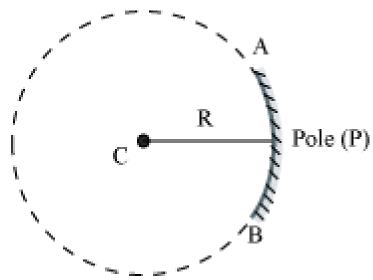
4Ray Optics And Optical Instruments

Image Formation by Spherical Mirrors and Mirror Formula

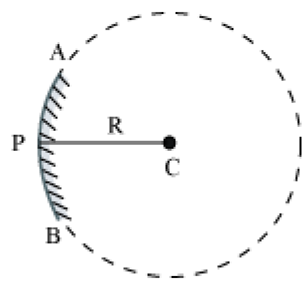
Spherical Mirror

- Concave spherical mirror – A spherical mirror whose reflecting surface is towards the centre of the sphere is called concave spherical mirror.



Concave mirror

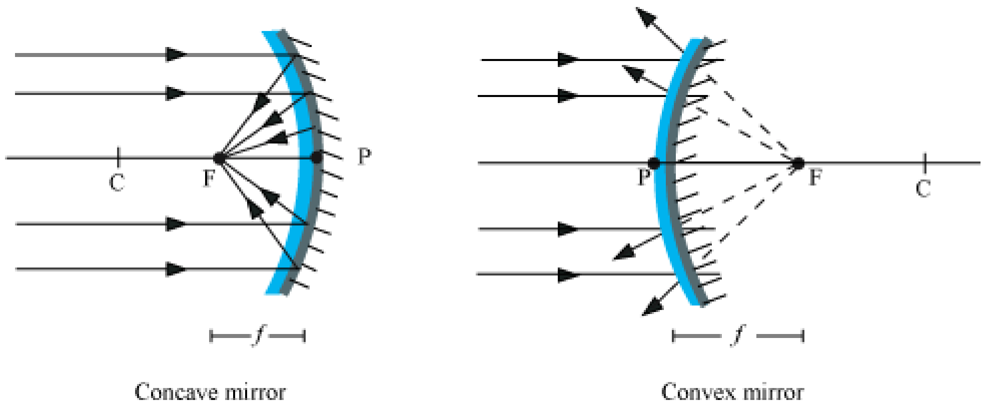
- Convex spherical mirror – A spherical mirror whose reflecting surface is away from the centre of the sphere is called convex spherical mirror.



Convex mirror

Focal Length of Spherical Mirror

- Principal focus (F) – The point at which a narrow beam of light incident on the mirror parallel to its principal axis after reflection from the mirror meets or appears to come from is called the principal focus of the mirror.



- Focal length – The distance between the pole and the principal focus of the mirror is called the focal length (*f)* of the mirror.
- For both concave and convex spherical mirrors,

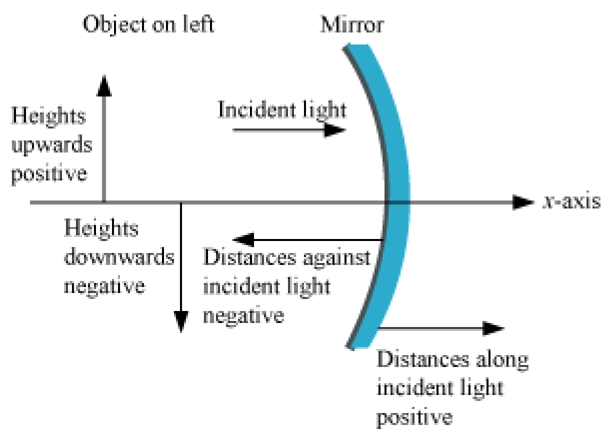
$$f = \frac{R}{2}$$

Where,

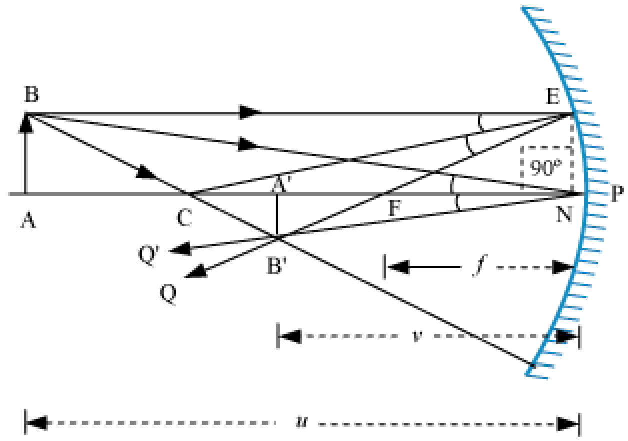
$f \rightarrow$ Focal length of the mirror

$R \rightarrow$ Radius of curvature of the spherical mirror

New Cartesian Sign Conventions



Mirror Formula



The above figure shows the ray diagram for image formation by a concave mirror.

In figure, triangles $A'B'F$ and ENF are similar.

$$\therefore \frac{A'B'}{NE} = \frac{A'F}{NF}$$

As the aperture of the concave mirror is small, the points N and P lie very close to each other.

$\therefore NF \approx PF$ and $NE = AB$

$$\frac{A'B'}{AB} = \frac{A'F}{PF}$$

Since all the distances are measured from the pole of the concave mirror, we have

$$A'F = PA' - PF$$

$$\therefore \frac{A'B'}{AB} = \frac{PA' - PF}{PF} \qquad \text{...(i)}$$

Also, triangles ABP and $A'B'P$ are similar.

$$\therefore \frac{A'B'}{AB} = \frac{PA'}{PA} \qquad \text{...(ii)}$$

From equations (i) and (ii), we obtain

$$\frac{PA' - PF}{PF} = \frac{PA'}{PA} \qquad \text{...(iii)}$$

Applying the new Cartesian sign conventions, we have

PA = − u ($\because$ distance of object is measured against incident ray)

PA' = − v ($\because$ distance of image is measured against incident ray)

PF = − f ($\because$ focal length of concave mirror is measured against incident ray)

Substituting these values in equation (iii),

We have

$$\frac{-v-(-f)}{-f}=\frac{-v}{-u}$$

$$\frac{v-f}{f}=\frac{v}{u}$$

$$\frac{v}{f}-1=\frac{v}{u} \quad \text{or} \quad \frac{1}{f}-\frac{1}{v}=\frac{1}{u}$$

$$\boxed{\frac{1}{u}+\frac{1}{v}=\frac{1}{f}}$$

The above relation is called mirror formula.

- Relation between u, v, and R

  $\because f=\frac{R}{2}$, we have

$$\frac{1}{u}+\frac{1}{v}=\frac{1}{\frac{R}{2}}$$

$$\boxed{\frac{1}{u}+\frac{1}{v}=\frac{2}{R}}$$

- Linear magnification − The ratio of the size of the image formed by a spherical mirror to the size of the object is called the linear magnification produced by the spherical mirror.

  It is denoted by m.

$$m=\frac{I}{O}$$

Where,

$I \rightarrow$ Size of the image

$O \rightarrow$ Size of the object

In the above figure, triangles ABP and $A'B'P$ are similar.

$$\therefore \frac{A'B'}{AB} = \frac{PA'}{PA}$$

Applying the new Cartesian sign conventions, we have

$A'B'$ = − I ($\because$ height is measured downwards)

AB = + O ($\because$ height is measured upwards)

PA = − u ($\because$ distance is measured against incident ray)

PA' = − v ($\because$ distance of image is measured against incident ray)

$\therefore$The above equation becomes

$$\frac{-I}{O} = \frac{-v}{-u}$$

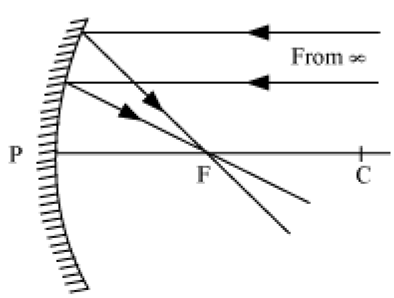
$$\text{Or, } \frac{I}{O} = \frac{-v}{u} \qquad \text{(iv)}$$

Note: The expression for magnification is same, both for the concave and convex mirrors.

