We note that $1 = 1 \times 1 \times 1 = 1^3$; $8 = 2 \times 2 \times 2 = 2^3$; $27 = 3 \times 3 \times 3 = 3^3$.
Since $5^3 = 5 \times 5 \times 5 = 125$, therefore 125 is a cube number.

Is 9 a cube number? No, as $9 = 3 \times 3$ and there is no natural number which multiplied by itself three times gives 9. We can see also that $2 \times 2 \times 2 = 8$ and $3 \times 3 \times 3 = 27$. This shows that 9 is not a perfect cube.

The following are the cubes of numbers from 1 to 10.

Table 1

The numbers 729, 1000, 1728 are also perfect cubes.

Number	Cube
1	$1^3 = 1$
2	$2^3 = 8$
3	$3^3 = 27$
4	$4^3 = 64$
5	$5^3 =$ _____
6	$6^3 =$ _____
7	$7^3 =$ _____
8	$8^3 =$ _____
9	$9^3 =$ _____
10	$10^3 =$ _____

Complete it.

There are only ten perfect cubes from 1 to 1000. (Check this). How many perfect cubes are there from 1 to 100?

Observe the cubes of even numbers. Are they all even? What can you say about the cubes of odd numbers?

Following are the cubes of the numbers from 11 to 20.

Table 2

We are even, so are our cubes

Number	Cube
11	1331
12	1728
13	2197
14	2744
15	3375
16	4096
17	4913
18	5832
19	6859
20	8000

We are odd so are our cubes

Consider a few numbers having 1 as the one's digit (or unit's). Find the cube of each of them. What can you say about the one's digit of the cube of a number having 1 as the one's digit?

Similarly, explore the one's digit of cubes of numbers ending in 2, 3, 4, ... , etc.

TRY THESE

Find the one's digit of the cube of each of the following numbers.

(i) 3331	(ii) 8888	(iii) 149	(iv) 1005
(v) 1024	(vi) 77	(vii) 5022	(viii) 53

7.2.1 Some interesting patterns

1. Adding consecutive odd numbers

Observe the following pattern of sums of odd numbers.

$$\begin{aligned}
1 &= 1 = 1^3\\
3 + 5 &= 8 = 2^3\\
7 + 9 + 11 &= 27 = 3^3\\
13 + 15 + 17 + 19 &= 64 = 4^3\\
21 + 23 + 25 + 27 + 29 &= 125 = 5^3
\end{aligned}$$

Is it not interesting? How many consecutive odd numbers will be needed to obtain the sum as 10^3?

TRY THESE

Express the following numbers as the sum of odd numbers using the above pattern?

(a) 6^3 (b) 8^3 (c) 7^3

Consider the following pattern.

$$2^3 - 1^3 = 1 + 2 \times 1 \times 3$$
$$3^3 - 2^3 = 1 + 3 \times 2 \times 3$$
$$4^3 - 3^3 = 1 + 4 \times 3 \times 3$$

Using the above pattern, find the value of the following.

(i) $7^3 - 6^3$ (ii) $12^3 - 11^3$ (iii) $20^3 - 19^3$ (iv) $51^3 - 50^3$

2. Cubes and their prime factors

Consider the following prime factorisation of the numbers and their cubes.

each prime factor appears three times in its cubes

Prime factorisation of a number	Prime factorisation of its cube
$4 = 2 \times 2$	$4^3 = 64 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 = 2^3 \times 2^3$
$6 = 2 \times 3$	$6^3 = 216 = 2 \times 2 \times 2 \times 3 \times 3 \times 3 = 2^3 \times 3^3$
$15 = 3 \times 5$	$15^3 = 3375 = 3 \times 3 \times 3 \times 5 \times 5 \times 5 = 3^3 \times 5^3$
$12 = 2 \times 2 \times 3$	$12^3 = 1728 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 3$ $= 2^3 \times 2^3 \times 3^3$

2	216
2	108
2	54
3	27
3	9
3	3
	1

Observe that each prime factor of a number appears three times in the prime factorisation of its cube.

Do you remember that $a^m \times b^m = (a \times b)^m$

In the prime factorisation of any number, if each factor appears three times, then, is the number a perfect cube? Think about it. Is 216 a perfect cube?

By prime factorisation, $216 = 2 \times 2 \times 2 \times 3 \times 3 \times 3$

Each factor appears 3 times. $216 = 2^3 \times 3^3 = (2 \times 3)^3$

$= 6^3$ which is a perfect cube!

Is 729 a perfect cube? $729 = \underline{3 \times 3 \times 3} \times \underline{3 \times 3 \times 3}$

factors can be grouped in triples

Yes, 729 is a perfect cube.

Now let us check for 500.

Prime factorisation of 500 is $2 \times 2 \times \underline{5 \times 5 \times 5}$.

So, 500 is not a perfect cube.

There are three 5's in the product but only two 2's.

Example 1: Is 243 a perfect cube?

Solution: $243 = \underline{3 \times 3 \times 3} \times 3 \times 3$

In the above factorisation 3×3 remains after grouping the 3's in triplets. Therefore, 243 is not a perfect cube.

TRY THESE

Which of the following are perfect cubes?

1. 400	2. 3375	3. 8000	4. 15625
5. 9000	6. 6859	7. 2025	8. 10648

7.2.2 Smallest multiple that is a perfect cube

Raj made a cuboid of plasticine. Length, breadth and height of the cuboid are 15 cm, 30 cm, 15 cm respectively.

Anu asks how many such cuboids will she need to make a perfect cube? Can you tell?

Raj said, Volume of cuboid is $15 \times 30 \times 15 = 3 \times 5 \times 2 \times 3 \times 5 \times 3 \times 5$

$= 2 \times \underline{3 \times 3 \times 3} \times \underline{5 \times 5 \times 5}$

Since there is only one 2 in the prime factorisation. So we need 2×2, i.e., 4 to make it a perfect cube. Therefore, we need 4 such cuboids to make a cube.

Example 2: Is 392 a perfect cube? If not, find the smallest natural number by which 392 must be multiplied so that the product is a perfect cube.

Solution: $392 = \underline{2 \times 2 \times 2} \times 7 \times 7$

The prime factor 7 does not appear in a group of three. Therefore, 392 is not a perfect cube. To make its a cube, we need one more 7. In that case

$392 \times 7 = \underline{2 \times 2 \times 2} \times \underline{7 \times 7 \times 7} = 2744$ which is a perfect cube.

Hence the smallest natural number by which 392 should be multiplied to make a perfect cube is 7.

Example 3: Is 53240 a perfect cube? If not, then by which smallest natural number should 53240 be divided so that the quotient is a perfect cube?

Solution: $53240 = \underline{2 \times 2 \times 2} \times \underline{11 \times 11 \times 11} \times 5$

The prime factor 5 does not appear in a group of three. So, 53240 is not a perfect cube. In the factorisation 5 appears only one time. If we divide the number by 5, then the prime factorisation of the quotient will not contain 5.

So, $53240 \div 5 = \underline{2 \times 2 \times 2} \times \underline{11 \times 11 \times 11}$

Hence the smallest number by which 53240 should be divided to make it a perfect cube is 5.

The perfect cube in that case is = 10648.

Example 4: Is 1188 a perfect cube? If not, by which smallest natural number should 1188 be divided so that the quotient is a perfect cube?

Solution: $1188 = 2 \times 2 \times \underline{3 \times 3 \times 3} \times 11$

The primes 2 and 11 do not appear in groups of three. So, 1188 is not a perfect cube. In the factorisation of 1188 the prime 2 appears only two times and the prime 11 appears once. So, if we divide 1188 by $2 \times 2 \times 11 = 44$, then the prime factorisation of the quotient will not contain 2 and 11.

Hence the smallest natural number by which 1188 should be divided to make it a perfect cube is 44.

And the resulting perfect cube is $1188 \div 44 = 27$ $(=3^3)$.

Example 5: Is 68600 a perfect cube? If not, find the smallest number by which 68600 must be multiplied to get a perfect cube.

Solution: We have, $68600 = 2 \times 2 \times 2 \times 5 \times 5 \times 7 \times 7 \times 7$. In this factorisation, we find that there is no triplet of 5.

So, 68600 is not a perfect cube. To make it a perfect cube we multiply it by 5.

Thus, $68600 \times 5 = 2 \times 2 \times 2 \times 5 \times 5 \times 5 \times 7 \times 7 \times 7$

$= 343000$, which is a perfect cube.

Observe that 343 is a perfect cube. From Example 5 we know that 343000 is also perfect cube.

THINK, DISCUSS AND WRITE

Check which of the following are perfect cubes. (i) 2700 (ii) 16000 (iii) 64000 (iv) 900 (v) 125000 (vi) 36000 (vii) 21600 (viii) 10,000 (ix) 27000000 (x) 1000. What pattern do you observe in these perfect cubes?

EXERCISE 7.1

1. Which of the following numbers are not perfect cubes?
   (i) 216 (ii) 128 (iii) 1000 (iv) 100
   (v) 46656
2. Find the smallest number by which each of the following numbers must be multiplied to obtain a perfect cube.
   (i) 243 (ii) 256 (iii) 72 (iv) 675
   (v) 100
3. Find the smallest number by which each of the following numbers must be divided to obtain a perfect cube.
   (i) 81 (ii) 128 (iii) 135 (iv) 192
   (v) 704
4. Parikshit makes a cuboid of plasticine of sides 5 cm, 2 cm, 5 cm. How many such cuboids will he need to form a cube?

7.3 Cube Roots

If the volume of a cube is 125 cm^3, what would be the length of its side? To get the length of the side of the cube, we need to know a number whose cube is 125.

Finding the square root, as you know, is the inverse operation of squaring. Similarly, finding the cube root is the inverse operation of finding cube.

We know that $2^3 = 8$; so we say that the cube root of 8 is 2.

We write $\sqrt[3]{8} = 2$. **The symbol $\sqrt[3]{\ }$ denotes 'cube-root.'**

Consider the following:

Statement	Inference
$1^3 = 1$	$\sqrt[3]{1} = 1$
$2^3 = 8$	$\sqrt[3]{8} = \sqrt[3]{2^3} = 2$
$3^3 = 27$	$\sqrt[3]{27} = \sqrt[3]{3^3} = 3$
$4^3 = 64$	$\sqrt[3]{64} = 4$
$5^3 = 125$	$\sqrt[3]{125} = 5$

Statement	Inference
$6^3 = 216$	$\sqrt[3]{216} = 6$
$7^3 = 343$	$\sqrt[3]{343} = 7$
$8^3 = 512$	$\sqrt[3]{512} = 8$
$9^3 = 729$	$\sqrt[3]{729} = 9$
$10^3 = 1000$	$\sqrt[3]{1000} = 10$

7.3.1 Cube root through prime factorisation method

Consider 3375. We find its cube root by prime factorisation:

$$3375 = \underline{3 \times 3 \times 3} \times \underline{5 \times 5 \times 5} = 3^3 \times 5^3 = (3 \times 5)^3$$

Therefore, cube root of 3375 = $\sqrt[3]{3375} = 3 \times 5 = 15$

Similarly, to find $\sqrt[3]{74088}$, we have,

$$74088 = \underline{2 \times 2 \times 2} \times \underline{3 \times 3 \times 3} \times \underline{7 \times 7 \times 7} = 2^3 \times 3^3 \times 7^3 = (2 \times 3 \times 7)^3$$

Therefore, $\sqrt[3]{74088} = 2 \times 3 \times 7 = 42$

Example 6: Find the cube root of 8000.

Solution: Prime factorisation of 8000 is $\underline{2 \times 2 \times 2} \times \underline{2 \times 2 \times 2} \times \underline{5 \times 5 \times 5}$

So, $\sqrt[3]{8000} = 2 \times 2 \times 5 = 20$

Example 7: Find the cube root of 13824 by prime factorisation method.

Solution:

$$13824 = \underline{2 \times 2 \times 2} \times \underline{2 \times 2 \times 2} \times \underline{2 \times 2 \times 2} \times \underline{3 \times 3 \times 3} = 2^3 \times 2^3 \times 2^3 \times 3^3.$$

Therefore, $\sqrt[3]{13824} = 2 \times 2 \times 2 \times 3 = 24$

THINK, DISCUSS AND WRITE

State true or false: for any integer m, $m^2 < m^3$. Why?

7.3.2 Cube root of a cube number

If you know that the given number is a cube number then following method can be used.

Step 1 Take any cube number say 857375 and start making groups of three digits starting from the right most digit of the number.

$$\underset{\text{second group}}{\underset{\downarrow}{\underline{857}}} \qquad \underset{\text{first group}}{\underset{\downarrow}{\underline{375}}}$$

We can estimate the cube root of a given cube number through a step by step process.

We get 375 and 857 as two groups of three digits each.

Step 2 First group, i.e., 375 will give you the one's (or unit's) digit of the required cube root.

The number 375 ends with 5. We know that 5 comes at the unit's place of a number only when it's cube root ends in 5.

So, we get 5 at the unit's place of the cube root.

Step 3 Now take another group, i.e., 857.

We know that $9^3 = 729$ and $10^3 = 1000$. Also, $729 < 857 < 1000$. We take the one's place, of the smaller number 729 as the ten's place of the required cube root. So, we get $\sqrt[3]{857375} = 95$.

Example 8: Find the cube root of 17576 through estimation.

Solution: The given number is 17576.

Step 1 Form groups of three starting from the rightmost digit of 17576.

17 576. In this case one group i.e., 576 has three digits whereas 17 has only two digits.

Step 2 Take 576.

The digit 6 is at its one's place.

We take the one's place of the required cube root as 6.

Step 3 Take the other group, i.e., 17.

Cube of 2 is 8 and cube of 3 is 27. 17 lies between 8 and 27.

The smaller number among 2 and 3 is 2.

The one's place of 2 is 2 itself. Take 2 as ten's place of the cube root of 17576.

Thus, $\sqrt[3]{17576} = 26$ (Check it!)

EXERCISE 7.2

1. Find the cube root of each of the following numbers by prime factorisation method.

(i) 64 (ii) 512 (iii) 10648 (iv) 27000

(v) 15625 (vi) 13824 (vii) 110592 (viii) 46656

(ix) 175616 (x) 91125

2. State true or false.

(i) Cube of any odd number is even.

(ii) A perfect cube does not end with two zeros.

(iii) If square of a number ends with 5, then its cube ends with 25.

(iv) There is no perfect cube which ends with 8.

(v) The cube of a two digit number may be a three digit number.

(vi) The cube of a two digit number may have seven or more digits.

(vii) The cube of a single digit number may be a single digit number.

3. You are told that 1,331 is a perfect cube. Can you guess without factorisation what is its cube root? Similarly, guess the cube roots of 4913, 12167, 32768.

WHAT HAVE WE DISCUSSED?

1. Numbers like 1729, 4104, 13832, are known as Hardy – Ramanujan Numbers. They can be expressed as sum of two cubes in two different ways.

2. Numbers obtained when a number is multiplied by itself three times are known as **cube numbers**. For example 1, 8, 27, ... etc.

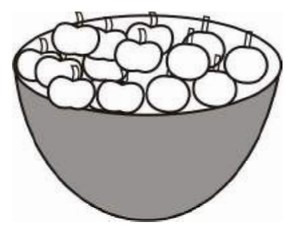
3. If in the prime factorisation of any number each factor appears three times, then the number is a perfect cube.

4. The symbol $\sqrt[3]{\ }$ denotes cube root. For example $\sqrt[3]{27} = 3$.

Comparing Quantities

CHAPTER 8

8.1 Recalling Ratios and Percentages



We know, ratio means comparing two quantities.
A basket has two types of fruits, say, 20 apples and 5 oranges.
Then, the ratio of the number of oranges to the number of apples = 5 : 20.

The comparison can be done by using fractions as, $\frac{5}{20} = \frac{1}{4}$

The number of oranges are $\frac{1}{4}$th the number of apples. In terms of ratio, this is 1 : 4, read as, "1 is to 4"

OR

Number of apples to number of oranges = $\frac{20}{5} = \frac{4}{1}$ which means, the number of apples are 4 times the number of oranges. This comparison can also be done using percentages.

There are 5 oranges out of 25 fruits. So percentage of oranges is $\frac{5}{25} \times \frac{4}{4} = \frac{20}{100} = 20\%$ [Denominator made 100].	OR	By unitary method: Out of 25 fruits, number of oranges are 5. So out of 100 fruits, number of oranges $= \frac{5}{25} \times 100 = 20$.

Since  contains only apples and oranges,

So, percentage of apples + percentage of oranges = 100

or percentage of apples + 20 = 100

or percentage of apples = 100 – 20 = 80

Thus the basket has 20% oranges and 80% apples.

Example 1: A picnic is being planned in a school for Class VII. Girls are 60% of the total number of students and are 18 in number.

The picnic site is 55 km from the school and the transport company is charging at the rate of ₹ 12 per km. The total cost of refreshments will be ₹ 4280.

Can you tell.

1. The ratio of the number of girls to the number of boys in the class?
2. The cost per head if two teachers are also going with the class?
3. If their first stop is at a place 22 km from the school, what per cent of the total distance of 55 km is this? What per cent of the distance is left to be covered?

Solution:

1. To find the ratio of girls to boys.

Ashima and John came up with the following answers.

They needed to know the number of boys and also the total number of students.

Ashima did this

Let the total number of students be x. 60% of x is girls.

Therefore, 60% of x = 18

$$\frac{60}{100} \times x = 18$$

or, $x = \frac{18 \times 100}{60} = 30$

Number of students = 30.

OR

John used the unitary method

There are 60 girls out of 100 students.

There is one girl out of $\frac{100}{60}$ students.

So, 18 girls are out of how many students?

Number of students $= \frac{100}{60} \times 18$

$= 30$

So, the number of boys = 30 – 18 = 12.

Hence, ratio of the number of girls to the number of boys is 18 : 12 or $\frac{18}{12} = \frac{3}{2}$.

$\frac{3}{2}$ is written as 3 : 2 and read as 3 is to 2.

2. To find the cost per person.

Transportation charge = Distance both ways × Rate

= ₹ (55 × 2) × 12

= ₹ 110 × 12 = ₹ 1320

Total expenses = Refreshment charge + Transportation charge

= ₹ 4280 + ₹ 1320

= ₹ 5600

Total number of persons = 18 girls + 12 boys + 2 teachers

= 32 persons

Ashima and John then used unitary method to find the cost per head.

For 32 persons, amount spent would be ₹ 5600.

The amount spent for 1 person = ₹ $\frac{5600}{32}$ = ₹ 175.

3. The distance of the place where first stop was made = 22 km.

To find the percentage of distance:

Ashima used this method:

$$\frac{22}{55} = \frac{22}{55} \times \frac{100}{100} = 40\%$$

$\left[\text{She is multiplying the ratio by } \frac{100}{100} = 1 \text{ and converting to percentage.}\right]$

OR

John used the unitary method:

Out of 55 km, 22 km are travelled.

Out of 1 km, $\frac{22}{55}$ km are travelled.

Out of 100 km, $\frac{22}{55} \times 100$ km are travelled.

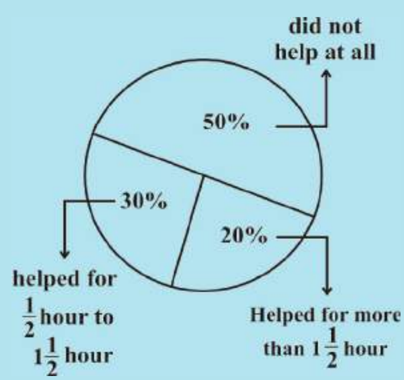
That is 40% of the total distance is travelled.

Both came out with the same answer that the distance from their school of the place where they stopped at was 40% of the total distance they had to travel.
Therefore, the percent distance left to be travelled = 100% – 40% = 60%.

TRY THESE

In a primary school, the parents were asked about the number of hours they spend per day in helping their children to do homework. There were 90 parents who helped for $\frac{1}{2}$ hour to $1\frac{1}{2}$ hours. The distribution of parents according to the time for which, they said they helped is given in the adjoining figure ; 20% helped for more than $1\frac{1}{2}$ hours per day;
30% helped for $\frac{1}{2}$ hour to $1\frac{1}{2}$ hours; 50% did not help at all.
Using this, answer the following:

(i) How many parents were surveyed?
(ii) How many said that they did not help?
(iii) How many said that they helped for more than $1\frac{1}{2}$ hours?



EXERCISE 8.1

1. Find the ratio of the following.
   (a) Speed of a cycle 15 km per hour to the speed of scooter 30 km per hour.
   (b) 5 m to 10 km (c) 50 paise to ₹ 5
2. Convert the following ratios to percentages.
   (a) 3 : 4 (b) 2 : 3
3. 72% of 25 students are good in mathematics. How many are not good in mathematics?
4. A football team won 10 matches out of the total number of matches they played. If their win percentage was 40, then how many matches did they play in all?
5. If Chameli had ₹ 600 left after spending 75% of her money, how much did she have in the beginning?

6. If 60% people in a city like cricket, 30% like football and the remaining like other games, then what per cent of the people like other games? If the total number of people are 50 lakh, find the exact number who like each type of game.

8.2 Finding the Increase or Decrease Per cent

We often come across such information in our daily life as.

(i) 25% off on marked prices (ii) 10% hike in the price of petrol

Let us consider a few such examples.

Example 2: The price of a scooter was ₹ 34,000 last year. It has increased by 20% this year. What is the price now?

