Mathematics

(Chapter – 13) (Probability) (Exercise 13.1)
(Class – XII)

Question 1:

Given that E and F are events such that P(E) = 0.6, P(F) = 0.3 and $P(E \cap F) = 0.2$, find P(E|F) and P(F|E).

Answer 1:

It is given that P(E) = 0.6, P(F) = 0.3 and $P(E \cap F) = 0.2$

$$P(E|F) = \frac{P(E \cap F)}{P(F)} = \frac{0.2}{0.3} = \frac{2}{3}$$

$$P(F|E) = \frac{P(E \cap F)}{P(E)} = \frac{0.2}{0.6} = \frac{1}{3}$$

Question 2:

Compute P(A|B), if P(B) = 0.5 and $P(A \cap B) = 0.32$

Answer 2:

It is given that $P(B) = 0.5$ and $P(A \cap B) = 0.32$

$$P\left(\frac{A}{B}\right) = \frac{P(A \cap B)}{P(B)} = \frac{0.32}{0.5} = \frac{16}{25}$$

Question 3:

If P(A) = 0.8, P(B) = 0.5 and P(B|A) = 0.4 find

(i) $P(A \cap B)$ **(ii)** $P(A|B)$ **(iii)** $P(A \cup B)$

Answer 3:

(i) It is given that P(A) = 0.8, P(B) = 0.5 and P(B|A) = 0.4

$$\Rightarrow \frac{P(A \cap B)}{P(A)} = 0.4$$

$$\Rightarrow \frac{P(A \cap B)}{0.8} = 0.4$$

$$\Rightarrow P(A \cap B) = 0.32$$

(ii) $P(A|B) = \frac{P(A \cap B)}{P(B)} \quad \Rightarrow P(A|B) = \frac{0.32}{0.5} = 0.64$

(iii) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$$\Rightarrow P(A \cup B) = 0.5 + 0.5 - 0.32 = 0.98$$

Question 4:

Evaluate $P(A \cup B)$, if $2P(A) = P(B) = \frac{5}{13}$ and $P(A|B) = \frac{2}{5}$

Answer 4:

It is given that $2P(A) = P(B) = \frac{5}{13}$

$$\Rightarrow P(A) = \frac{5}{26} \text{ and } P(B) = \frac{5}{13}$$

$$P(A|B) = \frac{2}{5} \qquad \Rightarrow \frac{P(A \cap B)}{P(B)} = \frac{2}{5}$$

$$P(A \cap B) = \frac{2}{5} \times P(B) = \frac{2}{5} \times \frac{5}{13} = \frac{2}{13}$$

It is known that $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

Therefore, $P(A \cup B) = \frac{5}{26} + \frac{5}{13} - \frac{2}{13} = \frac{5 + 10 - 4}{26} = \frac{11}{26}$

Question 5:

If $P(A) = \frac{6}{11}, P(B) = \frac{5}{11}$ and $P(A \cup B) = \frac{7}{11}$, find

(i) $P(A \cap B)$ **(ii)** $P(A|B)$ **(iii)** $P(B|A)$

Answer 5:

It is given that $P(A) = \frac{6}{11},\ P(B) = \frac{5}{11}$ and $P(A \cup B) = \frac{7}{11}$

(i) $P(A \cup B) = \frac{7}{11} \quad \Rightarrow P(A) + P(B) - P(A \cap B) = \frac{7}{11}$

$$\Rightarrow \frac{6}{11} + \frac{5}{11} - P(A \cap B) = \frac{7}{11} \quad \Rightarrow P(A \cap B) = \frac{11}{11} - \frac{7}{11} = \frac{4}{11}$$

(ii) It is known that $P(A|B) = \frac{P(A \cap B)}{P(B)}$

$$\Rightarrow P(A|B) = \frac{\frac{4}{11}}{\frac{5}{11}} = \frac{4}{5}$$

(iii) It is known that $P(B|A) = \frac{P(A \cap B)}{P(A)}$

$$\Rightarrow P(B|A) = \frac{\frac{4}{11}}{\frac{6}{11}} = \frac{4}{6} = \frac{2}{3}$$

Question 6:

Determine P(E|F), if a coin is tossed three times, where

(i) E: head on third toss, F: heads on first two tosses

(ii) E: at least two heads, F: at most two heads

(iii) E: at most two tails, F: at least one tail.

Answer 6:

If a coin is tossed three times, then the sample space S is

S = {HHH, HHT, HTH, HTT, THH, THT, TTH, TTT}

It can be seen that the sample space has 8 elements.

(i) E = {HHH, HTH, THH, TTH} and F = {HHH, HHT}, therefore E ∩ F = {HHH}

$$P(F) = \frac{2}{8} = \frac{1}{4} \text{ and } P(E \cap F) = \frac{1}{8} \quad \Rightarrow P(E|F) = \frac{P(E \cap F)}{P(F)} = \frac{\frac{1}{8}}{\frac{1}{4}} = \frac{4}{8} = \frac{1}{2}$$

(ii) E = {HHH, HHT, HTH, THH}

F = {HHT, HTH, HTT, THH, THT, TTH, TTT} and E ∩ F = {HHT, HTH, THH}

Clearly, $P(E \cap F) = \frac{1}{8}$ and $P(F) = \frac{7}{8}$

$$P(E|F) = \frac{P(E \cap F)}{P(F)} = \frac{\frac{1}{8}}{\frac{7}{8}} = \frac{3}{7}$$

(iii) E = {HHH, HHT, HTH, HTT, THH, THT, TTH}

F = {HHT, HTH, HTT, THH, THT, TTH, TTT}

∴ E ∩ F = {HHT, HTH, THH, HTT, THT, TTH}

$$P(F) = \frac{7}{8} \text{ and } P(E \cap F) = \frac{6}{8}$$

Thus,

$$P(E|F) = \frac{P(E \cap F)}{P(F)} = \frac{\frac{6}{8}}{\frac{7}{8}} = \frac{6}{7}$$

Question 7:

Determine P(E|F), if two coins are tossed once, where

(i) E: tail appears on one coin, F: one coin shows head

(ii) E: no tail appears, F: no head appears

Answer 7:

If two coins are tossed once, then the sample space S = {HH, HT, TH, TT}

(i) E = {HT, TH}, F = {HT, TH} and E ∩ F = { HT, TH}

Therefore, $P(F) = \frac{2}{8} = \frac{1}{4}$ and $P(E \cap F) = \frac{2}{8} = \frac{1}{4}$

$$\Rightarrow P(E|F) = \frac{P(E \cap F)}{P(F)} = \frac{2}{2} = 1$$

(ii) E = {HH}, F = {TT}, $(E \cap F) = \phi$, therefore $P(F) = 1$ and $P(E \cap F) = 0$

$$\Rightarrow P(E|F) = \frac{P(E \cap F)}{P(F)} = \frac{0}{1} = 0$$

Question 8:

Determine P(E|F), if A die is thrown three times,

E: 4 appears on the third toss, F: 6 and 5 appears respectively on first two tosses

Answer 8:

If a die is thrown three times, then the number of element in the sample space will be 6×6×6 = 216

E = (1, 1, 4), (1, 2, 4),.....(1, 6, 4)
(2, 1, 4), (2, 2, 4),......(2, 6, 4)
(3, 1, 4), (3, 2, 4),......(3, 6, 4)
(4, 1, 4), (4, 2, 4),......(4, 6, 4)
(5, 1, 4), (5, 2, 4),......(5, 6, 4)
(6, 1, 4), (6, 2, 4),.......(6, 6, 4)
F = {(6, 5, 1), (6, 5, 2), (6, 5, 3), (6, 5, 4), (6, 5, 5), (6, 5, 6)}
Therefore, E ∩ F = {(6,5,4)}

$$P(F) = \frac{6}{216} \text{ and } P(E \cap F) = \frac{1}{216} \Rightarrow P(E|F) = \frac{P(E \cap F)}{P(F)} = \frac{\frac{1}{216}}{\frac{6}{216}} = \frac{1}{6}$$

Question 9:

Determine P(E|F), if Mother, father and son line up at random for a family picture.

E: son on one end, F: father is middle

Answer 9:

If mother (M), father (F), and son (S) line up for the family picture, then the sample space will be

S' = {(MFS, MSF, FMS, FSM, SMF, SFM}

E = {MFS, FMS, SMF, SFM} and F = {MFS, SFM}

Here, $P(E \cap F) = \frac{2}{6} = \frac{1}{3}$ and $P(F) = \frac{2}{6} = \frac{1}{3}$

$$P(E|F) = \frac{P(E \cap F)}{P(F)} = \frac{\frac{1}{3}}{\frac{1}{3}} = 1$$

Question 10:

A black and a red dice are rolled.

(a) Find the conditional probability of obtaining a sum greater than 9, given that the black die resulted in a 5.

(b) Find the conditional probability of obtaining the sum 8, given that the red die resulted in a number less than 4.

Answer 10:

Let the first observation be from the black die and second from the red die.

When two dice (one black another red) are rolled, the sample space S has 6×6 = 36 number of elements.