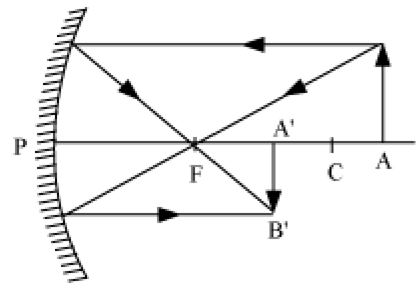
Formation of Image by Spherical Mirrors

- **Formation of image by a concave mirror**

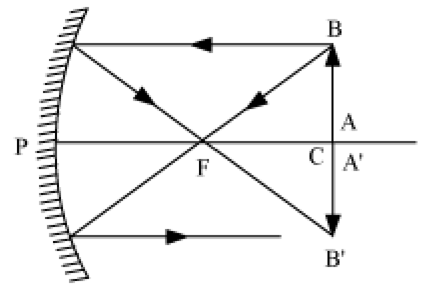
When object lies at infinity:



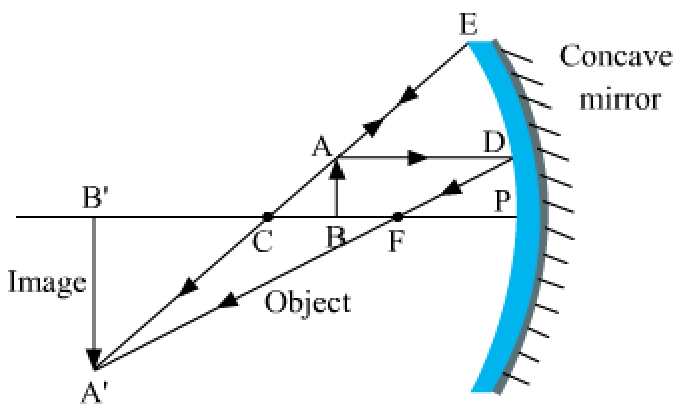
When object lies beyond C:



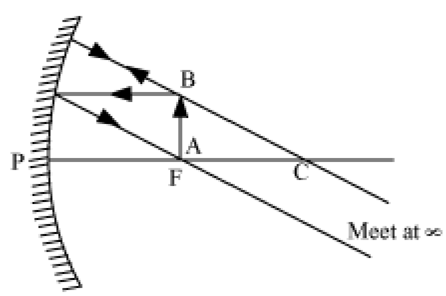
When object lies at C:



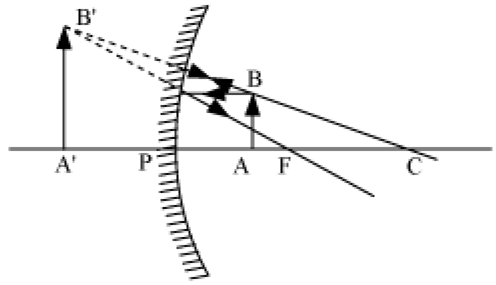
When object lies between F and C:



When object lies at F:

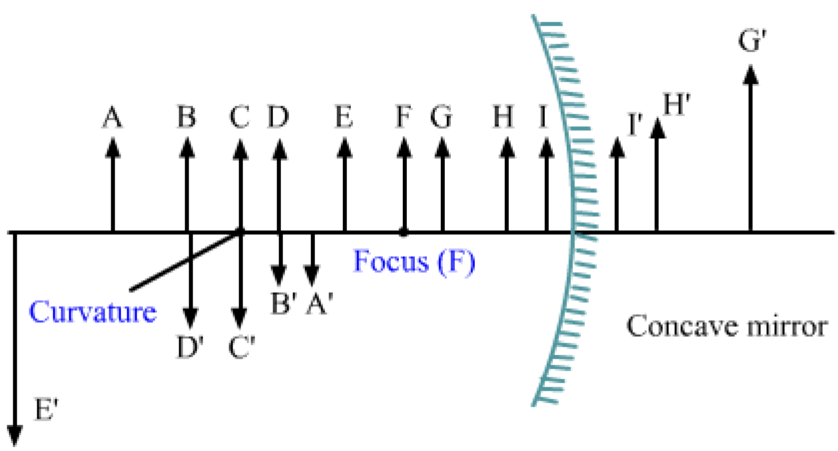


When object lies between P and F:



The discussion is summarised in the table given below.

Object position	Image position	Size of image	Nature of image
At infinity	At F	Point-sized	Real
Beyond C	Between F and C	Small	Real and inverted
At C	At C	Same as that of the object	Real and inverted
Between C and F	Behind C	Enlarged	Real and inverted
At F	At infinity	Highly enlarged	Real and inverted
Between F and P	Behind the mirror	Enlarged	Virtual and erect

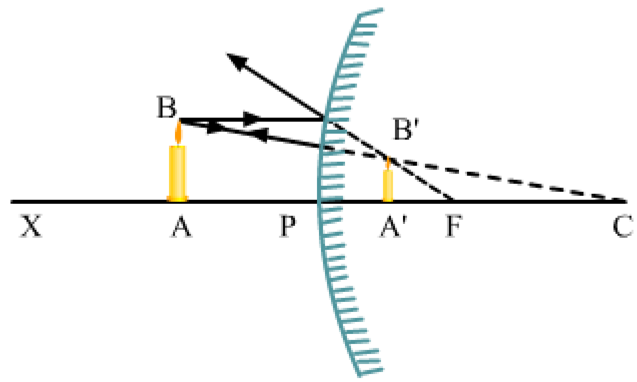


- **Formation of image by a convex mirror**

  A convex mirror always produces virtual and erect images of very small size. The images formed by a convex mirror are primarily classified in two ways.

I. When the object is at infinity

In this case, the image appears to form at the focus. This image is virtual, erect and very small in size.

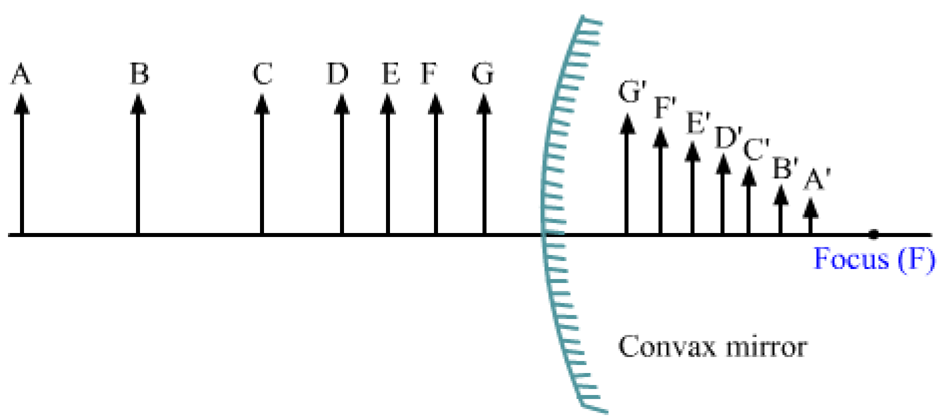


II. When the object is between the pole (P) and a point X (X lies beyond C)

In this case, the image is formed between the pole (P) and the focus (F), behind the mirror. This image is virtual, erect and small in size.

These results are summarised in the following table.

Object position	**Image position**	**Size of image**	**Nature of image**
At infinity	At F	Extremely small	Virtual and erect
Between P and X (X lies beyond C)	Between P and F	Small	Virtual and erect



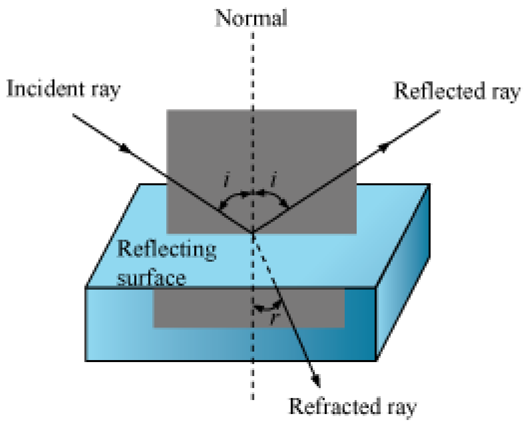
Difference Between a Real Image and a Virtual Image

S. No.	Real Image	Virtual Image
1.	Can be obtained on a screen or wall	Cannot be obtained on a screen or wall
2.	Can be touched	Cannot be touched
3.	Formed in front of the mirror	Formed behind the mirror
4.	Formed by concave mirrors only	Formed by all types of mirrors i.e., plane, convex, and concave
5.	These images are always inverted	These images are always erect

You can distinguish between real and virtual images by checking the orientation (erect or inverted) of images and also by touching them.

Refraction

Refraction of monochromatic light:
The phenomenon of change in the path of light, as it goes from one medium to another, is called refraction.



The angle between the incident ray and normal is called **angle of incidence,** while the angle between the refracted ray and normal is called **angle of refraction**. The angle between the incident ray and reflecting surface is called **glancing angle**.

Laws of refraction:

- The incident ray, the normal to the refracting surface at the point of incidence and the refracted ray lie in the same plane.
- The ratio of the sine of the angle of incidence to the sine of the angle of refraction is constant for two given media. This constant is denoted by μbaμbaand is called the relative refractive index of medium b with respect to medium a.

$$\therefore \frac{\sin i}{\sin r} = {}^{a}\mu_{b}$$

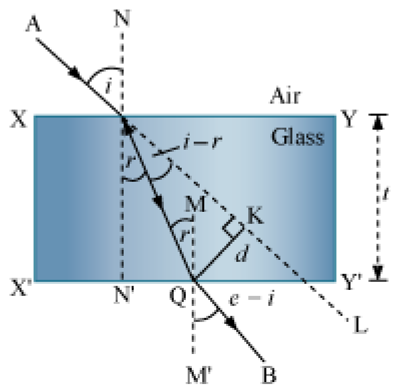
This law is also called Snell's Law of Refraction.