Solution:

Amita said that she would first find the increase in the price, which is 20% of ₹ 34,000, and then find the new price.

$$20\% \text{ of } ₹\, 34000 = ₹\, \frac{20}{100} \times 34000$$
$$= ₹\, 6800$$
$$\text{New price} = \text{Old price} + \text{Increase}$$
$$= ₹\, 34,000 + ₹\, 6,800$$
$$= ₹\, 40,800$$

OR

Sunita used the unitary method.
20% increase means,
₹ 100 increased to ₹ 120.
So, ₹ 34,000 will increase to?

$$\text{Increased price} = ₹\, \frac{120}{100} \times 34000$$
$$= ₹\, 40,800$$

Similarly, a percentage decrease in price would imply finding the actual decrease followed by its subtraction the from original price.

Suppose in order to increase its sale, the price of scooter was decreased by 5%. Then let us find the price of scooter.

$$\text{Price of scooter} = ₹\, 34000$$
$$\text{Reduction} = 5\% \text{ of } ₹\, 34000$$
$$= ₹\, \frac{5}{100} \times 34000 = ₹\, 1700$$
$$\text{New price} = \text{Old price} - \text{Reduction}$$
$$= ₹\, 34000 - ₹\, 1700 = ₹\, 32300$$

We will also use this in the next section of the chapter.

8.3 Finding Discounts

Discount is a reduction given on the Marked Price (MP) of the article.

This is generally given to attract customers to buy goods or to promote sales of the goods. You can find the discount by subtracting its sale price from its marked price.

So, Discount = Marked price – Sale price

Example 3: An item marked at ₹ 840 is sold for ₹ 714. What is the discount and discount %?

Solution:

Discount = Marked Price – Sale Price
= ₹ 840 – ₹ 714
= ₹ 126

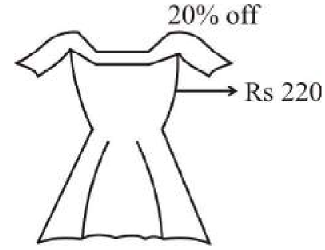
Since discount is on marked price, we will have to use marked price as the base.

On marked price of ₹ 840, the discount is ₹ 126.
On MP of ₹ 100, how much will the discount be?

$$\text{Discount} = \frac{126}{840} \times 100 = 15\%$$

You can also find discount when discount % is given.

Example 4: The list price of a frock is ₹ 220. A discount of 20% is announced on sales. What is the amount of discount on it and its sale price.



Solution: Marked price is same as the list price.
20% discount means that on ₹ 100 (MP), the discount is ₹ 20.

By unitary method, on ₹1 the discount will be ₹ $\frac{20}{100}$.

On ₹ 220, discount = ₹ $\frac{20}{100} \times 220$ = ₹ 44

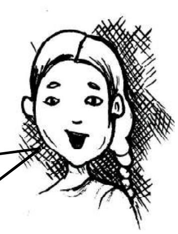
The sale price = (₹ 220 – ₹ 44) or ₹ 176

Rehana found the sale price like this —

A discount of 20% means for a MP of ₹ 100, discount is ₹ 20. Hence the sale price is ₹ 80. Using unitary method, when MP is ₹ 100, sale price is ₹ 80;

When MP is ₹ 1, sale price is ₹ $\frac{80}{100}$.

Hence when MP is ₹ 220, sale price = ₹ $\frac{80}{100} \times 220$ = ₹ 176.



Even though the discount was not found, I could find the sale price directly.

TRY THESE

1. A shop gives 20% discount. What would the sale price of each of these be?
   (a) A dress marked at ₹ 120 (b) A pair of shoes marked at ₹ 750
   (c) A bag marked at ₹ 250
2. A table marked at ₹ 15,000 is available for ₹ 14,400. Find the discount given and the discount per cent.
3. An almirah is sold at ₹ 5,225 after allowing a discount of 5%. Find its marked price.

8.3.1 Estimation in percentages

Your bill in a shop is ₹ 577.80 and the shopkeeper gives a discount of 15%. How would you estimate the amount to be paid?

(i) Round off the bill to the nearest tens of ₹ 577.80, i.e., to ₹ 580.

(ii) Find 10% of this, i.e., ₹ $\frac{10}{100} \times 580 =$ ₹ 58 .

(iii) Take half of this, i.e., $\frac{1}{2} \times 58 =$ ₹ 29 .

(iv) Add the amounts in (ii) and (iii) to get ₹ 87.

You could therefore reduce your bill amount by ₹ 87 or by about ₹ 85, which will be ₹ 495 approximately.

1. Try estimating 20% of the same bill amount.   2. Try finding 15% of ₹ 375.

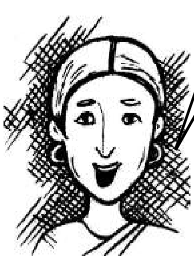
8.4 Prices Related to Buying and Selling (Profit and Loss)

For the school fair (mela) I am going to put a stall of lucky dips. I will charge ₹ 10 for one lucky dip but I will buy items which are worth ₹ 5.

So you are making a profit of 100%.

No, I will spend ₹ 3 on paper to wrap the gift and tape. So my expenditure is ₹ 8.

This gives me a profit of ₹ 2, which is, $\frac{2}{8} \times 100 = 25\%$ only.



Sometimes when an article is bought, some additional expenses are made while buying or before selling it. These expenses have to be included in the cost price.

These expenses are sometimes referred to as **overhead charges**. These may include expenses like amount spent on repairs, labour charges, transportation etc.

8.4.1 Finding cost price/selling price, profit %/loss%

Example 5: Sohan bought a second hand refrigerator for ₹ 2,500, then spent ₹ 500 on its repairs and sold it for ₹ 3,300. Find his loss or gain per cent.

Solution: Cost Price (CP) = ₹ 2500 + ₹ 500 (overhead expenses are added to give CP)

= ₹ 3000

Sale Price (SP) = ₹ 3300

As SP > CP, he made a profit = ₹ 3300 – ₹ 3000 = ₹ 300

His profit on ₹ 3,000, is ₹ 300. How much would be his profit on ₹ 100?

$$\text{Profit} = \frac{300}{3000} \times 100\% = \frac{30}{3}\% = 10\%$$

$$\boxed{P\% = \frac{P}{CP} \times 100}$$

TRY THESE

1. Find selling price (SP) if a profit of 5% is made on
   (a) a cycle of ₹ 700 with ₹ 50 as overhead charges.
   (b) a lawn mower bought at ₹ 1150 with ₹ 50 as transportation charges.
   (c) a fan bought for ₹ 560 and expenses of ₹ 40 made on its repairs.

Example 6: A shopkeeper purchased 200 bulbs for ₹ 10 each. However 5 bulbs were fused and had to be thrown away. The remaining were sold at ₹ 12 each. Find the gain or loss %.

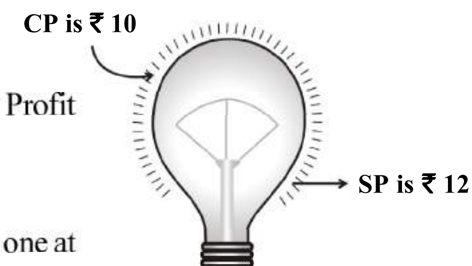
Solution: Cost price of 200 bulbs = ₹ 200 × 10 = ₹ 2000

5 bulbs were fused. Hence, number of bulbs left = 200 – 5 = 195

These were sold at ₹ 12 each.

The SP of 195 bulbs = ₹ 195 × 12 = ₹ 2340

He obviously made a profit (as SP > CP).



Profit = ₹ 2340 – ₹ 2000 = ₹ 340

On ₹ 2000, the profit is ₹ 340. How much profit is made on ₹ 100? Profit

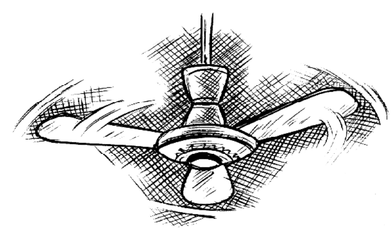
$$= \frac{340}{2000} \times 100 = 17\%.$$

Example 7: Meenu bought two fans for ₹ 1200 each. She sold one at a loss of 5% and the other at a profit of 10%. Find the selling price of each. Also find out the total profit or loss.

Solution: Overall CP of each fan = ₹ 1200. One is sold at a loss of 5%.

This means if CP is ₹ 100, SP is ₹ 95.

Therefore, when CP is ₹ 1200, then SP = ₹ $\frac{95}{100} \times 1200$ = ₹ 1140



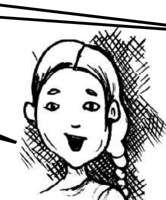
Also second fan is sold at a profit of 10%.

It means, if CP is ₹ 100, SP is ₹ 110.

Therefore, when CP is ₹ 1200, then SP = ₹ $\frac{110}{100} \times 1200$ = ₹ 1320

Was there an overall loss or gain?

We need to find the combined CP and SP to say whether there was an overall profit or loss.



Total CP = ₹ 1200 + ₹ 1200 = ₹ 2400

Total SP = ₹ 1140 + ₹ 1320 = ₹ 2460

Since total SP > total CP, a profit of ₹ (2460 – 2400) or ₹ 60 has been made.

TRY THESE

1. A shopkeeper bought two TV sets at ₹ 10,000 each. He sold one at a profit 10% and the other at a loss of 10%. Find whether he made an overall profit or loss.

8.5 Sales Tax/Value Added Tax

The teacher showed the class a bill in which the following heads were written.

Bill No.			**Date**	
Menu				
S.No.	Item	Quantity	Rate	Amount
		Bill amount + ST (5%)		
	Total			

ST means Sales Tax, which we pay when we buy items.

This sales tax is charged by the government on the sale of an item.
It is collected by the shopkeeper from the customer and given to the government.
This is, therefore, always on the selling price of an item and is added to the value of the bill.
These days however, the prices include the tax known as **Value Added Tax (VAT)**.

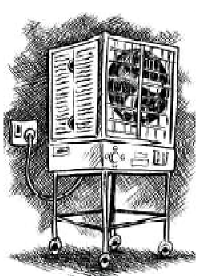
Example 8: (**Finding Sales Tax**) The cost of a pair of roller skates at a shop was ₹ 450. The sales tax charged was 5%. Find the bill amount.

Solution: On ₹ 100, the tax paid was ₹ 5.

On ₹ 450, the tax paid would be $= ₹\ \frac{5}{100} \times 450$

$= ₹\ 22.50$

Bill amount = Cost of item + Sales tax = ₹ 450 + ₹ 22.50 = ₹ 472.50.



Example 9: (**Value Added Tax (VAT)**) Waheeda bought an air cooler for ₹ 3300 including a tax of 10%. Find the price of the air cooler before VAT was added.

Solution: The price includes the VAT, i.e., the value added tax. Thus, a 10% VAT means if the price without VAT is ₹ 100 then price including VAT is ₹ 110.

Now, when price including VAT is ₹ 110, original price is ₹ 100.

Hence when price including tax is ₹ 3300, the original price $= ₹\ \frac{100}{110} \times 3300 = ₹\ 3000$.

TRY THESE

1. Find the buying price of each of the following when 5% ST is added on the purchase of
   (a) A towel at ₹ 50 (b) Two bars of soap at ₹ 35 each
   (c) 5 kg of flour at ₹ 15 per kg

2. If 8% VAT is included in the prices, find the original price of
   (a) A TV bought for ₹ 13,500 (b) A shampoo bottle bought for ₹ 180

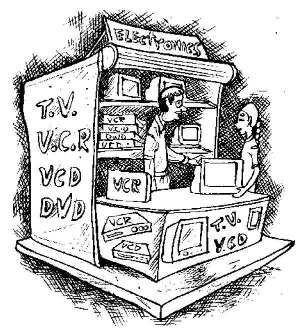
THINK, DISCUSS AND WRITE

1. Two times a number is a 100% increase in the number. If we take half the number what would be the decrease in per cent?
2. By what per cent is ₹ 2,000 less than ₹ 2,400? Is it the same as the per cent by which ₹ 2,400 is more than ₹ 2,000?

EXERCISE 8.2

1. A man got a 10% increase in his salary. If his new salary is ₹ 1,54,000, find his original salary.

2. On Sunday 845 people went to the Zoo. On Monday only 169 people went. What is the per cent decrease in the people visiting the Zoo on Monday?



3. A shopkeeper buys 80 articles for ₹ 2,400 and sells them for a profit of 16%. Find the selling price of one article.

4. The cost of an article was ₹ 15,500. ₹ 450 were spent on its repairs. If it is sold for a profit of 15%, find the selling price of the article.

5. A VCR and TV were bought for ₹ 8,000 each. The shopkeeper made a loss of 4% on the VCR and a profit of 8% on the TV. Find the gain or loss percent on the whole transaction.

6. During a sale, a shop offered a discount of 10% on the marked prices of all the items. What would a customer have to pay for a pair of jeans marked at ₹ 1450 and two shirts marked at ₹ 850 each?

7. A milkman sold two of his buffaloes for ₹ 20,000 each. On one he made a gain of 5% and on the other a loss of 10%. Find his overall gain or loss. (**Hint:** Find CP of each)

8. The price of a TV is ₹ 13,000. The sales tax charged on it is at the rate of 12%. Find the amount that Vinod will have to pay if he buys it.

9. Arun bought a pair of skates at a sale where the discount given was 20%. If the amount he pays is ₹ 1,600, find the marked price.

10. I purchased a hair-dryer for ₹ 5,400 including 8% VAT. Find the price before VAT was added.

8.6 Compound Interest

You might have come across statements like "one year interest for FD (fixed deposit) in the bank @ 9% per annum" or 'Savings account with interest @ 5% per annum'.

Interest is the extra money paid by institutions like banks or post offices on money deposited (kept) with them. Interest is also paid by people when they borrow money. We already know how to calculate **Simple Interest**.

Example 10: A sum of ₹ 10,000 is borrowed at a rate of interest 15% per annum for 2 years. Find the simple interest on this sum and the amount to be paid at the end of 2 years.

Solution: On ₹ 100, interest charged for 1 year is ₹ 15.

So, on ₹ 10,000, interest charged $= \frac{15}{100} \times 10000 =$ ₹ 1500

Interest for 2 years = ₹ 1500 × 2 = ₹ 3000

Amount to be paid at the end of 2 years = Principal + Interest

= ₹ 10000 + ₹ 3000 = ₹ 13000

TRY THESE

Find interest and amount to be paid on ₹ 15000 at 5% per annum after 2 years.

My father has kept some money in the post office for 3 years. Every year the money increases as more than the previous year.

We have some money in the bank. Every year some interest is added to it, which is shown in the passbook. This interest is not the same, each year it increases.

Normally, the interest paid or charged is never simple. The interest is calculated on the amount of the previous year. This is known as interest compounded or **Compound Interest** (C.I.).

Let us take an example and find the interest year by year. Each year our sum or principal changes.

Calculating Compound Interest

A sum of ₹ 20,000 is borrowed by Heena for 2 years at an interest of 8% compounded annually. Find the Compound Interest (C.I.) and the amount she has to pay at the end of 2 years.

Aslam asked the teacher whether this means that they should find the interest year by year. The teacher said 'yes', and asked him to use the following steps :

1. Find the Simple Interest (S.I.) for one year.
   Let the principal for the first year be P_1. Here, P_1 = ₹ 20,000

   SI_1 = SI at 8% p.a. for 1st year = ₹ $\frac{20000 \times 8}{100}$ = ₹ 1600

2. Then find the amount which will be paid or received. This becomes principal for the next year.

   Amount at the end of 1st year = $P_1 + SI_1$ = ₹ 20000 + ₹ 1600

   = ₹ 21600 = P_2 (Principal for 2nd year)

3. Again find the interest on this sum for another year.

$$SI_2 = \text{SI at 8\% p.a. for 2nd year} = ₹\ \frac{21600 \times 8}{100}$$

$$= ₹\ 1728$$

4. Find the amount which has to be paid or received at the end of second year.

$$\text{Amount at the end of 2nd year} = P_2 + SI_2$$
$$= ₹\ 21600 + ₹\ 1728$$
$$= ₹\ 23328$$
$$\text{Total interest given} = ₹\ 1600 + ₹\ 1728$$
$$= ₹\ 3328$$

Reeta asked whether the amount would be different for simple interest. The teacher told her to find the interest for two years and see for herself.

$$\text{SI for 2 years} = ₹\ \frac{20000 \times 8 \times 2}{100} = ₹\ 3200$$

Reeta said that when compound interest was used Heena would pay ₹ 128 more.

Let us look at the difference between simple interest and compound interest. We start with ₹ 100. Try completing the chart.

		Under Simple Interest	Under Compound Interest
First year	Principal	₹ 100.00	₹ 100.00
	Interest at 10%	₹ 10.00	₹ 10.00
	Year-end amount	₹ 110.00	₹ 110.00
Second year	Principal	₹ 100.00	₹ 110.00
	Interest at 10%	₹ 10.00	₹ 11.00
	Year-end amount	₹(110 + 10) = ₹ 120	₹ 121.00
Third year	Principal	₹ 100.00	₹ 121.00
	Interest at 10%	₹ 10.00	₹ 12.10
	Year-end amount	₹(120 + 10) = ₹ 130	₹ 133.10

Which means you pay interest on the interest accumulated till then!

Note that in 3 years,

Interest earned by Simple Interest = ₹ (130 – 100) = ₹ 30, whereas,

Interest earned by Compound Interest = ₹ (133.10 – 100) = ₹ 33.10

Note also that the Principal remains the same under Simple Interest, while it changes year after year under compound interest.

8.7 Deducing a Formula for Compound Interest

Zubeda asked her teacher, 'Is there an easier way to find compound interest?' The teacher said 'There is a shorter way of finding compound interest. Let us try to find it.'

Suppose P_1 is the sum on which interest is compounded annually at a rate of R% per annum.

Let P_1 = ₹ 5000 and R = 5% per annum. Then by the steps mentioned above

1. $SI_1 = ₹\ \frac{5000 \times 5 \times 1}{100}$ or $SI_1 = ₹\ \frac{P_1 \times R \times 1}{100}$

so, $A_1 = ₹\ 5000 + \frac{5000 \times 5 \times 1}{100}$ or $A_1 = P_1 + SI_1 = P_1 + \frac{P_1 R}{100}$

$= ₹\ 5000\left(1+\frac{5}{100}\right) = P_2$ $\quad = P_1\left(1+\frac{R}{100}\right) = P_2$

2. $SI_2 = ₹\ 5000\left(1+\frac{5}{100}\right) \times \frac{5 \times 1}{100}$ or $SI_2 = \frac{P_2 \times R \times 1}{100}$

$= ₹\ \frac{5000 \times 5}{100}\left(1+\frac{5}{100}\right)$ $\quad = P_1\left(1+\frac{R}{100}\right) \times \frac{R}{100}$

$$= \frac{P_1 R}{100}\left(1+\frac{R}{100}\right)$$

$A_2 = ₹\ 5000\left(1+\frac{5}{100}\right) + ₹\ \frac{5000 \times 5}{100}\left(1+\frac{5}{100}\right)$ $\quad A_2 = P_2 + SI_2$

$= ₹\ 5000\left(1+\frac{5}{100}\right)\left(1+\frac{5}{100}\right)$ $\quad = P_1\left(1+\frac{R}{100}\right) + P_1\frac{R}{100}\left(1+\frac{R}{100}\right)$

$= ₹\ 5000\left(1+\frac{5}{100}\right)^2 = P_3$ $\quad = P_1\left(1+\frac{R}{100}\right)\left(1+\frac{R}{100}\right)$

$$= P_1\left(1+\frac{R}{100}\right)^2 = P_3$$

Proceeding in this way the amount at the end of n years will be

$$A_n = P_1\left(1+\frac{R}{100}\right)^n$$

Or, we can say $\quad A = P\left(1+\frac{R}{100}\right)^n$

So, Zubeda said, but using this we get only the formula for the amount to be paid at the end of n years, and not the formula for compound interest.

Aruna at once said that we know CI = A – P, so we can easily find the compound interest too.

Example 11: Find CI on ₹ 12600 for 2 years at 10% per annum compounded annually.

Solution: We have, $A = P\left(1+\frac{R}{100}\right)^n$, where Principal (P) = ₹ 12600, Rate (R) = 10, Number of years (n) = 2

$$= ₹\ 12600\left(1+\frac{10}{100}\right)^2 = ₹\ 12600\left(\frac{11}{10}\right)^2$$

$$= ₹\ 12600 \times \frac{11}{10} \times \frac{11}{10} = ₹\ 15246$$

CI = A – P = ₹ 15246 – ₹ 12600 = ₹ 2646

TRY THESE

1. Find CI on a sum of ₹ 8000 for 2 years at 5% per annum compounded annually.

8.8 Rate Compounded Annually or Half Yearly (Semi Annually)

You may want to know why 'compounded annually' was mentioned after 'rate'. Does it mean anything?

It does, because we can also have interest rates compounded half yearly or quarterly. Let us see what happens to ₹ 100 over a period of one year if an interest is compounded annually or half yearly.

Time period and rate when interest not compounded annually

The time period after which the interest is added each time to form a new principal is called the **conversion period**. When the interest is compounded half yearly, there are two conversion periods in a year each after 6 months. In such situations, the half yearly rate will be half of the annual rate. What will happen if interest is compounded quarterly? In this case, there are 4 conversion periods in a year and the quarterly rate will be one-fourth of the annual rate.

P = ₹ 100 at 10% per annum compounded annually	P = ₹ 100 at 10% per annum compounded half yearly
The time period taken is 1 year	The time period is 6 months or $\frac{1}{2}$ year
$I = ₹\ \frac{100 \times 10 \times 1}{100} = \text{Rs } 10$	$I = ₹\ \frac{100 \times \boxed{10 \times \frac{1}{2}}}{100} = ₹\ 5$
A = ₹ 100 + ₹ 10 = ₹ 110	A = ₹ 100 + ₹ 5 = ₹ 105 Now for next 6 months the P = ₹ 105
	So, $I = ₹\ \frac{105 \times 10 \times \frac{1}{2}}{100} = ₹\ 5.25$ and A = ₹ 105 + ₹ 5.25 = ₹ 110.25

Rate becomes half

Do you see that, if interest is compounded half yearly, we compute the interest two times. So time period becomes twice and rate is taken half.

