EXERCISE 13.1 / PROBABILITY

Q No 1: Given that E and F are events such that $P(E) = 0.6$, $P(F) = 0.3$ and $P(E \cap F) = 0.2$. Find $P(E|F)$ and $P(F|E)$

Soln:

$$P(E|F) = \frac{P(E \cap F)}{P(F)} = \frac{0.2}{0.3} = \frac{2}{3}$$

$$P(F|E) = \frac{P(E \cap F)}{P(E)} = \frac{0.2}{0.6} = \frac{2}{6} = \frac{1}{3}$$

Q No. 2. Compute $P(A|B)$ if $P(B) = 0.5$ and $P(A \cap B) = 0.32$

Sol.

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{0.32}{0.5} = \frac{32}{50} = \frac{16}{25}$$

Q No 3 If $P(A) = 0.8$, $P(B) = 0.5$ and $P(B|A) = 0.4$, find

(i) $P(A \cap B)$ (ii) $P(A|B)$ (iii) $P(A \cup B)$

Sol.

(i) $$P(B|A) = \frac{P(A \cap B)}{P(A)} \Rightarrow 0.4 = \frac{P(A \cap B)}{0.8}$$

$$\Rightarrow P(A \cap B) = 0.4 \times 0.8 = 0.32.$$

(ii) $$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{0.32}{0.5} = \frac{32}{50} = \frac{16}{25}$$

(iii) $$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$= 0.8 + 0.5 - 0.32$$

$$= 1.3 - 0.32 = 0.98.$$

Q No 4 Evaluate $P(A \cup B)$ if $2P(A) = P(B) = \frac{5}{13}$ and $P(A|B) = \frac{2}{5}$

Sol.

$$2(P(A)) = P(B) = \frac{5}{13} \Rightarrow P(A) = \frac{5}{26}$$

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

$$\Rightarrow \frac{2}{5} = \frac{P(A \cap B)}{5/26} \Rightarrow P(A \cap B) = \frac{2}{5} \times \frac{5}{26} = \frac{2}{13}$$

$$\therefore P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$= \frac{5}{26} + \frac{5}{13} - \frac{1}{13} = \frac{5+10-4}{26} = \frac{11}{26}$$

Q.No5. If $P(A) = \frac{6}{11}$ $P(B) = \frac{5}{11}$ and $P(A \cup B) = \frac{7}{11}$, find

(i) $P(A \cap B)$ (ii) $P(A|B)$ (iii) $P(B|A)$

Sol.

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

(i) $\Rightarrow$ $P(A \cap B) = P(A) + P(B) - P(A \cup B)$

$$= \frac{6}{11} + \frac{5}{11} - \frac{7}{11} = \frac{4}{11}$$

(ii) $$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{4/11}{5/11} = 4/5$$

(iii) $$P(B|A) = \frac{P(A \cap B)}{P(A)} = \frac{4/11}{6/11} = 4/6 = 2/3.$$

Determine $P(E|F)$ in Question No 6 to Q.No.9.

Q.No 6: A coin is tossed 3 times, where.

(i) E: Head on Third toss, F: heads on first two tosses

(ii) E: At least two heads, F: At most 2 heads.

(iii) E: A most 2 tails, F: A least one tail.

Sol. When a coin is tossed three times, Sample Space

$$S = \{HHH, HHT, HTH, THH, HTT, THT, TTH, TTT\}$$

(i) Here E = head on third toss = {HHH, HTH, THH, TTH}

$$\therefore P(E) = \frac{4}{8} = \frac{1}{2}$$

F = Heads on first two tosses = {HHH, HHT}

$$\therefore P(F) = \frac{2}{8} = \frac{1}{4}$$

$E \cap F = \{HHH\}$ $\therefore$ $P(E \cap H) = \frac{1}{8}$.

$$\therefore P(E|F) = \frac{P(E \cap F)}{P(F)} = \frac{\frac{1}{8}}{\frac{2}{8}} = \frac{1}{2}$$

(ii) E: at least two heads = {HHH, HHT, HTH, THH}

F: At most two heads = {HHT, HTH, THH, THT, HTT, TTH, TTT}

$E \cap F$ = {HHT, HTH, THH}

$$\therefore \quad P(E|F) = \frac{P(E\cap F)}{P(F)} = \frac{3/8}{7/8} = \frac{3}{7}$$