The absolute refractive index of a medium is given by,

$$\mu = \frac{\text{Velocity of light in vacuum}(c)}{\text{Velocity of light in medium}(v)}$$

A medium with a larger value of refractive index is called optically denser, while a medium with a smaller value is called optically rarer.

Lateral Shift:



The perpendicular distance between the incident ray and the emergent ray when light is incident obliquely on a parallel-sided refracting slab is called lateral shift.

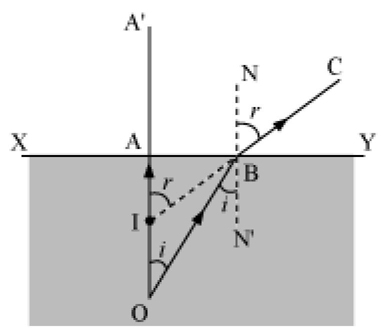
Refractive index of medium b with respect to medium a is equal to the reciprocal of the refractive index of medium a with respect to medium b.

$$\therefore {}^{a}\mu_{b} = \frac{1}{{}^{b}\mu_{a}}$$

Real and Apparent Depths:

The depth of an object immersed in water appears to be less than its actual depth.

Let O be a point object at an actual depth OA below the free surface of water, XY.



A ray of light incident along OA passes straight along OAA'. Another ray of light from O, incident at $\angle i$ on XY along OB, deviates away from the normal. It is refracted at $\angle r$ along BC. On producing backwards, BC meets OA at *I*. Therefore, *I* is the virtual image of O.

Apparent depth = AI

Real depth = OA

Clearly, AI < OA.

Now, ∠BOA = ∠OB N' = *i* [Alternate $\angle s'$]

∠AIB = ∠CBN = *r* (Corresponding angles)

In ΔOAB, $\sin i = \frac{AB}{OB}$

In ΔIAB, $\sin r = \frac{AB}{IB}$

As light is travelling from a denser medium to a rarer medium,

$$\therefore {}^{a}\mu_{\omega} = \frac{\sin r}{\sin i}$$
$${}^{a}\mu_{\omega} = \frac{AB}{IB} \times \frac{OB}{AB} = \frac{OB}{IB}$$

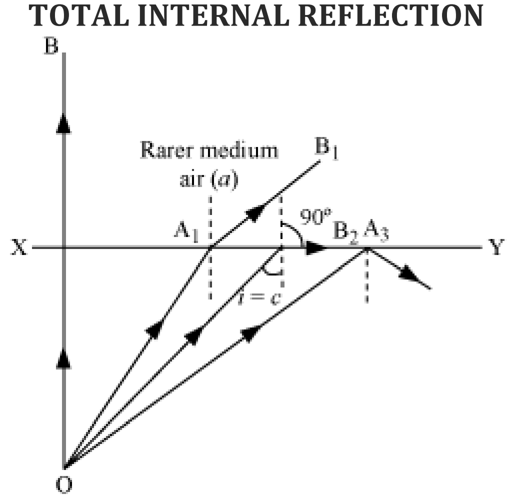
B is close to A (As the angles are very close).

$$\therefore {}^{a}\mu_{\omega} = \frac{OA}{IA} = \frac{\text{Real depth}}{\text{Apparent depth}}$$

Principal of reversibility of light:
According to the principle of reversibility of light, when a ray of light undergoes a number of reflections and retractions, its final path gets reversed. The ray retraces its entire path.

Total Internal Reflection

TOTAL INTERNAL REFLECTION



Total internal reflection is the phenomenon of reflection of light into a denser medium from an interface of the denser medium and the rarer medium.

Two essential conditions for total internal reflection:

- Incident ray should travel in the denser medium and refracted ray should travel in the rarer medium.

- Angle of incidence (*i*) should be greater than the critical angle for the pair of media in contact.

Relation between refractive index and critical angle (*C*):

When, $i = C$ and $r = 90°$:

Applying Snell's law at A_2,

$\mu_b \sin C = \mu_a \sin 90° = \mu_a \times 1$

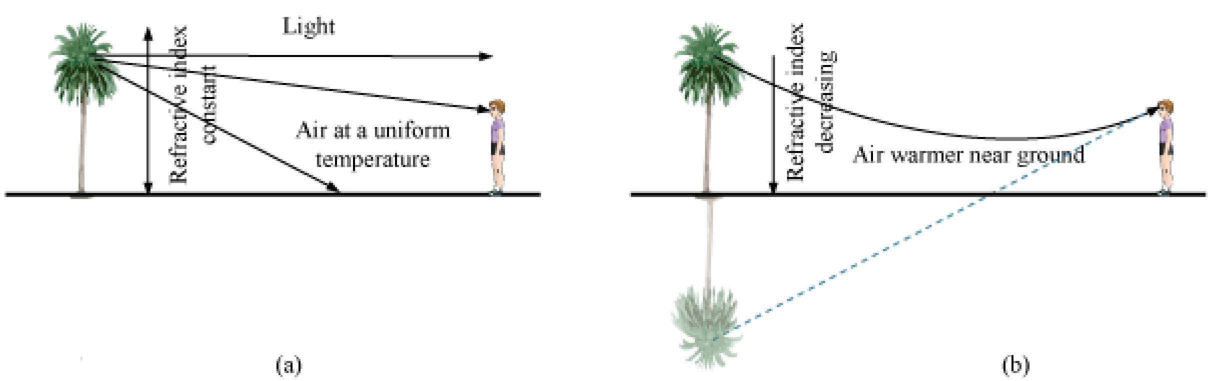
$$\frac{\mu_b}{\mu_a} = \frac{1}{\sin C}$$

$$^a\mu_b = \frac{1}{\sin C}$$

Some applications of total internal reflection:

- **Brilliance of diamond** – The critical angle for diamond-air interface is 24.4°. The diamond is cut suitably, so that light entering the diamond from any face falls at an angle greater than 24.4°, suffers multiple total internal reflections at the various faces and remains within the diamond. Hence, the diamond sparkles.

- **Mirage** → It is an optical illusion in which an object, such as a tree, appears to be inverted.



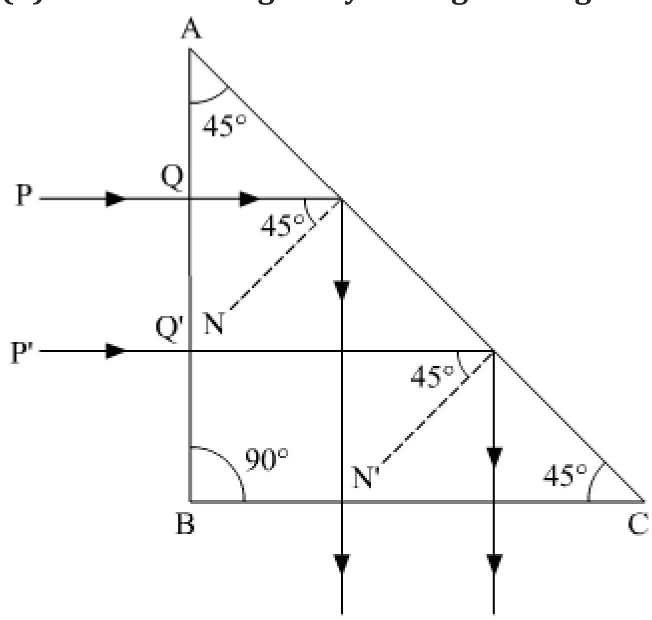
This happens due to uneven heating of the different layers of air due to which density and refractive index of air go on increasing slightly with height above the surface of the earth.

As a result of this, light from a tall object, such as a tree, passes through a medium whose refractive index decreases towards the ground. Thus, a ray of light undergoes total internal reflection. To a distant observer, the light appears to be coming from somewhere below the ground.

Total internal reflection through prism – When a ray of light falls normally on one of the surfaces of a right angled isosceles prism or an equilateral prism, then it is able to totally reflect the light ray internally.