TRY THESE

Find the time period and rate for each .

1. A sum taken for $1\frac{1}{2}$ years at 8% per annum is compounded half yearly.
2. A sum taken for 2 years at 4% per annum compounded half yearly.

THINK, DISCUSS AND WRITE

A sum is taken for one year at 16% p.a. If interest is compounded after every three months, how many times will interest be charged in one year?

Example 12: What amount is to be repaid on a loan of ₹ 12000 for $1\frac{1}{2}$ years at 10% per annum compounded half yearly.

Solution:

Principal for first 6 months = ₹ 12,000	**Principal for first 6 months = ₹ 12,000**
There are 3 half years in $1\frac{1}{2}$ years. Therefore, compounding has to be done 3 times. Rate of interest = half of 10% = 5% half yearly $A = P\left(1+\frac{R}{100}\right)^n$ = ₹ $12000\left(1+\frac{5}{100}\right)^3$ = ₹ $12000 \times \frac{21}{20} \times \frac{21}{20} \times \frac{21}{20}$ = ₹ 13,891.50	Time = 6 months = $\frac{6}{12}$ year = $\frac{1}{2}$ year Rate = 10% I = ₹ $\frac{12000 \times 10 \times \frac{1}{2}}{100}$ = ₹ 600 A = P + I = ₹ 12000 + ₹ 600 = ₹12600. It is principal for next 6 months. I = ₹ $\frac{12600 \times 10 \times \frac{1}{2}}{100}$ = ₹ 630 Principal for third period = ₹ 12600 + ₹ 630 = ₹ 13,230. I = ₹ $\frac{13230 \times 10 \times \frac{1}{2}}{100}$ = ₹ 661.50 A = P + I = ₹ 13230 + ₹ 661.50 = ₹ 13,891.50

TRY THESE

Find the amount to be paid

1. At the end of 2 years on ₹ 2,400 at 5% per annum compounded annually.
2. At the end of 1 year on ₹ 1,800 at 8% per annum compounded quarterly.

Example 13: Find CI paid when a sum of ₹ 10,000 is invested for 1 year and 3 months at $8\frac{1}{2}$% per annum compounded annually.

Solution: Mayuri first converted the time in years.

$$1 \text{ year } 3 \text{ months} = 1\frac{3}{12} \text{ year} = 1\frac{1}{4} \text{ years}$$

Mayuri tried putting the values in the known formula and came up with:

$$A = ₹\ 10000\left(1+\frac{17}{200}\right)^{1\frac{1}{4}}$$

Now she was stuck. She asked her teacher how would she find a power which is fractional? The teacher then gave her a hint:

Find the amount for the whole part, i.e., 1 year in this case. Then use this as principal to get simple interest for $\frac{1}{4}$ year more. Thus,

$$A = ₹\ 10000\left(1+\frac{17}{200}\right)$$

$$= ₹\ 10000 \times \frac{217}{200} = ₹\ 10,850$$

Now this would act as principal for the next $\frac{1}{4}$ year. We find the SI on ₹ 10,850 for $\frac{1}{4}$ year.

$$SI = ₹\ \frac{10850 \times \frac{1}{4} \times 17}{100 \times 2}$$

$$= ₹\ \frac{10850 \times 1 \times 17}{800} = ₹\ 230.56$$

Interest for first year = ₹ 10850 – ₹ 10000 = ₹ 850

And, interest for the next $\frac{1}{4}$ year = ₹ 230.56

Therefore, total compound Interest = 850 + 230.56 = ₹ 1080.56.

8.9 Applications of Compound Interest Formula

There are some situations where we could use the formula for calculation of amount in CI. Here are a few.

(i) Increase (or decrease) in population.

(ii) The growth of a bacteria if the rate of growth is known.

(iii) The value of an item, if its price increases or decreases in the intermediate years.

Example 14: The population of a city was 20,000 in the year 1997. It increased at the rate of 5% p.a. Find the population at the end of the year 2000.

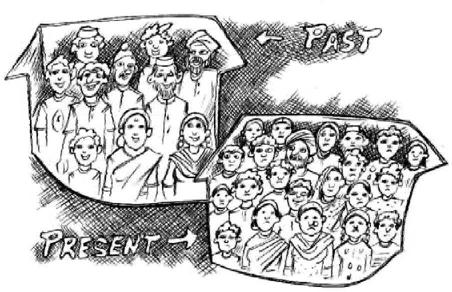
Solution: There is 5% increase in population every year, so every new year has new population. Thus, we can say it is increasing in compounded form.

Population in the beginning of 1998 = 20000 (we treat this as the principal for the 1st year)

Increase at 5% = $\frac{5}{100} \times 20000 = 1000$

Population in 1999 = 20000 + 1000 = 21000

Treat as the Principal for the 2nd year.



Increase at 5% = $\frac{5}{100} \times 21000 = 1050$

Population in 2000 = 21000 + 1050

= 22050

Treat as the Principal for the 3rd year.

Increase at 5% = $\frac{5}{100} \times 22050$

= 1102.5

At the end of 2000 the population = 22050 + 1102.5 = 23152.5

or, Population at the end of 2000 = $20000\left(1+\frac{5}{100}\right)^3$

$= 20000 \times \frac{21}{20} \times \frac{21}{20} \times \frac{21}{20}$

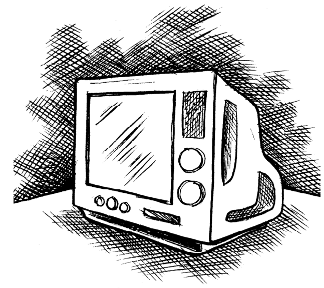
= 23152.5

So, the estimated population = 23153.

Aruna asked what is to be done if there is a decrease. The teacher then considered the following example.

Example 15: A TV was bought at a price of ₹ 21,000. After one year the value of the TV was depreciated by 5% (Depreciation means reduction of value due to use and age of the item). Find the value of the TV after one year.

Solution:



$$\text{Principal} = ₹\ 21{,}000$$

$$\text{Reduction} = 5\%\ \text{of}\ ₹\ 21000\ \text{per year}$$

$$= ₹\ \frac{21000 \times 5 \times 1}{100} = ₹\ 1050$$

$$\text{value at the end of 1 year} = ₹\ 21000 - ₹\ 1050 = ₹\ 19{,}950$$

Alternately, We may directly get this as follows:

$$\text{value at the end of 1 year} = ₹\ 21000 \left(1 - \frac{5}{100}\right)$$

$$= ₹\ 21000 \times \frac{19}{20} = ₹\ 19{,}950$$

TRY THESE

1. A machinery worth ₹ 10,500 depreciated by 5%. Find its value after one year.
2. Find the population of a city after 2 years, which is at present 12 lakh, if the rate of increase is 4%.

EXERCISE 8.3

1. Calculate the amount and compound interest on

(a) ₹ 10,800 for 3 years at $12\frac{1}{2}\%$ per annum compounded annually.

(b) ₹ 18,000 for $2\frac{1}{2}$ years at 10% per annum compounded annually.

(c) ₹ 62,500 for $1\frac{1}{2}$ years at 8% per annum compounded half yearly.

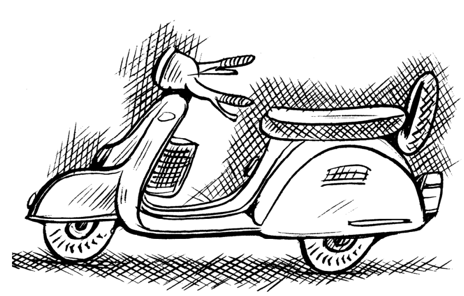
(d) ₹ 8,000 for 1 year at 9% per annum compounded half yearly.
(You could use the year by year calculation using SI formula to verify).

(e) ₹ 10,000 for 1 year at 8% per annum compounded half yearly.

2. Kamala borrowed ₹ 26,400 from a Bank to buy a scooter at a rate of 15% p.a. compounded yearly. What amount will she pay at the end of 2 years and 4 months to clear the loan?

(**Hint:** Find A for 2 years with interest is compounded yearly and then find SI on the 2nd year amount for $\frac{4}{12}$ years).

3. Fabina borrows ₹ 12,500 at 12% per annum for 3 years at simple interest and Radha borrows the same amount for the same time period at 10% per annum, compounded annually. Who pays more interest and by how much?
4. I borrowed ₹ 12,000 from Jamshed at 6% per annum simple interest for 2 years. Had I borrowed this sum at 6% per annum compound interest, what extra amount would I have to pay?
5. Vasudevan invested ₹ 60,000 at an interest rate of 12% per annum compounded half yearly. What amount would he get
   (i) after 6 months?
   (ii) after 1 year?
6. Arif took a loan of ₹ 80,000 from a bank. If the rate of interest is 10% per annum, find the difference in amounts he would be paying after $1\frac{1}{2}$ years if the interest is
   (i) compounded annually.
   (ii) compounded half yearly.
7. Maria invested ₹ 8,000 in a business. She would be paid interest at 5% per annum compounded annually. Find
   (i) The amount credited against her name at the end of the second year.
   (ii) The interest for the 3rd year.
8. Find the amount and the compound interest on ₹ 10,000 for $1\frac{1}{2}$ years at 10% per annum, compounded half yearly. Would this interest be more than the interest he would get if it was compounded annually?
9. Find the amount which Ram will get on ₹ 4096, if he gave it for 18 months at $12\frac{1}{2}\%$ per annum, interest being compounded half yearly.
10. The population of a place increased to 54,000 in 2003 at a rate of 5% per annum
    (i) find the population in 2001.
    (ii) what would be its population in 2005?
11. In a Laboratory, the count of bacteria in a certain experiment was increasing at the rate of 2.5% per hour. Find the bacteria at the end of 2 hours if the count was initially 5, 06,000.
12. A scooter was bought at ₹ 42,000. Its value depreciated at the rate of 8% per annum. Find its value after one year.



WHAT HAVE WE DISCUSSED?

1. **Discount** is a reduction given on marked price.
   Discount = Marked Price – Sale Price.
2. Discount can be calculated when discount percentage is given.
   Discount = Discount % of Marked Price
3. Additional expenses made after buying an article are included in the cost price and are known as **overhead expenses**.
   CP = Buying price + Overhead expenses
4. Sales tax is charged on the sale of an item by the government and is added to the Bill Amount.
   Sales tax = Tax% of Bill Amount
5. Compound interest is the interest calculated on the previous year's amount (A = P + I)
6. (i) Amount when interest is compounded annually

   $$= P\left(1+\frac{R}{100}\right)^n; \qquad \text{P is principal, R is rate of interest, } n \text{ is time period}$$

   (ii) Amount when interest is compounded half yearly

   $$= P\left(1+\frac{R}{200}\right)^{2n} \qquad \begin{cases} \dfrac{R}{2} \text{ is half yearly rate and} \\ 2n = \text{number of 'half-years'} \end{cases}$$

NOTES

CHAPTER

9

Algebraic Expressions and Identities

9.1 What are Expressions?

In earlier classes, we have already become familiar with what algebraic expressions (or simply expressions) are. Examples of expressions are:

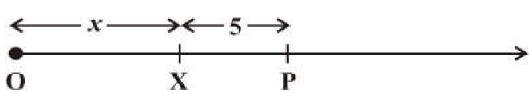
$$x + 3,\ 2y - 5,\ 3x^2,\ 4xy + 7 \text{ etc.}$$

You can form many more expressions. As you know expressions are formed from variables and constants. The expression $2y - 5$ is formed from the variable y and constants 2 and 5. The expression $4xy + 7$ is formed from variables x and y and constants 4 and 7.

We know that, the value of y in the expression, $2y - 5$, may be anything. It can be $2, 5, -3, 0, \frac{5}{2}, -\frac{7}{3}$ etc.; actually countless different values. The value of an expression changes with the value chosen for the variables it contains. Thus as y takes on different values, the value of $2y - 5$ goes on changing. When $y = 2$, $2y - 5 = 2(2) - 5 = -1$; when $y = 0$, $2y - 5 = 2 \times 0 - 5 = -5$, etc. Find the value of the expression $2y - 5$ for the other given values of y.

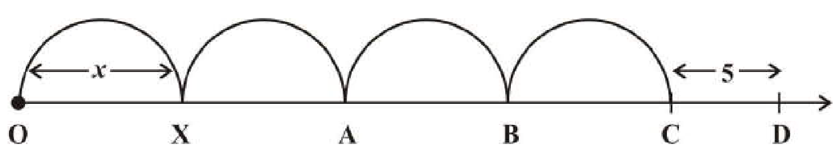
Number line and an expression:

Consider the expression $x + 5$. Let us say the variable x has a position X on the number line;



X may be anywhere on the number line, but it is definite that the value of $x + 5$ is given by a point P, 5 units to the right of X. Similarly, the value of $x - 4$ will be 4 units to the left of X and so on.

What about the position of $4x$ and $4x + 5$?



The position of $4x$ will be point C; the distance of C from the origin will be four times the distance of X from the origin. The position D of $4x + 5$ will be 5 units to the right of C.

TRY THESE

1. Give five examples of expressions containing one variable and five examples of expressions containing two variables.
2. Show on the number line x, $x - 4$, $2x + 1$, $3x - 2$.

9.2 Terms, Factors and Coefficients

Take the expression $4x + 5$. This expression is made up of two terms, $4x$ and 5. **Terms are added to form expressions**. **Terms** themselves **can be** formed as **the product of factors**. The term $4x$ is the product of its factors 4 and x. The term 5 is made up of just one factor, i.e., 5.

The expression $7xy - 5x$ has two terms $7xy$ and $-5x$. The term $7xy$ is a product of factors 7, x and y. The numerical factor of a term is called its **numerical coefficient or simply coefficient**. The coefficient in the term $7xy$ is 7 and the coefficient in the term $-5x$ is -5.

TRY THESE

Identify the coefficient of each term in the expression $x^2y^2 - 10x^2y + 5xy^2 - 20$.

9.3 Monomials, Binomials and Polynomials

Expression that contains only one term is called a **monomial**. Expression that contains two terms is called a **binomial**. An expression containing three terms is a **trinomial** and so on. In general, an expression containing, one or more terms with non-zero coefficient (with variables having non negative exponents) is called a **polynomial**. A polynomial may contain any number of terms, one or more than one.

Examples of monomials: $4x^2$, $3xy$, $-7z$, $5xy^2$, $10y$, -9, $82mnp$, etc.

Examples of binomials: $a + b$, $4l + 5m$, $a + 4$, $5 - 3xy$, $z^2 - 4y^2$, etc.

Examples of trinomials: $a + b + c$, $2x + 3y - 5$, $x^2y - xy^2 + y^2$, etc.

Examples of polynomials: $a + b + c + d$, $3xy$, $7xyz - 10$, $2x + 3y + 7z$, etc.

TRY THESE

1. Classify the following polynomials as monomials, binomials, trinomials.
   $-z + 5$, $x + y + z$, $y + z + 100$, $ab - ac$, 17
2. Construct
   (a) 3 binomials with only x as a variable;
   (b) 3 binomials with x and y as variables;
   (c) 3 monomials with x and y as variables;
   (d) 2 polynomials with 4 or more terms.

9.4 Like and Unlike Terms

Look at the following expressions:

$7x$, $14x$, $-13x$, $5x^2$, $7y$, $7xy$, $-9y^2$, $-9x^2$, $-5yx$

Like terms from these are:

(i) $7x$, $14x$, $-13x$ are like terms.

(ii) $5x^2$ and $-9x^2$ are like terms.

(iii) $7xy$ and $-5yx$ are like terms.

Why are $7x$ and $7y$ not like?

Why are $7x$ and $7xy$ not like?

Why are $7x$ and $5x^2$ not like?

TRY THESE

Write two terms which are like

(i) $7xy$ (ii) $4mn^2$ (iii) $2l$

9.5 Addition and Subtraction of Algebraic Expressions

In the earlier classes, we have also learnt how to add and subtract algebraic expressions. For example, to add $7x^2 - 4x + 5$ and $9x - 10$, we do

$$\begin{array}{rrrr} & 7x^2 & -\ 4x & +\ 5 \\ + & & 9x & -\ 10 \\ \hline & 7x^2 & +\ 5x & -\ 5 \end{array}$$

Observe how we do the addition. We write each expression to be added in a separate row. While doing so we write like terms one below the other, and add them, as shown. Thus $5 + (-10) = 5 - 10 = -5$. Similarly, $-4x + 9x = (-4 + 9)x = 5x$. Let us take some more examples.

Example 1: Add: $7xy + 5yz - 3zx$, $4yz + 9zx - 4y$, $-3xz + 5x - 2xy$.

Solution: Writing the three expressions in separate rows, with like terms one below the other, we have

$$\begin{array}{llllll} & 7xy & +\ 5yz & -3zx & & \\ + & & 4yz & +\ 9zx & & -\ 4y \\ + & -2xy & & -\ 3zx & +\ 5x & \\ \hline & 5xy & +\ 9yz & +3zx & +\ 5x & -\ 4y \end{array}$$

(Note xz is same as zx)

Thus, the sum of the expressions is $5xy + 9yz + 3zx + 5x - 4y$. Note how the terms, $-4y$ in the second expression and $5x$ in the third expression, are carried over as they are, since they have no like terms in the other expressions.

Example 2: Subtract $5x^2 - 4y^2 + 6y - 3$ from $7x^2 - 4xy + 8y^2 + 5x - 3y$.

Solution:

$$\begin{array}{llllll} 7x^2 & -\ 4xy & +\ 8y^2 & +\ 5x & -\ 3y & \\ 5x^2 & & -\ 4y^2 & & +\ 6y & -\ 3 \\ (-) & & (+) & & (-) & (+) \\ \hline 2x^2 & -\ 4xy & +\ 12y^2 & +\ 5x & -\ 9y & +\ 3 \end{array}$$

> Note that subtraction of a number is the same as addition of its additive inverse. Thus subtracting –3 is the same as adding +3. Similarly, subtracting $6y$ is the same as adding $-6y$; subtracting $-4y^2$ is the same as adding $4y^2$ and so on. The signs in the third row written below each term in the second row help us in knowing which operation has to be performed.

EXERCISE 9.1

1. Identify the terms, their coefficients for each of the following expressions.

(i) $5xyz^2 - 3zy$ (ii) $1 + x + x^2$ (iii) $4x^2y^2 - 4x^2y^2z^2 + z^2$

(iv) $3 - pq + qr - rp$ (v) $\frac{x}{2} + \frac{y}{2} - xy$ (vi) $0.3a - 0.6ab + 0.5b$

2. Classify the following polynomials as monomials, binomials, trinomials. Which polynomials do not fit in any of these three categories?

$x + y$, 1000, $x + x^2 + x^3 + x^4$, $7 + y + 5x$, $2y - 3y^2$, $2y - 3y^2 + 4y^3$, $5x - 4y + 3xy$, $4z - 15z^2$, $ab + bc + cd + da$, pqr, $p^2q + pq^2$, $2p + 2q$

3. Add the following.

(i) $ab - bc$, $bc - ca$, $ca - ab$ (ii) $a - b + ab$, $b - c + bc$, $c - a + ac$

(iii) $2p^2q^2 - 3pq + 4$, $5 + 7pq - 3p^2q^2$ (iv) $l^2 + m^2$, $m^2 + n^2$, $n^2 + l^2$, $2lm + 2mn + 2nl$

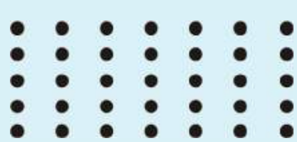
4. (a) Subtract $4a - 7ab + 3b + 12$ from $12a - 9ab + 5b - 3$

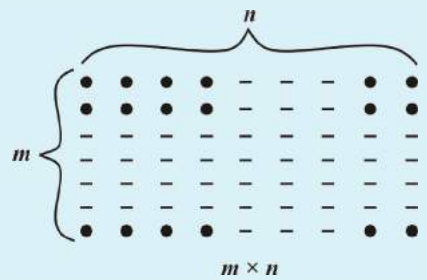
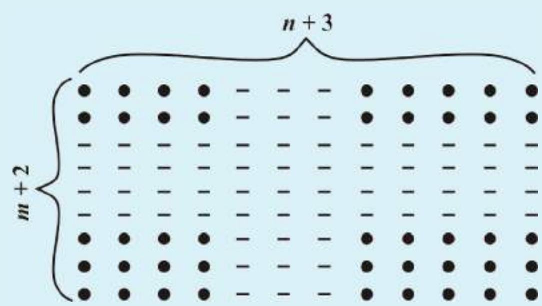
(b) Subtract $3xy + 5yz - 7zx$ from $5xy - 2yz - 2zx + 10xyz$

(c) Subtract $4p^2q - 3pq + 5pq^2 - 8p + 7q - 10$ from $18 - 3p - 11q + 5pq - 2pq^2 + 5p^2q$

9.6 Multiplication of Algebraic Expressions: Introduction

(i) Look at the following patterns of dots.

Pattern of dots	Total number of dots
	4×9
	5×7

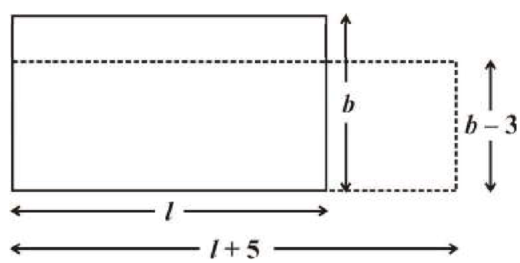
	$m \times n$
	$(m + 2) \times (n + 3)$

To find the number of dots we have to multiply the expression for the number of rows by the expression for the number of columns.