Let A be obtaining a sum greater than 9 = {(4, 6), (5, 5), (5, 6), (6, 4), (6, 5), (6, 6)}

B: Black die result in 5 = {(5, 1), (5, 2), (5, 3), (5, 4), (5, 5), (5, 6)}

$A \cap B = \{(5,5),(5,6)\}$

(a) The conditional probability of obtaining a sum greater than 9.
Given that the black die resulted in a 5, is given by $P(A|B)$

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{\frac{2}{36}}{\frac{6}{35}} = \frac{2}{6} = \frac{1}{3}$$

(b) E: Sum of observation is 8. = {(2, 6), (3, 5), (4, 4), (5, 3), (6, 2)}

F: Red die resulted in a number less than 4

$$= \left\{\begin{matrix} (1,1),(1,2),(1,3),(2,1)(2,2),(2,3), \\ (3,1),(3,2),(3,3),(4,1),(4,2)(4,3), \\ (5,1),(5,2),(5,3),(6,1),(6,2),(6,3), \end{matrix}\right\}$$

$E \cap F = \{(5,3),(6,2)\}, P(F) = \frac{18}{36}$ and $P(E \cap F) = \frac{2}{36}$

The conditional probability of obtaining the sum equal to 8, given that the red die resulted in a number less than 4, is given by P(E|F) therefore

$$P(E|F) = \frac{P(E \cap F)}{P(F)} = \frac{\frac{2}{36}}{\frac{18}{36}} = \frac{2}{18} = \frac{1}{9}$$

Question 11:

A fair die is rolled consider event E = {1, 3, 5}, F = {2, 3} and G = {2, 3, 4, 5} Find

(i) $P(E|F)$ and $P(F|E)$ **(ii)** $P(E|G)$ and $P(G|E)$

(iii) $P((E \cup F)|G)$ and $P((E \cap F)|G)$

Answer 11:

When a fair die is rolled ,the sample space S will be S = {1, 2, 3, 4, 5, 6}

It is given that E = {1, 3, 5}, F = {2, 3}, and G = {2, 3, 4, 5}

$P(E) = \frac{3}{6} = \frac{1}{2}$, $P(F) = \frac{2}{6} = \frac{1}{3}$ and $P(G) = \frac{4}{6} = \frac{2}{3}$

(i) $P(E \cap F) = \frac{1}{6}$

$$P(E|F) = \frac{P(E \cap F)}{P(F)} = \frac{\frac{1}{6}}{\frac{1}{3}} = \frac{1}{2} \quad \text{and} \quad P(F|E) = \frac{P(E \cap F)}{P(E)} = \frac{\frac{1}{6}}{\frac{1}{2}} = \frac{1}{3}$$

(ii) $E \cap G = \frac{2}{6} = \frac{1}{3}$

$$P(E|G) = \frac{P(E \cap G)}{P(G)} = \frac{\frac{1}{3}}{\frac{2}{3}} = \frac{1}{2} \quad \text{and} \quad P(G|E) = \frac{P(E \cap G)}{P(E)} = \frac{\frac{1}{3}}{\frac{1}{2}} = \frac{2}{3}$$

(iii) $E \cup F = \{1, 2, 3, 5\}$

$(E \cup F) \cap G = \{1, 2, 3, 5\} \cap \{2, 3, 4, 5\} = \{2, 3, 5\}$

$E \cap F = \{3\}$

$(E \cap F) \cap G = \{3\} \cap \{2, 3, 4, 5\} = \{3\}$

$$\therefore P(E \cup G) = \frac{4}{6} = \frac{2}{3}$$

$$P((E \cup F) \cap G) = \frac{3}{6} = \frac{1}{2}$$

$$P(E \cap F) = \frac{1}{6}$$

$$P((E \cap F) \cap G) = \frac{1}{6}$$

$$\therefore P((E \cup F)|G) = \frac{P((E \cup F) \cap G)}{P(G)} = \frac{\frac{1}{2}}{\frac{2}{3}} = \frac{1}{2} \times \frac{3}{2} = \frac{3}{4}$$

$$P((E \cap F)|G) = \frac{P((E \cap G) \cap G)}{P(G)} = \frac{\frac{1}{6}}{\frac{2}{3}} = \frac{1}{6} \times \frac{3}{2} = \frac{1}{4}$$

Question 12:

Assume that each born child is equally like to be a boy or a girl. If a family has two children, what is the conditional probability that both are girls given that (i) the youngest is a girl, (ii) at least one is a girl?

Answer 12:

Let B and G represent the boy and the girl child respectively. If a family has two children, the sample space will be S = {(b, b), (b, g), (g, b), (g, g)}.

(i) Let A be the event that both children are girl.

Hence A = {g, g} (i) let B the event that the youngest child is a girl.

$B = [(b, g), (g, g)]$

$A \cap B = \{(g, g)\}$, $P(B) = \frac{2}{4} = \frac{1}{2}$ and $P(A \cap B) = \frac{1}{4}$

The conditional probability that both are girls, given that the youngest child is girl, is given by:

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{\frac{1}{2}}{\frac{1}{4}} = \frac{1}{2}$$

Therefore, the required probability is $\frac{1}{2}$.

(ii) Let C be the event at least one child is a girl.

$C = \{(b, g), (g, b), (g, g)\}$ and $A \cap C = \{g, g\}$

$$P(C) = \frac{3}{4} \text{ and } P(A \cap C) = \frac{1}{4}$$

The conditional probability that both are girl, given that at least one child is a girl, is given by $P(A|C)$, therefore

$$P(A|C) = \frac{P(A \cap C)}{P(C)} = \frac{\frac{1}{4}}{\frac{3}{4}} = \frac{1}{3}$$

Question 13:

An instructor has a question beck consisting of 300 easy true/False question, 200 difficult true/False question, 500 easy multiple choice question and 400 difficult multiple choice question. If a question is selected at random from the question bank, what is the probability that it will be an easy question given that it is a multiple choice question?

Answer 13:

Let us denote E = easy question,

M = multiple choice question,

Total number of question = 1400

Total number of multiple choice question = 900

Therefore, probability of selecting an easy multiple choice question is

$$P(E \cap M) = \frac{500}{1400} = \frac{5}{14}$$

The probability of selecting a multiple choice question is $P(M) = \frac{900}{1400} = \frac{900}{1400}$

Therefore, $$P(E|M) = \frac{P(E \cap M)}{P(M)} = \frac{\frac{5}{14}}{\frac{9}{14}} = \frac{5}{9}$$

Hence, the required probability is $\frac{5}{9}$.

Question 14:

Given that the two numbers appearing on throwing the two dice are different. Find the probability of the event the sum of numbers on the dice is 4.

Answer 14:

When dice is thrown, number of observation in the sample place $6 \times 6 = 36$ let A be the event that the sum of the numbers on the dice is 4 and B be the event that the two numbers appearing on throwing the two dice are different.

A = {(1, 3), (2, 2), (3, 1)}

B = {(1, 2)(1, 3), (1, 4), (1, 5), (1, 6), (2, 1), (2, 2), (2, 3), (2, 4)(2, 5), (2, 6),
(3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (3, 6), (4, 1), (4, 2), (4, 3), (4, 4), (4, 5), (4, 6)
(5, 1), (5, 2), (5, 3), (5, 4), (5, 5), (5, 6), (6, 1),(6, 2),(6, 3),(6, 4),(6,5),(6,6)}

$(A \cap B) = \{(1,3), (3,1)\}$

$P(B) = \frac{30}{36} = \frac{5}{6}$ and $P(A \cap B) = \frac{2}{36} = \frac{1}{18}$

Let P(A|B) respectively the probability that the sum of the number on the dice is 4, given that the two numbers appearing on throwing the two dice are different, so

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{\frac{1}{18}}{\frac{5}{6}} = \frac{1}{15}$$

Hence, the probability is $\frac{1}{15}$.

Question 15:

Consider the experiment of throwing a die, if a multiple of 3 comes up, throw the die again and if any other number comes, toss a coin. Find the conditional probability of the event 'the coin shows a tail', given that 'at least one die shows a 3'.

Answer 15:

The sample space of the experiment is,

$$S = \begin{Bmatrix} (1,H),(1,T)(2,H),(2,\ T),(3,1),(3,2),(3,3),(3,4)(3,4),(3,5),(3,6), \\ (4,H\ ),(4,T),(5,H),(5,T),(6,1),(6,2),(6,3),(6,4),(6,5),(6,6) \end{Bmatrix}$$

Let A be the event that the coin shows a tail and B be the event that at least one die shows 3.

$A = \{(1,T),(2,T),(4,T),(5,T)\}$

$B = \{(3,1),(3,2),(3,3),(3,4),(3,5),(3,6),(6,3)\}$

A∩B = ∅ therefore P(A∩B) = 0, then

$P(B) = P\{(3,1)\} + \{(3,2)\} + \{(3,3)\} + \{(3,4)\} + \{(3,5)\} + \{(3,6)\} + \{(6,3)\}$

$= \frac{1}{36} + \frac{1}{36} + \frac{1}{36} + \frac{1}{36} + \frac{1}{36} + \frac{1}{36} + \frac{1}{36}$

$= \frac{7}{36}$

Probability of the event that the coin shows a tail, given that at least one die shows 3, is given by P (A| B).