Case I: Total internal reflection through an isosceles right angled prism: The prism can be used
(1) to deviate a ray of light through 90°
(2) to bend a ray of light through 180°
(3) to erect an inverted image without producing a deviation in its path

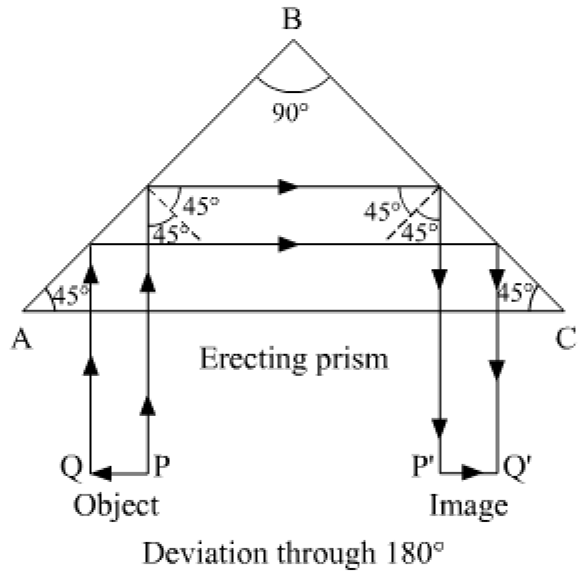
(1) Deviation of light ray through 90 degrees:



Deviation through 90°

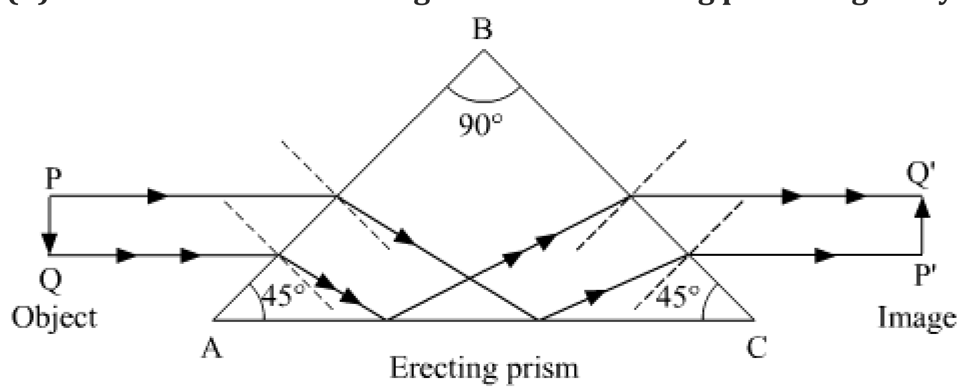
A total reflecting prism ABC is used for deviating light through 90 degrees. A ray of light incident normally on the face AB, passes un-deviated into the prism and strikes the face AC at angle of incidence of 45 degrees. Now, the light suffers total internal reflection at face AC because the incidence angle is greater than the critical angle (for glass-air interface, critical angle is approximately equal to 42 degrees). Now, the reflected beam inside the prism strike the face BC normally and therefore passes undeviated. Hence, the incident ray gets deviated by 90 degrees.

(2) Deviation of light ray through 180 degrees:



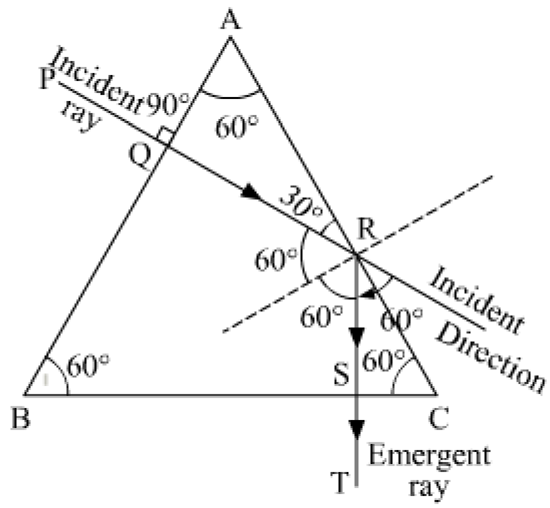
In this case, when an object is placed in front of face AC, the incident rays fall normally on it and enter the prism un-deviated. Now, these rays incident on face AB at an angle of incidence of 45 degrees greater than critical angle of glass-air interface and suffer total internal reflection. The reflected beam inside the prism strikes the face BC with an angle of incidence equal to 45 degree greater than the critical angle and again suffers total internal reflection. The beam now falls normally on the face AC and therefore passes un-deviated out of the prism, forming an image P'Q' of the object PQ. Hence, the incident ray gets deviated by 90 degrees at each reflection and therefore total deviation due to two reflection becomes 180 degrees.

(3) Erection of inverted image without deviating path of light ray:

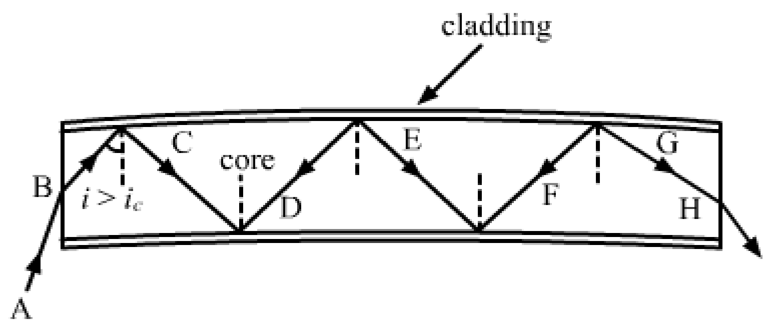


Here, the incident beam of an object PQ falls parallel to the face AC. It bends towards the normal at the air-glass interface AB and strikes the face AC of the prism. The incident angle of the beam at face AC is 45 degrees which is greater than the critical angle of the glass-air interface and thus the beam suffers total internal reflection at this face. The totally reflected beam strikes the face BC at an angle less than the critical angle of glass-air interface and bends away from the normal at the interface. Hence, the beam emerges parallel to face AC as result of refraction. On emergence, the rays coming from the object PQ get inverted and thus an erect image P'Q' is obtained for PQ.

Case II: Total internal reflection through an equilateral prism: In this case, the incident ray bends through an angle of 60 degrees from its initail direction. This is shown in the figure below.



- **Optical fibres -** Optical fibres consist of thousands of very long thin fibres of fine quality glass or quartz. The central part of the fibres is made up of material with refractive index 1.7; it is called **core**.The fibres are coated with a material of refractive index 1.5 called **cladding.** Thus, the core is denser then the cladding.



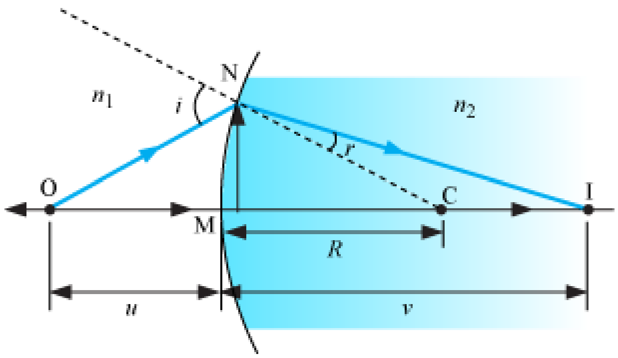
Working:
When a ray of light is incident at one end of the fibre, it undergoes repeated total internal reflection inside the fibre. The only condition is that the light must be incident on the wall of the fibre at an angle greater then the critical angle. Thus, light finally comes out at the other end of the fibre without any loss in intensity.

Uses:

- Optical fibres are extensively used for transmitting audio and video signals through long distances.
- They are used in endoscopes for medical examinations of inner parts of the body of a patient.

Refraction at Spherical Surface

- A refracting surface which forms a part of a sphere of transparent refracting material is called a spherical refracting surface.



- The above figure shows the geometry of formation of image I of an object O and the principal axis of a spherical surface with centre of curvature C and radius of curvature R.

Assumptions:

(i) The aperture of the surface is small compared to other distance involved.

(ii) NM will be taken to be nearly equal to the length of the perpendicular from the point N on the principal axis.

$$\tan \angle NOM = \frac{MN}{OM}$$
$$\tan \angle NCM = \frac{MN}{MC}$$
$$\tan \angle NIM = \frac{MN}{MI}$$

For ΔNOC, i is the exterior angle.

∴ i = ∠NOM + ∠NCM

$$i = \frac{MN}{OM} + \frac{MN}{MC} \qquad \text{...(i)}$$

Similarly, r = ∠NCM − ∠NIM

i.e., $$r = \frac{MN}{MC} - \frac{MN}{MI} \qquad \text{...(ii)}$$

BySnell's law,

$n_1 \sin i = n_2 \sin r$

For small angles,

$n_1 i = n_2 r$

Substitutingthe values of *i*and *r* from equations (i) and (ii), we obtain

$$n_1\left(\frac{MN}{OM}+\frac{MN}{MC}\right)=n_2\left(\frac{MN}{MC}-\frac{MN}{MI}\right)$$

$$\text{Or, } \frac{n_1}{OM}+\frac{n_2}{MI}=\frac{n_2-n_1}{MC} \qquad \text{...(iii)}$$

Applying new Cartesian sign conventions,

OM = − u, MI = + v, MC = + R

Substituting these in equation (iii), we obtain

$$\boxed{\frac{n_2}{v}-\frac{n_1}{u}=\frac{n_2-n_1}{R}} \qquad \text{...(iv)}$$

This equation holds for any curved spherical surface.

