1. PROPERTIES OF METALS, STRESS AND STRAIN

IMPORTANT MECHANICAL PROPERTIES

- Elasticity

  It is the property by virtue of which a material deformed under the load is ***enabled*** to return to its original dimension when the load is removed.

> If body regains ***completely*** its original shape then it is called ***perfectly*** elastic body
>
> ***Elastic limit*** marks the ***partial*** break down of elasticity beyond which removal of load result in a degree of ***permanent deformation***.
>
> Steel, Aluminium, Copper, may be considered to be perfectly elastic ***within certain limit***.

- Plasticity

  The characteristics of the material by which it undergoes ***inelastic strain*** beyond those at the ***elastic limit*** is known as plasticity.

> This property is particularly useful in operation of ***pressing*** and ***forging***.
>
> When large deformation occurs in a ***ductile*** material loaded in ***plastic*** region, the material is said to undergo ***plastic flow***.

- Ductility

  It is the property which permits a material to be drawn out ***longitudinally*** to a reduced section, under the action of ***tensile force***.

> A ductile material must posses a high degree of plasticity and strength.
>
> Ductile material must have ***low*** degree of elasticity.
>
> This is useful in ***wire drawing***.

- Brittleness

  It is lack of ductility. Brittleness implies that it can ***not*** be drawn out by tension to smaller section

> In brittle material failure take place under load ***without*** significant deformation.

> Ordinary ***Glass*** is nearly ***ideal*** brittle material.
>
> Cast iron, ***concrete*** and ceramic material are brittle material.

- Malleability

  It is the property of a material which permits the material to be ***extended*** in ***all direction*** without rupture.

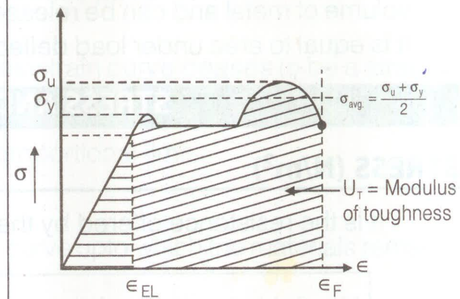
> A malleable material posses a ***high degree*** of plasticity, but ***not*** necessarily ***great strength***.

- Toughness

  It is the property of material which enables it to absorb energy ***without fracture***.

  Modulus of toughness $U_T = \left(\dfrac{\sigma_u + \sigma_y}{2}\right)\epsilon_f$

> It is desirable in material which is subjected to ***cyclic*** or ***shock loading***.
>
> It is represented by area under ***stress-strain*** curve for material upto fracture.
>
> ***Bend test*** used for common comparative test for toughness.



- Hardness

  It is the ability of a material to resist ***Indentation*** or ***surface abrasion***.

> Brinnell hardness test is used to check hardness.
>
> $$\text{Brinnell hardness number} = \frac{P}{\dfrac{\pi D}{2}\left[D - \sqrt{D^2 - d^2}\right]}$$
>
> Here, P = Standard load
>
> D = Diameter of steel ball (mm)
>
> d = Diameter of indent (mm)

- Strength

  This property enables material to resist fracture under load.

This is most important property from ***design*** point of view. Load required to cause fracture, divided by area of test specimen, is termed as ***ultimate strength***.

- **Creep**
  Creep is a permanent deformation which is recorded with passage of time at constant loading. It is plastic deformation (permanent and non-recoverable) in nature.
  Note: The temperature at which creep is uncontrolable is called *Homologous Temperature.*
- **Fatigue**
  Due to cyclic or reverse cyclic loading fracture failure may occur if total accumulated strain energy exceeds the toughness. Fatigue causes rough fracture surface even in ductile metals.
- **Resilience**
  It is the total elastic strain energy which can be stored in the given volume of metal and can be released after unloading.
  It is equal to area under load deflection curve within ***elastic limit.***

STRESS AND STRAIN

STRESS (N/m²)

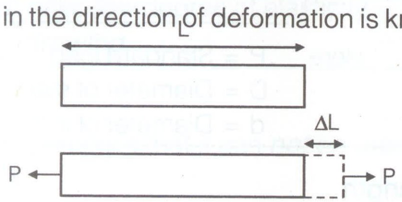
It is the resistance offered by the body to deformation

- $$\text{Nominal stress (Engineering stress)} = \frac{\text{Load}}{\text{Original Area}}$$
- $$\text{Actual/True stress} = \frac{\text{Load}}{\text{Changed (Actual) Area}}$$

STRAIN

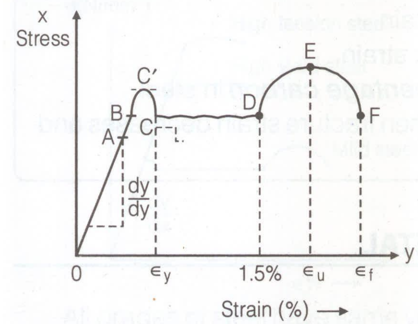
Deformation per unit length in the direction of deformation is known as strain.

$$\text{Strain} = \frac{\Delta L}{L}$$



It is a ***dimensionless*** quantity.