Here the number of rows is increased by 2, i.e., $m + 2$ and number of columns increased by 3, i.e., $n + 3$.

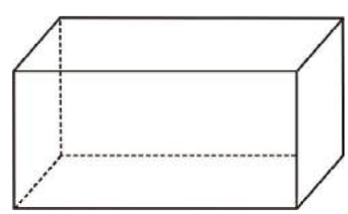
(ii) Can you now think of similar other situations in which two algebraic expressions have to be multiplied?

Ameena gets up. She says, "We can think of area of a rectangle." The area of a rectangle is $l \times b$, where l is the length, and b is breadth. If the length of the rectangle is increased by 5 units, i.e., $(l + 5)$ and breadth is decreased by 3 units , i.e., $(b - 3)$ units, the area of the new rectangle will be $(l + 5) \times (b - 3)$.



To find the area of a rectangle, we have to multiply algebraic expressions like $l \times b$ or $(l + 5) \times (b - 3)$.

(iii) Can you think about volume? (The volume of a rectangular box is given by the product of its length, breadth and height).



(iv) Sarita points out that when we buy things, we have to carry out multiplication. For example, if

price of bananas per dozen = ₹ p

and for the school picnic bananas needed = z dozens,

then we have to pay = ₹ $p \times z$

Suppose, the price per dozen was less by ₹ 2 and the bananas needed were less by 4 dozens.

Then, price of bananas per dozen = ₹ $(p - 2)$

and bananas needed = $(z - 4)$ dozens,

Therefore, we would have to pay = ₹ $(p - 2) \times (z - 4)$

TRY THESE

Can you think of two more such situations, where we may need to multiply algebraic expressions?

[**Hint:**
- Think of speed and time;
- Think of interest to be paid, the principal and the rate of simple interest; etc.]

In all the above examples, we had to carry out multiplication of two or more quantities. If the quantities are given by algebraic expressions, we need to find their product. This means that we should know how to obtain this product. Let us do this systematically. To begin with we shall look at the multiplication of two monomials.

9.7 Multiplying a Monomial by a Monomial

9.7.1 Multiplying two monomials

We begin with

$$4 \times x = x + x + x + x = 4x \text{ as seen earlier.}$$

Similarly, $4 \times (3x) = 3x + 3x + 3x + 3x = 12x$

Now, observe the following products.

(i) $x \times 3y = x \times 3 \times y = 3 \times x \times y = 3xy$

(ii) $5x \times 3y = 5 \times x \times 3 \times y = 5 \times 3 \times x \times y = 15xy$

(iii) $5x \times (-3y) = 5 \times x \times (-3) \times y$

$= 5 \times (-3) \times x \times y = -15xy$

Notice that all the three products of monomials, $3xy$, $15xy$, $-15xy$, are also monomials.

Some more useful examples follow.

(iv) $5x \times 4x^2 = (5 \times 4) \times (x \times x^2)$

$= 20 \times x^3 = 20x^3$

(v) $5x \times (-4xyz) = (5 \times -4) \times (x \times xyz)$

$= -20 \times (x \times x \times yz) = -20x^2yz$

Note that $5 \times 4 = 20$

i.e., coefficient of product = coefficient of first monomial × coefficient of second monomial;

and $x \times x^2 = x^3$

i.e., algebraic factor of product = algebraic factor of first monomial × algebraic factor of second monomial.

Observe how we collect the powers of different variables in the algebraic parts of the two monomials. While doing so, we use the rules of exponents and powers.

9.7.2 Multiplying three or more monomials

Observe the following examples.

(i) $2x \times 5y \times 7z = (2x \times 5y) \times 7z = 10xy \times 7z = 70xyz$

(ii) $4xy \times 5x^2y^2 \times 6x^3y^3 = (4xy \times 5x^2y^2) \times 6x^3y^3 = 20x^3y^3 \times 6x^3y^3 = 120x^3y^3 \times x^3y^3$

$= 120\,(x^3 \times x^3) \times (y^3 \times y^3) = 120x^6 \times y^6 = 120x^6y^6$

It is clear that we first multiply the first two monomials and then multiply the resulting monomial by the third monomial. This method can be extended to the product of any number of monomials.

TRY THESE

Find $4x \times 5y \times 7z$
First find $4x \times 5y$ and multiply it by $7z$;
or first find $5y \times 7z$ and multiply it by $4x$.
Is the result the same? What do you observe?
Does the order in which you carry out the multiplication matter?

We can find the product in other way also.
$4xy \times 5x^2y^2 \times 6x^3y^3$
$= (4 \times 5 \times 6) \times (x \times x^2 \times x^3) \times (y \times y^2 \times y^3)$
$= 120\ x^6y^6$

Example 3: Complete the table for area of a rectangle with given length and breadth.

Solution:

length	breadth	area
$3x$	$5y$	$3x \times 5y = 15xy$
$9y$	$4y^2$	....
$4ab$	$5bc$	....
$2l^2m$	$3lm^2$	....

Example 4: Find the volume of each rectangular box with given length, breadth and height.

	length	breadth	height
(i)	$2ax$	$3by$	$5cz$
(ii)	m^2n	n^2p	p^2m
(iii)	$2q$	$4q^2$	$8q^3$

Solution: Volume = length × breadth × height

Hence, for (i) volume $= (2ax) \times (3by) \times (5cz)$
$= 2 \times 3 \times 5 \times (ax) \times (by) \times (cz) = 30abcxyz$

for (ii) volume $= m^2n \times n^2p \times p^2m$
$= (m^2 \times m) \times (n \times n^2) \times (p \times p^2) = m^3n^3p^3$

for (iii) volume $= 2q \times 4q^2 \times 8q^3$
$= 2 \times 4 \times 8 \times q \times q^2 \times q^3 = 64q^6$

EXERCISE 9.2

1. Find the product of the following pairs of monomials.
   (i) $4, 7p$ (ii) $-4p, 7p$ (iii) $-4p, 7pq$ (iv) $4p^3, -3p$
   (v) $4p, 0$
2. Find the areas of rectangles with the following pairs of monomials as their lengths and breadths respectively.
   (p, q); $(10m, 5n)$; $(20x^2, 5y^2)$; $(4x, 3x^2)$; $(3mn, 4np)$

3. Complete the table of products.

First monomial → / Second monomial ↓	$2x$	$-5y$	$3x^2$	$-4xy$	$7x^2y$	$-9x^2y^2$
$2x$	$4x^2$	...	...	...	...	...
$-5y$	...	...	$-15x^2y$	...	...	...
$3x^2$	...	...	...	...	...	...
$-4xy$	...	...	...	...	...	...
$7x^2y$	...	...	...	...	...	...
$-9x^2y^2$	...	...	...	...	...	...

4. Obtain the volume of rectangular boxes with the following length, breadth and height respectively.

(i) $5a, 3a^2, 7a^4$ (ii) $2p, 4q, 8r$ (iii) $xy, 2x^2y, 2xy^2$ (iv) $a, 2b, 3c$

5. Obtain the product of

(i) xy, yz, zx (ii) $a, -a^2, a^3$ (iii) $2, 4y, 8y^2, 16y^3$

(iv) $a, 2b, 3c, 6abc$ (v) $m, -mn, mnp$

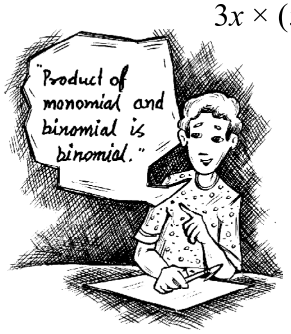
9.8 Multiplying a Monomial by a Polynomial

9.8.1 Multiplying a monomial by a binomial

Let us multiply the monomial $3x$ by the binomial $5y + 2$, i.e., find $3x \times (5y + 2) = ?$

Recall that $3x$ and $(5y + 2)$ represent numbers. Therefore, using the distributive law,

$3x \times (5y + 2) = (3x \times 5y) + (3x \times 2) = 15xy + 6x$



We commonly use distributive law in our calculations. For example:

$7 \times 106 = 7 \times (100 + 6)$

$= 7 \times 100 + 7 \times 6$ (Here, we used distributive law)

$= 700 + 42 = 742$

$7 \times 38 = 7 \times (40 - 2)$

$= 7 \times 40 - 7 \times 2$ (Here, we used distributive law)

$= 280 - 14 = 266$

Similarly, $(-3x) \times (-5y + 2) = (-3x) \times (-5y) + (-3x) \times (2) = 15xy - 6x$

and $5xy \times (y^2 + 3) = (5xy \times y^2) + (5xy \times 3) = 5xy^3 + 15xy$.

What about a binomial × monomial? For example, $(5y + 2) \times 3x = ?$

We may use commutative law as : $7 \times 3 = 3 \times 7$; or in general $a \times b = b \times a$

Similarly, $(5y + 2) \times 3x = 3x \times (5y + 2) = 15xy + 6x$ as before.

TRY THESE

Find the product (i) $2x(3x + 5xy)$ (ii) $a^2(2ab - 5c)$

9.8.2 Multiplying a monomial by a trinomial

Consider $3p \times (4p^2 + 5p + 7)$. As in the earlier case, we use distributive law;

$$3p \times (4p^2 + 5p + 7) = (3p \times 4p^2) + (3p \times 5p) + (3p \times 7)$$
$$= 12p^3 + 15p^2 + 21p$$

Multiply each term of the trinomial by the monomial and add products.

> Observe, by using the distributive law, we are able to carry out the multiplication term by term.

TRY THESE

Find the product:
$(4p^2 + 5p + 7) \times 3p$

Example 5: Simplify the expressions and evaluate them as directed:

(i) $x (x - 3) + 2$ for $x = 1$, (ii) $3y (2y - 7) - 3 (y - 4) - 63$ for $y = -2$

Solution:

(i) $x (x - 3) + 2 = x^2 - 3x + 2$

For $x = 1$, $x^2 - 3x + 2 = (1)^2 - 3 (1) + 2$
$= 1 - 3 + 2 = 3 - 3 = 0$

(ii) $3y (2y - 7) - 3 (y - 4) - 63 = 6y^2 - 21y - 3y + 12 - 63$
$= 6y^2 - 24y - 51$

For $y = -2$, $6y^2 - 24y - 51 = 6 (-2)^2 - 24(-2) - 51$
$= 6 \times 4 + 24 \times 2 - 51$
$= 24 + 48 - 51 = 72 - 51 = 21$

Example 6: Add

(i) $5m (3 - m)$ and $6m^2 - 13m$ (ii) $4y (3y^2 + 5y - 7)$ and $2 (y^3 - 4y^2 + 5)$

Solution:

(i) First expression $= 5m (3 - m) = (5m \times 3) - (5m \times m) = 15m - 5m^2$

Now adding the second expression to it, $15m - 5m^2 + 6m^2 - 13m = m^2 + 2m$

(ii) The first expression $= 4y (3y^2 + 5y - 7) = (4y \times 3y^2) + (4y \times 5y) + (4y \times (-7))$
$= 12y^3 + 20y^2 - 28y$

The second expression $= 2 (y^3 - 4y^2 + 5) = 2y^3 + 2 \times (-4y^2) + 2 \times 5$
$= 2y^3 - 8y^2 + 10$

Adding the two expressions,

$$\begin{array}{rrrrr} & 12y^3 & + & 20y^2 - 28y & \\ + & 2y^3 & - & 8y^2 & + 10 \\ \hline & 14y^3 & + & 12y^2 - 28y & + 10 \end{array}$$

Example 7: Subtract $3pq (p - q)$ from $2pq (p + q)$.

Solution: We have $3pq (p - q) = 3p^2q - 3pq^2$ and

$2pq (p + q) = 2p^2q + 2pq^2$

Subtracting,

$$\begin{array}{rrcr} & 2p^2q & + & 2pq^2 \\ & 3p^2q & - & 3pq^2 \\ - & & + & \\ \hline & -p^2q & + & 5pq^2 \end{array}$$

EXERCISE 9.3

1. Carry out the multiplication of the expressions in each of the following pairs.

   (i) $4p, q + r$ (ii) $ab, a - b$ (iii) $a + b, 7a^2b^2$ (iv) $a^2 - 9, 4a$

   (v) $pq + qr + rp, 0$

2. Complete the table.

	First expression	Second expression	Product
(i)	a	$b + c + d$	...
(ii)	$x + y - 5$	$5xy$	...
(iii)	p	$6p^2 - 7p + 5$	...
(iv)	$4p^2q^2$	$p^2 - q^2$	...
(v)	$a + b + c$	abc	...

3. Find the product.

   (i) $(a^2) \times (2a^{22}) \times (4a^{26})$ (ii) $\left(\frac{2}{3}xy\right) \times \left(\frac{-9}{10}x^2y^2\right)$

   (iii) $\left(-\frac{10}{3}pq^3\right) \times \left(\frac{6}{5}p^3q\right)$ (iv) $x \times x^2 \times x^3 \times x^4$

4. (a) Simplify $3x(4x - 5) + 3$ and find its values for (i) $x = 3$ (ii) $x = \frac{1}{2}$.

   (b) Simplify $a(a^2 + a + 1) + 5$ and find its value for (i) $a = 0$, (ii) $a = 1$ (iii) $a = -1$.

5. (a) Add: $p(p - q), q(q - r)$ and $r(r - p)$

   (b) Add: $2x(z - x - y)$ and $2y(z - y - x)$

   (c) Subtract: $3l(l - 4m + 5n)$ from $4l(10n - 3m + 2l)$

   (d) Subtract: $3a(a + b + c) - 2b(a - b + c)$ from $4c(-a + b + c)$

9.9 Multiplying a Polynomial by a Polynomial

9.9.1 Multiplying a binomial by a binomial

Let us multiply one binomial $(2a + 3b)$ by another binomial, say $(3a + 4b)$. We do this step-by-step, as we did in earlier cases, following the distributive law of multiplication,

$$(3a + 4b) \times (2a + 3b) = 3a \times (2a + 3b) + 4b \times (2a + 3b)$$

Observe, every term in one binomial multiplies every term in the other binomial.

$$= (3a \times 2a) + (3a \times 3b) + (4b \times 2a) + (4b \times 3b)$$

$$= 6a^2 + 9ab + 8ba + 12b^2$$

$$= 6a^2 + 17ab + 12b^2 \qquad \text{(Since } ba = ab\text{)}$$

When we carry out term by term multiplication, we expect $2 \times 2 = 4$ terms to be present. But two of these are like terms, which are combined, and hence we get 3 terms. **In multiplication of polynomials with polynomials, we should always look for like terms, if any, and combine them**.

Example 8: Multiply

(i) $(x - 4)$ and $(2x + 3)$ (ii) $(x - y)$ and $(3x + 5y)$

Solution:

(i) $(x - 4) \times (2x + 3) = x \times (2x + 3) - 4 \times (2x + 3)$
$= (x \times 2x) + (x \times 3) - (4 \times 2x) - (4 \times 3) = 2x^2 + 3x - 8x - 12$
$= 2x^2 - 5x - 12$ (Adding like terms)

(ii) $(x - y) \times (3x + 5y) = x \times (3x + 5y) - y \times (3x + 5y)$
$= (x \times 3x) + (x \times 5y) - (y \times 3x) - (y \times 5y)$
$= 3x^2 + 5xy - 3yx - 5y^2 = 3x^2 + 2xy - 5y^2$ (Adding like terms)

Example 9: Multiply

(i) $(a + 7)$ and $(b - 5)$ (ii) $(a^2 + 2b^2)$ and $(5a - 3b)$

Solution:

(i) $(a + 7) \times (b - 5) = a \times (b - 5) + 7 \times (b - 5)$
$= ab - 5a + 7b - 35$

Note that there are no like terms involved in this multiplication.

(ii) $(a^2 + 2b^2) \times (5a - 3b) = a^2\,(5a - 3b) + 2b^2 \times (5a - 3b)$
$= 5a^3 - 3a^2b + 10ab^2 - 6b^3$

9.9.2 Multiplying a binomial by a trinomial

In this multiplication, we shall have to multiply each of the three terms in the trinomial by each of the two terms in the binomial. We shall get in all $3 \times 2 = 6$ terms, which may reduce to 5 or less, if the term by term multiplication results in like terms. Consider

$$\underbrace{(a+7)}_{\text{binomial}} \times \underbrace{(a^2+3a+5)}_{\text{trinomial}} = a \times (a^2 + 3a + 5) + 7 \times (a^2 + 3a + 5)$$
[using the distributive law]
$$= a^3 + 3a^2 + 5a + 7a^2 + 21a + 35$$
$$= a^3 + (3a^2 + 7a^2) + (5a + 21a) + 35$$
$$= a^3 + 10a^2 + 26a + 35$$
(Why are there only 4 terms in the final result?)

Example 10: Simplify $(a + b)\,(2a - 3b + c) - (2a - 3b)\,c$.

Solution: We have

$(a + b)\,(2a - 3b + c) = a\,(2a - 3b + c) + b\,(2a - 3b + c)$
$= 2a^2 - 3ab + ac + 2ab - 3b^2 + bc$
$= 2a^2 - ab - 3b^2 + bc + ac$ (Note, $-3ab$ and $2ab$ are like terms)

and $(2a - 3b)\,c = 2ac - 3bc$

Therefore,

$(a + b)\,(2a - 3b + c) - (2a - 3b)\,c = 2a^2 - ab - 3b^2 + bc + ac - (2ac - 3bc)$
$= 2a^2 - ab - 3b^2 + bc + ac - 2ac + 3bc$
$= 2a^2 - ab - 3b^2 + (bc + 3bc) + (ac - 2ac)$
$= 2a^2 - 3b^2 - ab + 4bc - ac$

EXERCISE 9.4

1. Multiply the binomials.
   (i) $(2x + 5)$ and $(4x - 3)$ (ii) $(y - 8)$ and $(3y - 4)$
   (iii) $(2.5l - 0.5m)$ and $(2.5l + 0.5m)$ (iv) $(a + 3b)$ and $(x + 5)$
   (v) $(2pq + 3q^2)$ and $(3pq - 2q^2)$
   (vi) $\left(\frac{3}{4}a^2 + 3b^2\right)$ and $4\left(a^2 - \frac{2}{3}b^2\right)$
2. Find the product.
   (i) $(5 - 2x)(3 + x)$ (ii) $(x + 7y)(7x - y)$
   (iii) $(a^2 + b)(a + b^2)$ (iv) $(p^2 - q^2)(2p + q)$
3. Simplify.
   (i) $(x^2 - 5)(x + 5) + 25$ (ii) $(a^2 + 5)(b^3 + 3) + 5$
   (iii) $(t + s^2)(t^2 - s)$
   (iv) $(a + b)(c - d) + (a - b)(c + d) + 2(ac + bd)$
   (v) $(x + y)(2x + y) + (x + 2y)(x - y)$ (vi) $(x + y)(x^2 - xy + y^2)$
   (vii) $(1.5x - 4y)(1.5x + 4y + 3) - 4.5x + 12y$
   (viii) $(a + b + c)(a + b - c)$

9.10 What is an Identity?

Consider the equality $(a + 1)(a + 2) = a^2 + 3a + 2$

We shall evaluate both sides of this equality for some value of a, say $a = 10$.

For $a = 10$, LHS $= (a + 1)(a + 2) = (10 + 1)(10 + 2) = 11 \times 12 = 132$

RHS $= a^2 + 3a + 2 = 10^2 + 3 \times 10 + 2 = 100 + 30 + 2 = 132$

Thus, the values of the two sides of the equality are equal for $a = 10$.

Let us now take $a = -5$

LHS $= (a + 1)(a + 2) = (-5 + 1)(-5 + 2) = (-4) \times (-3) = 12$

RHS $= a^2 + 3a + 2 = (-5)^2 + 3(-5) + 2$

$= 25 - 15 + 2 = 10 + 2 = 12$

Thus, for $a = -5$, also LHS = RHS.

We shall find that for any value of a, LHS = RHS. **Such an equality, true for every value of the variable in it, is called an identity**. Thus,

$$(a + 1)(a + 2) = a^2 + 3a + 2 \text{ is an identity.}$$

An equation is true for only certain values of the variable in it. It is not true for all values of the variable. For example, consider the equation

$$a^2 + 3a + 2 = 132$$

It is true for $a = 10$, as seen above, but it is not true for $a = -5$ or for $a = 0$ etc.

Try it: Show that $a^2 + 3a + 2 = 132$ is not true for $a = -5$ and for $a = 0$.

9.11 Standard Identities

We shall now study three identities which are very useful in our work. These identities are obtained by multiplying a binomial by another binomial.

Let us first consider the product $(a + b)(a + b)$ or $(a + b)^2$.

$$\begin{aligned}(a + b)^2 &= (a + b)(a + b)\\ &= a(a + b) + b(a + b)\\ &= a^2 + ab + ba + b^2\\ &= a^2 + 2ab + b^2 \qquad \text{(since } ab = ba\text{)}\end{aligned}$$

Thus $$\boxed{(a + b)^2 = a^2 + 2ab + b^2} \qquad \text{(I)}$$

Clearly, this is an identity, since the expression on the RHS is obtained from the LHS by actual multiplication. One may verify that for any value of a and any value of b, the values of the two sides are equal.

- Next we consider $(a - b)^2 = (a - b)(a - b) = a(a - b) - b(a - b)$

  We have $= a^2 - ab - ba + b^2 = a^2 - 2ab + b^2$

  or $$\boxed{(a - b)^2 = a^2 - 2ab + b^2} \qquad \text{(II)}$$

- Finally, consider $(a + b)(a - b)$. We have $(a + b)(a - b) = a(a - b) + b(a - b)$

  $= a^2 - ab + ba - b^2 = a^2 - b^2$ (since $ab = ba$)

  or $$\boxed{(a + b)(a - b) = a^2 - b^2} \qquad \text{(III)}$$

The identities (I), (II) and (III) are known as **standard identities**.

