Derivatives of Logarithmic Function

Q.1. If y = e^x log x, find dy/dx at x = 1.

Solution : 1

We have, y = e^{x} log x

Therefore, dy/dx = e^{x}. d/dx(log x) + log x. d/dx(e^{x})

= ex . 1/x + log x . e^{x}

= e^{x} (1/x + log x)

Therefore, [dy/dx] x =1 = e1 (1/1 + log 1)

= e.

Q.2. If y = e^x log tan 2x , find dy / dx .

Solution : 2

We have, y = e^{x}. log (tan 2x) ,

dy /dx = e^{x} . d/dx{log (tan 2x)}+ log (tan 2x) . d/dx (e^{x})

= e^{x} {1/tan 2x . sec2 2x . 2}+ log (tan 2x) . e^{x}

= e^{x} {(2 sec 2 2x) / tan 2x}+ e^{x} . log (tan 2x)

= 2 e^{x} {1 / (sin 2x cos 2x)}+ e^{x} log (tan 2x)

= 2 e^{x} {2 / (2 sin 2x cos 2x)}+ e^{x} log (tan 2x)

= 4 e ^{x} / sin 4x + e^{x} log (tan 2x)

= e^{x} {4 cosec 4x + log (tan 2x)}.

Q.3. If $y = \log \sqrt{(1 - \cos x) / (1 + \cos x)}$, find dy / dx.

Solution : 3

$y = \log \sqrt{\{(1 - \cos x) / (1 + \cos x)\}}$

$= (1/2) \log \{(1 - \cos x) / (1 + \cos x)\}$

$= (1/2) \{\log (1 - \cos x) - \log (1 + \cos x)$

Hence, $dy/dx = (1/2) [\sin x / (1 - \cos x) + \sin x / (1 + \cos x)]$

$= (\sin x / 2) [1 / (1 - \cos x) + 1/ (1 + \cos x)]$

$= (\sin x / 2) [(1 + \cos x + 1 - \cos x) / (1 - \cos^2 x)]$

$= (\sin x / 2)[2 / (1 - \cos^2 x)]$

$= (\sin x / 2).(2/\sin^2 x)$

$= 1 / \sin x = \operatorname{cosec} x$.

Q.4. If $y = - \cot^2 (x/2) - 2 \log \sin (x/2)$, prove that : $dy/dx = \cot^3 (x/2)$.

Solution : 4

$y = - \cot^2 (x/2) - 2 \log \sin (x/2)$

$= - \{\cot (x/2)\}^2 - 2 \log \sin (x/2)$

Hence , $dy / dx = - 2 \cot (x/2) \{- \operatorname{cosec}^2 (x/2)\} \times 1/2 - 2 / \sin (x/2) \times \cos (x/2) \times 1/2$

$= \cot (x/2) \operatorname{cosec}^2 (x/2) - \cot (x/2)$

$= \cot (x/2) [\operatorname{cosec}^2 (x/2) - 1]$

$= \cot (x/2) . \cot^2 (x/2) = \cot^3 (x/2)$. **[Proved.]**

Q.5. If $y = \log (\tan x)$, find dy / dx.

Solution : 5

y = log (tan x) => dy / dx = (1/tan x) . d / dx (tan x)

= (1 / tan x) . $\sec^2$ x

= cosec x × sec x.

Q.6. If y = log (sec x + tan x) , find dy / dx .

Solution : 6

y = log (sec x + tan x) => dy / dx = 1 / (sec x + tan x). d / dx (sec x + tan x)

= (sec x tan x + $\sec^2$x) / (sec x + tan x)

= {sec x (sec x + tan x)}/ (sec x + tan x)

= sec x.

Q.7. If y = log tan (π / 4 + x / 2) , find dy / dx.

Solution : 7

y = log tan (π / 4 + x / 2)

Then dy / dx = d / dx [log tan (π / 4 + x / 2)]

= [1 / tan (π / 4 + x / 2)] . d / dx [tan (π / 4 + x / 2)]

= [1 / tan (π /4 + x / 2)] . sec 2 (π / 4 + x / 2) . d / dx (π / 4 + x / 2)

= [$\sec^2$ (π / 4 + x / 2) / tan (π / 4 + x / 2)] . (1/2)

= 1/ [2 sin (π / 4 + x / 2) cos (π / 4 + x / 2)

= 1/ sin (π / 2 + x) = 1 / cos x = sec x .

Q.8. Differentiate x tan x $\log_5$ x w.r.t. x.

Solution : 8

Let $y = x \tan x \log_5 x$,

Then $dy/dx = x \tan x. \, d/dx (\log_5 x) + \log_5 x \, .d/dx (x \tan x)$

$= x \tan x \, . \, 1/x \log_5 + \log_5 x (x \, . \, \sec^2 x + \tan x \, .1)$

$= \tan x/\log_5 + (x \sec^2 x + \tan x) \log_5 x.$