(iii) Here E : At most 2 tails = {HHH, HHT, HTH, THH, HTT, THT, TTH}

F : At least one tail = {HHT, HTH, THH, HTT, THT, TTH, TTT}

$E\cap F$ = {HHT, HTH, HTT, THH, THT, TTH}

Here $P(E) = \frac{7}{8}$, $P(F) = \frac{7}{8}$, $P(E\cap F) = \frac{6}{8}$.

$$\therefore \quad P(E|F) = \frac{P(E\cap F)}{P(F)} = \frac{6/8}{7/8} = 6/7.$$

QNo. 7 Two coins are tosses once, where

(i) E : tail appears on one coin F: one coin shows head.

(ii) E : No tail appears, F: No Head appears.

Sol. When two coins are tossed, then sample space

S = {HH, HT, TH, TT}

(i) E = tail appears on one coin = {HT, TH}

F = One coin shows head = {TH, HT}

$\therefore$ $E\cap F$ = {HT, TH} = F.

$\therefore$ $P(E) = \frac{2}{4} = \frac{1}{2}$, $P(F) = \frac{2}{4} = \frac{1}{2}$.

Now But $E\cap F = F \Rightarrow P(E\cap F) = P(F)$

$$\therefore \quad P(E|F) = \frac{P(E\cap F)}{P(F)} = \frac{P(F)}{P(F)} = 1.$$

(ii) E : No tail appears = {HH}

F = No Head appears = {TT}

$E\cap F = \phi$

$\therefore$ $P(E\cap F) = P(\phi) = 0$ and $P(F) = \frac{1}{4}$.

$$\therefore \quad P(E|F) = \frac{P(E\cap F)}{P(F)} = \frac{0}{1/4} = 0$$

Q No. 8. A die is thrown three times.
E : 4 appears on third toss. F : 6 and 5 appears resp. on first two tosses.

Sol. When a die is thrown thrice, the sample space contains $6 \times 6 \times 6 = 216$ sample points and

$$S = \{(x, y, z) \text{ where } x, y, z \in \{1, 2, 3, 4, 5, 6\}\}$$

Now E : 4 appears on third toss $= \{(x, y, 4);\ x, y \in \{1,2,3,4,5,6\}\}$

F : 6 and 5 appears resp. on first two tosses

$= \{(6, 5, z);\ z \in \{1, 2, 3, 4, 5, 6\}\}$.

Clearly E contains $6 \times 6 \times 1 = 36$ sample points
F contains $1 \times 1 \times 6 = 6$ sample points

and $E \cap F = \{(6, 5, 4)\}$

$\therefore\ P(E) = 36/216 \quad P(F) = 6/216 \quad P(E \cap F) = 1/216$.

$$\therefore\ P(E|F) = \frac{P(E \cap F)}{P(F)} = \frac{1/216}{6/216} = \frac{1}{6}$$

Q No. 9 Mother, Father and Sone line up for a family picture. E : Son on one end, F : Father in middle

Sol. Let M, F and S denote mother, father and son resp. then sample space $S = \{MFS, MSF, FMS, FSM, SMF, SFM\}$

E : Son at one end $= \{MFS, FMS, SMF, SFM\}$

F : Father in middle $= \{MFS, SFM\}$

$\therefore\ E \cap F = \{MFS, SFM\} = F$.

$$\therefore\ P(E|F) = \frac{P(E \cap F)}{P(F)} = \frac{P(F)}{P(F)} = 1.$$

Q No 10 : A black and a red dice are rolled.

(a) Find the conditional probability of obtaining a sum greater than 9, given that the black die result in 5.

(b) Find the conditional probability of obtaining sum 8 given that the red die resulted in number less than 4.