TRY THESE

1. Put $-b$ in place of b in Identity (I). Do you get Identity (II)?

- We shall now work out one more useful identity.

$$\begin{aligned}(x + a)(x + b) &= x(x + b) + a(x + b)\\ &= x^2 + bx + ax + ab\end{aligned}$$

or $$\boxed{(x + a)(x + b) = x^2 + (a + b)x + ab} \qquad \text{(IV)}$$

TRY THESE

1. Verify Identity (IV), for $a = 2, b = 3, x = 5$.
2. Consider, the special case of Identity (IV) with $a = b$, what do you get? Is it related to Identity (I)?
3. Consider, the special case of Identity (IV) with $a = -c$ and $b = -c$. What do you get? Is it related to Identity (II)?
4. Consider the special case of Identity (IV) with $b = -a$. What do you get? Is it related to Identity (III)?

We can see that Identity (IV) is the general form of the other three identities also.

9.12 Applying Identities

We shall now see how, for many problems on multiplication of binomial expressions and also of numbers, use of the identities gives a simple alternative method of solving them.

Example 11: Using the Identity (I), find (i) $(2x + 3y)^2$ (ii) 103^2

Solution:

(i) $(2x + 3y)^2 = (2x)^2 + 2(2x)(3y) + (3y)^2$ [Using the Identity (I)]

$= 4x^2 + 12xy + 9y^2$

We may work out $(2x + 3y)^2$ directly.

$$(2x + 3y)^2 = (2x + 3y)(2x + 3y)$$
$$= (2x)(2x) + (2x)(3y) + (3y)(2x) + (3y)(3y)$$
$$= 4x^2 + 6xy + 6\,yx + 9y^2 \qquad (\text{as } xy = yx)$$
$$= 4x^2 + 12xy + 9y^2$$

Using Identity (I) gave us an alternative method of squaring $(2x + 3y)$. Do you notice that the Identity method required fewer steps than the above direct method? You will realise the simplicity of this method even more if you try to square more complicated binomial expressions than $(2x + 3y)$.

(ii) $(103)^2 = (100 + 3)^2$

$= 100^2 + 2 \times 100 \times 3 + 3^2$ (Using Identity I)

$= 10000 + 600 + 9 = 10609$

We may also directly multiply 103 by 103 and get the answer. Do you see that Identity (I) has given us a less tedious method than the direct method of squaring 103? Try squaring 1013. You will find in this case, the method of using identities even more attractive than the direct multiplication method.

Example 12: Using Identity (II), find (i) $(4p - 3q)^2$ (ii) $(4.9)^2$

Solution:

(i) $(4p - 3q)^2 = (4p)^2 - 2(4p)(3q) + (3q)^2$ [Using the Identity (II)]

$= 16p^2 - 24pq + 9q^2$

Do you agree that for squaring $(4p - 3q)^2$ the method of identities is quicker than the direct method?

(ii) $(4.9)^2 = (5.0 - 0.1)^2 = (5.0)^2 - 2(5.0)(0.1) + (0.1)^2$

$= 25.00 - 1.00 + 0.01 = 24.01$

Is it not that, squaring 4.9 using Identity (II) is much less tedious than squaring it by direct multiplication?

Example 13: Using Identity (III), find

(i) $\left(\frac{3}{2}m + \frac{2}{3}n\right)\left(\frac{3}{2}m - \frac{2}{3}n\right)$ (ii) $983^2 - 17^2$ (iii) 194×206

Solution:

(i) $\left(\frac{3}{2}m + \frac{2}{3}n\right)\left(\frac{3}{2}m - \frac{2}{3}n\right) = \left(\frac{3}{2}m\right)^2 - \left(\frac{2}{3}n\right)^2$

$= \frac{9}{4}m^2 - \frac{4}{9}n^2$

Try doing this directly. You will realise how easy our method of using Identity (III) is.

(ii) $983^2 - 17^2 = (983 + 17)(983 - 17)$

[Here $a = 983$, $b = 17$, $a^2 - b^2 = (a + b)(a - b)$]

Therefore, $983^2 - 17^2 = 1000 \times 966 = 966000$

(iii) $194 \times 206 = (200 - 6) \times (200 + 6) = 200^2 - 6^2$
$= 40000 - 36 = 39964$

Example 14: Use the Identity $(x + a)(x + b) = x^2 + (a + b)x + ab$ to find the following:

(i) 501×502 (ii) 95×103

Solution:

(i) $501 \times 502 = (500 + 1) \times (500 + 2) = 500^2 + (1 + 2) \times 500 + 1 \times 2$
$= 250000 + 1500 + 2 = 251502$

(ii) $95 \times 103 = (100 - 5) \times (100 + 3) = 100^2 + (-5 + 3) \times 100 + (-5) \times 3$
$= 10000 - 200 - 15 = 9785$

EXERCISE 9.5

1. Use a suitable identity to get each of the following products.
   (i) $(x + 3)(x + 3)$ (ii) $(2y + 5)(2y + 5)$ (iii) $(2a - 7)(2a - 7)$
   (iv) $(3a - \frac{1}{2})(3a - \frac{1}{2})$ (v) $(1.1m - 0.4)(1.1m + 0.4)$
   (vi) $(a^2 + b^2)(-a^2 + b^2)$ (vii) $(6x - 7)(6x + 7)$ (viii) $(-a + c)(-a + c)$
   (ix) $\left(\frac{x}{2} + \frac{3y}{4}\right)\left(\frac{x}{2} + \frac{3y}{4}\right)$ (x) $(7a - 9b)(7a - 9b)$
2. Use the identity $(x + a)(x + b) = x^2 + (a + b)x + ab$ to find the following products.
   (i) $(x + 3)(x + 7)$ (ii) $(4x + 5)(4x + 1)$
   (iii) $(4x - 5)(4x - 1)$ (iv) $(4x + 5)(4x - 1)$
   (v) $(2x + 5y)(2x + 3y)$ (vi) $(2a^2 + 9)(2a^2 + 5)$
   (vii) $(xyz - 4)(xyz - 2)$
3. Find the following squares by using the identities.
   (i) $(b - 7)^2$ (ii) $(xy + 3z)^2$ (iii) $(6x^2 - 5y)^2$
   (iv) $\left(\frac{2}{3}m + \frac{3}{2}n\right)^2$ (v) $(0.4p - 0.5q)^2$ (vi) $(2xy + 5y)^2$
4. Simplify.
   (i) $(a^2 - b^2)^2$ (ii) $(2x + 5)^2 - (2x - 5)^2$
   (iii) $(7m - 8n)^2 + (7m + 8n)^2$ (iv) $(4m + 5n)^2 + (5m + 4n)^2$
   (v) $(2.5p - 1.5q)^2 - (1.5p - 2.5q)^2$
   (vi) $(ab + bc)^2 - 2ab^2c$ (vii) $(m^2 - n^2m)^2 + 2m^3n^2$
5. Show that.
   (i) $(3x + 7)^2 - 84x = (3x - 7)^2$ (ii) $(9p - 5q)^2 + 180pq = (9p + 5q)^2$
   (iii) $\left(\frac{4}{3}m - \frac{3}{4}n\right)^2 + 2mn = \frac{16}{9}m^2 + \frac{9}{16}n^2$
   (iv) $(4pq + 3q)^2 - (4pq - 3q)^2 = 48pq^2$
   (v) $(a - b)(a + b) + (b - c)(b + c) + (c - a)(c + a) = 0$

6. Using identities, evaluate.

(i) 71^2 (ii) 99^2 (iii) 102^2 (iv) 998^2

(v) 5.2^2 (vi) 297×303 (vii) 78×82 (viii) 8.9^2

(ix) 1.05×9.5

7. Using $a^2 - b^2 = (a + b)(a - b)$, find

(i) $51^2 - 49^2$ (ii) $(1.02)^2 - (0.98)^2$ (iii) $153^2 - 147^2$

(iv) $12.1^2 - 7.9^2$

8. Using $(x + a)(x + b) = x^2 + (a + b)x + ab$, find

(i) 103×104 (ii) 5.1×5.2 (iii) 103×98 (iv) 9.7×9.8

WHAT HAVE WE DISCUSSED?

1. Expressions are formed from **variables** and **constants**.

2. Terms are added to form **expressions**. Terms themselves are formed as product of **factors**.

3. Expressions that contain exactly one, two and three terms are called **monomials**, **binomials** and **trinomials** respectively. In general, any expression containing one or more terms with non-zero coefficients (and with variables having non- negative exponents) is called a **polynomial**.

4. **Like** terms are formed from the same variables and the powers of these variables are the same, too. Coefficients of like terms need not be the same.

5. While adding (or subtracting) polynomials, first look for like terms and add (or subtract) them; then handle the unlike terms.

6. There are number of situations in which we need to multiply algebraic expressions: for example, in finding area of a rectangle, the sides of which are given as expressions.

7. A monomial multiplied by a monomial always gives a monomial.

8. While multiplying a polynomial by a monomial, we multiply every term in the polynomial by the monomial.

9. In carrying out the multiplication of a polynomial by a binomial (or trinomial), we multiply term by term, i.e., every term of the polynomial is multiplied by every term in the binomial (or trinomial). Note that in such multiplication, we may get terms in the product which are like and have to be combined.

10. An **identity** is an equality, which is true for all values of the variables in the equality.

On the other hand, an equation is true only for certain values of its variables. An equation is not an identity.

11. The following are the standard identities:

$(a + b)^2 = a^2 + 2ab + b^2$ (I)

$(a - b)^2 = a^2 - 2ab + b^2$ (II)

$(a + b)(a - b) = a^2 - b^2$ (III)

12. Another useful identity is $(x + a)(x + b) = x^2 + (a + b)x + ab$ (IV)

13. The above four identities are useful in carrying out squares and products of algebraic expressions. They also allow easy alternative methods to calculate products of numbers and so on.

CHAPTER

10

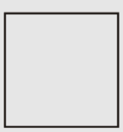
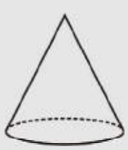
Visualising Solid Shapes

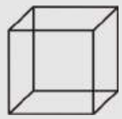
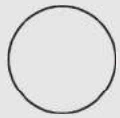
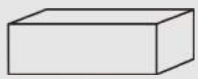
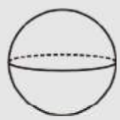
10.1 Introduction

In Class VII, you have learnt about plane shapes and solid shapes. Plane shapes have two measurements like length and breadth and therefore they are called two-dimensional shapes whereas a solid object has three measurements like length, breadth, height or depth. Hence, they are called three-dimensional shapes. Also, a solid object occupies some space. Two-dimensional and three-dimensional figures can also be briefly named as 2-D and 3-D figures. You may recall that triangle, rectangle, circle etc., are 2-D figures while cubes, cylinders, cones, spheres etc. are three-dimensional figures.

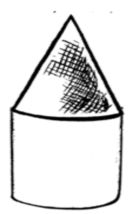
DO THIS

Match the following: (First one is done for you)

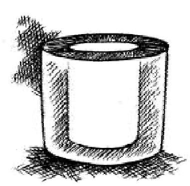
Shape	Type of Shape	Name of the shape
	3-dimensional	Sphere
	2-Dimensional	Cylinder
	3-dimensional	Square
	2-dimensional	Circle

	3-dimensional	Cuboid
	3- dimensional	Cube
	2-dimensional	Cone
	3-dimensional	Triangle

Note that all the above shapes are single. However, in our practical life, many a times, we come across combinations of different shapes. For example, look at the following objects.

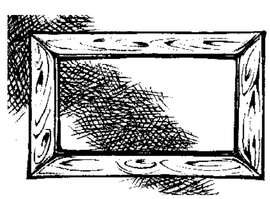


A tent
A cone surmounted on a cylinder

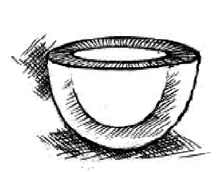


A tin
A cylinderical shell

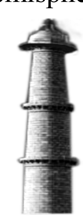
Softy (ice-cream)
A cone surmounted by a hemisphere



A photoframe
A rectangular path



A bowl
A hemispherical shell

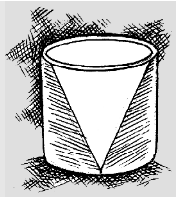
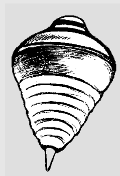
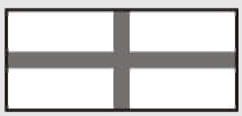


Tomb on a pillar
Cylinder surmounted by a hemisphere

DO THIS

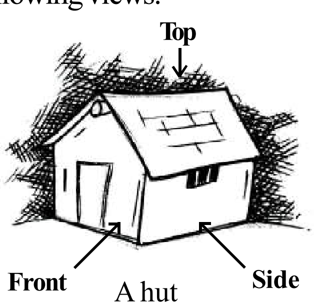
Match the following pictures (objects) with their shapes:

Picture (object)		**Shape**
(i) An agricultural field		Two rectangular cross paths inside a rectangular park.

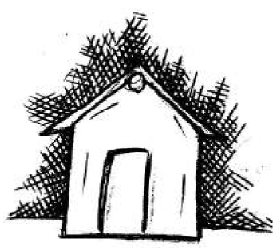
(ii)	A groove		A circular path around a circular ground.	
(iii)	A toy		A triangular field adjoining a square field.	
(iv)	A circular park		A cone taken out of a cylinder.	
(v)	A cross path		A hemisphere surmounted on a cone.	

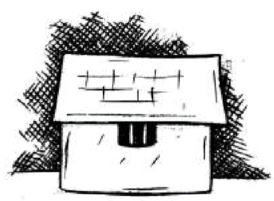
10.2 Views of 3D-Shapes

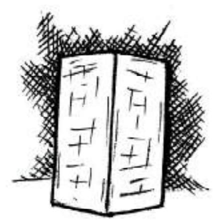
You have learnt that a 3-dimensional object can look differently from different positions so they can be drawn from different perspectives. For example, a given hut can have the following views.



A hut

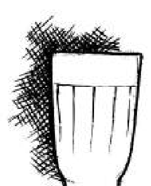

Front view

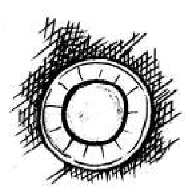

Side view


Top view

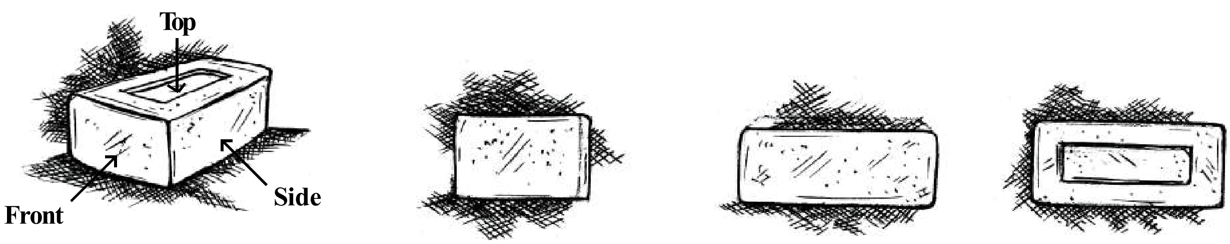
similarly, a glass can have the following views.


A glass


Side view

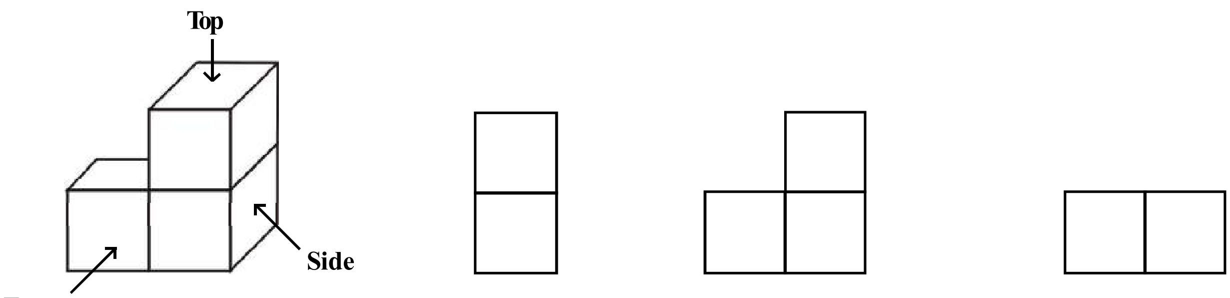

Top view

Why is the top view of the glass a pair of concentric circles? Will the side view appear different if taken from some other direction? Think about this! Now look at the different views of a brick.

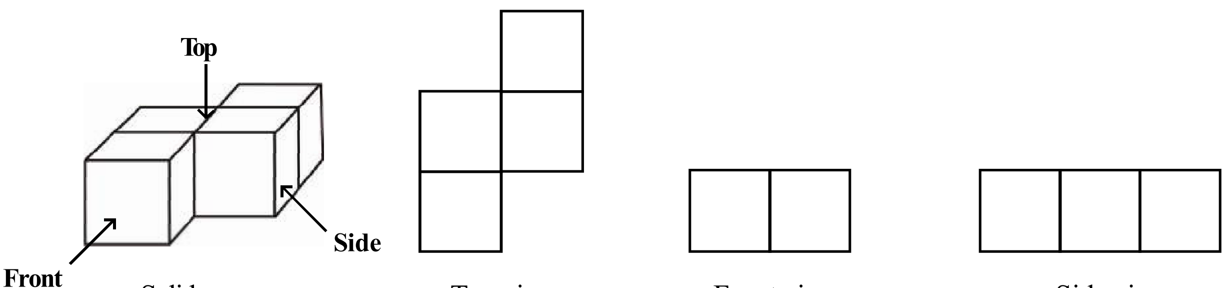


A brick | Front view | Side view | Top view

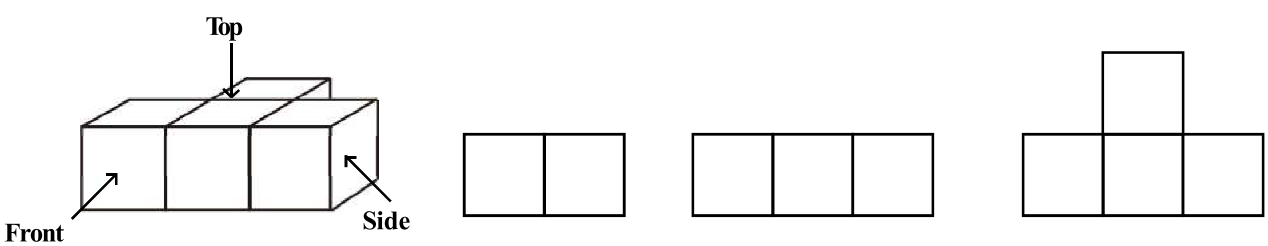
We can also get different views of figures made by joining cubes. For example.



Solid made of three cubes | Side view | Front view | Top view



Solid made of four cubes | Top view | Front view | Side view



Solid made of four cubes | Side view | Front view | Top view

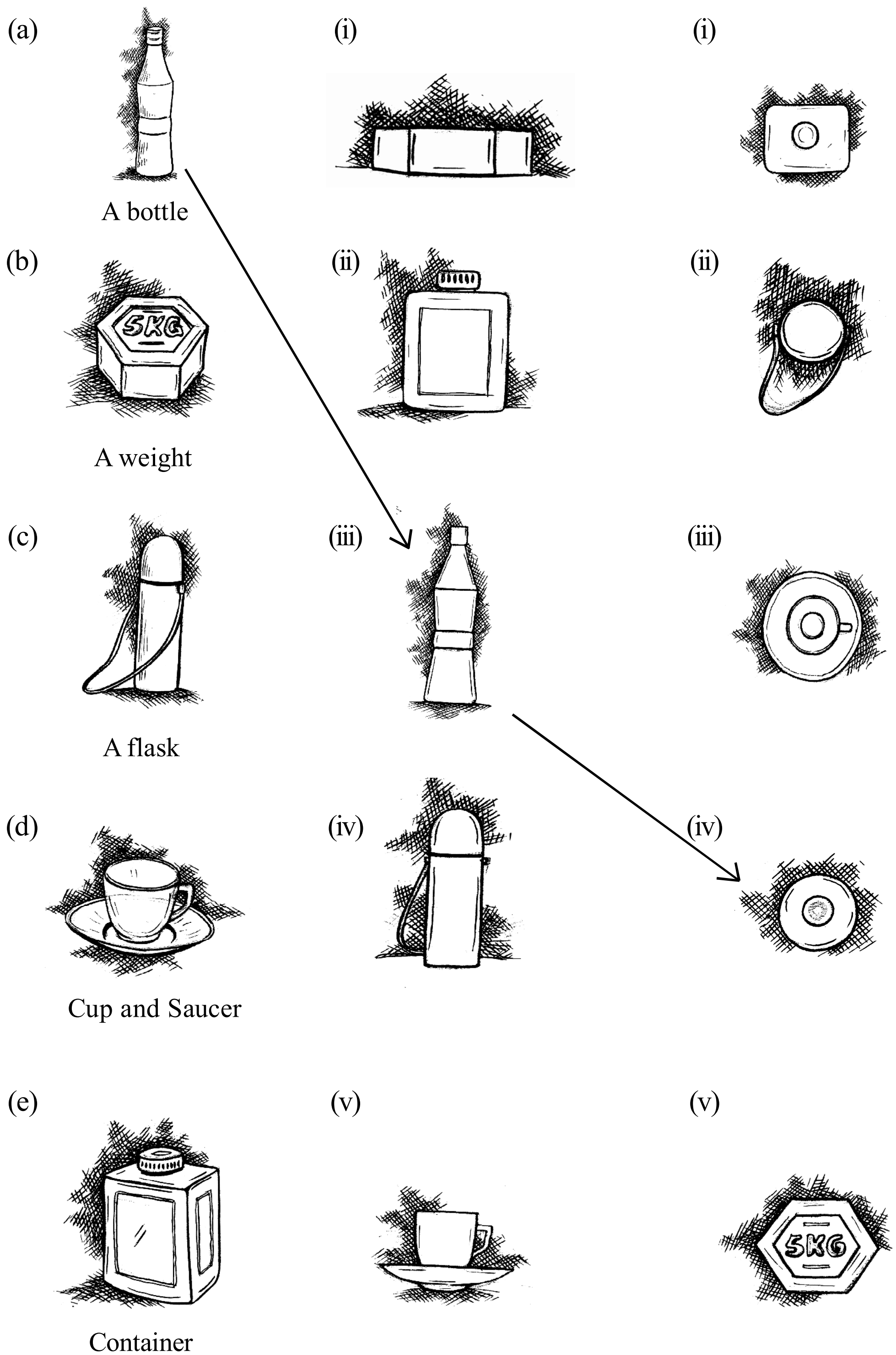
DO THIS

Observe different things around you from different positions. Discuss with your friends their various views.