Therefore

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{0}{\frac{7}{36}} = 0$$

In each of the Exercises 16 and 17 choose the correct answer:

Question 16:

If $P(A) = \frac{1}{2},\ P(B) = 0,$ then $P(A|B)$ is

(A) 0

(B) $\frac{1}{2}$

(C) not defined

(D) 1

Answer 16:

It is given that $P(A) = \frac{1}{2},\ P(B) = 0$

Now, $P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{P(A \cap B)}{0}$

Therefore, $P(A|B)$ is not defined.

Hence, the correct answer is (C).

Question 17:

If A and B are events such that, $P(A|B) = P(B|A)$, then

(A) $A \subset B$ but $A \neq B$

(B) A = B

(C) $A \cap B = \emptyset$

(D) P(A) = P(B)

Answer 17:

It is given that, P(A|B) = P(B|A)

$$\Rightarrow \frac{P(A \cap B)}{P(B)} = \frac{P(A \cap B)}{P(A)}$$

$\Rightarrow$ P(A) = P(B)

Hence, the correct answer is (D).

Mathematics

(Chapter – 13) (Probability) (Exercise 13.2)
(Class – XII)

Question 1:

If $P(A) = \frac{3}{5}$ and $P(B) = \frac{1}{5}$, find $P(A \cap B)$ if A and B are independent events.

Answer 1:

It is given that $P(A) = \frac{3}{5}$ and $P(B) = \frac{1}{5}$

A and B are independent events. Therefore $P(\text{A} \cap \text{B}) = \text{P(A)}.\text{P(B)} = \frac{3}{5}.\frac{1}{5} = \frac{3}{25}$

Question 2:

Two cards are drawn at random and without replacement from a pack of 52 playing cards. Find the probability that both the cards are black.

Answer 2:

There are 26 black cards in a deck of 52 cards.

Let P(A) = The probability of getting a black card on the first draw.

P(B) = The probability of getting a black card on the second draw.

Therefore, $P(A) = \frac{26}{52} = \frac{1}{2}$

Since the card is not replaced, $P(B) = \frac{25}{51}$

Thus, probability of getting both the cards black $= P(A).P(B) = \frac{1}{2} \times \frac{25}{51} = \frac{25}{102}$

Question 3:

A box of oranges is inspected by examining three randomly selected oranges drawn without replacement. If all the three oranges are good, the box is approved for sale, otherwise, it is rejected. Find the probability that a box containing 15 oranges out of which 12 are good and 3 are bad ones will be approved for sale.

Answer 3:

Let A, B. and C be the respective events that the first, second and third dawn orange is good.

Therefore, probability that first drawn orange is good, $P(A) = \frac{12}{15}$

The orange are not replaced, therefore, probability of getting second orange good, $P(B) = \frac{11}{14}$

Similarly, probability of getting third orange is good, $P(C) = \frac{10}{13}$

The box is approved for sale, if all the three orange are good.

Thus, the probability of getting all the orange good $= \frac{12}{15} \times \frac{11}{14} \times \frac{10}{13} = \frac{44}{91}$

Therefore, the probability that the box is approved for sale is $\frac{44}{91}$.

Question 4:

A fair coin and an unbiased die tossed. Let A be the event 'head appear on the coin' and B be the event '3 on the die'. Check whether A and B are independent events or not.

Answer 4:

If affair coin and an unbiased die are tossed, then the sample space S is given by,

$$S = \begin{Bmatrix} (H,1)(H,2)(H,3)(H,4)(H,5)(H,6) \\ (T,1)(T,2)(T,3)(T,4)(T,5)(T,6) \end{Bmatrix}$$

Let A: Head appears on the coin

$A = \{(H,1)(H,2)(H,3)(H,4)(H,5)(H,6)\}$, therefore $P(A) = \frac{6}{12} = \frac{1}{2}$

B: 3 on die $= \{(H,3)(T,3)\}$, therefore $P(B) = \frac{2}{12} = \frac{1}{6}$

$A \cap B = \{(H,3)\}$ therefore, $P(A \cap B) = \frac{1}{12}$

$\mathrm{P(A).P(B)} = \frac{1}{2} \times \frac{1}{6} = \frac{1}{12} = P(A \cap B)$

Therefore, A and B are endependent event.

Question 5:

A die marked 1, 2, 3 in red and 4, 5, 6 in green is tossed. Let A be the Event, the number is even, and B be the event, the number is red. Are A and B is independent?

Answer 5:

When a die is thrown, the sample space $S = \{1,2,3,4,5,6\}$

Here, A is the event that the number is even, therefore $\mathrm{A} = \{2,4,6\}$ and $P(A) = \frac{3}{6} = \frac{1}{2}$

B: the number is red, therefore $\mathrm{B} = \{1,2,3\}$ and $P(B) = \frac{3}{6} = \frac{1}{2}$

$A \cap B = \{2\}$ therefore, $P(A \cap B) = \frac{1}{6}$

Here, $P(A).P(B) \neq P(A \cap B)$. Therefore A and B are not independent.

Question 6:

Let E and F be event with $P(E) = \frac{3}{5}$ $P(F) = \frac{3}{10}$ and $P(E \cap F) = \frac{1}{5}$. Are E and F independent?

Answer 6:

It is given that $P(E) = \frac{3}{5}$, $P(F) = \frac{3}{10}$ and $P(E \cap F) = \frac{1}{5}$

$P(E).P(F) = \frac{3}{5}.\frac{3}{10} = \frac{9}{50} \neq \frac{1}{5}$

Here, $P(E).P(F) \neq P(E \cap F)$. Therefore E and F are not independent.

Question 7:

Given that the event A and B are such that $P(A) = \frac{1}{2}$, $P(A \cup B) = \frac{3}{5}$, and $P(B) = p$. Find P if they are

(i) Mutually exclusive **(ii)** Independent.

Answer 7:

It is given that $P(A) = \frac{1}{2}$, $P(A \cap B) = \frac{3}{5}$, and $P(B) = p$.

(i) When A and B are mutually exclusive $(A \cap B) = \phi$, therefore, $P(A \cap B) = 0$

It is known that, $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$\Rightarrow \frac{3}{5} = \frac{1}{2} + p - 0 \quad \Rightarrow P = \frac{3}{5} - \frac{1}{2} = \frac{1}{10}$

(ii) When A and B are independent, $P(A \cap B) = P(A).P(B) = \frac{1}{2}p$ It is known that $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$\Rightarrow \frac{3}{5} = \frac{1}{2} + p - \frac{1}{2}p \quad \Rightarrow \frac{3}{5} = \frac{1}{2} + \frac{p}{2} \quad \Rightarrow \frac{p}{2} = \frac{3}{5} - \frac{1}{2} = \frac{1}{10} \quad \Rightarrow p = \frac{1}{5}$

Question 8:

Let A and B be independent events with P(A) = 0.3 and P(B) = 0.4. Find

(i) $P(A \cap B)$ **(ii)** $P(A \cup B)$

(iii) $P(A|B)$ **(iv)** $P(B|A)$

Answer 8:

(i) It is given that P(A) = 0.3 and P(B) = 0.4

If A and B are independent event, then $P(A \cap B) = P(A).P(B) = 0.3 \times 0.4 = 0.12$

(ii) It is known that $P(A \cup B) = P(A) + P(B) - P(A \cap B) \Rightarrow P(A \cup B) = 0.3 + 0.4 - 0.12 = 0.58$

(iii) It is known that $P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{0.12}{0.4} = 0.3$

(iv) It is known that $P(B|A) = \frac{P(A \cap B)}{P(A)} = \frac{0.12}{0.3} = 0.4$

Question 9:

If A and B are two events such that $P(A) = \frac{1}{4}, P(B) = \frac{1}{2}$ and $P(A \cap B) = \frac{1}{8}$, Find P(not A and not B).

Answer 9:

It is given that $P(A) = \frac{1}{4}, P(B) = \frac{1}{2}$ and $P(A \cap B) = \frac{1}{8}$

We know that P(not A and not B) = $P(A' \cap B') = P(A \cup B)'$ [Since $A' \cap B' = (A \cup B)'$]

$= 1 - P(A \cup B)$

$= 1 - [P(A) + P(B) - P(A \cap B)]$

$= 1 - \left[\frac{1}{4} + \frac{1}{2} - \frac{1}{8}\right] = 1 - \frac{5}{8} = \frac{3}{8}$

Question 10:

Event A and B are such that $P(A) = \frac{1}{2}, P(B) = \frac{7}{12}$ and $P(\text{not } A \text{ or not } B) = \frac{1}{4}$. State whether A and B are independent?