ENGINEERING STRESS-STRAIN CURVE OF MILD STEEL FOR TENSION UNDER STATIC-LOADING



OA — Straight line (proportional region, ***Hooke's law is valid***)
OB — Elastic region
BC — Elasto plastic region
CD — Perfectly plastic region
DE — Strain hardening
EF — Necking region
A — Limit of proportionality
B — Elastic limit
C — Lower yield point
F — Fracture point
C′ — Upper yield point
D — Strain hardening starts
E — Ultimate point or maximum stress point

- **Limit of Proportionality**
  It is the stress at which the stress-strain curve ***ceases*** to be a straight line.

Hooke's law is valid upto proportional limit.

- **Elastic Limit**
  It is the point on the stress-strain curve upto which the materials remains elastic.

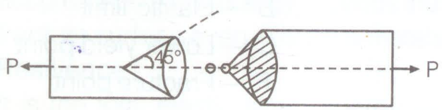
Upto this point there is ***no permanent*** deformation after removal of load.

- **Plastic Range**
  It is the region of the stress-strain curve between the elastic limit and point of rupture.
- **Yield Point**
  This point is just beyond the elastic limit, at which the specimen undergoes an appreciable increase in length ***without*** further increase in the load.
- **Rupture Strength**
  It is the stress corresponding to the failure point 'F' of the stress-strain curve.
- **Proof Stress**
  It is the stress necessary to cause a ***permanent extension*** equal to defined percentage of gauge length.

Slope of OA = Modulus of elasticity (*Young's Modulus*).
It is constant of proportionality which is defined as the intensity of stress that causes unit strain.
Plastic strain is 10 to 15 times elastic strain.
Fracture strain (ϵ_f) depends on ***percentage carbon*** in steel.
When carbon percentage increases then fracture strain decreases and yield stress increases.

TYPE OF TENSION FAILURE IN METAL

A. Ductile metal (*Shear* failure)

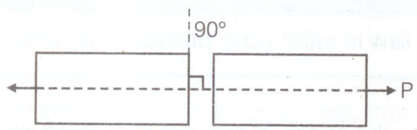


Failure plane is at ***45°***

Cup-cone fracture

Shear strength < Tensile strength ≤ Compressive strength

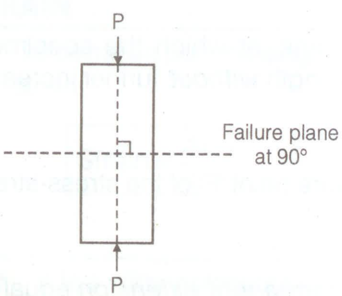
B. Brittle metal



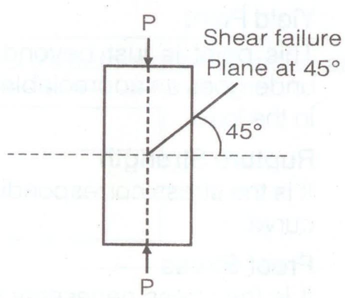
Failure plane at ***90°*** with longitudinal direction
Necking is not formed and failure is due to tension failure.
Tensile strength < Shear strength < Compressive strength

TYPE OF FAILURE IN COMPRESSION

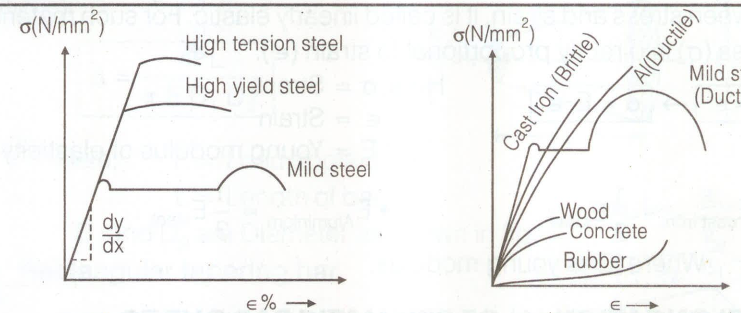
A. Ductile material



B. Brittle material



STRESS-STRAIN DIAGRAM FOR VARIOUS TYPE OF STEEL/MATERIAL



All grades of steel have same young's modulus but different yield stress.