EXERCISE 10.1

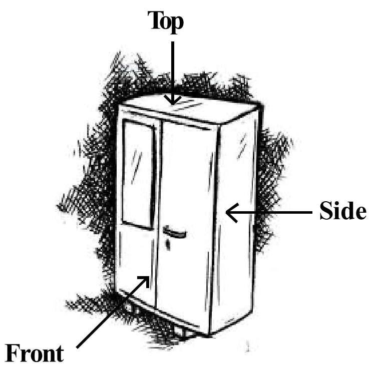
1. For each of the given solid, the two views are given. Match for each solid the corresponding top and front views. The first one is done for you.

Object **Side view** **Top view**

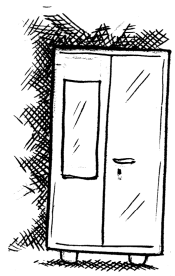


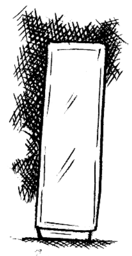
2. For each of the given solid, the three views are given. Identify for each solid the corresponding top, front and side views.

(a) **Object** (i) (ii) (iii)

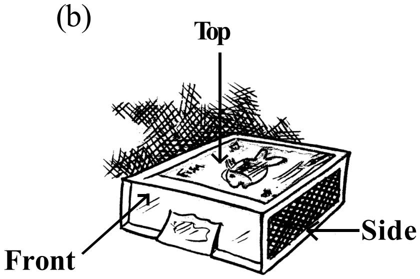


An almirah

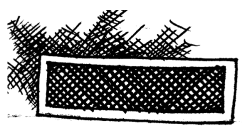


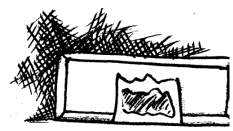


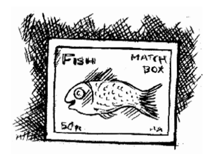
(b)



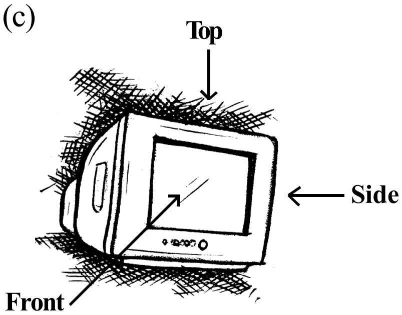
A Match box



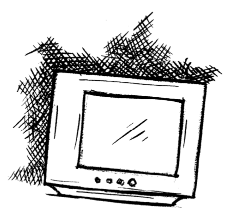


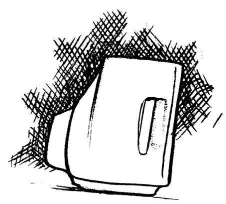


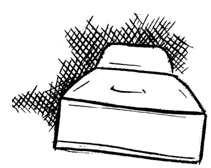
(c)



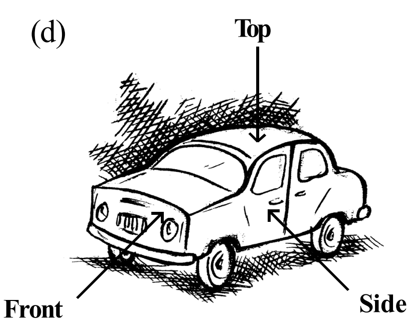
A Television



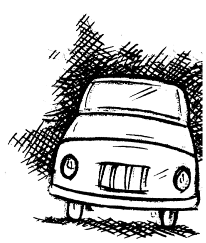


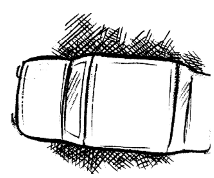


(d)



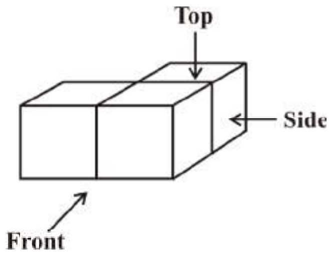
A car

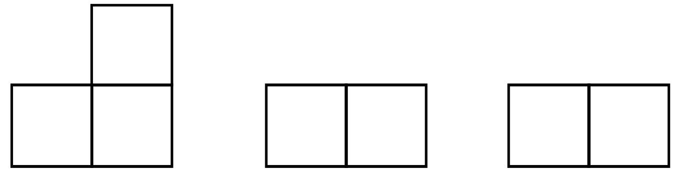




3. For each given solid, identify the top view, front view and side view.

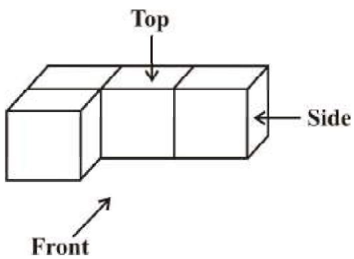
(a)

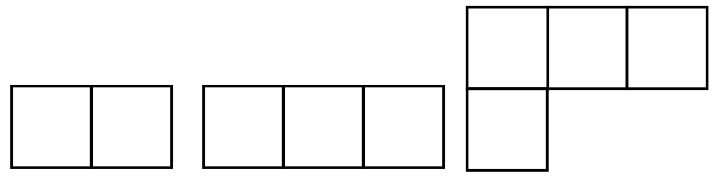




(i) (ii) (iii)

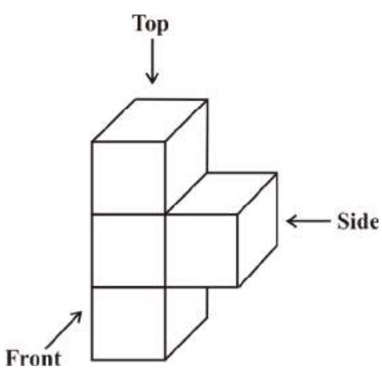
(b)

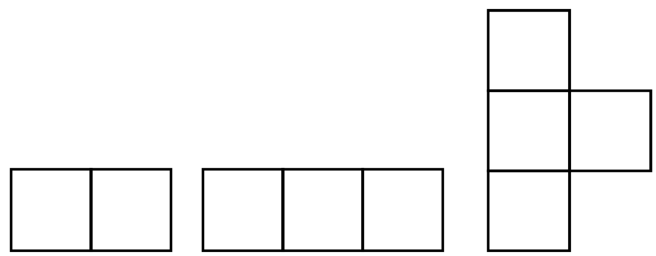




(i) (ii) (iii)

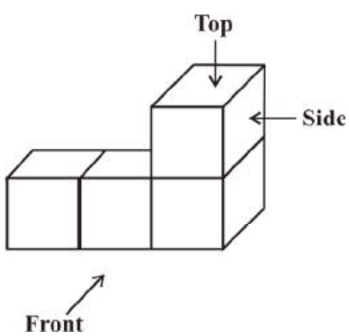
(c)

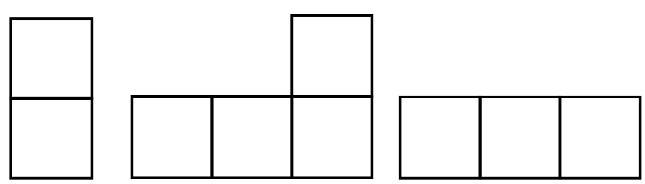




(i) (ii) (iii)

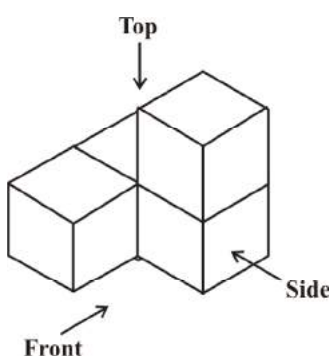
(d)

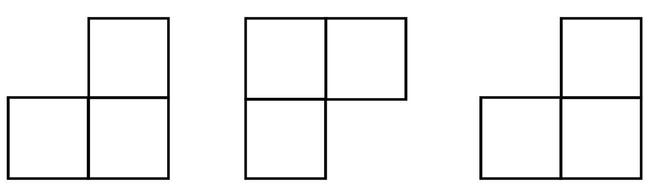




(i) (ii) (iii)

(e)

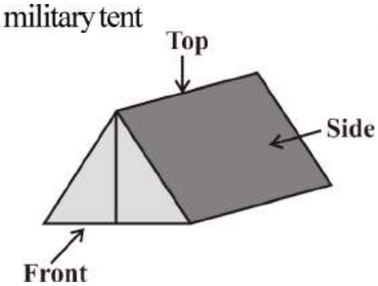




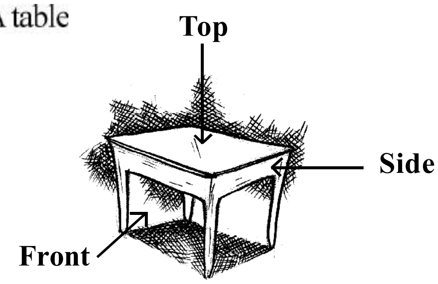
(i) (ii) (iii)

4. Draw the front view, side view and top view of the given objects.

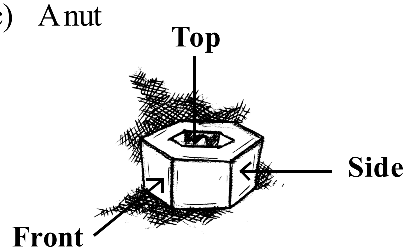
(a) A military tent



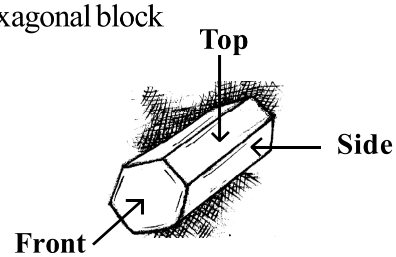
(b) A table



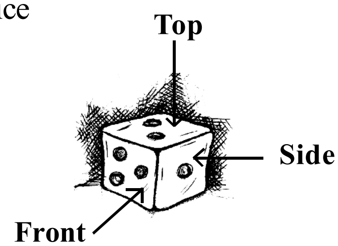
(c) A nut



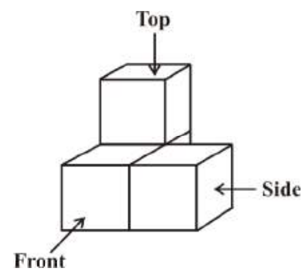
(d) A hexagonal block



(e) A dice



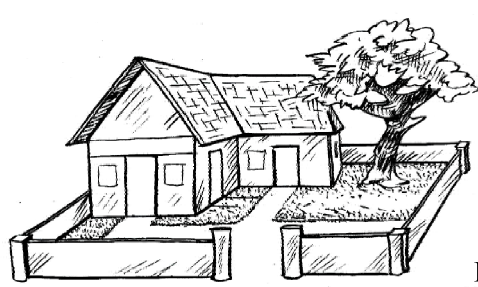
(f) A solid



10.3 Mapping Space Around Us

You have been dealing with maps since you were in primary, classes. In Geography, you have been asked to locate a particular State, a particular river, a mountain etc., on a map. In History, you might have been asked to locate a particular place where some event had occured long back. You have traced routes of rivers, roads, railwaylines, traders and many others.

How do we read maps? What can we conclude and understand while reading a map? What information does a map have and what it does not have? Is it any different from a picture? In this section, we will try to find answers to some of these questions. Look at the map of a house whose picture is given alongside (Fig 10.1).



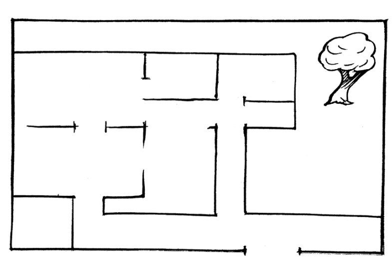


Fig 10.1

What can we conclude from the above illustration? When we draw a picture, we attempt to represent reality as it is seen with all its details, whereas, a map depicts only the location of an object, in relation to other objects. Secondly, different persons can give descriptions of pictures completely different from one another, depending upon the position from which they are looking at the house. But, this is not true in the case of a map. The map of the house remains the same irrespective of the position of the observer. In other words, **perspective is very important for drawing a picture but it is not relevant for a map.**

Now, look at the map (Fig 10.2), which has been drawn by seven year old Raghav, as the route from his house to his school:

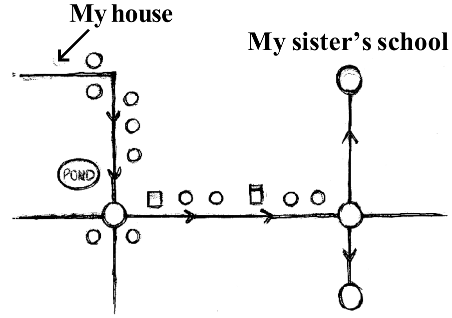


Fig 10.2

From this map, can you tell –

(i) how far is Raghav's school from his house?

(ii) would every circle in the map depict a round about?

(iii) whose school is nearer to the house, Raghav's or his sister's?

It is very difficult to answer the above questions on the basis of the given map. Can you tell why?

The reason is that we do not know if the distances have been drawn properly or whether the circles drawn are roundabouts or represent something else.

Now look at another map drawn by his sister, ten year old Meena, to show the route from her house to her school (Fig 10.3).

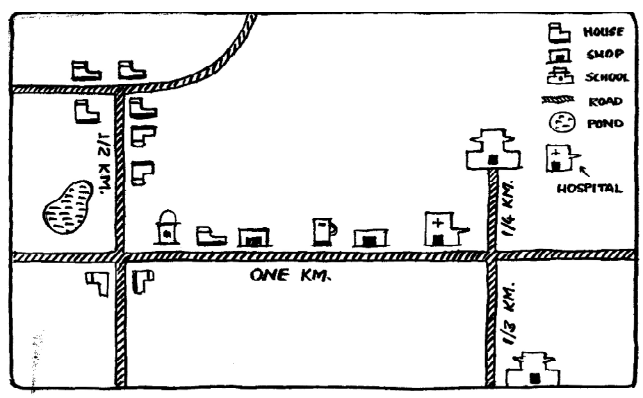


Fig 10.3

This map is different from the earlier maps. Here, Meena has used different symbols for different landmarks. Secondly, longer line segments have been drawn for longer distances and shorter line segments have been drawn for shorter distances, i.e., she has drawn the map to a scale.

Now, you can answer the following questions:

- How far is Raghav's school from his residence?
- Whose school is nearer to the house, Raghav's or Meena's?
- Which are the important landmarks on the route?

Thus we realise that, use of certain symbols and mentioning of distances has helped us read the map easily. Observe that the distances shown on the map are proportional to the actual distances on the ground. This is done by considering a proper scale. While drawing (or reading) a map, one must know, to what scale it has to be drawn (or has been drawn), i.e., how much of actual distance is denoted by 1mm or 1cm in the map. This means, that if one draws a map, he/she has to decide that 1cm of space in that map shows a certain fixed distance of say 1 km or 10 km. This scale can vary from map to map but not within a map. For instance, look at the map of India alongside the map of Delhi.

You will find that when the maps are drawn of same size, scales and the distances in the two maps will vary. That is 1 cm of space in the map of Delhi will represent smaller distances as compared to the distances in the map of India.

The larger the place and smaller the size of the map drawn, the greater is the distance represented by 1 cm.

Thus, we can summarise that:

1. A map depicts the location of a particular object/place in relation to other objects/places.
2. Symbols are used to depict the different objects/places.
3. There is no reference or perspective in map, i.e., objects that are closer to the observer are shown to be of the same size as those that are farther away. For example, look at the following illustration (Fig 10.4).

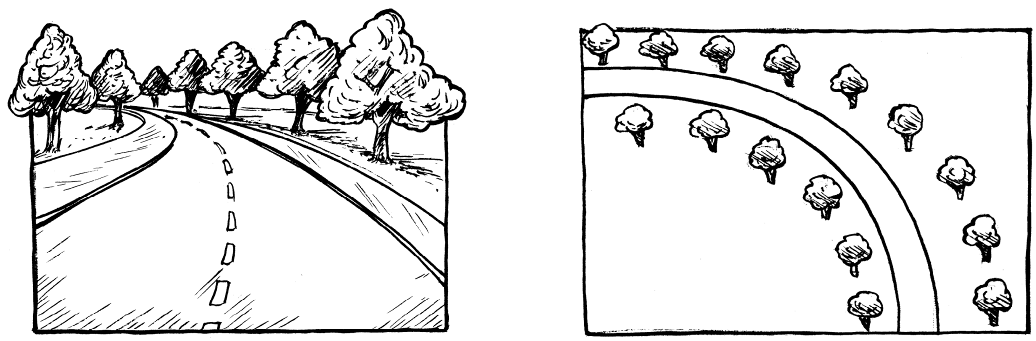


Fig 10.4

4. Maps use a scale which is fixed for a particular map. It reduces the real distances proportionately to distances on the paper.

DO THIS

1. Look at the following map of a city (Fig 10.5).

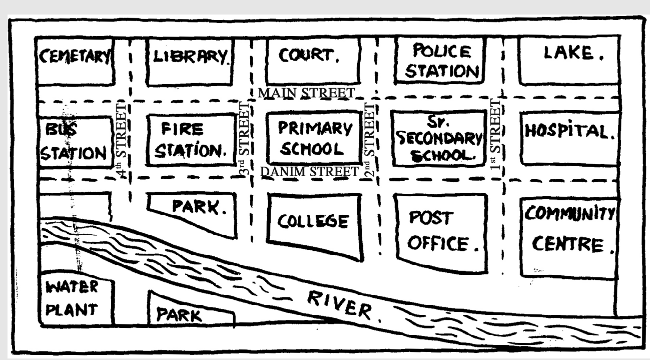


Fig 10.5

(a) Colour the map as follows: Blue-water, Red-fire station, Orange-Library, Yellow-schools, Green-Parks, Pink-Community Centre, Purple-Hospital, Brown-Cemetry.

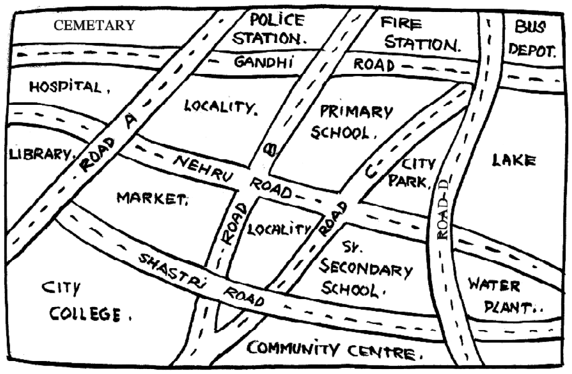
(b) Mark a Green 'X' at the intersection of 2nd street and Danim street. A Black 'Y' where the river meets the third street. A red 'Z' at the intersection of main street and 1st street.

(c) In magenta colour, draw a short street route from the college to the lake.

2. Draw a map of the route from your house to your school showing important landmarks.

EXERCISE 10.2

1. Look at the given map of a city.



Answer the following.

(a) Colour the map as follows: Blue-water, red-fire station, orange-library, yellow - schools, Green - park, Pink - College, Purple - Hospital, Brown - Cemetery.

(b) Mark a green 'X' at the intersection of Road 'C' and Nehru Road, Green 'Y' at the intersection of Gandhi Road and Road A.

(c) In red, draw a short street route from Library to the bus depot.

(d) Which is further east, the city park or the market?

(e) Which is further south, the primary school or the Sr. Secondary School?

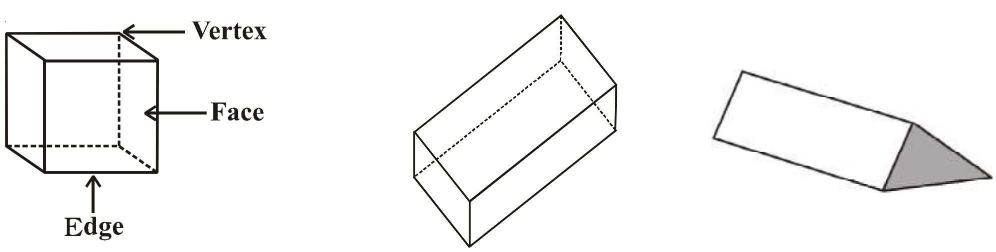
2. Draw a map of your class room using proper scale and symbols for different objects.

3. Draw a map of your school compound using proper scale and symbols for various features like play ground main building, garden etc.