Answer 10:

It is given that $P(A) = \frac{1}{2}, P(B) = \frac{7}{12}$ and $P(\text{not } A \text{ or not } B) = \frac{1}{4}$

$\Rightarrow P(A' \cup B') = \frac{1}{4} \quad \Rightarrow P\{(A \cap B)'\} = \frac{1}{4}$ [Since $A' \cup B' = (A \cap B)'$]

$\Rightarrow 1 - P(A \cap B) = \frac{1}{4}$

$\Rightarrow P(A \cap B) = \frac{3}{4}$ ... (1)

However, $P(A).P(B) = \frac{1}{2}.\frac{7}{12}$ ... (2)

Here, $\frac{3}{4} \neq \frac{7}{24} \quad \Rightarrow P(A \cap B) \neq P(A).P(B)$

Therefore, A and B are not independent events.

Question 11:

Given two independent events A and B such that P(A) = 0.3, P(B) = 0.6. Find

(i) P(A and B)

(ii) P(A and not B)

(iii) P(A or B)

(iv) P(neither A nor B)

Answer 11:

It is given that P(A) = 0.3, P(B) = 0.6 and A, B are Independent Events.

(i) $P(A \text{ and } B) = P(A).P(B) \Rightarrow P(A \cap B) = 0.3 \times 0.6 = 0.18$

(ii) $P(A \text{ and not } B) = P(A \cap B') = P(A) - P(A \cap B) = 0.3 - 0.18 = 0.12$

(iii) P(A or B) = P(A∪B) = $[P(A) + P(B) - P(A \cap B)]$ = 0.3+0.6-0.18 = 0.72

(iv) P(neither A nor B) = $P(A' \cap B') = P\left((A \cup B)'\right) = 1 - (A \cup B) = 1 - 0.72 = 0.28$

Question 12:
A die is tossed thrice. Find the probability of getting an odd number at least once.

Answer 12:
Probability of getting an odd number in a single throw of a die $= \frac{3}{6} = \frac{1}{2}$

Similarly , probability of getting an even number $= \frac{3}{6} = \frac{1}{2}$

Probability of getting an even number three times $= \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{8}$

Therefore, probability of getting an odd number at least once
= 1 - Probability of getting an odd number in none of the throws
= 1- Probability of getting an even number thrice
$= 1 - \frac{1}{8} = \frac{7}{8}$

Question 13:
Two balls are drawn at random with replacement from a box containing 10 black and 8 red balls. Find the probability that
(i) both balls are red.
(ii) first ball is black and second is red.
(iii) one of them is black and other is red.

Answer 13:
Total number of balls = 18, number of red balls = 8 and number of black balls =10

(i) Probability of getting a red balls in the first draw $= \frac{8}{18} = \frac{4}{9}$

The ball is replaced after the first draw.

Therefore, the probability of getting a red balls in the second draw $= \frac{8}{18} = \frac{4}{9}$

The probability of getting both the balls red $= \frac{4}{9} \times \frac{4}{9} = \frac{16}{81}$

(ii) Probability of getting first ball black $= \frac{10}{18} = \frac{5}{9}$

The ball is replaced after the first draw.

So, the probability of getting second ball as red $= \frac{8}{18} = \frac{4}{9}$

The probability of getting first ball black and second ball as red $= \frac{5}{9} \times \frac{4}{9} = \frac{20}{81}$

(iii) Probability of getting first ball as red $= \frac{8}{18} = \frac{4}{9}$

The ball is replaced after the first draw. Probability of getting second ball as black $= \frac{10}{18} = \frac{5}{9}$

Therefore, the probability of getting first ball as black and second ball as red $= \frac{4}{9} \times \frac{5}{9} = \frac{20}{81}$

Therefore, probability that one of them is black and other is red
= Probability of getting first ball black and second as red + Probability of getting first ball red and second ball black $= \frac{20}{81} + \frac{20}{81} = \frac{40}{81}$

Question 14:
Probability of solving specific problem independently by A and B are $\frac{1}{2}$ and $\frac{1}{3}$ respectively. If both try to solve the problem independently, find the probability that,
(i) the problem is solved **(ii)** exactly one of them solve the problem

Answer 14:

Probability of solving problem by A, P(A) $= \frac{1}{2}$

Probability of solving problem by B, P(B) $= \frac{1}{3}$

Since the problem is solved independently by A and B, therefore $P(A \cap B) = P(A).P(B) = \frac{1}{2} \times \frac{1}{3} = \frac{1}{6}$

$P(A') = 1 - P(A) = 1 - \frac{1}{2}$

$P(B') = 1 - P(B) = 1 - \frac{1}{3} = \frac{2}{3}$

(i) Probability that problem is solved = P(A∪B) = P(A) + P(B) – P(A∩B) $= \frac{1}{2} + \frac{1}{3} - \frac{1}{6} = \frac{4}{6} = \frac{2}{3}$

(ii) Probability that exactly one of them solve the problem is given by

$$P(A).P(B') + P(B)P(A') = \frac{1}{2} \times \frac{2}{3} + \frac{1}{2} \times \frac{1}{3} = \frac{1}{2}$$

Question 15:

One card is drawn at random from a well shuffled deck of 52 cards. In which of the following cases are the events E and F independent?

(i) E: 'the card drawn is a spade' F: 'the card drawn is an ace'

(ii) E: 'the card drawn is black' F: 'the card drawn is a king'

(iii) E: 'the card drawn is a king or queen' F: 'the card drawn is a queen or jack'.

Answer 15:

(i) In a deck of 52 cards, 13 cards are spades and 4 card are aces

∴ P(E) = P(the card drawn is spade) $= \frac{13}{52} = \frac{1}{4}$

∴ P(F) = P(the card drawn is an ace)$= \frac{4}{52} = \frac{1}{13}$

In the deck of card, only 1 cards is an ace of spade.

P(E∩F) = P(the card drawn is spade and an ace) $= \frac{1}{52}$

$P(E) \times P(F) = \frac{1}{4} \times \frac{1}{13} = \frac{1}{52} = P(E \cap F) \quad \Rightarrow P(E).P(F) = P(E \cap F)$

Therefore, the events E and F are independent.

(ii) In a deck of 52 cards, 26 cards are black and 4 cards are kings.

∴ P(E) = P(the card drawn is a black) = $\frac{26}{52} = \frac{1}{2}$

∴ P(F) = P(the card drawn is a king) = $\frac{4}{52} = \frac{1}{13}$

In the pack of 52 cards, 2 cards are black as well as king

∴ P(E∩F) = P(the card drawn is black king) = $\frac{2}{52} = \frac{1}{26}$ and P(E) × P(F) = $\frac{1}{2} \times \frac{1}{13}$

$\Rightarrow P(E).P(F) = P(E \cap F)$

Therefore, given events E and F are independent.

(iii) In a deck of 52 cards, 4 cards are king, 4 cards are queen, and 4 cards are jacks.

∴ P(E) = P(the cards drawn is a king or a queen) = $\frac{8}{52} = \frac{2}{13}$

∴ P(cards drawn is a queen or a jack) = $\frac{8}{52} = \frac{2}{13}$

There are 4 cards which are king or queen and queen or jack

∴ P(E∩F) = P(the card drawn is a king or a queen ,or queen or jack) = $\frac{8}{52} = \frac{1}{13}$

$\Rightarrow P(E) \times P(F) \neq P(EF)$. Therefore, the events E and F are not independent.

Question 16:

In a hostel, 60% of the students read Hindi newspaper, 40% read English newspaper and 20% read both Hindi and English newspapers. A student is selected at random.

(a) Find the probability that she reads neither Hindi nor English newspapers.

(b) If she reads Hindi newspaper, find the probability that she reads English newspaper.

(c) If she reads English newspaper, find the probability that she reads Hindi newspaper.

Answer 16:

Let H denote the student who read Hindi newspaper and E denote the student who read English newspaper.

It is given that $P(H) = 60\% = \frac{6}{10} = \frac{3}{5}$ and $P(E) = 40\% = \frac{4}{10} = \frac{2}{5}$

$P(H \cap E) = 20\% = \frac{2}{10} = \frac{1}{5}$

$P(H \cup E)' = 1 - P(H \cup E)$

$= 1 - \{P(H) + P(E) - P(H \cap E)\}$

$= 1 - \left(\frac{3}{5} + \frac{2}{5} - \frac{1}{5}\right) = 1 - \frac{4}{5} = \frac{1}{5}$

Probability that a randomly chosen student reads English newspaper, if she reads Hindi newspaper, is given by P(E|H)

$$P(E|H) = \frac{P(E \cap H)}{P(H)} = \frac{\frac{1}{5}}{\frac{3}{5}} = \frac{1}{3}$$

Probability that a randomly chosen student reads Hindi newspaper, if she reads English newspaper, is given by P(H|E)

$$P(H|E) = \frac{P(H \cap E)}{P(E)} = \frac{\frac{1}{5}}{\frac{2}{5}} = \frac{1}{2}$$

Choose the correct answer in Exercises 17 and 18.

Question 17:

The probability of obtaining an even prime number on each die, when a pair of dice is rolled is

(A) 0 (B) $\frac{1}{3}$ (C) $\frac{1}{12}$ (D) $\frac{1}{36}$

Answer 17:

When two dice are rolled the number of outcomes is 36.

The only even prime number is 2.

Let E be the event of getting an even prime number on each die.

∴ E = {(2, 2)}, therefore

$$P(E) = \frac{1}{36}$$

Hence, the correct answer is (D).