Sol: Let x denote the outcome on black die and y denotes the outcome on Red die., Clearly sample space has $6 \times 6 = 36$ sample space where $S = \{(x,y);\ x,y \in \{1,2,3,4,5,6\}\}$

(a) E = Sum greater than $9 = \{(4,6)(6,4)(5,5)(5,6)(6,5)(6,6)\}$

F = Black die Resulted in $5 = \{(5,1)(5,2)(5,3)(5,4)(5,5)(5,6)\}$

$$E \cap F = \{(5,5),(5,6)\}$$

Required probability $= P(E|F) = \frac{P(E\cap F)}{P(F)} = \frac{2/36}{6/36} = \frac{2}{6} = \frac{1}{3}$

(b) E: A total of $8 = \{(2,6)(6,2)(3,5)(5,3)(4,4)\}$

F: Red die resulted in number than less than 4.

$= \{(x,y):\ x \in \{1,2,3,4,5,6\};\ y \in \{1,2,3\}\}$

Clearly F contains $6 \times 3 = 18$ sample points

Also $E \cap F = \{(6,2),(5,3)\}$

$\therefore$ Required probability $= P(E|F) = \frac{P(E\cap F)}{P(F)} = \frac{2/36}{18/36} = \frac{2}{18} = \frac{1}{9}$

Q No 11 A fair die is rolled. Consider event $E = \{1,3,5\}$, $F = \{2,3\}$ $G = \{2,3,4,5\}$ find.

(i) $P(E|F)$ and $P(F|E)$ (ii) $P(E|G)$ and $P(G|E)$ (iii) $P(E\cup F|G)$ and $P(E\cap F|G)$.

Sol. A fair die is rolled $\therefore$ Sample space $S = \{1,2,3,4,5,6\}$

$E = \{1,3,5\}$ $F = \{2,3\}$, $G = \{2,3,4,5\}$

$E \cap F = F \cap E = \{3\}$

$E \cap G = G \cap E = \{3,5\}$ $E \cup F = \{1,2,3,5\}$

$(E \cup F) \cap G = \{2,3,5\}$ $(E \cap F) \cap G = \{3\}$

(i) $P(E|F) = \frac{P(E\cap F)}{P(F)} = \frac{1/6}{2/6} = \frac{1}{2}$ and $P(F|E) = \frac{P(F\cap E)}{P(E)} = \frac{1/6}{3/6} = \frac{1}{3}$

(ii) $P(E|G) = \frac{P(E\cap G)}{P(G)} = \frac{2/6}{4/6} = \frac{1}{2}$ $P(G|E) = \frac{P(E\cap G)}{P(E)} = \frac{2/6}{3/6} = \frac{2}{3}$

(iii) $P(E\cup F|G) = \frac{P((E\cup F)\cap G)}{P(G)} = \frac{3/6}{4/6} = \frac{3}{4}$

$P(E\cap F|G) = \frac{P((E\cap F)\cap G)}{P(G)} = \frac{1/6}{4/6} = 1/4.$

Q.No. 12 Assume that each born child is equally likely to be a boy or a girl. If a family had two Children, what is Conditional probability that both are girls given that (i) The youngest is a girl (ii) A least one is girl.

Sol. Let the boy be denoted by 'B' and younger boy by 'b', Elder girl by 'G' and younger girl by 'g'.

Then the sample space will be $S = \{Bb, Bg, Gb, Gg\}$ Which are equally likely sample points.

Let E: Both children are girls, then $E = \{Gg\}$

(i) Let F: The youngest is Girl, $\therefore F = \{Bg, Gg\}$

$$\therefore E \cap F = \{Gg\} = E$$

Now $P(E) = \frac{1}{4}$, $P(F) = \frac{2}{4}$

$$\therefore P(E|F) = \frac{P(E \cap F)}{P(F)} = \frac{P(E)}{P(F)} = \frac{1/4}{2/4} = \frac{1}{2}$$

(ii) Let F': At least one girl. Then $F' = \{Bg, Gb, Gg\}$

$$\therefore E \cap F' = \{Gg\} = E$$

and $P(F') = 3/4$

$$\therefore P(E|F') = \frac{P(E \cap F')}{P(F')} = \frac{P(E)}{P(F')} = \frac{1/4}{3/4} = \frac{1}{3}$$

Q.No. 13. An instructor has a question bank consisting of 300 easy True/False questions, 200 difficult True/False questions. 500 easy MCQ and 400 difficult MCQ. If a question is selected at random, what is the probability that it will be an easy question given that it is a MCQ.

Sol. Let E: Easy question.

F: MCQ.