DUCTILE MATERIAL

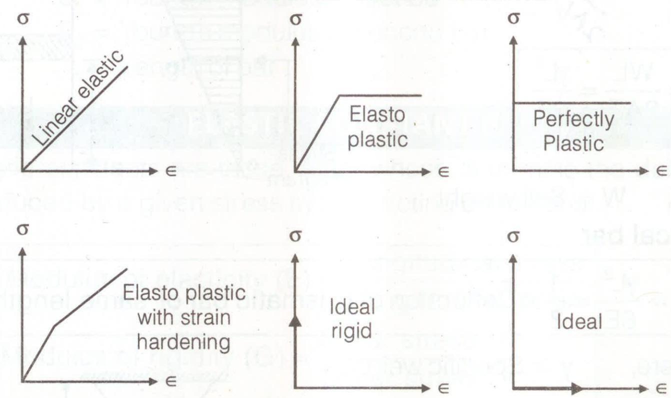
If post elastic strain is greater than 5%, it is called ductile material.
It undergoes large permanent strains before failure,
Large reduction in area before fracture
e.g. ***lead***, mild steel, copper

BRITTLE MATERIAL

If post elastic strain is less than 5%. It is called brittle material.

It fails with only little elongation after the proportional limit is exceeded.
Very less reduction in area before fracture, e.g. Bronze, ***Rubber***, Glass

Behaviour of Various Material



Where σ = Stress, ∈ = Strain

'Mild steel' is ***more*** elastic than *'Rubber'*.

HOOKE'S LAW

When a material behaves elastically and exhibits a linear relationship between stress and strain, it is called linearly elastic. For such materials stress (σ) is directly proportional to strain ($\in$).

$\boxed{\sigma \propto \in} \rightarrow \boxed{\sigma = E.\in}$

Here, σ = Stress
$\in$ = Strain
E = Young modulus of elasticity

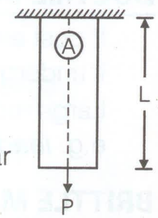
- $E_{\text{cast iron}} \approx \frac{1}{2} E_{\text{steel}}$.
- $E_{\text{Aluminium}} \approx \frac{1}{3} E_{\text{steel}}$

Where 'E' is young modulus.

AXIAL ELONGATION (Δ) OF PRISMATIC BAR DUE TO EXTERNAL LOAD

$$\boxed{\Delta = \frac{PL}{AE}}$$

Here, P = Load applied
L = Length of bar
A = Area of bar
E = Young modulus

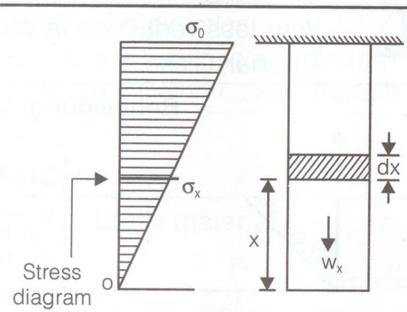


$$\boxed{\Delta = \frac{P}{\frac{EA}{L}} = \frac{P}{K}}$$

Here, K = AE/L = Axial stiffness of bar
AE = Axial rigidity
EI/L = Flexural stiffness
EI = Flexural rigidity

DEFLECTION OF BAR (Δ) DUE TO SELF-WEIGHT

A. Prismatic bar



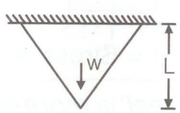
$$\boxed{\Delta = \frac{WL}{2AE} = \frac{\gamma L^2}{2E}}$$

Here, W = Self weight

B. Conical bar

$$\boxed{\Delta = \frac{\gamma L^2}{6E} = \frac{1}{3} \times \text{Deflection of prismatic bar of same length}}$$

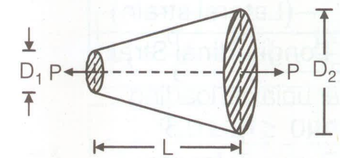
Here, γ = Specific weight
L = Length of bar
E = Young's modulus



DEFLECTION (Δ) OF TAPERED BAR

A. Circular tapering bar

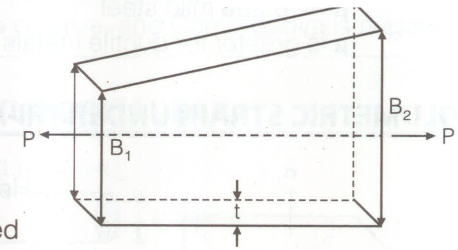
$$\boxed{\Delta = \frac{4PL}{\pi E D_1 D_2}}$$



Here, P = Load applied
L = Length of bar
D_1 and D_2 are Diameter as shown in fig.

