EXERCISE 6.3 (TANGENTS & NORMALS)

QNo.1 Find the slope of tangent to curve $y = 3x^4 - 4x$ at $x = 4$.

Sol. Given Curve is $y = 3x^4 - 4x$.

$$\Rightarrow \frac{dy}{dx} = 3 \times 4x^3 - 4 = 12x^3 - 4.$$

$\therefore$ Slope of tangent to given curve at $x = 4$

$$= \left(\frac{dy}{dx}\right)_{x=4} = 12(4)^3 - 4 = 768 - 4 = 764.$$

QNo.2 Find the slope of tangent to the curve $y = \frac{x-1}{x-2}$; $x \neq 2$ at $x = 10$

Sol. Given $y = \frac{x-1}{x-2}$; $x \neq 2$

$$\Rightarrow \frac{dy}{dx} = \frac{(x-1)(1-0) - (x-1)(1-0)}{(x-2)^2} = \frac{-1}{(x-2)^2}.$$

$\therefore$ Slope of tangent at $x = 10 = \left(\frac{dy}{dx}\right)_{x=10} = \frac{-1}{(10-2)^2} = \frac{-1}{64}$.

QNo.3 Find the slope of the tangent to curve $y = x^3 - x + 1$ at point whose x-coordinate is 2.

Sol. Given Curve is $y = x^3 - x + 1$

$$\therefore \frac{dy}{dx} = 3x^2 - 1.$$

$\therefore$ Slope of tangent where x-coordinate is $x = 2$.

$$= \left(\frac{dy}{dx}\right)_{x=2} = 3(2)^2 - 1 = 12 - 1 = 11.$$

QNo 4. Find the slope of the tangent to the curve $y = x^3 - 3x + 2$ at point whose x-coordinate is $x = 3$.

Sol. Given Curve is $y = x^3 - 3x + 2$

$$\therefore \frac{dy}{dx} = 3x^2 - 3$$

$\therefore$ Slope of tangent where x-coordinate is 3

$$= \left(\frac{dy}{dx}\right)_{x=2} = 3(3)^2 - 3 = 27 - 3 = 24.$$

Q No 5 Find the slope of normal to curve $x = a\cos^3\theta$ $y = a\sin^3\theta$ at $\theta = \frac{\pi}{4}$

Sol:

$x = a\cos^3\theta$; $y = a\sin^3\theta$.

$$\frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{\frac{d}{d\theta}[a\sin^3\theta]}{\frac{d}{d\theta}[a\cos^3\theta]} = \frac{a(3\sin^2\theta\cos\theta)}{a(3.\cos^2\theta(-\sin\theta))} = -\tan\theta.$$

$\therefore$ Slope of Normal at $\theta = \frac{\pi}{4} = \frac{-1}{\left(\frac{dy}{dx}\right)_{\theta=\pi/4}} = \frac{-1}{(-\tan\theta)_{\pi/4}} = 1.$

Q No.6 find the slope of normal to the curve at $x = 1 - a\sin\theta$ $y = b\cos^2\theta$ at $\theta = \pi/2$.

Sol Given $x = 1 - a\sin\theta$; $y = b\cos^2\theta$.

$$\frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{\frac{d}{d\theta}(b\cos^2\theta)}{\frac{d}{d\theta}(1-a\sin\theta)} = \frac{b(2\cos\theta(-\sin\theta))}{0 - a\cos\theta} = \frac{2b}{a}(\sin\theta)$$

$\therefore$ Slope of Normal at $\theta = \frac{\pi}{2} = \frac{-1}{\left(\frac{dy}{dx}\right)_{\theta=\pi/2}} = \frac{-1}{\frac{2b}{a}\left(\sin\frac{\pi}{2}\right)} = \frac{-a}{2b}$

Q No 7. Find point at which tangent to the curve $y = x^3 - 3x^2 - 9x + 7$ is parallel to x-axis.

Sol: Given Curve is $y = x^3 - 3x^2 - 9x + 7$ ---- (1)

$\Rightarrow \frac{dy}{dx} = 3x^2 - 6x - 9 = 3(x^2 - 2x - 3)$

Now tangents will be parallel to x-axis if slope is zero

ie if $\frac{dy}{dx} = 0$.

$\Rightarrow 3x^2 - 6x - 9 = 0$ ie $3(x^2 - 2x - 3) = 0$ ie $x^2 - 2x - 3 = 0$

ie $(x-3)(x+1) = 0 \Rightarrow x = 3, -1$.