Refraction by a Lens

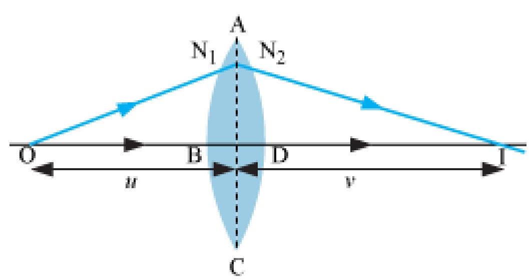


Figure (a)

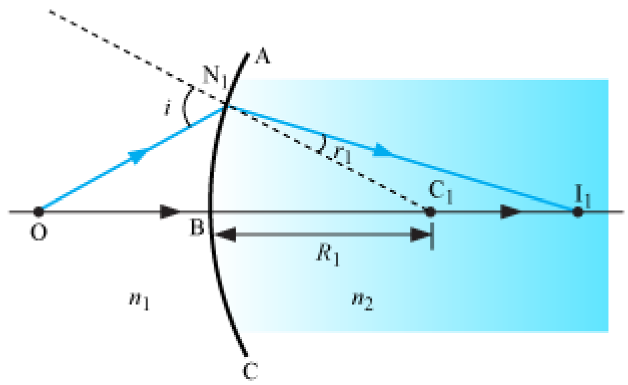


Figure (b)

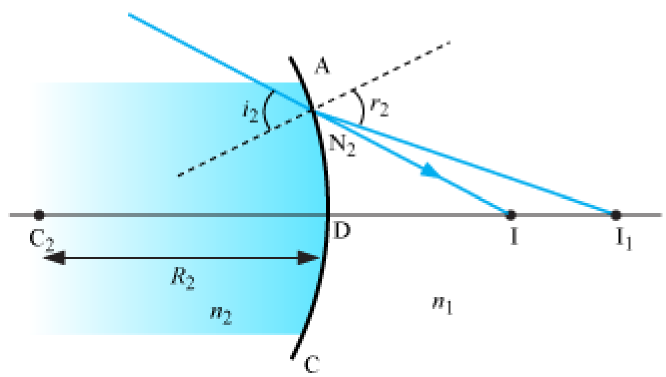


Figure (c)

The above figure shows the image formation by a convex lens.

Assumptions made in the derivation:

- The lens is thin so that distances measured from the poles of its surfaces can be taken as equal to the distances from the optical centre of the lens.
- The aperture of the lens is small.
- The object consists only of a point lying on the principle axis of the lens.
- The incident ray and refracted ray make small angles with the principle axis of the lens.

A convex lens is made up of two convex spherical refracting surfaces.

The first refracting surface forms image I of the object O [figure (b)].

Image I_1acts as virtual object for the second surface that forms the image at I [figure (c)]. Applying the equation for spherical refracting surface to the first interface ABC, we obtain

$$\frac{n_1}{OB} + \frac{n_2}{BI_1} = \frac{n_2 - n_1}{BC_1} \quad ...(i)$$

A similar procedure applied to the second interface ADC gives

$$-\frac{n_2}{DI_1} + \frac{n_1}{DI} = \frac{n_2 - n_1}{DC_2} \quad ...(ii)$$

For a thin lens, $BI_1 = DI_1$

Adding equations (i) and (ii), we obtain

$$\frac{n_1}{OB} + \frac{n_1}{DI} = (n_2 - n_1)\left(\frac{1}{BC_1} + \frac{1}{DC_2}\right) \quad ...(iii)$$

Suppose the object is at infinity i.e.,

$OB \to \infty$ and $DI \to f$

Equation (iii) gives

$$\frac{n_1}{f} = (n_2 - n_1)\left[\frac{1}{BC_1} + \frac{1}{DC_2}\right] \quad ...(iv)$$

The point where image of an object placed at infinity is formed is called the focus (*F*) of the lens and the distance *f* gives its focal length. A lens has two foci, *F* and F', on either side of it by the sign convention.

$BC_1 = R_1$

$CD_2 = -R_2$

Therefore, equation (iv) can be written as

$$\frac{1}{f} = (n_{21} - 1)\left[\frac{1}{R_1} - \frac{1}{R_2}\right] \left[\because n_{21} = \frac{n_2}{n_1}\right]$$

Equation (v) is known as the lens maker's formula.

From equations (iii) and (iv), we obtain

$$\frac{n_1}{OB} + \frac{n_1}{DI} = \frac{n_1}{f} \qquad ...(vi)$$

As B and D both are close to the optical centre of the lens,

BO = − u, DI = + v, we obtain

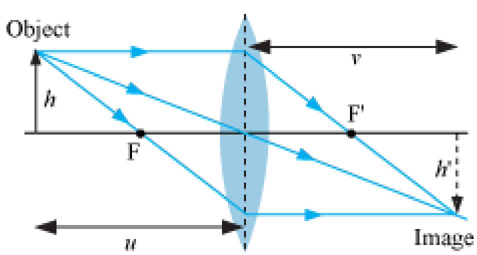
$$\boxed{\frac{1}{v} - \frac{1}{u} = \frac{1}{f}} \qquad ...(vii)$$

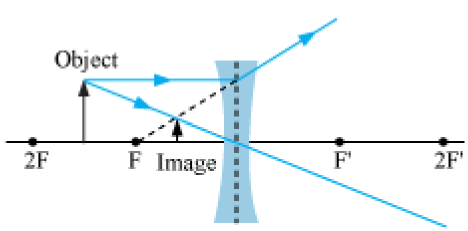
Equation (vii) is known as thin lens formula.

Rules for Image Formation by a Lens

To find the image of an object by a lens, we can in principle take any two rays emanating from a point on an object; trace their paths using the laws of refraction, and find the point where the refracted rays meet (or appear to meet). In practice, however, it is convenient to choose any two of the following rays:

(i) A ray emanating from the object parallel to the principal axis of the lens after refraction passes through the second principal focus F' (in a convex lens) or appears to diverge (in a concave lens) from the first principal focus *F.*





(ii) A ray of light passing through the first principal focus (for a convex lens) or appearing to meet at it (for a concave lens) emerges parallel to the principal axis after refraction.

Power of Lens & Combination of Thin Lenses in Contact

Power of Lens

The ability of a lens to converge or diverge the rays of light incident on it is called the power of the lens.

The power of a lens is measured as the reciprocal of its focal length (in metre).

i.e. $P = \frac{1}{f(\text{metre})}$

If f= 1 m, then

P= 1/m= 1 dioptre (D)

According to the lens maker's formula for a lens,

$$\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

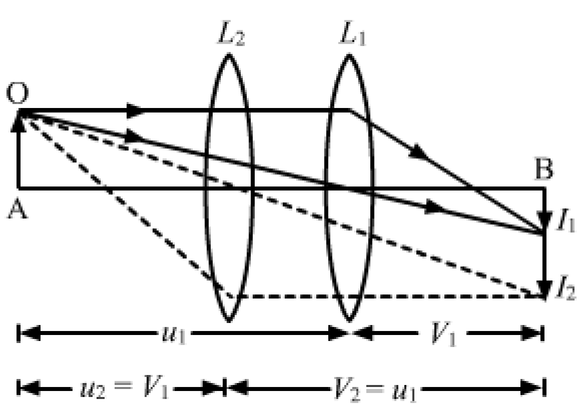
$$\because P = \frac{1}{f},$$

$$P = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

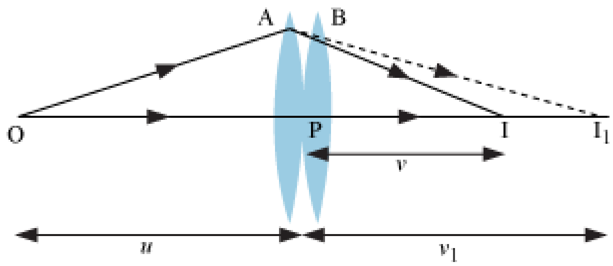
Here, R_1 and R_2 are to be measured in metre.

Concept of conjugate foci

Conjugate foci are the two points on the axis of a lens, corresponding to positions of the object and image, such that their positions are interchangeable.
Let L_1 be the initial position of the lens and u_1 and V_1 be the distances of the object and image from the lens. Now, if the lens is moved to position L_2 and if the object distance is now $u_2 = V_1$, then the image will be formed at $V_2 = u_1$ as shown.



Combination of Thin Lenses in Contact



Consider two lenses A and B of focal length f_1 and f_2 placed in contact with each other. An object is placed at a point O beyond the focus of the first lens A. The first lens produces an image at I_1(real image), which serves as a virtual object for the second lens B, producing the final image at I.

Since the lenses are thin, we assume the optical centres (P) of the lenses to be co-incident.