B. Rectangular tapering bar

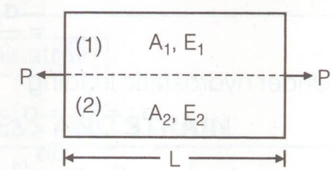
$$\boxed{\Delta = \frac{PL \log_e\left(\frac{B_2}{B_1}\right)}{E \cdot t(B_2 - B_1)}}$$



Here, t = thickness
P = Load applied
E = Young modulus

EQUIVALENT YOUNG'S MODULUS OF PARALLEL COMPOSITE BAR

$$\boxed{E_{\text{equivalent}} = \frac{A_1 E_1 + A_2 E_2}{A_1 + A_2}}$$



Here, P = Load
A_1 = Area of first bar
A_2 = Area of second bar
E_1 = Young's modulus of first bar
E_2 = Young's modulus of second bar
L = Length of bar

ELASTIC CONSTANT'S

Elastic constants are those factor whose determine the deformation produced by a given stress system acting on material.

- Modulus of elasticity (E) = $\frac{\text{Longitudinal stress}}{\text{Longitudinal strain}}$

  Modulus of rigidity (G) = $\frac{\text{Shear stress}}{\text{Shear strain}}$

  Bulk modulus (K) = $\frac{\text{Direct stress}}{\text{Volumetric strain}}$

POISSON'S RATIO (μ)

$$\mu = \frac{-(\text{Lateral strain})}{\text{Longitudinal Strain}} \qquad \mu = \frac{|\text{Lateral strain}|}{\text{Longitudinal strain}}$$

Under uniaxial loading

$0 \le \mu \le 0.5$

μ = 0 for cork

μ = 0.5 For perfectly plastic body ***(Rubber)***

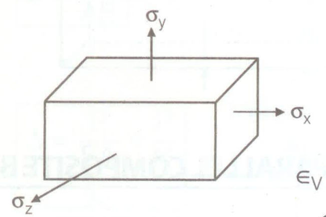
μ = 0.25 to 0.42 for elastic metals

μ = 0.1 to 0.2 for concrete

μ = 0.286 mild steel

μ is greater for ductile metals than for brittle metals.

VOLUMETRIC STRAIN UNDER TRI-AXIAL LOADING



Here, σ_x = Stress in x-direction

σ_y = Stress in y-direction

σ_z = Stress in z-direction

ϵ_v = Volumetric strain

$$\epsilon_V = \epsilon_x + \epsilon_y + \epsilon_z$$

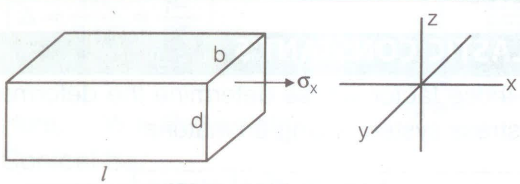
$$= \frac{\sigma_x + \sigma_y + \sigma_z}{E}(1-2\mu)$$

Under hydrostatic loading

$$\sigma_x = \sigma_y = \sigma_z = \sigma$$

$$\therefore \quad \epsilon_V = \frac{3\sigma}{E}(1-2\mu)$$

UNI-AXIAL LOADING ON RECTANGULAR PARALLELEPIPED

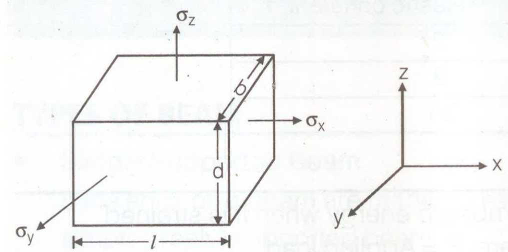


$$\epsilon_x = \frac{\Delta l}{l} = \frac{\sigma_x}{E}$$

$$\epsilon_y = \frac{\Delta b}{b} = -\frac{\mu\sigma_x}{E} \qquad \epsilon_z = \frac{\Delta d}{d} = -\frac{\mu\sigma_x}{E}$$

Here, ϵ_x, ϵ_y and ϵ_z are strain in x, y and z directions respectively.
Δl, Δb and Δd are change in length, width and depth respectively.
l, b and d are original length, width and depth respectively.

