

MATHEMATICS PAPER IIB - MAY 2009

COORDINATE GEOMETRY & CALCULUS

TIME: 3hrs

Max. Marks.75

Note: This question paper consists of three sections A,B and C.

SECTION A

Very Short Answer Type Questions.

$10 \times 2 = 20$

Note : Attempt all questions. Each question carries 2 marks.

1. If the center of the circle $x^2 + y^2 + ax + by - 12 = 0$ is (2,3) find the values of a, b and the radius of the circle.
2. Find the equation of the sphere that passes through the point (4, 3, -1) and having its centre (3,8,1).
3. Find the coordinates of the points on the parabola $y^2 = 2x$ whose focal distance is $5/2$.
4. Find the equations of the tangents to the hyperbola $3x^2 - 4y^2 = 12$ which is parallel to the line $y = x - 7$.
5. Find the n^{th} derivative of $f(x) = \log(8x^3 + 36x^2 + 54x + 27)$.
6. Evaluate $\int \sec^2 x \cos e^x dx$.
7. Evaluate $\int \frac{e^x (1+x)}{(2+x)^2} dx$.
8. Evaluate $\int_{-\pi/2}^{\pi/2} \sin^2 x \cos^4 x dx$.
9. Find the area enclosed by the curve $f(x) = \sin x$ in the interval $[0, 2\pi]$.
10. Form the differential equation corresponding to $y = cx - 2c^2$, where c is a parameter.

SECTION B

Short Answer Type Questions.

$5 \times 4 = 20$

Note: Answer any FIVE questions. Each question carries 4 marks.

11. Show that $x + y + 1 = 0$ touches the circle $x^2 + y^2 - 3x + 7y + 14 = 0$ and find the point of contact.
12. Prove that the poles of tangents to the parabola $y^2 = 4ax$ w.r.t the parabola $y^2 = 4bx$ lie on a parabola.
13. One focus of hyperbola located at (1, -3) and corresponding directrix in the line $y = 2$. Find the equation of hyperbola if its eccentricity is $3/2$.

14. If PSQ is chord passing through the focus S of a conic and 'l' is semi lotus rectum, show that

$$\frac{1}{SP} + \frac{1}{SQ} = \frac{2}{l}$$

15. Evaluate $\int \frac{1}{(1-x)(4+x^2)} dx$

16. Solve $(x^2 - y^2) dx - xy dy = 0$

17. Solve $(1 + y^2) dx = (\tan^{-1} y - x) dy$

SECTION C

Long Answer Type Questions.

5X7=35

Note: Answer any Five of the following. Each question carries 7 marks.

18. Find the equation of the circle whose centre lies on X-axis and passing through the points (-2,3), (4,5).

19. In the limiting points of the coaxial system determined by the circles $x^2 + y^2 + 2x - 6y = 0$ and $2x^2 + 2y^2 - 10y + 5 = 0$.

20. Find eccentricity, coordinates of foci and equations of directories of the ellipse

$$9x^2 + 16y^2 - 36x + 32y - 92 = 0$$

21. If $y = \frac{\sinh^{-1} x}{\sqrt{1+x^2}}$, then show that $(1+x^2)y_2 + 3xy_1 + y = 0$ and hence deduce that

$$(1+x^2)y_{n+2} + (2n+3)xy_{n+1} + (n+1)^2 y_n = 0$$

22. Obtain the reduction formula for $I_n = \int \sin^n x dx$, n being a positive integer, $n \geq 2$ and deduce the value of $\int \sin^4 x dx$

23. Show that $\int_0^{\pi/2} \frac{x}{\sin x + \cos x} dx = \frac{\pi}{2\sqrt{2}} \log(\sqrt{2} + 1)$

24. Dividing [0,6] into 6 equal parts evaluate $\int_0^6 x^3 dx$ approximately by using Trapezoidal rule and Simpson's rule.