Question 18:

Two events A and B will be independent, if

(A) A and B are mutually exclusive (B) $P(A' \cap B') = [1 - P(A)][1 - P(B)]$

(C) P(A) = P(B) (D) P(A) + P(B) = 1

Answer 18:

We know that two events A and B are said to be independent, if P(A∩B)= P(A) × P(**B**).

Now, discuss the option (A).

Let P(A) = m , P(B) = n , 0<m, n<1

A and B are mutually exclusive,

$\therefore A \cap B = \Phi \Rightarrow P(A \cap B) = 0$

However, $P(A).P(B) = mn \neq P(A \cap B)$

Thus, the option (A) is incorrect.

Consider the result given in alternative (B).

$P(A' \cap B') = [1 - P(A)][1 - P(B)]$

$\Rightarrow P(A' \cap B') = 1 - P(A) - P(B) + P(A).P(B)$

$\Rightarrow 1 - P(A \cup B) = 1 - P(A) - P(B) + P(A).P(B)$

$\Rightarrow P(A \cup B) = P(A) - P(B) + P(A).P(B)$

$\Rightarrow P(A) + P(B) - P(A \cap B) = P(A) + P(B) - P(A).P(B)$

$\Rightarrow P(A \cap B) = P(A).P(B)$

This implies that A and B are independent, if $P(A' \cap B') = [1 - P(A)][1 - P(B)]$

Thus, the option (B) is correct.

Let A Event of getting an odd number on throw of a die = {1, 3, 5}

$P(A) = \frac{3}{6} = \frac{1}{2}$

B: Event of getting an even number on throw of die = {2, 4, 6}

$P(B) = \frac{3}{6} = \frac{1}{2}$

Here, $A \cap B = \Phi \therefore P(A \cap B) = 0$ and $P(A).P(B) = \frac{1}{4} \neq 0 \Rightarrow P(A).P(B) \neq P(A \cap B)$

Thus, the option (C) is incorrect.

From the above example, it can be seen that,

$$P(A) + P(B) = \frac{1}{2} + \frac{1}{2} = 1$$

However, it cannot be inferred that A and B are independent.

Thus, the option (D) is incorrect.

Hence, the correct answer is (B).

Mathematics

(Chapter – 13) (Probability) (Exercise 13.3)
(Class – XII)

Question 1:

An urn contains 5 red and 5 black balls. A ball is drawn at random, its colour is noted and is returned to the urn. Moreover, 2 additional balls of the colour drawn are put in the urn and then a ball is drawn at random. What is the probability that the second ball is red?

Answer 1:

The urn contains 5 red and 5 black balls. Let a red ball be drawn in the first attempt.

$\therefore$ P (drawing a red ball) $= \frac{5}{10} = \frac{1}{2}$

If two red balls are added to the urn, then the urn contains 7 red and 5 black balls.

P (drawing a red ball) $= \frac{7}{12}$

Let a black ball be drawn in the first attempt,

$\therefore$ P(drawing a black ball in the first attempt) $= \frac{5}{10} = \frac{1}{2}$

If two black balls are added to the urn, then the urn contain 5 red and 7 black balls.

P (drawing a red ball) $= \frac{5}{12}$, therefore

Probability of drawing second ball as red $= \frac{1}{2} \times \frac{7}{12} + \frac{1}{2} \times \frac{5}{12} = \frac{1}{2}\left(\frac{7}{12} + \frac{5}{12}\right) = \frac{1}{2} \times 1 = \frac{1}{2}$

Question 2:

A bag contains 4 red and 4 black balls, another bag contains 2 red and 6 black balls. One of the two bags is selected at random and a ball is drawn from the bag which is found to be red. Find the probability that the ball is drawn from the first bag.

Answer 2:

Let E_1 and E_2 be the event of selecting first bag and second bag respectively.

$$\therefore P(E_1) = P(E_2) = \frac{1}{2}$$

Let A be the event of getting a red ball.

$P(A|E_1) =$ P(drawing a red ball from first bag) $= \frac{4}{8} = \frac{1}{2}$

$P(A|E_2) =$ P(drawing a red ball from second bag) $= \frac{2}{8} = \frac{1}{4}$

The probability of drawing a ball from the first bag, given that it is red, is given by $P(E_2|A)$ by using Bayes' theorem, we have

$$P(E_1|A) = \frac{P(E_1).P(A|E_1)}{P(E_1).P(A|E_1) + P(E_2).P(A|E_2)} = \frac{\frac{1}{2}\cdot\frac{1}{2}}{\frac{1}{2}\cdot\frac{1}{2} + \frac{1}{2}\cdot\frac{1}{4}} = \frac{\frac{1}{4}}{\frac{1}{4} + \frac{1}{8}} = \frac{\frac{1}{4}}{\frac{3}{8}} = \frac{2}{3}$$

Question 3:

Of the students in a college, it is known that 60% reside in hostel and 40% are day scholars (not residing in hostel). Previous year results report that 30% of all students who reside in hostel attain A grade and 20% of day scholars attain A grade in their annual examination. At the end of the year, one student is chosen at random from the college and he has an A grade, what is the probability that the student is a hostlier?

Answer 3:

Let E_1 and E_2 be the event that the student is a hustler and a day scholar respectively and A be the event that the chosen students gets grade A.

$\therefore P(E_1) = 60\% = \frac{60}{100} = 0.6$ and $P(E_2) = 40\% = \frac{40}{100} = 0.4$

$\therefore P(A|E_1) =$ P(student getting an A grade is a hostler) $= 30\% = 0.3$

$P(A|E_2) =$ P(student getting an A grade is a day scholar) $= 20\% = 0.2$

The probability that a randomly chosen student is a hostlier, given that he has an A grade, is given by $P(E_1|A)$
By using Bayes' theorem, we have

$$P(E_1|A) = \frac{P(E_1).P(A|E_1)}{P(E_1).P(A|E_1) + P(E_2).P(A|E_2)} = \frac{0.6 \times 0.3}{0.6 \times 0.3 + 0.4 \times 0.2} = \frac{0.18}{0.26} = \frac{9}{13}$$

Question 4:

In answering a question in a multiple choice question test, a student either known the answer or guesses. Let $\frac{3}{4}$ be the probability that he knows the answer and $\frac{1}{4}$ be the probability that he guess. Assuming that the student who guesses at the answer will be correct with probability $\frac{1}{4}$. What is the probability that the student knows the answer given that he answered it correctly?

Answer 4:

Let E_1 and E_2 be the respective events that the student knows the answer and he guesses the answer.

$$\therefore P(E_1) = \frac{3}{4} \quad \text{and} \quad P(E_2) = \frac{1}{4}$$

Let A be the event that the answer is correct.
The probability that the student answered correctly, given that he knows, is 1 i.e. $P(A|E_1) = 1$
Probability that the student answered correctly, given that he guessed, is $\frac{1}{4}$ i.e. $P(A|E_2) = \frac{1}{4}$
The probability that the student knows the answer, given that he answered it correctly, is given by $P(E_1|A)$
Using Bayes' theorem, we have

$$P(E_1|A) = \frac{P(E_1).P(A|E_1)}{P(E_1).P(A|E_1) + P(E_2).P(A|E_2)} = \frac{\frac{3}{4} \times 1}{\frac{3}{4} \times 1 + \frac{1}{4} \times \frac{1}{4}} = \frac{\frac{3}{4}}{\frac{3}{4} + \frac{1}{16}} = \frac{\frac{3}{4}}{\frac{13}{16}} = \frac{12}{13}$$

Question 5:

A laboratory blood test is 99% effective in detecting a certain disease when it is in fact, present. However, the test also yields a false positive result for 0.5% of the healthy person tested (i.e. if a healthy person is tested, then, with probability 0.005, the test will imply he has the disease). If 0.1 percent of the population actually has the disease, what is the probability that a person has the disease given that his test result is positive?

Answer 5:

Let E_1 and E_2 be the respective events that a person has a disease and a person has no disease.
Since E_1 and E_2 are the event complimentary to each other,
$\therefore P(E_1) + P(E_2) = 1 \quad \Rightarrow \quad P(E_2) = 1 - P(E_1) = 1 - 0.001 = 0.999$

$$P(E_1) = 0.1\% = \frac{0.1}{100} = 0.001$$

Let A be the event that the blood test result is positive.
$P(A|E_1) = \text{P(result is positive given the person has disease)} = 99\% = 0.99$
$P(A|E_2) = \text{P(result is positive given the person has no disease)} = 0.5\% = 0.005$
Probability that a person has a disease, given that his test result is positive, is given by $P(E_1|A)$
Using Bayes' theorem, we have

$$P(E_1|A) = \frac{P(E_1).P(A|E_1)}{P(E_1).P(A|E_1) + P(E_2).P(A|E_2)} = \frac{0.001 \times 0.99}{0.001 \times 0.99 + 0.999 \times 0.005}$$

$$= \frac{0.00099}{0.00099 + 0.004995} = \frac{990}{5985} = \frac{110}{665} = \frac{22}{133}$$

Question 6:

There are three coins. One is a two headed coin (having head on both faces), another is a biased coin that comes up heads 75% of the time and third is an unbiased coin. One of the three coins is chosen at random and tossed, it shows heads, what is the probability that it was the two headed coin?