TRIAXIAL LOADING ON RECTANGULAR PARALLELEPIPED



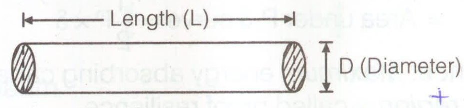
$$\epsilon_x = \frac{\sigma_x}{E} - \frac{\mu\sigma_y}{E} - \frac{\mu\sigma_z}{E} = \frac{\delta l}{l}$$

$$\epsilon_y = \frac{\sigma_y}{E} - \frac{\mu\sigma_x}{E} - \frac{\mu\sigma_z}{E} = \frac{\delta b}{b}$$

$$\epsilon_z = \frac{\sigma_z}{E} - \frac{\mu\sigma_x}{E} - \frac{\mu\sigma_y}{E} = \frac{\delta d}{d}$$

> Remember: **Sign convention:** Tensile is positive, and Compressive is negative.

VOLUMETRIC STRAIN OF CYLINDRICAL BAR



$$\epsilon_v = \text{Longitudinal Strain} + (2 \times \text{Diametric strain})$$

VOLUMETRIC STRAIN OF SPHERE

$$\epsilon_V = 3 \times \text{Diametric strain}$$

MATRIX REPRESENTATION OF STRESS AND STRAIN

3-D stress matrix $\begin{bmatrix} \sigma_{xx} & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \sigma_{yy} & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \sigma_{zz} \end{bmatrix}$ 3-D strain matrix $\begin{bmatrix} \epsilon_{xx} & \frac{\phi_{xy}}{2} & \frac{\phi_{xz}}{2} \\ \frac{\phi_{yx}}{2} & \epsilon_{yy} & \frac{\phi_{yz}}{2} \\ \frac{\phi_{zx}}{2} & \frac{\phi_{zy}}{2} & \epsilon_{zz} \end{bmatrix}$

RELATION BETWEEN E, G, K, μ

- $E = 3K(1-2\mu)$
- $E = 2G(1+\mu)$
- $E = \dfrac{9KG}{3K+G}$
- $\mu = \dfrac{3K-2G}{6K+2G}$

Here, E = Young's modulus, G = shear modulus
K = Bulk modulus, μ = Poisson ratio

Material	Number of Independent elastic constant
Homogeneous & Isotropic	2
Orthotropic (Wood)	9
Anisotropic	21

STRAIN ENERGY

It is the ability of material to absorb energy when it is strained

$$U = \frac{1}{2}P \times \delta = \frac{1}{2}T \times \theta$$

Here, P = Applied load
δ = Elongation due to applied load
T = Applied torque
θ = Angle of twist due to applied torque

- **Resilience:** Ability of a material to absorb energy in the ***elastic region*** when it is strained.

$$= \text{Area under P-}\delta\text{ curve} = \frac{1}{2}P \times \delta$$

- **Proof Resilience: *Maximum*** energy absorbing capacity of a material in the ***elastic region*** is called proof resilience.

$$= \text{Area under P-}\delta\text{ curve} = \frac{1}{2}P_{EL} \times \delta_{EL}$$

Here P_{EL} = Load at elastic limit
δ_{EL} = Elongation upto elastic limit

$$\text{Modulus of Resilience} = \frac{\text{Proof Resilience}}{\text{Volume}} = \frac{\sigma_{EL}^2}{2E}$$

Here σ_{EL} = Strain at elastic limit
E = Modulus of elasticity

THERMAL STRESS AND STRAIN

$$\sigma_{Th\cdot stress} = E\alpha T$$

$$\Delta = L\alpha T$$

$$\text{Strain} = \frac{L\alpha T}{L} = \alpha T$$

Here, σ = Thermal stress
α = Coefficient of thermal expansion
T = Temperature change
Δ = Change in length

$$\alpha_{steel} = \alpha_{concrete} = 12 \times 10^{-6}/°C$$

$$\alpha_{Aluminium} > \alpha_{Brass} > \alpha_{Copper} > \alpha_{Steel}$$

Remember: When bar is ***free*** to expand than there will be ***no thermal*** stress due to change in temperature.