For the image formed by the first lens A,

$$\frac{1}{v_1} - \frac{1}{u} = \frac{1}{f_1} \qquad \text{...(i)}$$

For the image formed by the second lens B,

$$\frac{1}{v} - \frac{1}{v_1} = \frac{1}{f_2} \qquad \text{...(ii)}$$

Adding equations (i) and (ii), we get:

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f_1} + \frac{1}{f_2} \qquad \text{...(iii)}$$

If the two lens system is regarded equivalent to a single lens of focal length f,

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f} \qquad \text{...(iv)}$$

From equations (iii) and (iv), we get:

$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} \qquad \text{...(v)}$$

For several thin lenses of focal length f_1, f_2, f_3, ..., the effective focal length,

$$\frac{1}{f}=\frac{1}{f_1}+\frac{1}{f_2}+\frac{1}{f_3}+\ldots \qquad \ldots\text{(vi)}$$

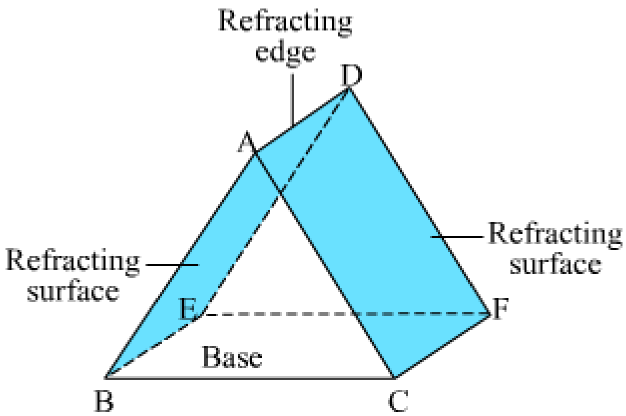
In terms of power, equation (vi) can be written as

$$P=P_1+P_2+P_3+\ldots$$

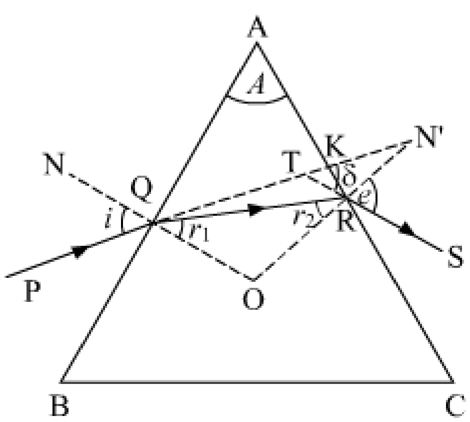
Refraction Through a Prism

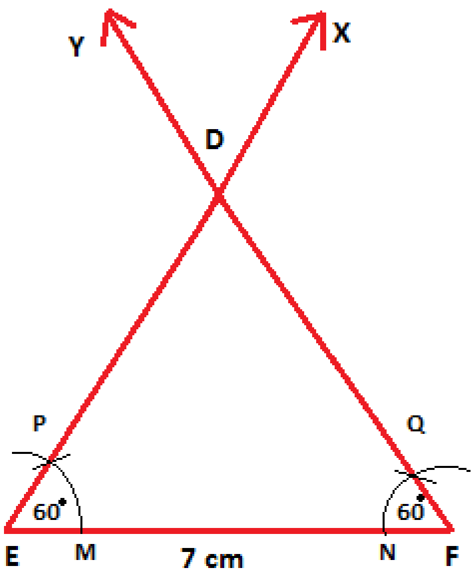
Prism

A transparent refracting medium which is bounded by five plane surfaces and having a triangular cross section is known as prism.



The figure below shows the passage of light through a triangular prism ABC.





The angles of incidence and refraction at first face AB are $\angle i$ and $\angle r_1$.

The angle of incidence at the second face AC is $\angle r_2$ and the angle of emergence $\angle e$.

δ is the angle between the emergent ray RS and incident ray PQ and is called the angle of deviation.

Here, $\angle PQN = i$

$\angle SRN' = e$

$\angle RQO = r_1$

$\angle QRO = r_2$

$\angle KTS = \delta$

$\because \angle TQO = i$ and $\angle RQO = r_1$, we have

$\angle TQR = i - r_1$

$\angle TRO = e$ and $\angle QRO = r_2$

$\angle TRQ = e - r_2$

In triangle TQR, the side QT has been produced outwards. Therefore, the exterior angle δ should be equal to the sum of the interior opposite angles.

i.e, $\delta = \angle TQR + \angle TRQ = (i - r_1) + (e - r_2)$

$\delta = (i + e) - (r_1 + r_2)$ ...(i)

In triangle QRO,

$r_1 + r_2 + \angle ROQ = 180°$ ...(ii)

From quadrilateral AROQ, we have the sum of angles ($\angle AQO + \angle ARO = 180°$). This means that the sum of the remaining two angles should be 180°.

i.e , $\angle A + \angle QOR = 180°$ [$\angle A$ is called the angle of prism]

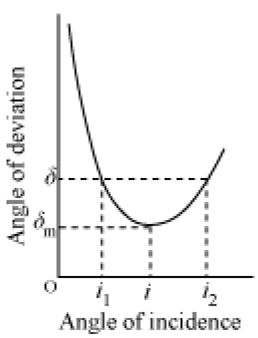
From equations (i) and (ii),

$r_1 + r_2 = A$ (iii)

Substituting (iii) in (i), we obtain

$\delta = (i + e) - A$

A+δ=i+eA+δ=i+e



If the angle of incidence is increased gradually, then the angle of deviation first decreases, attains a minimum value (δ_m), and then again starts increasing.

When angle of deviation is minimum, the prism is said to be placed in the minimum deviation position.

There is only one angle of incidence for which the angle of deviation is minimum.

When

$\delta = \delta_m$ [prism in minimum deviation position],

$e = i$ and $r_2 = r_1 = r$ ...(iv)

∵ r1+r2=A∵ r1+r2=A

From equation (iv), $r + r = A$

r=A2r=A2

Also, we have

$A + \delta = i + e$

Setting,

$\delta = \delta_m$ and $e = i$

$A + \delta_m = i + i$

$$i = \frac{(A+\delta_m)}{2}$$

$$\because \mu = \frac{\sin\ i}{\sin\ r}$$

$$\therefore \mu = \frac{\sin\left(\frac{A+\delta_m}{2}\right)}{\sin\left(\frac{A}{2}\right)}$$

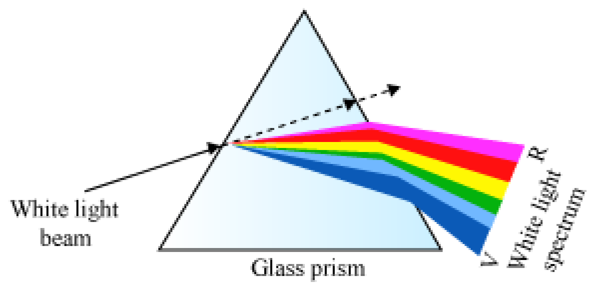
Factors affecting angle of deviation

- Angle of incidence (i)
- Angle of prism (A)
- Refractive index (μμ) of the material of the prism
- Colour or wavelength (λλ) of light

Dispersion Through a Prism

- Dispersion is the phenomenon of splitting of light into its component colours.

- When a narrow beam of sunlight is incident on a glass prism, the emergent light comprises seven colours, namely, violet (V), indigo (I), blue (B), green (G), yellow (Y), orange (O) and red (R).



- Red light bends the least, while violet light bends the most.

- Cause of dispersion: Different colours of white light have different wavelengths. The wavelength of violet light is smaller than that of red light. The refractive index of a material in terms of the wavelength of the light is given by Cauchy's expression.

  $\mu = a + b/\lambda^2 + c/\lambda^4$

  Here, *a, b* and *c* are constants for the material.

  ∴ Refractive index for violet light > Refractive index for red light

  i.e., $\mu_v > \mu_r$

  For a small-angled prism, we have:

  $\delta = A(\mu - 1)$

  Since $\mu_v > \mu_r$, violet light will have greater deviation than red light.

  $\delta_v > \delta_r$

Angular Dispersion
Angular dispersion is the difference in the angle of deviation of two extreme colours in the dispersed beam of light.
If violet and red are the extreme colours and yellow is the mean colour for white light, then the angular dispersion will be:

$\delta_v - \delta_r = A(\mu_v - 1) - A(\mu_r - 1)$

$\delta_v - \delta_r = A(\mu_v - \mu_r)$

The unit for angular dispersion is degree or radian.

Dispersive Power
Dispersive power of a prism is the ratio of angular dispersion between the extreme colours to the deviation of the mean colour produced by the prism.

For a thin prism,

$$\omega = \frac{\delta_v - \delta_r}{\delta_y}$$

$$\therefore\ \omega = \frac{A(\mu_v - \mu_r)}{(\mu_y - 1)}$$

Also,

$$\mu_y = \mu_m = \frac{\mu_v + \mu_r}{2} \text{ and } \delta_y = \delta_m = \frac{\delta_v + \delta_r}{2}$$

$$\therefore\ \omega = \frac{\delta_v - \delta_r}{[(\delta_v + \delta_r)/2]}$$

$$\Rightarrow\ \omega = \frac{\mu_v - \mu_r}{[(\mu_v + \mu_r)/2] - 1}$$

Scattering of Light

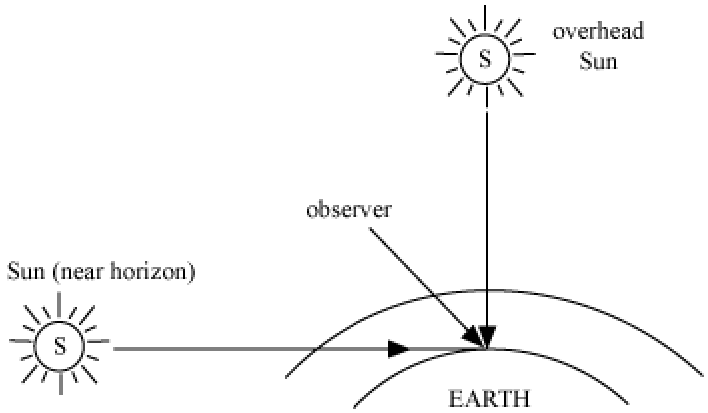
- Scattering of light takes place when the size of the scattering object is smaller compared to the wavelength of the light.