Answer 6:

Let E_1, E_2 and E_3 be the respective event of choosing a two headed coin, biased coin and an unbiased coin.

$$\therefore P(E_1) = P(E_2) = P(E_3) = \frac{1}{3}$$

Let A be the event that the coin shows heads. A two–headed coin will always show heads. Therefore,

$P(A|\text{E}_1) = \text{P(coin shows heads, given that it is a two headed coin)} = 1$

Probability of heads coming up, given that it is biased coin = 75%

$$P(A|\text{E}_2) = \text{P(coin showing heads, given that it is a biased coin)} = \frac{75}{100} = \frac{3}{4}$$

Since the third coin is unbiased, the probability that it shows heads is always $\frac{1}{2}$.

$$P(A|\text{E}_3) = P(\text{coin showing heads, given that it is an unbiased coin}) = \frac{1}{2}$$

The probability that the coin is two –headed, given that it shows heads, is given by $\text{P}(\text{E}_1|\text{A})$

Using Bayes' theorem, we have

$$\text{P}(\text{E}_1|\text{A}) = \frac{P(\text{E}_1).\text{P}(A|\text{E}_1)}{\text{P}(\text{E}_1).P(A|\text{E}_1) + \text{P}(\text{E}_2).P(A|\text{E}_2) + P(\text{E}_3)\text{P}(A|\text{E}_3)}$$

$$= \frac{\frac{1}{3}\times 1}{\frac{1}{3}\times 1 + \frac{1}{3}\times\frac{3}{4} + \frac{1}{3}\times\frac{1}{2}} = \frac{\frac{1}{3}}{\frac{1}{3}\left(1 + \frac{3}{4} + \frac{1}{2}\right)} = \frac{1}{\frac{9}{4}} = \frac{4}{9}$$

Question 7:

An insurance company insured 2000 scooter drivers, 4000 car drivers and 6000 truck drivers. The probability of an accidents are 0.01, 0.03 and 0.15 respectively. One of the insured persons meets with an accident. What is the probability that he is a scooter driver?

Answer 7:

Let E_1, E_2 and E_3 be the respective events that the driver is a scooter driver, a car driver, and a truck driver. Let A be the event that the person meets with an accident.

There are 2000 scooter driver, 4000 car drivers, and 6000 truck drivers.

Total number of drivers = 2000 + 4000 + 6000 = 12000

$$\text{P}(\text{E}_1) = \text{P(driver is a scooter driver)} = \frac{2000}{12000} = \frac{1}{6}$$

$$\text{P}(\text{E}_2) = \text{P(driver is a car driver)} = \frac{4000}{12000} = \frac{1}{3}$$

$$\text{P}(\text{E}_3) = \text{P(driver is a truck driver)} = \frac{6000}{12000} = \frac{1}{2}$$

$$P(A|\text{E}_1) = P(\text{scooter driver met with an accident}) = 0.01 = \frac{1}{100}$$

$$P(A|\text{E}_2) = P(\text{car driver met with an accident}) = 0.03 = \frac{3}{100}$$

$$P(A|\text{E}_3) = P(\text{truck driver met with an accident}) = 0.15 = \frac{15}{100}$$

The probability that the driver is a scooter driver, given that he met with an accident, is given by $\text{P}(\text{E}_1|\text{A})$.

Using Bayes' theorem, we have

$$\text{P}(\text{E}_1|\text{A}) = \frac{P(\text{E}_1).\text{P}(A|\text{E}_1)}{\text{P}(\text{E}_1).P(A|\text{E}_1) + \text{P}(\text{E}_2).P(A|\text{E}_2) + P(\text{E}_3)\text{P}(A|\text{E}_3)}$$

$$= \frac{\frac{1}{6}\times\frac{1}{100}}{\frac{1}{6}\times\frac{1}{100} + \frac{1}{3}\times\frac{3}{100} + \frac{1}{2}\times\frac{15}{100}} = \frac{\frac{1}{6}\times\frac{1}{100}}{\frac{1}{100}\left(\frac{1}{6} + 1 + \frac{15}{2}\right)} = \frac{\frac{1}{6}}{\frac{104}{12}} = \frac{1}{52}$$

Question 8:

A factory has two machines A and B. Past record shows that machine A produced 60% of the items of output and machine B produced 40% of the items. Further, 2% of the items produced by machine A and 1% produced by machine B were defective. All the items are put into one stockpile and then one item is chosen at random from this and is found to be defective. What is the probability that it was produced by machine B?

Answer 8:

Let E_1 and E_2 be the respective events of items produced by machine A and B.
Let X be the event that the produced item was found to be defective.

$\therefore$ Probability of items produced by machine By A, $P(E_1) = 60\% = \frac{3}{5}$

Probability of items produced by machine By B, $P(E_2) = 40\% = \frac{2}{5}$

Probability machine A produced defective items, $P(X|E_1) = 2\% = \frac{2}{100}$

Probability machine B produced defective items, $P(X|E_2) = 2\% = \frac{1}{100}$

The probability that the randomly selected item was from machine B, given that it is defective, is given by $P(E_2|X)$.
Using Bayes' theorem, we have,

$$P(E_2|A) = \frac{P(E_2).P(A|E_2)}{P(E_1).P(A|E_1) + P(E_2).P(A|E_2)} = \frac{\frac{2}{5}\times\frac{1}{100}}{\frac{3}{5}\times\frac{2}{100}+\frac{2}{5}\times\frac{1}{100}} = \frac{\frac{2}{500}}{\frac{6}{500}+\frac{2}{500}} = \frac{2}{8} = \frac{1}{4}$$

Question 9:

Two groups are competing for the position on the Board of directors of a corporation. The probabilities that the first and the second groups will win are 0.6 and 0.4 respectively. Further, if the first group wins, the probability of introducing a new product is 0.7 and the corresponding probability is 0.3 if the second group wins. Find the probability that the new product introduced was by the second group.

Answer 9:

Let E_1 and E_2 be the respective events that the first group and the second group win the competition.
Let A be the event of introducing a new product. Therefore,
$P(E_1)$ = Probability that the first group wins the competition = 0.6,
$P(E_2)$ = Probability that the second group wins the competition = 0.4
$P(A|E_1)$ =Probability of introducing a new product if the first group win = 0.7
$P(A|E_2)$ = Probability of introducing a new product if the second group win = 0.3
The probability that the new product is introduced by the second group is given by P (E2|A),
Using Bayes' theorem, we have

$$P(E_2|A) = \frac{P(E_2).P(A|E_2)}{P(E_1).P(A|E_1) + P(E_2).P(A|E_2)} = \frac{0.4\times 0.3}{0.6\times\ 0.7\ + 0.4\times 0.3} = \frac{0.12}{0.42+0.12} = \frac{0.12}{0.54} = \frac{2}{9}$$

Question 10:

Suppose a girl throws a die. If she gets a 5 or 6, she tosses a coin three times and notes the number of heads. If she gets 1, 2, 3 or 4, she tosses a coin once and notes whether a head or tail is obtained. If she obtained exactly one head, what is the probability that she threw 1, 2, 3 or 4 with the die?

Answer 10:

Let E_1 be the event that the outcome on the die is 5 or 6 and E_1 be the event that the outcome on the die is 1, 2, 3, or 4. Therefore,

$$P(E_1) = \frac{2}{6} = \frac{1}{3} \text{ and } P(E_2) = \frac{4}{6} = \frac{2}{3}$$

Let A be the event of getting exactly one head

$P(A|E_1)$ = probability of getting exactly one head by tossing the coin three times if she gets 5 or 6 = $\frac{3}{8}$

P(A|E₂) = probability of getting exactly one head by tossing the coin three times if she gets 1, 2, 3 or 4 = $\frac{1}{2}$

The probability that the girl threw 1, 2, 3, or 4 with the die, if she obtained exactly one head, is given by $P(E_2|A)$. Using Bayes' theorem, we have

$$P(E_2|A) = \frac{P(E_2).P(A|E_2)}{P(E_1).P(A|E_1)+P(E_2).P(A|E_2)} = \frac{\frac{2}{3}\times\frac{1}{2}}{\frac{2}{3}\times\frac{3}{8}+\frac{2}{3}\times\frac{1}{2}} = \frac{\frac{1}{3}}{\frac{1}{3}\left(\frac{3}{2}+1\right)} = \frac{1}{\frac{11}{8}} = \frac{8}{11}$$

Question 11:

A manufacturer has three machine operators A, B and C. The first operator A produces 1% defective items, whereas the other two operators B and C produce 5% and 7% defective items respectively. A is on the job for 50% of the time, B is on the job for 30% of the time and C is on the job for 20% of the time. A defective item is produced, what is the probability that it was produced by A?

