

Practical: Use of spreadsheet

Graphs of an exponential function, demand and supply functions on Excel and study the nature of function at various points, maxima/minima, Matrix operations using Excel

Suggested practical using the spreadsheet

- Plot the graphs of functions on excel and study the graph to find out the point of maxima/minima
- Probability and dice roll simulation
- Matrix multiplication and the inverse of a matrix
- Stock Market data sheet on excel
- Collect the data on weather, price, inflation, and pollution analyze the data and make meaningful inferences
- Collect data from newspapers on traffic, sports activities and market trends and use excel to study future trends

List of Suggested projects (Class XI /XII)

- Use of prime numbers in coding and decoding of messages
- Prime numbers and divisibility rules
- Logarithms for financial calculations such as interest, present value, future value, profit/loss etc. with large values)
- The cardinality of a set and orders of infinity
- Comparing sets of Natural numbers, rational numbers, real numbers and others
- Use of Venn diagram in solving practical problems

- vii) Fibonacci sequence: Its' history and presence in nature
- viii) Testing the validity of mathematical statements and framing truth tables
- ix) Investigating Graphs of functions for their properties
- x) Visit the census site of India
http://www.censusindia.gov.in/Census_Data_2001/Census_Data_Online/Language/State ment3.htm Depict the information given there in a pictorial form
- xi) Prepare a questionnaire to collect information about money spent by your friends in a month on activities like travelling, movies, recharging of the mobiles, etc. and draw interesting conclusions
- xii) Check out the local newspaper and cut out examples of information depicted by graphs. Draw your own conclusions from the graph and compare it with the analysis given in the report
- xiii) Analysis of population migration data – positive and negative influence on urbanization
- xiv) Each day newspaper tells us about the maximum temperature, minimum temperature, and humidity. Collect the data for a period of 30 days and represent it graphically. Compare it with the data available for the same time period for the previous year
- xv) Analysis of career graph of a cricketer (batting average for a batsman and bowling average for a bowler). Conclude the best year of his career. It may be extended for other players also – tennis, badminton, athlete
- xvi) Vehicle registration data – correlating with pollution and the number of accidents
- xvii) Visit a village near Delhi and collect data of various crops over the past few years from the farmers. Also, collect data about temperature variation and rain over the period for a particular crop. Try to find the effect of temperature and rain variations on various crops
- xviii) Choose any week of your ongoing semester. Collect data for the past 10 – 15 years for the amount of rainfall received in Delhi during that week. Predict the amount of rainfall for the current year
- xix) Weather prediction (prediction of monsoon from past data)
- xx) Visit Kirana shops near your home and collect the data regarding the sales of certain commodities over a month. Try to figure out the stock of a particular commodity which should be in the store in order to maximize the profit
- xxi) Stock price movement
- xxii) Risk assessments by insurance firms from data
- xxiii) Predicting stock market crash
- xxiv) Predicting the outcome of an election – exit polls
- xxv) Predicting mortality of infants

Assessment Plan

1. Overall Assessment of the course is out of 100 marks.
2. The assessment plan consists of an External Exam and Internal Assessment.
3. External Exam will be of 03 hours duration Pen/ Paper Test consisting of 80 marks.
4. The weightage of the Internal Assessment is 20 marks. Internal Assessment can be a combination of activities spread throughout the semester/ academic year. Internal Assessment activities include projects and excel based practical. Teachers can choose activities from the suggested list of practical or they can plan activities of a similar nature. For data-based practical, teachers are encouraged to use data from local sources to make it more relevant for students.
5. Weightage for each area of internal assessment may be as under:

Sl. No.	Area and Weightage	Assessment Area	Marks allocated
1	Project work (10 marks)	Project work and record	5
		Year-end Presentation/ Viva of the Project	5
2	Practical work (10 marks)	Performance of practical and record	5
		Year-end test of any one practical	5
Total			20