Hyperbola

Q.1. Find the equation of tangents to the hyperbola $3x^2 - y^2 = 3$ which are perpendicular to the line x + 3y = 2.

Solution : 1

The given equation of hyperbola can be written as , $x^2 - y^2/3 = 1$ and line is x + 3y = 5.

Slope of given line is – 1/3 and slope of required line is 3.

Equation of tangents are : $y = 3x \pm \sqrt{\{1.(3)^2 - 3\}}$

Or, $y = 3x \pm \sqrt{6}$.

Q.2. Find the foci and latus rectum for the hyperbola $x^2/4 - y^2/5 = 1/2$.

Solution : 2

We have equation of hyperbola as

$x^2/4 - y^2/5 = 1/2$.

Or, $x^2/2 - y^2/(5/2) = 1$.

Thus, $a^2 = 2$ and $b^2 = 5/2$ => $a = \sqrt{2}$ and $b = (5\sqrt{2})/^2$ => $2a = 2\sqrt{2}$, $2b = 5\sqrt{2}$.

Now, $b^2 = a^2 (e^2 - 1)$

Or, $5/2 = 2(e^2 - 1)$

Or, $5/4 = e^2 - 1$

Or, $5/4 + 1 = e^2$

Or, $e = \pm 3/2$.

Foci are $(\pm a\, e, 0)$ or, $[\pm(3\sqrt{2})/2, 0]$

Length of latus rectum = $(2b^2)/a = \{2.(5/2)\}/\sqrt{2} = 5/\sqrt{2} = (5\sqrt{2})/2$.

Q.3. Find the equation of the hyperbola whose eccentricity is $\sqrt{5}$ and the sum of whose semi-axes is 9.

Solution : 3

We have, $e = \sqrt{5}$ and $a + b = 9 \Rightarrow b = 9 - a$.

As, $b^2 = a^2 (e^2 - 1)$

Or, $(9 - a)^2 = a^2 \{(\sqrt{5})^2 - 1\}$

Or, $81 - 18a + a^2 = 4a^2$

Or, $3a^2 + 18a - 81 = 0$

Or, $a^2 + 6a - 27 = 0$

Or, $(a - 3)(a + 9) = 0$

Or, $a = 3, -9$ but $a > 0$

Therefore, $a = 3$ and hence $b = 9 - 3 = 6$.

The equation of the hyperbola is thus $x^2/3^2 - y^2/6^2 = 1$

Or, $x2/9 - y^2/36 = 1$.

Q.4. Find the equation of the hyperbola whose directrix is $2x + y = 1$, focus is $(1, 1)$ and eccentricity is $\sqrt{3}$.

Solution : 4

We have, focus of the hyperbola at $F(1, 1)$, $e = \sqrt{3}$, directrix is $2x + y = 1$.

Let $P(x, y)$ be any point on the hyperbola and MP be the perpendicular distance from

P to the directrix, then by definition of hyperbola

$| FP | = e | MP|$

Or, $\sqrt{[(x - 1)^2 + (y - 1)^2]} = \sqrt{3}. | 2x + y - 1|/\sqrt{(2^2 + 1^2)}$

Or, $x^2 - 2x + 1 + y^2 - 2y + 1 = 3/5.(4x^2 + y^2 + 1 + 4xy - 4x - 2y)$

Or, $5x^2 - 10x + 5 + 5y^2 - 10y + 5 = 12x^2 + 3y^2 + 3 + 12xy - 12x - 6y$

Or, $7x^2 + 12xy - 2y^2 - 2x + 4y - 7 = 0$.

This is the equation of the hyperbola.