Answer 11:

Let E_1, E_2 and E_3 be the respective events of the time consumed by the machine A, B and C for the job. So,

$$P(E_1) = 50\% = \frac{50}{100} = \frac{1}{2}, \quad P(E_2) = 30\% = \frac{30}{100} = \frac{3}{10} \quad \text{and} \quad P(E_3) = 20\% = \frac{20}{100} = \frac{1}{5}$$

Let X be the event of producing the defective item. Therefore,

$$P(X|E_1) = 1\% = \frac{1}{100}, \quad P(X|E_2) = 5\% = \frac{5}{100} \quad \text{and} \quad P(X|E_3) = 7\% = \frac{7}{100}$$

The probability that the defective item was produced by A is given by $P(E_1|A)$. Using Bayes' theorem

$$P(E_1|A) = \frac{P(E_1).P(A|E_1)}{P(E_1).P(A|E_1)+P(E_2).P(A|E_2)+P(E_3)P(A|E_3)}$$

$$= \frac{\frac{1}{2}\times\frac{1}{100}}{\frac{1}{2}\times\frac{1}{100}+\frac{3}{10}\times\frac{5}{100}+\frac{1}{5}\times\frac{7}{100}} = \frac{\frac{1}{2}\times\frac{1}{100}}{\frac{1}{100}\left(\frac{1}{2}+\frac{3}{2}+\frac{7}{5}\right)} = \frac{\frac{1}{2}}{\frac{17}{5}} = \frac{5}{54}$$

Question 12:

A card from a pack of 52 cards is lost. From the remaining cards of the pack, two cards are drawn and are found to be both diamonds. Find the probability of the lost card being a diamond.

Answer 12:

Let E_1 and E_2 be the respective event of choosing a diamond cards and a card which is not a diamond.

Let A denote the lost card, out of 52 cards. Here, 13 cards are diamond and 39 cards are not diamond.

$$P(E_1) = \frac{13}{52} = \frac{1}{4} \quad \text{and} \quad P(E_2) = \frac{39}{52} = \frac{3}{4}$$

When one diamond cards is lost, there are 12 diamond cards out of 51 cards, two cards can be drawn out of 12 diamonds cards in ${}^{12}C_2$ ways similarly, 2 diamond cards can be drawn out of 51 cards in ${}^{51}C_2$ ways. The probability of getting two cards, when one diamond cards is lost, is given by $P(A|E_1)$

$$P(A|E_1) = \frac{12C_2}{51C_2} = \frac{12!}{2!\times 10!}\times\frac{2!\times 49!}{51!} = \frac{11\times 12}{50\times 51} = \frac{22}{425}$$

When the lost cards is not a diamond, there are 13 diamonds cards out of 51 cards. Two cards can be drawn out of 13 diamond cards in $13C_2$ ways and 2 cards can be drawn out of 51 cards in $51C_2$ ways. The probability of getting two cards, when one cards is lost which is not a diamond, is given by $P(A|E_2)$

$$P(A|E_2) = \frac{13C_2}{51C_2} = \frac{13!}{2!\times 11!}\times\frac{2!\times 49!}{51!} = \frac{12\times 13}{50\times 51} = \frac{26}{425}$$

The probability that the lost card is diamond is given by $P(E_1|A)$

$$P(E_1|A) = \frac{P(E_1).P(A|E_1)}{P(E_1).P(A|E_1)+P(E_2).P(A|E_2)} = \frac{\frac{1}{4}\times\frac{22}{425}}{\frac{1}{4}\times\frac{22}{425}+\frac{3}{4}\times\frac{26}{425}} = \frac{\frac{1}{425}\left(\frac{22}{4}\right)}{\frac{1}{425}\left(\frac{22}{4}+\frac{26\times 3}{4}\right)} = \frac{\frac{11}{2}}{25} = \frac{11}{50}$$

Question 13:

Probability that A speaker truth is $\frac{4}{5}$. A coin is tossed A reports that a head appears the probability that actually there was head is

(A) $\frac{4}{5}$ (B) $\frac{1}{2}$ (C) $\frac{1}{5}$ (D) $\frac{2}{5}$

Answer 13:

Let E_1 and E_2 be the event such that E_1: A speak truth E_2: A speak false.

Let X be the event that a head appears. Therefore,

$$P(E_1) = \frac{4}{5} \text{ and } P(E_2) = 1 - P(E_1) = 1 - \frac{4}{5} = \frac{1}{5}$$

If a coin tossed, then it may result either head (H) or tail (T). The probability of getting a head is $\frac{1}{2}$ whether A speak truth or not.

$$\therefore\ P(X|E_1) = P(X|E_2) = \frac{1}{2}$$

The probability that there is actually a head is given by $P(E_1|X)$

$$P(E_1|X) = \frac{P(E_1).P(A|E_1)}{P(E_1).P(A|E_1) + P(E_2).P(A|E_2)}$$

$$= \frac{\frac{4}{5}\times\frac{1}{2}}{\frac{4}{5}\times\frac{1}{2}+\frac{4}{5}\times\frac{1}{2}} = \frac{\frac{4}{5}\times\frac{1}{2}}{\frac{1}{2}\left(\frac{4}{5}+\frac{1}{5}\right)} = \frac{\frac{4}{5}}{\frac{5}{5}} = \frac{4}{5}$$

Question 14:

If A and B are two events such that $A \subset B$ and $P(B) \neq 0$, then which of the following is correct?

(A) $P(A|B) = \frac{P(B)}{P(A)}$ (B) $P(A|B) < P(A)$ (C) $P(A|B) \geq P(A)$ (D) None of these

Answer 14:

If $A \subset B$, then $A \cap B = A$, therefore $P(A\cap B) = P(A)$ also $P(A) < P(B)$.

Consider, $$P(A|B) = \frac{P(A\cap B)}{P(B)} = \frac{P(A)}{P(B)} \neq \frac{P(B)}{P(A)}$$

$$\Rightarrow P(A|B) = \frac{P(A\cap B)}{P(B)} = \frac{P(A)}{P(B)} \qquad \text{... (1)}$$

We know that, $P(B) \leq 1 \quad \Rightarrow \frac{1}{P(B)} \geq 1$

Therefore, from equation (1), we have $\frac{P(A)}{P(B)} \geq P(A)$

$$\Rightarrow P(A|B) \geq P(A) \qquad \text{... (2)}$$

$\Rightarrow P(A|B)$ is not less than $P(A)$.

Hence, from equation (2), it can be concluded that the relation given in alternative (C) is correct.

Mathematics

(Chapter – 13) (Probability) (Miscellaneous Exercise)
(Class – XII)

Question 1:

A and B are two events such that $P(A) \neq 0$. Find P(B|A), if

(i) A is a subset of B **(ii)** $A \cap B = \emptyset$

Answer 1:

It is given that, $P(A) \neq 0$

(i) A is a subset of B. $\Rightarrow A \cap B = A$

$$\therefore P(A \cap B) = P(B \cap A) = P(A) \quad \Rightarrow \quad \mathrm{P(B|A)} = \frac{P(B \cap A)}{P(A)} = \frac{P(A)}{P(A)} = 1$$

(ii) $A \cap B = \emptyset$

$$\Rightarrow (A \cap B) = 0 \Rightarrow \mathrm{P(B|A)} = \frac{P(A \cap B)}{P(A)} = 0$$

Question 2:

A couple has two children,

(i) Find the probability that both children are males, if it is known that at least one of the children is male.

(ii) Find the probability that both children are females, if it is known that the elder child is a female.

Answer 2:

If a couple has two children, then the sample space $S = \{(\mathrm{b,b}), (\mathrm{b,g}), (\mathrm{g,b}), (\mathrm{g,g})\}$

(i) Let E and F respectively denotes the event that both children are male and at least one of the children is a male. Therefore,

$$E \cap F = \{(b,b)\} \Rightarrow P(E \cap F) = \frac{1}{4}$$

Here, $P(E) = \frac{1}{4}$ and $P(F) = \frac{3}{4} \quad \Rightarrow \mathrm{P(E|F)} = \frac{P(E \cap F)}{P(F)} = \frac{\frac{1}{4}}{\frac{3}{4}} = \frac{1}{3}$

(ii) Let A and B respectively denote the events that both children are female and the elder child is female.

$A = \{(\mathrm{g,g})\} \Rightarrow P(A) = \frac{1}{4}$ and $B = \{(\mathrm{g,b}), (\mathrm{g,g})\} \Rightarrow P(B) = \frac{2}{4} = \frac{1}{2}$

$A \cap B = \{(\mathrm{g,g})\} \Rightarrow P(B) = \frac{1}{4} \quad \Rightarrow \quad \mathrm{P(A|B)} = \frac{P(A \cap B)}{P(B)} = \frac{\frac{1}{4}}{\frac{2}{4}} = \frac{1}{2}$

Question 3:

Suppose that 5% of men and 0.25% of women have grey hair. A grey haired person is selected at random. What is the probability of this person being male? Assume that there are equal number of males and females.

Answer 3:

Given that 5% of men and 0.25% of women have grey hair. Therefore, percentage of people with grey hair = (5 + 0.25) % = 5.25%

Probability that the selected haired person is a male $= \frac{5}{5.25} = \frac{20}{21}$

Question 4:

Suppose that 90% of people are right-handed. What is the probability that at most 6 of a random sample of 10 people are right-handed?