Then $E \cap F$: Easy MCQ

Now Total No. of questions $= 300 + 200 + 500 + 400 = 1400$

$$\therefore P(E \cap F) = \frac{500}{1400} = \frac{5}{14}, \quad P(F) = \frac{500 + 400}{1400} = \frac{900}{1400} = \frac{9}{14}$$

Hence Required Probability $= P(E|F) = \frac{P(E\cap F)}{P(F)} = \frac{5/14}{9/14} = \frac{5}{9}$.

QNo. 14 Given that the two numbers appearing on throwing two dice are different. Find the probability of the event "The Sum of numbers on dice is 4"

Sol. Sample Space of Throwing two dice Simultaneously.

$S = \{(x, y);\ x, y \in \{1,2,3,4,5,6\}\}$.

Which contains $6\times 6 = 36$ equally likely Sample points.

Let E : The Sum of numbers on dice is 4

Then $E = \{(1,3)\ (2,2)\ (3,1)\}$

F: Two numbers on both dice are different.

$F = \{(x,y);\ x \neq y \text{ and } x, y \in \{1,2,3,4,5,6\}\}$

Then clearly F has $36 - 6 = 30$ sample points.

Also $E\cap F = \{(1,3)\ (3,1)\}$ $\quad \therefore P(E\cap F) = \frac{2}{36}$.

$\therefore$ Required Probability $= P(E|F) = \frac{P(E\cap F)}{P(F)} = \frac{2/36}{30/36} = \frac{2}{30} = \frac{1}{15}$

QNo15 Consider the experiment of throwing of a die. If a multiple of 3 comes up, throw the die again and if any other No. comes, toss a coin. Find the Conditional probability of the event 'The coin shows a tale' given that at least one die shows a 3.

Sol. Sample Space of Random Experiment is

$$S = \left\{\begin{matrix} (3,1) & (3,2) & (3,3) & (3,4) & (3,5) & (3,6) \\ (6,1) & (6,2) & (6,3) & (6,4) & (6,5) & (6,6) \\ (1,H) & (1,T) & (2,H) & (2,T) & (4,H) & (4,T) \\ (5,H) & (5,T) & & & & \end{matrix}\right\}$$

Now clearly out comes are Not equally likely.

firsts 12 outcomes are equally likely and are such that

Sum of their probabilities is $\frac{2}{6} = \frac{1}{3}$.

$\therefore$ First 12 outcomes have probabilities $= \frac{1}{12} \times \frac{1}{3} = \frac{1}{36}$

Remaining 8 outcomes are equally likely and are such that the sum of their probabilities $= \frac{4}{6} = \frac{2}{3}$.

$\therefore$ 8 outcomes have probabilities $= \frac{1}{8} \times \frac{2}{3} = \frac{2}{24} = \frac{1}{12}$

Now Let E: The coin shows a tail.

F: At least one die shows up a 3.

$\therefore E = \{(1,T)\ (2,T)\ (4,T)\ (5,T)\}$

and $F = \{(3,1)\ (3,2)\ (3,3)\ (3,4)\ (3,5)\ (3,6)\ (6,3)\}$

$E \cap F = \phi$

$\therefore P(E \cap F) = 0$

$$\therefore P(E|F) = \frac{P(E \cap F)}{P(F)} = \frac{0}{P(F)} = 0$$

Choose the correct answer.

Q No 16. If $P(A) = \frac{1}{2}$ $P(B) = 0$ then $P(A|B)$ is

(A) 0 (B) ½ (C) Non defined (D) 1.

Sol: (C) is correct option as $P(A|B) = \frac{P(A \cap B)}{P(B)}$,

which is not defined as $P(B) = 0$ and $B = \phi$.

Q No 17. If A and B are events such that $P(A|B) = P(B|A)$ then

(A) $A \subset B$ by $A \neq B$ (B) $A = B$ (C) $A \cap B = \phi$ (D) $P(A) = P(B)$

Sol. Since $P(A|B) = P(B|A)$

$$\Rightarrow \frac{P(A \cap B)}{P(B)} = \frac{P(B \cap A)}{P(A)}$$

$$\Rightarrow P(A) = P(B)$$

$\therefore$ (D) is correct option.

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