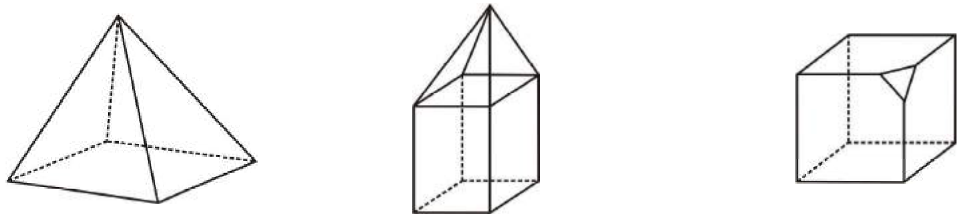
4. Draw a map giving instructions to your friend so that she reaches your house without any difficulty.

10.4 Faces, Edges and Vertices

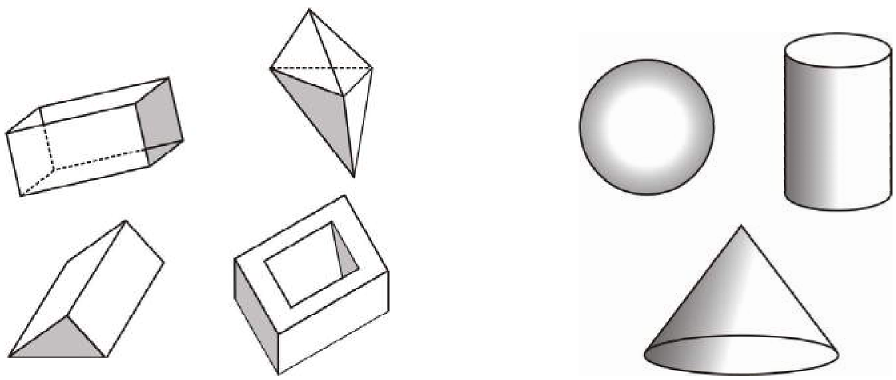
Look at the following solids!



Riddle
I have no vertices.
I have no flat faces. Who am I?

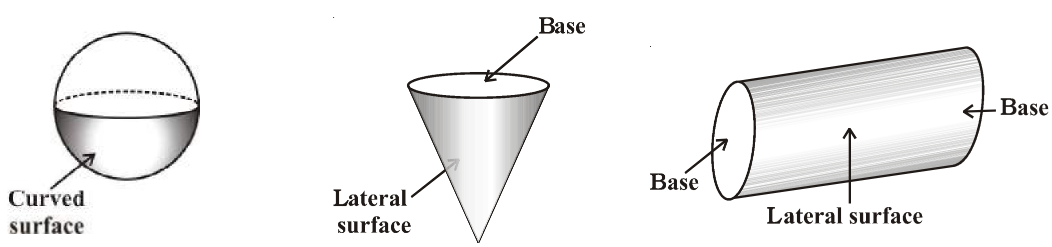


Each of these solids is made up of polygonal regions which are called its **faces**; these faces meet at **edges** which are line segments; and the edges meet at vertices which are **points**. Such solids are called **polyhedrons**.



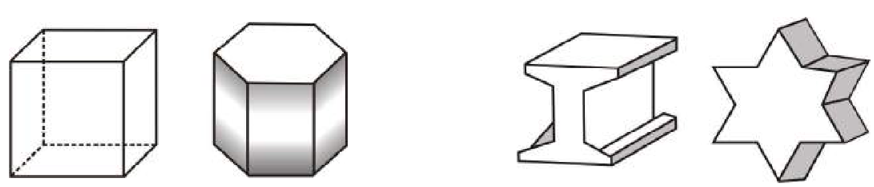
These are polyhedrons **These are not polyhedrons**

How are the polyhedrons different from the non-polyhedrons? Study the figures carefully. You know three other types of common solids.



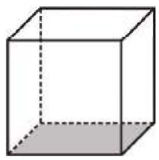
Sphere **Cone** **Cylinder**

Convex polyhedrons: You will recall the concept of convex polygons. The idea of convex polyhedron is similar.

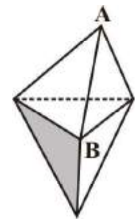


These are convex polyhedrons **These are not convex polyhedrons**

Regular polyhedrons: A polyhedron is said to be **regular** if its faces are made up of regular polygons and the same number of faces meet at **each** vertex.

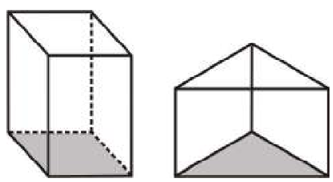


This polyhedron is regular.
Its faces are congruent, regular polygons. Vertices are formed by the same number of faces

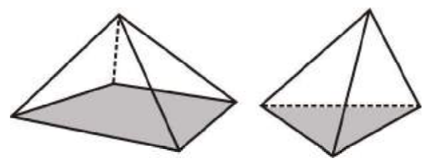


This polyhedon is not regular. All the sides are congruent; but the vertices are not formed by the same number of faces.
3 faces meet at A but
4 faces meet at B.

Two important members of polyhedron family around are prisms and pyramids.



These are prisms



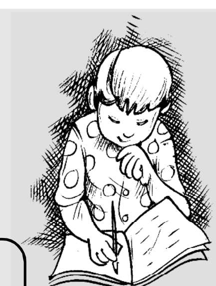
These are pyramids

We say that **a prism** is a polyhedron whose base and top are congruent polygons and whose other faces, i.e., lateral faces are parallelograms in shape.

On the other hand, **a pyramid** is a polyhedron whose base is a polygon (of any number of sides) and whose lateral faces are triangles with a common vertex. (If you join all the corners of a polygon to a point not in its plane, you get a model for pyramid).

A prism or a pyramid is named after its base. Thus a hexagonal prism has a hexagon as its base; and a triangular pyramid has a triangle as its base. What, then, is a rectangular prism? What is a square pyramid? Clearly their bases are rectangle and square respectively.

DO THIS



Tabulate the number of faces, edges and vertices for the following polyhedrons: (Here 'V' stands for number of vertices, 'F' stands for number of faces and 'E' stands for number of edges).

Solid	**F**	**V**	**E**	**F+V**	**E+2**
Cuboid					
Triangular pyramid					
Triangular prism					
Pyramid with square base					
Prism with square base					

What do you infer from the last two columns? In each case, do you find $F + V = E + 2$, i.e., $F + V - E = 2$? This relationship is called **Euler's formula.** In fact this formula is true for any polyhedron.

THINK, DISCUSS AND WRITE

What happens to F, V and E if some parts are sliced off from a solid? (To start with, you may take a plasticine cube, cut a corner off and investigate).

EXERCISE 10.3

1. Can a polyhedron have for its faces
   (i) 3 triangles? (ii) 4 triangles?
   (iii) a square and four triangles?

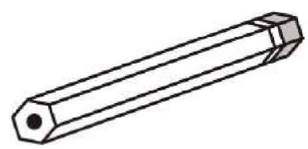
2. Is it possible to have a polyhedron with any given number of faces? (**Hint:** Think of a pyramid).

3. Which are prisms among the following?

(i)

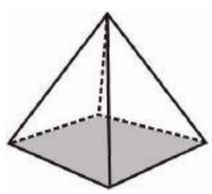
A nail

(ii)



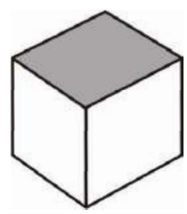
Unsharpened pencil

(iii)



A table weight

(iv)

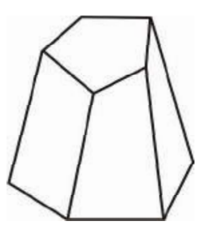


A box

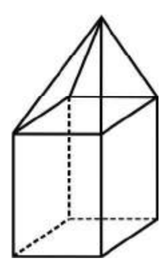
4. (i) How are prisms and cylinders alike?
   (ii) How are pyramids and cones alike?

5. Is a square prism same as a cube? Explain.

6. Verify Euler's formula for these solids.



(i)



(ii)

7. Using Euler's formula find the unknown.

Faces	?	5	20
Vertices	6	?	12
Edges	12	9	?

8. Can a polyhedron have 10 faces, 20 edges and 15 vertices?

WHAT HAVE WE DISCUSSED?

1. Recognising 2D and 3D objects.

2. Recognising different shapes in nested objects.

3. 3D objects have different views from different positions.

4. A map is different from a picture.

5. A map depicts the location of a particular object/place in relation to other objects/places.

6. Symbols are used to depict the different objects/places.

7. There is no reference or perspective in a map.

8. Maps involve a scale which is fixed for a particular map.

9. For any polyhedron,

$$F + V - E = 2$$

where 'F' stands for number of faces, V stands for number of vertices and E stands for number of edges. This relationship is called **Euler's formula**.

NOTES

Mensuration

CHAPTER 11

11.1 Introduction

We have learnt that for a closed plane figure, the perimeter is the distance around its boundary and its area is the region covered by it. We found the area and perimeter of various plane figures such as triangles, rectangles, circles etc. We have also learnt to find the area of pathways or borders in rectangular shapes.

In this chapter, we will try to solve problems related to perimeter and area of other plane closed figures like quadrilaterals.

We will also learn about surface area and volume of solids such as cube, cuboid and cylinder.

11.2 Let us Recall

Let us take an example to review our previous knowledge.

This is a figure of a rectangular park (Fig 11.1) whose length is 30 m and width is 20 m.

(i) What is the total length of the fence surrounding it? To find the length of the fence we need to find the perimeter of this park, which is 100 m. (Check it)

(ii) How much land is occupied by the park? To find the land occupied by this park we need to find the area of this park which is 600 square meters (m^2) (How?).

(iii) There is a path of one metre width running inside along the perimeter of the park that has to be cemented. If 1 bag of cement is required to cement 4 m^2 area, how many bags of cement would be required to construct the cemented path?

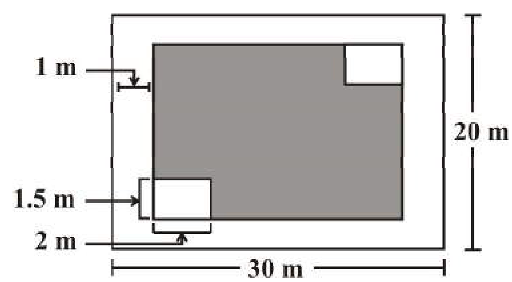


Fig 11.1

We can say that the number of cement bags used = $\dfrac{\text{area of the path}}{\text{area cemented by 1 bag}}$.

Area of cemented path = Area of park – Area of park not cemented.

Path is 1 m wide, so the rectangular area not cemented is $(30 - 2) \times (20 - 2)$ m^2.

That is 28×18 m^2.

Hence number of cement bags used = ------------------

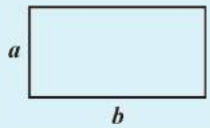
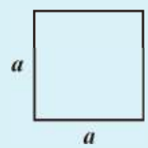
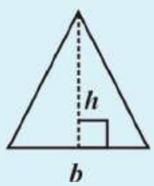
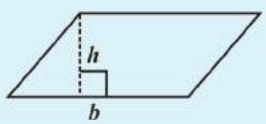
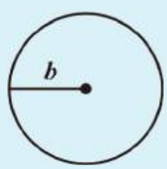
(iv) There are two rectangular flower beds of size 1.5 m × 2 m each in the park as shown in the diagram (Fig 11.1) and the rest has grass on it. Find the area covered by grass.

Area of rectangular beds = ------------------

Area of park left after cementing the path = ------------------

Area covered by the grass = ------------------

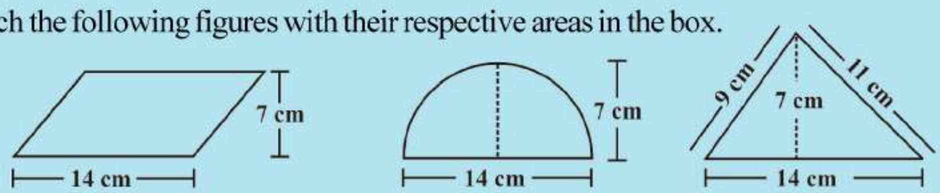
We can find areas of geometrical shapes other than rectangles also if certain measurements are given to us . Try to recall and match the following:

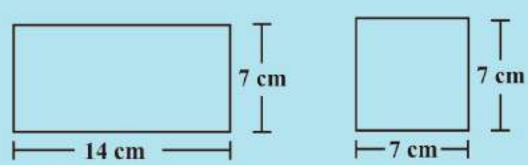
Diagram	Shape	Area
	rectangle	$a \times a$
	square	$b \times h$
	triangle	πb^2
	parallelogram	$\frac{1}{2} b \times h$
	circle	$a \times b$

Can you write an expression for the perimeter of each of the above shapes?

TRY THESE

(a) Match the following figures with their respective areas in the box.



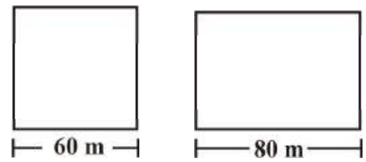


49 cm^2
77 cm^2
98 cm^2

(b) Write the perimeter of each shape.

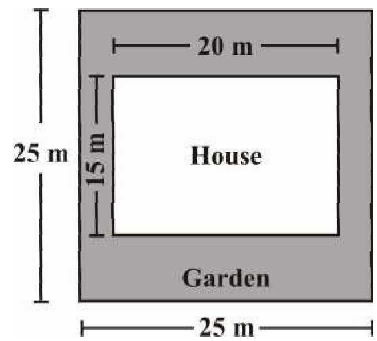
EXERCISE 11.1

1. A square and a rectangular field with measurements as given in the figure have the same perimeter. Which field has a larger area?

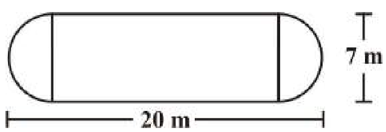


(a) (b)

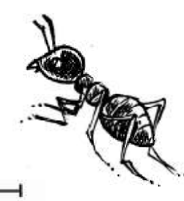
2. Mrs. Kaushik has a square plot with the measurement as shown in the figure. She wants to construct a house in the middle of the plot. A garden is developed around the house. Find the total cost of developing a garden around the house at the rate of ₹ 55 per m^2.



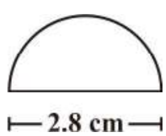
3. The shape of a garden is rectangular in the middle and semi circular at the ends as shown in the diagram. Find the area and the perimeter of this garden [Length of rectangle is 20 – (3.5 + 3.5) metres].



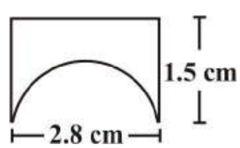
4. A flooring tile has the shape of a parallelogram whose base is 24 cm and the corresponding height is 10 cm. How many such tiles are required to cover a floor of area 1080 m^2? (If required you can split the tiles in whatever way you want to fill up the corners).
5. An ant is moving around a few food pieces of different shapes scattered on the floor. For which food-piece would the ant have to take a longer round? Remember, circumference of a circle can be obtained by using the expression $c = 2\pi r$, where r is the radius of the circle.



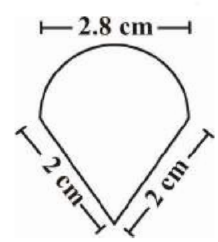
(a)



(b)



(c)



11.3 Area of Trapezium

Nazma owns a plot near a main road (Fig 11.2). Unlike some other rectangular plots in her neighbourhood, the plot has only one pair of parallel opposite sides. So, it is nearly a trapezium in shape. Can you find out its area?

Let us name the vertices of this plot as shown in Fig 11.3.

By drawing EC || AB, we can divide it into two parts, one of rectangular shape and the other of triangular shape, (which is right angled at C), as shown in Fig 11.3.

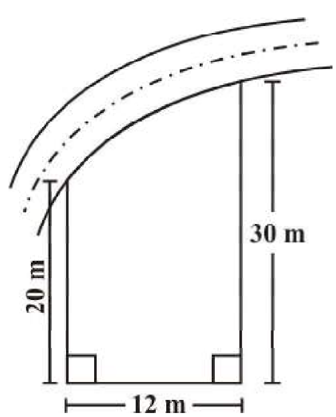


Fig 11.2

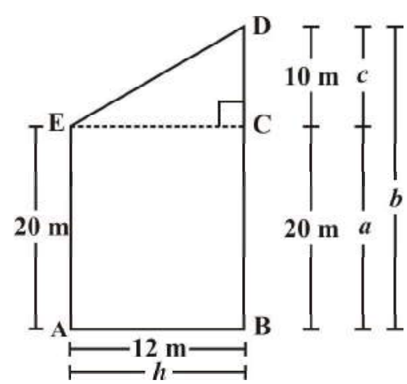


Fig 11.3

($b = c + a = 30$ m)

Area of Δ ECD $= \frac{1}{2} h \times c = \frac{1}{2} \times 12 \times 10 = 60 \text{ m}^2$.

Area of rectangle ABCE $= h \times a = 12 \times 20 = 240 \text{ m}^2$.

Area of trapezium ABDE = area of Δ ECD + Area of rectangle ABCE = 60 + 240 = 300 m^2.

We can write the area by combining the two areas and write the area of trapezium as

$$\text{area of ABDE} = \frac{1}{2} h \times c + h \times a = h\left(\frac{c}{2} + a\right)$$

$$= h\left(\frac{c + 2a}{2}\right) = h\left(\frac{c + a + a}{2}\right)$$

$$= h\frac{(b+a)}{2} = \text{height } \frac{(\text{sum of parallel sides})}{2}$$

By substituting the values of h, b and a in this expression, we find $h\frac{(b+a)}{2} = 300 \text{ m}^2$.

TRY THESE

1. Nazma's sister also has a trapezium shaped plot. Divide it into three parts as shown (Fig 11.4). Show that the area of trapezium WXYZ $= h\frac{(a+b)}{2}$.

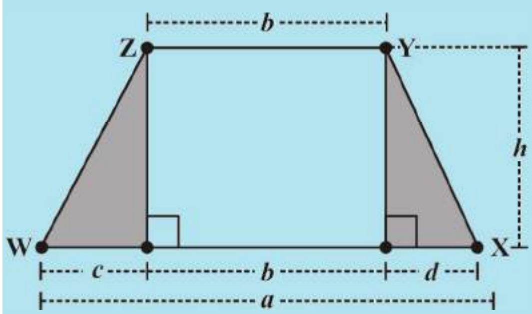
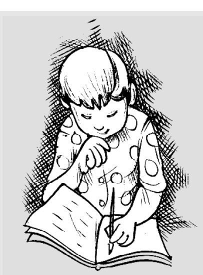


Fig 11.4

2. If $h = 10$ cm, $c = 6$ cm, $b = 12$ cm, $d = 4$ cm, find the values of each of its parts separetely and add to find the area WXYZ. Verify it by putting the values of h, a and b in the expression $\frac{h(a+b)}{2}$.

DO THIS



1. Draw any trapezium WXYZ on a piece of graph paper as shown in the figure and cut it out (Fig 11.5).

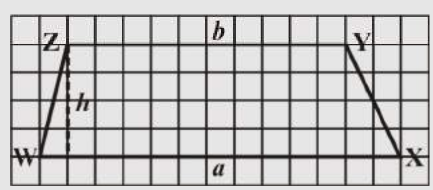


Fig 11.5

2. Find the mid point of XY by folding the side and name it A (Fig 11.6).

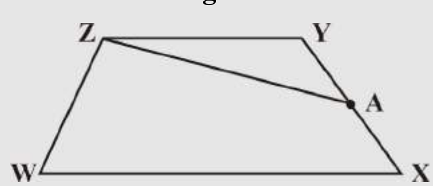


Fig 11.6

3. Cut trapezium WXYZ into two pieces by cutting along ZA. Place Δ ZYA as shown in Fig 11.7, where AY is placed on AX.

   What is the length of the base of the larger triangle? Write an expression for the area of this triangle (Fig 11.7).

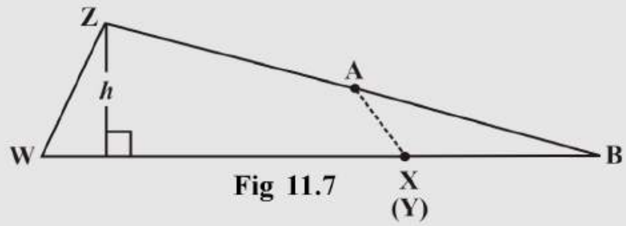


Fig 11.7

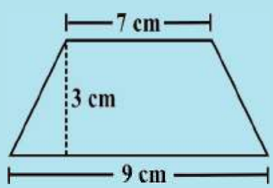
4. The area of this triangle and the area of the trapezium WXYZ are same (How?). Get the expression for the area of trapezium by using the expression for the area of triangle.

So to find the area of a trapezium we need to know the length of the parallel sides and the perpendicular distance between these two parallel sides. Half the product of the sum of the lengths of parallel sides and the perpendicular distance between them gives the area of trapezium.

TRY THESE

Find the area of the following trapeziums (Fig 11.8).

(i)



(ii)

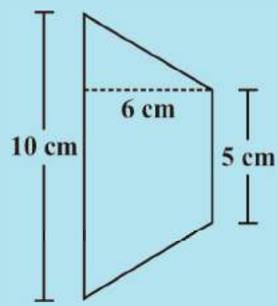


Fig 11.8

DO THIS

In Class VII we learnt to draw parallelograms of equal areas with different perimeters. Can it be done for trapezium? Check if the following trapeziums are of equal areas but have different perimeters (Fig 11.9).

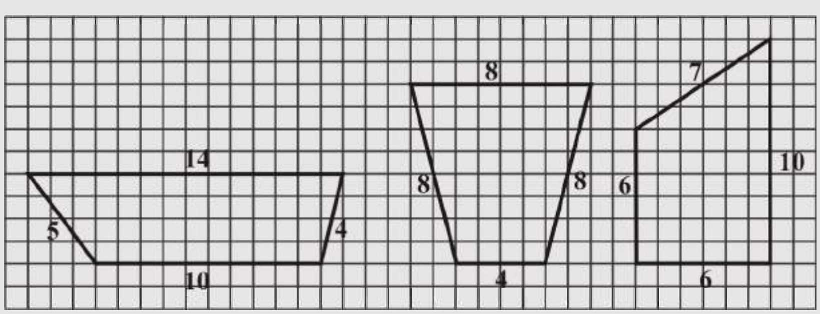


Fig 11.9

We know that all congruent figures are equal in area. Can we say figures equal in area need to be congruent too? Are these figures congruent?