Answer 4:

A person can be either right handed or left handed. It is given that 90% of the people are right handed.

$$\therefore p = P\,(\text{right} - \text{handed}) = \frac{9}{10} \text{ and } q = P(\text{left} - \text{handed}) = 1 - \frac{9}{10} = \frac{1}{10}$$

Using binomial distribution, the probability that more than 6 people are right handed is given by,

$$= \sum_{r=7}^{10} {}^{10}\mathrm{C}_r P^r q^{n-r} \qquad = \sum_{r=7}^{10} {}^{10}\mathrm{C}_r \left(\frac{9}{10}\right)^r \left(\frac{1}{10}\right)^{10-r}$$

Therefore, the probability that at most 6 people are right handed = 1 – P(more than 6 are right handed)

$$= 1 - \sum_{r=7}^{10} {}^{10}\mathrm{C}_r (0.9)^r (0.1)^{10-r}$$

Question 5:
If a leap year is selected at random, what is the chance that it will contain 53 Tuesdays?

Answer 5:
In a leap year, there are 366 days i.e., 52 weeks and 2 days.

In 52 weeks, there are 52 Tuesdays.

Therefore, the probability that the leap year will contain 53 Tuesdays is equal to the probability that the remaining 2 days will be Tuesdays.

The remaining 2 days can be

Monday and Tuesday, Tuesday and Wednesday, Wednesday and Thursday,
Thursday and Friday, Friday and Saturday, Saturday and Sunday,
Sunday and Monday

Total number of cases = 7 and favourable cases = 2

∴ Probability that a leap year will have 53 Tuesdays = $\frac{2}{7}$

Question 6:
Suppose we have four boxes A, B, C and D containing coloured marbles as given below:

Box	**Marble Colour**		
	Red	White	Black
A	1	6	3
B	6	2	2
C	8	1	1
D	0	6	4

One of the boxes has been selected at random and a single marble is drawn from it. If the marble is red, what is the probability that it was drawn from box A?, box B?, box C?

Answer 6:
Let R be the event of drawing the red marble. Let E_A, E_B, and E_C respectively denote the events of selecting the box A, B, and C.

Total number of marbles = 40, Number of red marble = 15

$$\therefore P\,(R) = \frac{15}{40} = \frac{3}{8}$$

Probability of drawing the red marble from box A is given by P (E_A|R).

$$\therefore P\,(E_A|R) = \frac{P\,(E_A \cap R)}{P\,(R)} = \frac{\frac{1}{40}}{\frac{3}{8}} = \frac{1}{15}$$

Probability that the red marble is from box B is P(E_B|R).

$$\therefore P\,(E_B|R) = \frac{P\,(E_B \cap R)}{P\,(R)} = \frac{\frac{6}{40}}{\frac{3}{8}} = \frac{2}{5}$$

Probability that the red marble is from box C is P(E_C|R).

$$\therefore P\,(E_C|R) = \frac{P\,(E_C \cap R)}{P\,(R)} = \frac{\frac{8}{40}}{\frac{3}{8}} = \frac{8}{15}$$

Question 7:
Assume that the chances of a patient having a heart attack is 40%. It is also assumed that a meditation and yoga course reduce the risk of heart attack by 30% and prescription of certain drug reduces its chances by 25%. At a time a patient can choose any one of the two options with equal probabilities. It is given that after going through one of the two options the patient selected at random suffers a heart attack. Find the probability that the patient followed a course of meditation and yoga?

Answer 7:

Let A, E_1, and E_2 respectively denote the events that a person has a heart attack, the selected person followed the course of yoga and meditation, and the person adopted the drug prescription.

$\therefore P(A) = 0.40, \ P(E_1) = P(E_2) = \frac{1}{2},$

$P(A|E_1) = 0.40 \times 0.70 = 0.28$ and $P(A|E_2) = 0.40 \times 0.75 = 0.30$

Probability that the patient suffering a heart attack followed a course of meditation and yoga is given by P (E_1|A).

$$P(E_1|A) = \frac{P(E_1).P(A|E_1)}{P(E_1).P(A|E_1) + P(E_2).P(A|E_2)} = \frac{\frac{1}{2} \times 0.28}{\frac{1}{2} \times 0.28 + \frac{1}{2} \times 0.30} = \frac{14}{29}$$

Question 8:

If each element of a second order determinant is either zero or one, what is the probability that the value of the determinant is positive? (Assume that the individual entries of the determinant are chosen independently, each value being assumed with probability 1/2).

Answer 8:

The total number of determinants of second order with each element being 0 or 1 is $(2)^4 = 16$.
The value of determinant is positive in the following cases:

$$\begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix}, \begin{vmatrix} 1 & 1 \\ 0 & 1 \end{vmatrix} \text{ and } \begin{vmatrix} 1 & 0 \\ 1 & 1 \end{vmatrix}$$

$\therefore$ The required probability $= \frac{3}{16}$

Question 9:

An electronic assembly consists of two subsystems, say, A and B. From previous testing procedures, the following probabilities are assumed to be known:

P(A fails) = 0.2
P(B fails alone) = 0.15
P(A and B fail) = 0.15

Evaluate the following probabilities

(i) P(A fails|B has failed) **(ii)** P(A fails alone)

Answer 9:

Let the event in which A fails and B fails be denoted by E_A and E_B.

$P(E_A) = 0.2$

$P(E_A \text{ and } E_B) = 0.15$

P(B fails alone) = $P(E_B) - P(E_A \text{ and } E_B) \Rightarrow 0.15 = P(E_B) - 0.15 \Rightarrow P(E_B) = 0.3$

(i) $P(E_A|E_B)$

$$= \frac{P(E_A \cap E_B)}{P(E_B)} = \frac{0.15}{0.3} = 0.5$$

(ii) P(A fails alone) = $P(E_A) - P(E_A \text{ and } E_B) = 0.2 - 0.15 = 0.05$

Question 10:

Bag I contains 3 red and 4 black balls and Bag II contains 4 red and 5 black balls. One ball is transferred from Bag I to Bag II and then a ball is drawn from Bag II. The ball so drawn is found to be red in colour. Find the probability that the transferred ball is black.

Answer 10:

Let E_1 and E_2 respectively denote the events that a red ball is transferred from bag I to II and a black ball is transferred from bag I to II.

$$P(E_1) = \frac{3}{7} \text{ and } P(E_2) = \frac{4}{7}$$

Let A be the event that the ball drawn is red.

When a red ball is transferred from bag I to bag II, then

$$P\ (A|E_1) = \frac{5}{10} = \frac{1}{2}$$

When a black ball is transferred from bag I to II, then

$$P\ (A|E_2) = \frac{4}{10} = \frac{2}{5}$$

Therefore, $$P\ (E_2|\text{A}\ ) = \frac{P\ (E_2)P\ (A|E_2)}{P\ (E_1)P\ (A|E_1) + P\ (E_2)\ P\ (A|E_2)} = \frac{\frac{4}{7}\times\frac{1}{2}}{\frac{3}{7}\times\frac{1}{2}\ + \frac{4}{7}\times\frac{2}{5}} = \frac{16}{31}$$

Choose the correct answer in each of the following:

Question 11:

If A and B are two events such that P (A) ≠ 0 and P(B|A) = 1, then

(A) A ⊂ B (B) B ⊂ A (C) B = ϕ (D) A = ϕ

Answer 11:

Given that: $P\ (A) \neq 0$ and $P\ (B|A) = 1$, therefore

$$P\ (B|A) = \frac{P\ (B \cap A)}{P\ (A)} \quad \Rightarrow \quad 1 = \frac{P\ (B \cap A)}{P\ (A)}$$

$\Rightarrow P\ (A) = P\ (B \cap A)$

$\Rightarrow$ A ⊂ B

Hence, the correct answer is (A).

Question 12:

If P(A|B) > P(A), then which of the following is correct:

(A) P(B|A) < P(B)

(B) P(A ∩ B) < P(A).P(B)

(C) P(B|A) > P(B)

(D) P(B|A) = P(B)

Answer 12:

Given that: $P\ (A|B) > P\ (A)$

$$\Rightarrow \frac{P\ (A \cap B)}{P\ (B)} > P\ (A)$$

$\Rightarrow P\ (A \cap B) > P\ (A).P\ (B)$

$\Rightarrow\ P\ (B|A) > P\ (A).P\ (B)$

Hence, the correct answer is (C).

Question 13:

If A and B are any two events such that P(A) + P(B) − P(A and B) = P(A), then

(A) P(B|A) = 1

(B) P(A|B) = 1

(C) P(B|A) = 0

(D) P(A|B) = 0

Answer 13:

Given that: $P(A) + P(B) - \ P(A \text{ and } B\ ) = P(A)$

$\Rightarrow P(A) + P(B) - P(A \cap B) = P(A)$

$\Rightarrow P(B) - P(A \cap B) = 0$

$\Rightarrow P(A \cap B) = P(B)$

$$\therefore P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{P(B)}{P(B)} = 1$$

Hence, the correct answer is (B).