- **Rayleigh's Law of Scattering:** The intensity of light corresponding to the wavelength in the scattered light varies inversely as the fourth power of the wavelength.

  Amount of scattering: $\propto \frac{1}{\lambda^4}$

- The sky appears blue on a clear day because of the scattering of blue light by the air molecules in the Earth's atmosphere. The air molecules are smaller compared to the wavelength of visible light. Therefore, they scatter light of shorter wavelengths more easily and effectively than light of longer wavelengths. Blue light gets scattered very easily because of its shorter wavelength compared to other colours of light such as red, orange and yellow.

- At sunrise or sunset, the Sun looks almost reddish. This is because at the time of sunset or sunrise, the Sun is near the horizon. The light from the Sun should traverse larger thickness of atmosphere than what it actually covers when the Sun is overhead. Blue and violet-coloured rays are scattered in a large amount than the red-coloured rays. Hence, the light reaching the observer is mostly red; as a result, the Sun appears reddish or orange-reddish at sunrise or sunset.

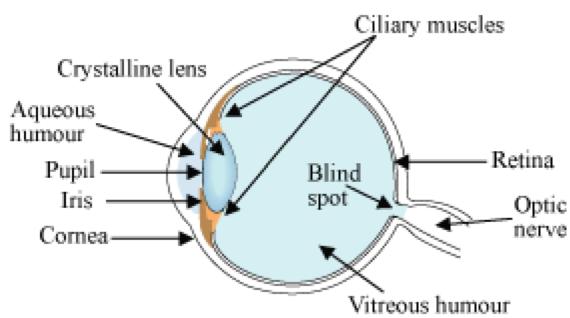


Strengthen this topic

SCROLL DOWN FOR THE NEXT TOPIC

The Eye

Part of the Eye



Human eye is nearly spherical shaped of diameter 2.5 cm.

Sclerotic – It is the outer covering of the eye ball. Its function is to protect the eye.

Cornea – In front of the eye, the sclerotic merges into a transparent curved tissue known as cornea. The light enters the eye through the cornea.

Choroid – It is the black coating of the sclerotic. Its main function is to absorb the superfluous light.

Iris – It is the circular diaphragm behind the cornea. It has a central hole known as the pupil of the eye.

Crystalline lens – Behind the iris is a converging lens, which is composed of microscopic glassy fibres. It is held behind the iris by external objects on the retina.

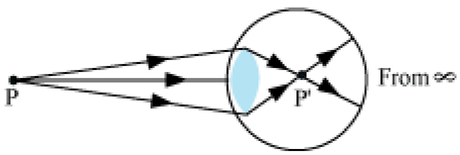
Retina – It is a light sensitive membrane on the back interior wall of the eye ball. It is composed of nerves and two types of light sensitive cells – called rods and cones.

Aqueous humour and vitreous humour – These are watery liquids of almost the same refractive index, filling up the anterior and posterior chambers of the eyeball, partitioned by the crystalline lens.

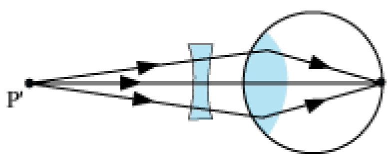
Accommodation – It is the process by which the eye adapts itself for objects at different distances.

Defects of Vision

- **Nearsightedness or Myopia –** A person suffering from myopia can see only nearby objects clearly, but cannot see the objects beyond a certain distance clearly.



Correction – In order to correct the eye for this defect, a concave lens of suitable focal length is placed close to the eye so that the parallel ray of light from an object at infinity after refraction through the lens appears to come from the far point P' of the myopic eye.



If *x* is the distance of the far point from the eye, then for the concave lens placed before the eye,

$u = \infty$

$v = -x$

Let '*f*' be the focal length of the required concave lens. From the lens formula,

$$-\frac{1}{u}+\frac{1}{v}=\frac{1}{f}$$

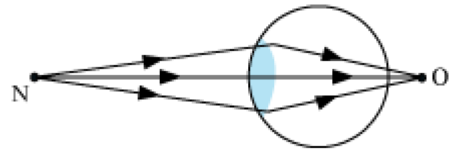
$$-\frac{1}{\infty}+\frac{1}{-x}=\frac{1}{f}$$

$$0-\frac{1}{x}=\frac{1}{f}$$

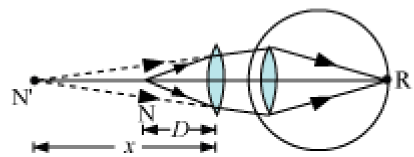
$$\boxed{f=-x}$$

Thus, myopic eye is cured against the defect by using a concave lens of focal length equal to the distance of its far point from the eye.

- **Farsightedness or Hypermetropia** − A person suffering from hypermetropia can see distant objects clearly, but cannot see nearby objects.



Correction − To correct this defect, a convex lens of suitable focal length is placed close to the eye so that the rays of light from an object placed at the point N after refraction through the lens appear to come from the near point N' of the hypermetropic eye.



Let

x → Distance of the near point N' from the eye

D → Least distance of distinct vision

$u = -D$ ($\because$ distance is measured against the incident rays)

$v = -x$ ($\because$ distance is measured against the incident rays)

If f is the focal length of the required convex lens, then from the lens formula,

$$-\frac{1}{u}+\frac{1}{v}=\frac{1}{f}$$

$$\frac{-1}{-D}+\frac{1}{-x}=\frac{1}{f}$$

$$\frac{1}{f}=\frac{1}{D}-\frac{1}{x}$$

$$f=\frac{xD}{x-D}$$

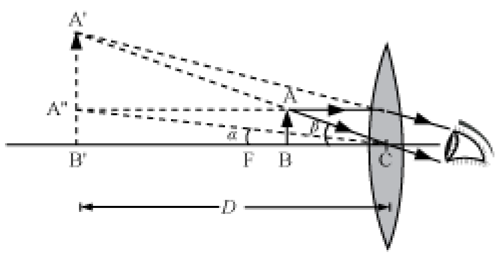
- **Presbyopia –** The farsightedness defect of vision occurring with age is called presbyopia. This defect can be cured by using a convex lens of focal length given by the equation,

$$f = \frac{xD}{x - D}$$

Microscope

Simple Microscope

When image is formed at the near point



The angular magnification of a simple microscope is the ratio of the angle β subtended at the eye by the image at the near point and the angle αsubtended at the unaided eye by the object at the near point.

$$\therefore M = \frac{\beta}{\alpha}$$

Now, $\tan\beta = \frac{A'B'}{D}$ and $\tan\alpha = \frac{\text{A''B'}}{D} = \frac{\text{AB}}{D}$

Since the angles are small,

$$\therefore \beta = \frac{\text{A'B'}}{D}$$

$$\alpha = \frac{\text{AB}}{D}$$

$$\therefore M = \frac{\text{A'B'}}{D} \times \frac{D}{\text{AB}} = \frac{\text{A'B'}}{\text{AB}}$$

This gives the linear magnification produced by the lens.

It can be proved that $\frac{A'B'}{AB} = \frac{v}{u}$

We know that,

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

$$\frac{v}{v} - \frac{v}{u} = \frac{v}{f} \Rightarrow 1 - M = \frac{v}{f}$$

$$\boxed{M = 1 - \frac{v}{f}}$$

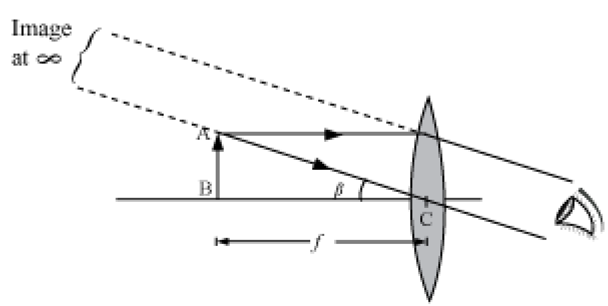
v = – *D*

$$\boxed{\therefore M = 1 + \frac{D}{f}}$$

In case the eye is placed behind the lens at a distance '*a*', then

$$\boxed{M = 1 + \frac{D - a}{f}}$$

When the image is formed at infinity



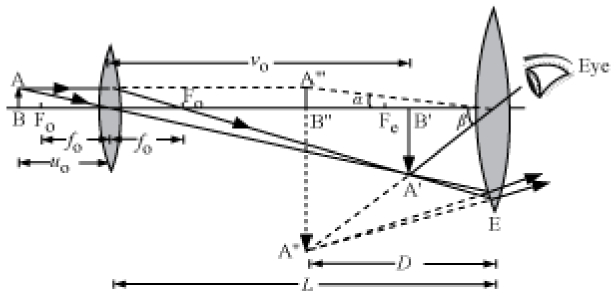
In this case, $\beta = \frac{AB}{f}$ and $\alpha = \frac{AB}{D}$

$$\therefore M = \frac{AB}{f} \times \frac{D}{AB} = \frac{D}{f}$$

$$\boxed{M = \frac{D}{f}}$$

Compound Microscope

A compound microscope consists of two convex lenses co-axially separated by some distance. The lens nearer to the object is called the objective. The lens through which the final image is viewed is called the eyepiece.