Draw at least three trapeziums which have different areas but equal perimeters on a squared sheet.

11.4 Area of a General Quadrilateral

A general quadrilateral can be split into two triangles by drawing one of its diagonals. This "triangulation" helps us to find a formula for any general quadrilateral. Study the Fig 11.10.

Area of quadrilateral ABCD

$= (\text{area of } \Delta \text{ABC}) + (\text{area of } \Delta \text{ADC})$

$= (\frac{1}{2} \text{AC} \times h_1) + (\frac{1}{2} \text{AC} \times h_2)$

$= \frac{1}{2} \text{AC} \times (h_1 + h_2)$

$= \frac{1}{2} d(h_1 + h_2)$ where d denotes the length of diagonal AC.

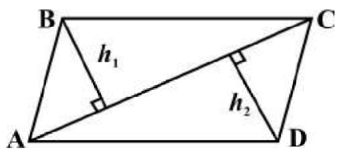


Fig 11.10

Example 1: Find the area of quadrilateral PQRS shown in Fig 11.11.

Solution: In this case, $d = 5.5$ cm, $h_1 = 2.5$cm, $h_2 = 1.5$ cm,

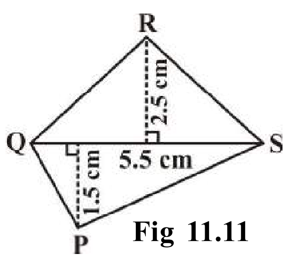


Fig 11.11

$$\text{Area} = \frac{1}{2} d(h_1 + h_2)$$

$$= \frac{1}{2} \times 5.5 \times (2.5 + 1.5) \text{ cm}^2$$

$$= \frac{1}{2} \times 5.5 \times 4 \text{ cm}^2 = 11 \text{ cm}^2$$

TRY THESE

We know that parallelogram is also a quadrilateral. Let us also split such a quadrilateral into two triangles, find their areas and hence that of the parallelogram. Does this agree with the formula that you know already? (Fig 11.12)

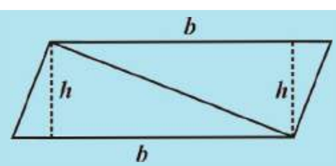


Fig 11.12

11.4.1 Area of special quadrilaterals

We can use the same method of splitting into triangles (which we called "triangulation") to find a formula for the area of a rhombus. In Fig 11.13 ABCD is a rhombus. Therefore, its diagonals are perpendicular bisectors of each other.

Area of rhombus ABCD = (area of Δ ACD) + (area of Δ ABC)

$$= \left(\frac{1}{2} \times AC \times OD\right) + \left(\frac{1}{2} \times AC \times OB\right) = \frac{1}{2} AC \times (OD + OB)$$

$$= \frac{1}{2} AC \times BD = \frac{1}{2} d_1 \times d_2 \quad \text{where } AC = d_1 \text{ and } BD = d_2$$

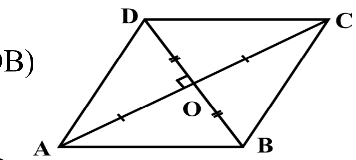


Fig 11.13

In other words, area of a rhombus is half the product of its diagonals.

Example 2: Find the area of a rhombus whose diagonals are of lengths 10 cm and 8.2 cm.

Solution: Area of the rhombus $= \frac{1}{2} d_1 d_2$ where d_1, d_2 are lengths of diagonals.

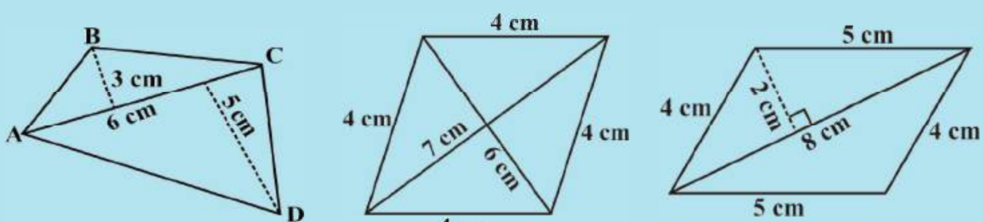
$$= \frac{1}{2} \times 10 \times 8.2 \text{ cm}^2 = 41 \text{ cm}^2.$$

THINK, DISCUSS AND WRITE

A parallelogram is divided into two congruent triangles by drawing a diagonal across it. Can we divide a trapezium into two congruent triangles?

TRY THESE

Find the area of these quadrilaterals (Fig 11.14).



(i) (ii) (iii)

Fig 11.14

11.5 Area of a Polygon

We split a quadrilateral into triangles and find its area. Similar methods can be used to find the area of a polygon. Observe the following for a pentagon: (Fig 11.15, 11.16)

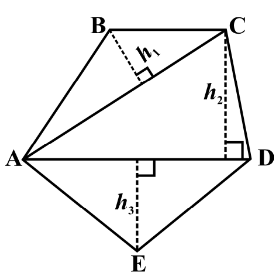


Fig 11.15

By constructing two diagonals AC and AD the pentagon ABCDE is divided into three parts. So, area ABCDE = area of Δ ABC + area of Δ ACD + area of Δ AED.

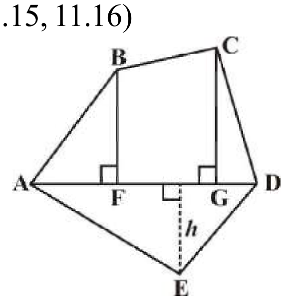
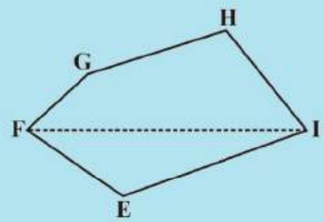


Fig 11.16

By constructing one diagonal AD and two perpendiculars BF and CG on it, pentagon ABCDE is divided into four parts. So, area of ABCDE = area of right angled Δ AFB + area of trapezium BFGC + area of right angled Δ CGD + area of Δ AED. (Identify the parallel sides of trapezium BFGC.)

TRY THESE

(i) Divide the following polygons (Fig 11.17) into parts (triangles and trapezium) to find out its area.



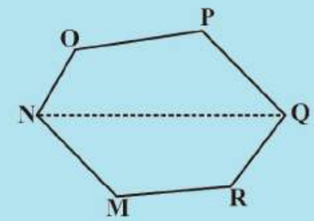


Fig 11.17

FI is a diagonal of polygon EFGHI   NQ is a diagonal of polygon MNOPQR

(ii) Polygon ABCDE is divided into parts as shown below (Fig 11.18). Find its area if AD = 8 cm, AH = 6 cm, AG = 4 cm, AF = 3 cm and perpendiculars BF = 2 cm, CH = 3 cm, EG = 2.5 cm.

Area of Polygon ABCDE = area of Δ AFB + ...

Area of Δ AFB = $\frac{1}{2} \times \text{AF} \times \text{BF} = \frac{1}{2} \times 3 \times 2 = ...$

Area of trapezium FBCH = FH $\times \frac{(\text{BF} + \text{CH})}{2}$

$= 3 \times \frac{(2+3)}{2}$ [FH = AH – AF]

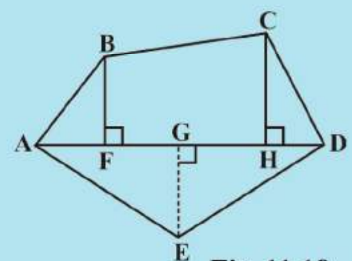


Fig 11.18

Area of ΔCHD = $\frac{1}{2} \times \text{HD} \times \text{CH}$ = ...;   Area of ΔADE = $\frac{1}{2} \times \text{AD} \times \text{GE}$ = ...

So, the area of polygon ABCDE = ...

(iii) Find the area of polygon MNOPQR (Fig 11.19) if MP = 9 cm, MD = 7 cm, MC = 6 cm, MB = 4 cm, MA = 2 cm

NA, OC, QD and RB are perpendiculars to diagonal MP.

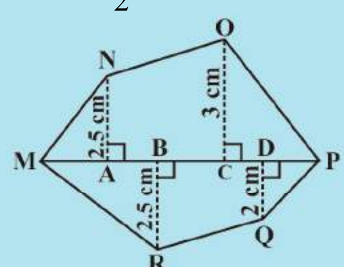


Fig 11.19

Example 1: The area of a trapezium shaped field is 480 m², the distance between two parallel sides is 15 m and one of the parallel side is 20 m. Find the other parallel side.

Solution: One of the parallel sides of the trapezium is a = 20 m, let another parallel side be b, height h = 15 m.

The given area of trapezium = 480 m².

$$\text{Area of a trapezium} = \frac{1}{2} h (a + b)$$

$$\text{So} \quad 480 = \frac{1}{2} \times 15 \times (20 + b) \quad \text{or} \quad \frac{480 \times 2}{15} = 20 + b$$

$$\text{or} \quad 64 = 20 + b \quad \text{or} \quad b = 44 \text{ m}$$

Hence the other parallel side of the trapezium is 44 m.

Example 2: The area of a rhombus is 240 cm^2 and one of the diagonals is 16 cm. Find the other diagonal.

Solution: Let length of one diagonal $d_1 = 16$ cm

and length of the other diagonal $= d_2$

$$\text{Area of the rhombus} = \frac{1}{2}\, d_1 \,.\, d_2 = 240$$

So, $$\frac{1}{2} 16 \cdot d_2 = 240$$

Therefore, $$d_2 = 30 \text{ cm}$$

Hence the length of the second diagonal is 30 cm.

Example 3: There is a hexagon MNOPQR of side 5 cm (Fig 11.20). Aman and Ridhima divided it in two different ways (Fig 11.21).

Find the area of this hexagon using both ways.

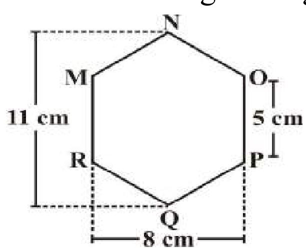
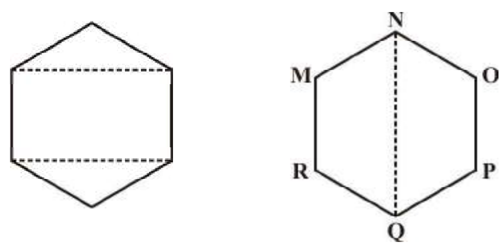


Fig 11.20



Ridhima's method

Aman's method

Fig 11.21

Solution: **Aman's method:**

Since it is a hexagon so NQ divides the hexagon into two congruent trapeziums. You can verify it by paper folding (Fig 11.22).

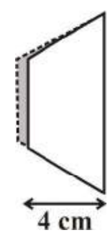


Fig 11.22

$$\text{Now area of trapezium MNQR} = 4 \times \frac{(11+5)}{2} = 2 \times 16 = 32 \text{ cm}^2.$$

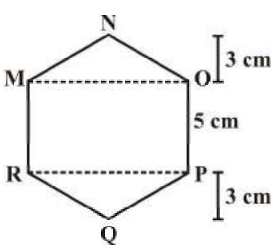


Fig 11.23

So the area of hexagon MNOPQR $= 2 \times 32 = 64$ cm^2.

Ridhima's method:

Δ MNO and Δ RPQ are congruent triangles with altitude 3 cm (Fig 11.23).

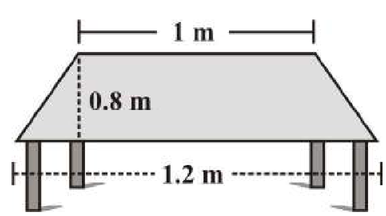
You can verify this by cutting off these two triangles and placing them on one another.

$$\text{Area of } \Delta \text{ MNO} = \frac{1}{2} \times 8 \times 3 = 12 \text{ cm}^2 = \text{Area of } \Delta \text{ RPQ}$$

Area of rectangle MOPR $= 8 \times 5 = 40$ cm^2.

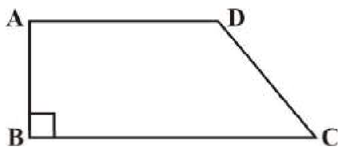
Now, area of hexagon MNOPQR $= 40 + 12 + 12 = 64$ cm^2.

EXERCISE 11.2



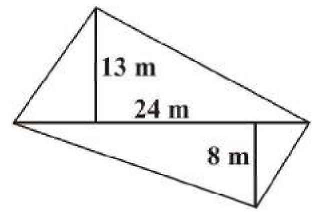
1. The shape of the top surface of a table is a trapezium. Find its area if its parallel sides are 1 m and 1.2 m and perpendicular distance between them is 0.8 m.

2. The area of a trapezium is 34 cm^2 and the length of one of the parallel sides is 10 cm and its height is 4 cm. Find the length of the other parallel side.



3. Length of the fence of a trapezium shaped field ABCD is 120 m. If BC = 48 m, CD = 17 m and AD = 40 m, find the area of this field. Side AB is perpendicular to the parallel sides AD and BC.

4. The diagonal of a quadrilateral shaped field is 24 m and the perpendiculars dropped on it from the remaining opposite vertices are 8 m and 13 m. Find the area of the field.

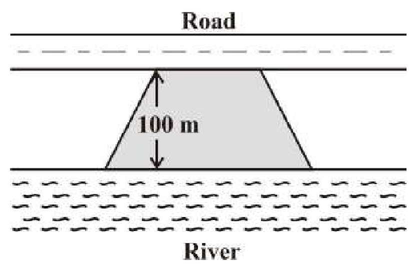


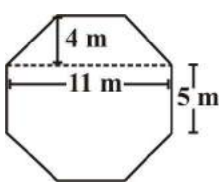
5. The diagonals of a rhombus are 7.5 cm and 12 cm. Find its area.

6. Find the area of a rhombus whose side is 5 cm and whose altitude is 4.8 cm. If one of its diagonals is 8 cm long, find the length of the other diagonal.

7. The floor of a building consists of 3000 tiles which are rhombus shaped and each of its diagonals are 45 cm and 30 cm in length. Find the total cost of polishing the floor, if the cost per m^2 is ₹ 4.

8. Mohan wants to buy a trapezium shaped field. Its side along the river is parallel to and twice the side along the road. If the area of this field is 10500 m^2 and the perpendicular distance between the two parallel sides is 100 m, find the length of the side along the river.

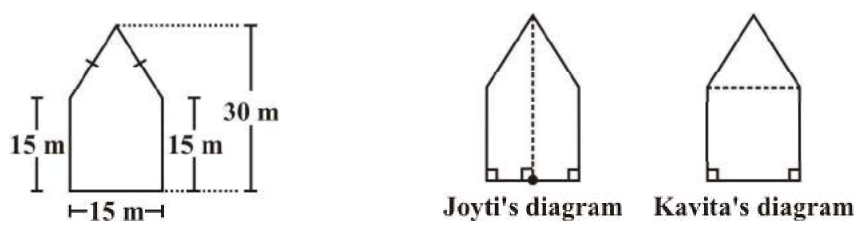




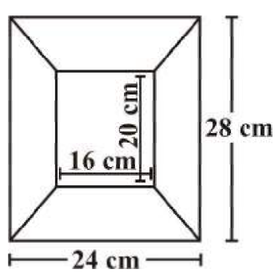
9. Top surface of a raised platform is in the shape of a regular octagon as shown in the figure. Find the area of the octagonal surface.

10. There is a pentagonal shaped park as shown in the figure.

For finding its area Jyoti and Kavita divided it in two different ways.



Find the area of this park using both ways. Can you suggest some other way of finding its area?



11. Diagram of the adjacent picture frame has outer dimensions = 24 cm × 28 cm and inner dimensions 16 cm × 20 cm. Find the area of each section of the frame, if the width of each section is same.

11.6 Solid Shapes

In your earlier classes you have studied that two dimensional figures can be identified as the faces of three dimensional shapes. Observe the solids which we have discussed so far (Fig 11.24).

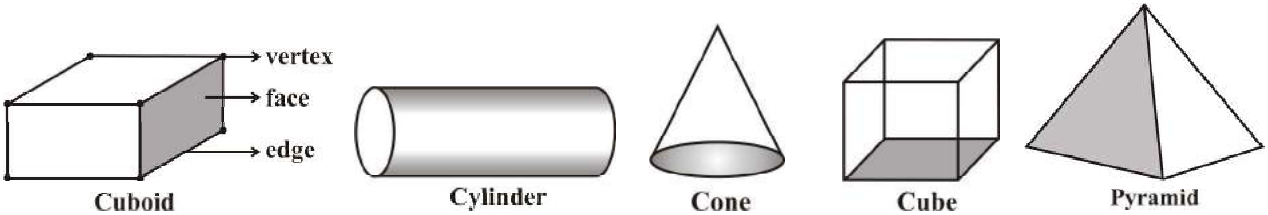


Fig 11.24

Observe that some shapes have two or more than two identical (congruent) faces. Name them. Which solid has all congruent faces?

DO THIS

Soaps, toys, pastes, snacks etc. often come in the packing of cuboidal, cubical or cylindrical boxes. Collect, such boxes (Fig 11.25).

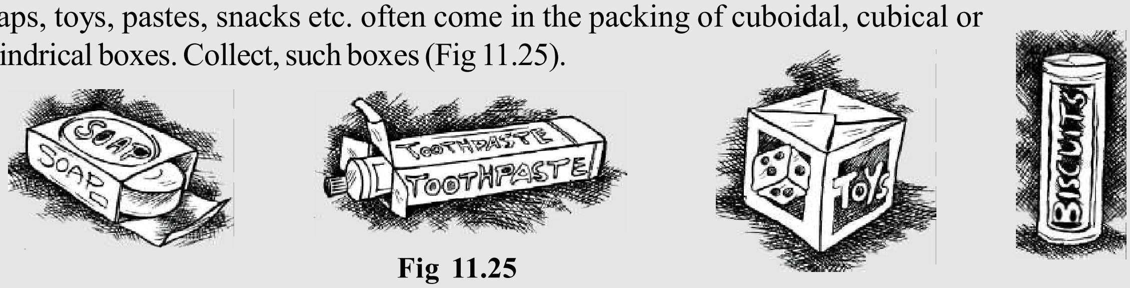
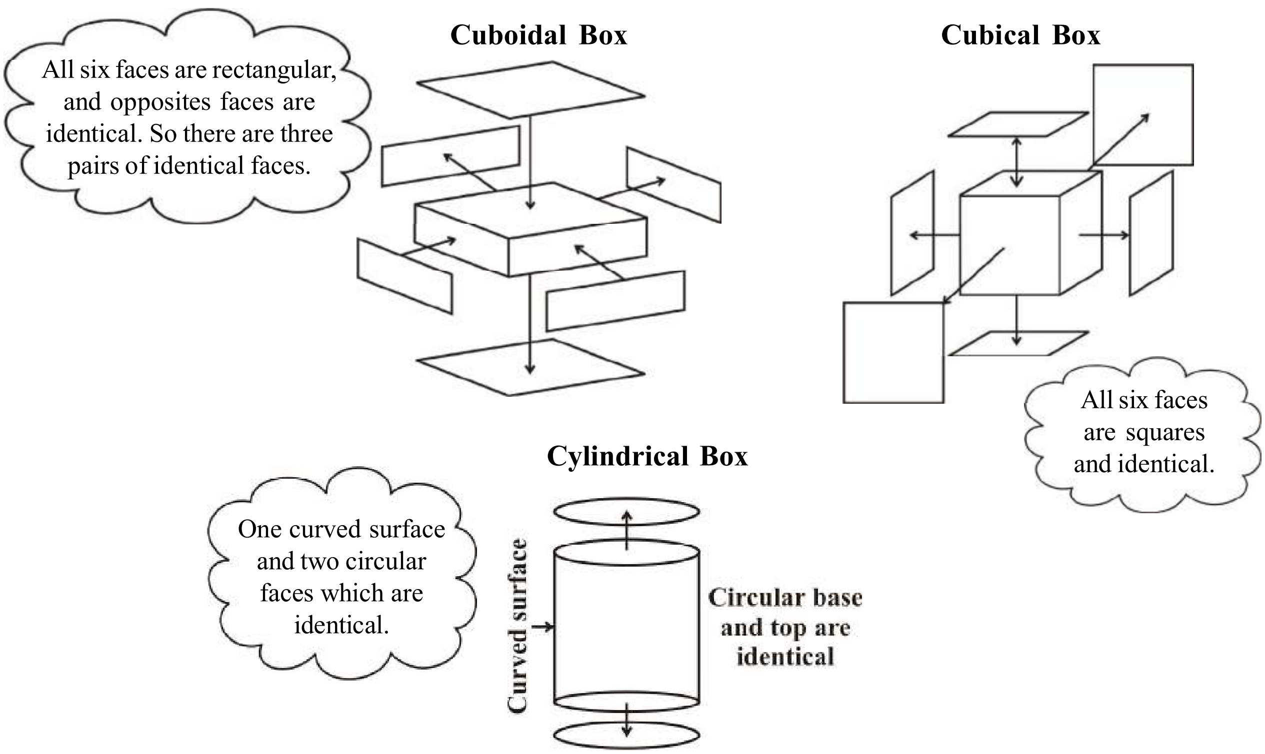


Fig 11.25



Now take one type of box at a time. Cut out all the faces it has. Observe the shape of each face and find the number of faces of the box that are identical by placing them on each other. Write down your observations.