Angular Magnification or Magnifying Power of the Compound Microscope

Angular magnification or magnifying power of a compound microscope is defined as the ratio of the angle β subtended by the final image at the eye to the angle αsubtended by the object seen directly, when both are placed at the least distance of distinct vision.

∴Angular magnification, $M = \frac{\beta}{\alpha}$

Since the angles are small,

$\alpha \approx \tan \alpha$

$\beta \approx \tan \beta$

$$\therefore M = \frac{\tan \beta}{\tan \alpha}$$

$$\tan \beta = \frac{\mathrm{A''B''}}{D}$$

$$\tan \alpha = \frac{\mathrm{A''B''}}{D} = \frac{\mathrm{AB}}{D}$$

$$\therefore M = \frac{\mathrm{A''B''}}{D} \times \frac{D}{\mathrm{AB}} = \frac{\mathrm{A''B''}}{\mathrm{AB}}$$

$$M = \frac{A''B''}{A'B'} \times \frac{A'B'}{AB}$$

Thus, the magnification produced by the compound microscope is the product of the magnifications produced by the eyepiece and objective.

$$\therefore M = M_e \times M_0 \qquad ...(i)$$

Where, M_e and M_0 are the magnifying powers of the eyepiece and objective respectively

The linear magnification of the real inverted image produced by the eyepiece is $\frac{A''B''}{A'B'}$.

Linear magnification, $M_e = 1 + \frac{D}{f_e} \qquad ...(ii)$

Where,

$f_e \rightarrow$ Focal length of the eye piece

$\frac{A'B'}{AB}$ is the linear magnification of the object produced by the objective.

$$M_0 = \frac{v_0}{u_0} \qquad ...(iii)$$

From (i), (ii), and (iii),

$$M = \frac{v_0}{u_0}\left(1 + \frac{D}{f_e}\right) \qquad ...(iv)$$

We know that, $\frac{1}{v_0} - \frac{1}{u_0} = \frac{1}{f_0}$

$$\frac{v_0}{v_0} - \frac{v_0}{u_0} = \frac{v_0}{f_0}$$

$$\frac{-v_0}{u_0} = -1 + \frac{v_0}{f_0}$$

$$\frac{v_0}{u_0} = 1 - \frac{v_0}{f_0}$$

$$\therefore M = \left(1 - \frac{v_0}{f_0}\right)\left(1 + \frac{D}{f_e}\right)$$

Magnifying power, when final image is at infinity:

If u_0 is the distance of the object from the objective and v_0 is the distance of the image from the objective, then the magnifying power of the objective is

$$M_0 = \frac{v_0}{u_0}$$

When the final image is at infinity, $M_e = \frac{D}{f_e}$

Magnifying power of compound microscope,

$$M = M_0 \times M_e = \frac{v_0}{u_0} \times \frac{D}{f_e}$$

If the object is very close to the principal focus of the objective and the image formed by the objective is very close to the eyepiece, then

$$M = \frac{-L}{f_0} \cdot \frac{D}{f_e}$$

Where,

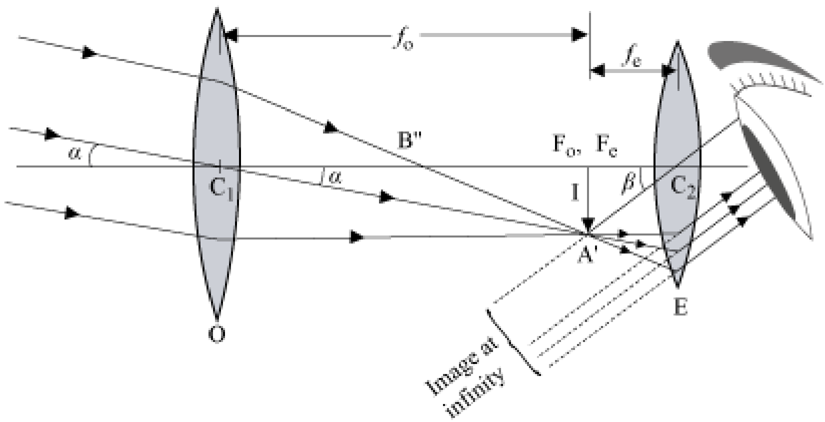
L = Length of the microscope

In this case, the microscope is said to be in normal adjustment.

Telescope

Astronomical Refracting Telescope

When the final image is formed at infinity



Angular magnification, $M = \frac{\beta}{\alpha}$

β and α are very small.

$\therefore \beta \approx \tan\beta$

$\alpha \approx \tan\alpha$

$$\Rightarrow M = \frac{\tan\beta}{\tan\alpha}$$

I is the image formed by the objective. f_0 and f_e are the focal lengths of the objective and eyepiece, respectively.

Here,

$$\tan\alpha = \frac{I}{f_0}$$

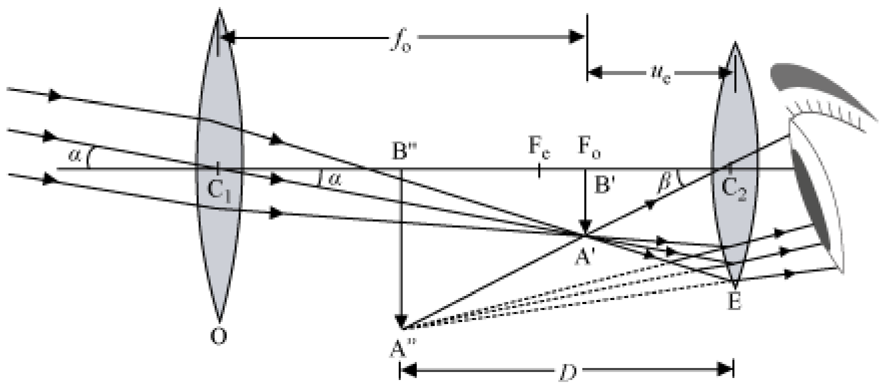
$$\tan\beta = \frac{I}{-f_e}$$

(Distance of the image from the eyepiece is taken as negative.)

$$\therefore M = \frac{\frac{-I}{f_e}}{\frac{I}{f_0}}$$

$$\boxed{M = \frac{-f_0}{f_e}}$$

When the final image is formed at the least distance of distinct vision



Magnifying power, $M = \frac{\beta}{\alpha}$

α and β are small.

$$\therefore M = \frac{\tan\beta}{\tan\alpha} \qquad ...(i)$$

In $\Delta A'B'C_2$,

$$\tan\beta = \frac{A'B'}{C_2B'}$$

In $\Delta A'B'C_1$,

$$\tan\alpha = \frac{A'B'}{C_1B'}$$

From equation (i), we have:

$$M = \frac{A'B'}{C_2B'} \times \frac{C_1B'}{A'B'}$$

$$M = \frac{C_1B'}{C_2B'}$$

Here,

$$C_1B' = +f_0$$

$$C_2B' = -u_e$$

$$M = \frac{f_0}{-u_e} \qquad \text{...(ii)}$$

Using the lens equation $\left(\frac{1}{v} - \frac{1}{u} = \frac{1}{f}\right)$ for the eyepieces, we get

$$\frac{1}{-D} - \frac{1}{-u_e} = \frac{1}{f_e}$$

$$-\frac{1}{D} + \frac{1}{u_e} = \frac{1}{f_e}$$

$$\Rightarrow \frac{1}{u_e} = \frac{1}{f_e} + \frac{1}{D}$$

$$\frac{f_0}{u_e} = \frac{f_0}{f_e}\left(1 + \frac{f_e}{D}\right)$$

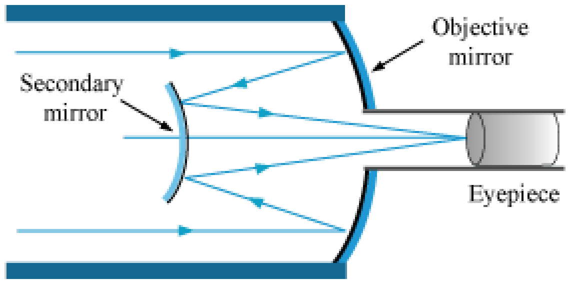
$$\frac{-f_0}{u_e} = \frac{-f_0}{f_e}\left(1 + \frac{f_e}{D}\right)$$

$$\boxed{M = -\frac{f_0}{f_e}\left(1 + \frac{f_e}{D}\right)}$$

Reflecting or Cassegrain Telescope

In the reflecting-type telescope, the objective lens is replaced by the concave parabolic mirror of a large aperture required for observing fainter objects. The use of parabolic mirror makes the resolving power of the telescope high. The parabolic mirrors are free from chromatic and spherical aberrations.

Schematic Diagram



Rays from distant objects enter the telescope and get reflected from the parabolic objective, as shown in the image. The reflected rays encounter the secondary convex mirror, which reflects them to the eyepiece.