When $x = 3$ $\quad y = x^3 - 3x^2 - 9x + 7 \Rightarrow y = (3)^3 - 3(3)^2 - 9(3) + 7$

$\Rightarrow y = 27 - 27 - 27 + 7 = -20$

When $x = -1$ $\quad y = (-1)^3 - 3(-1)^2 - 9(-1) + 7 = -1 - 3 + 9 + 7 = 12$

$\therefore$ Required points are $(3, -20)$ and $(-1, 12)$.

Q No. 8. Find a point on the curve $y=(x-2)^2$ at which tangent is parallel to the chord joining the points $(2,0),(4,4)$

Sol: Given Curve is $y=(x-2)^2$.

$$\therefore \frac{dy}{dx} = 2(x-2)$$

Also slope of chord joining $(2,0)$ and $(4,4)$

$$= \frac{4-0}{4-2} = \frac{4}{2} = 2 \qquad \left[\because \text{slope} = \frac{y_2-y_1}{x_2-x_1}\right]$$

Now for the point at which tangent is parallel to chord joining given points, we must have.

slope of tangent = slope of chord.

ie $2(x-2) = 2 \Rightarrow x = 3.$

When $x=3$, $y=(x-2)^2 \Rightarrow y=(3-2)^2=1.$

$\therefore$ Required point is $(3,1)$

Q No 9. Find the point on the curve $y=x^3-11x+5$ at which tangent is $y=x-11$.

Sol. Given Curve is $y=x^3-11x+5$

$$\therefore \frac{dy}{dx} = 3x^2-11. \quad \dots (1)$$

Given tangent is $y=x-11$

$\therefore$ slope of line $y=x-11$ is 1 ... (2) $\quad \left[\because y=mx+c \text{ form}\right]$

Now for required point from (1) and (2)

$$3x^2-11=1$$

ie $3x^2=12 \Rightarrow x^2=12/3 \Rightarrow x^2=4 \Rightarrow x=\pm 2$

When $x=2$; $y=(2)^3-11(2)+5 = 8-22+5=-9$

When $x=-2$; $y=(-2)^3-11(-2)+5 = -8+22+5=19$

$\therefore$ At points slope of tangent is 1 are $(2,-9),(-2,19)$

But only point $(2,-9)$ satisfies the equation of tangent

$y = x - 11$ $\therefore$ Required point is $(2, -9)$.

Q No 10 Find the equation of lines having slope -1 that are tangents to the curve $y = \frac{1}{x-1}$; $x \neq 1$

Sol Given Curve is $y = \frac{1}{x-1}$; $x \neq 1$ ---(1)

$\Rightarrow \frac{dy}{dx} = \frac{-1}{(x-1)^2}$

For tangents having slope $= -1$, we must have

$-\frac{1}{(x-1)^2} = -1 \Rightarrow (x-1)^2 = 1. \Rightarrow x - 1 = \pm 1.$

$\Rightarrow x = 2, 0.$

When $x = 2$

$y = \frac{1}{x-1}$

$\Rightarrow y = \frac{1}{2-1} = 1$

$\therefore$ Eq. of line tangent passing through $(2, 1)$ is

$y - 1 = -1(x - 2)$

ie $x + y - 3 = 0$

When $x = 0$

$y = \frac{1}{x-1}$

$\Rightarrow y = \frac{1}{0-1} = -1.$

$\therefore$ Equation of tangent passing through $(0, -1)$ is

$y + 1 = -1(x - 0)$

ie $x + y + 1 = 0.$

$\therefore$ Required tangents are $x + y - 3 = 0$ and $x + y + 1 = 0.$

Q No 11 Find the equations of all lines having slope 2 which are tangents to the curve $y = \frac{1}{x-3}$; $x \neq 3$

Sol: Given Curve is $y = \frac{1}{x-3}$

$\therefore \frac{dy}{dx} = \frac{-1}{(x-3)^2}$

For tangents having slope 2, we must have

$\frac{-1}{(x-3)^2} = 2$ ie $(x-3)^2 = -\frac{1}{2}.$

Which is not possible as square of any real No. can not be negative.

Hence there is no point on given curve whose slope of tangent is 2.

Q No 12 Find the equations of all lines having slope zero which are tangent to the curve $y = \frac{1}{x^2-2x+3}$.

Sol. Given curve is $y = \frac{1}{x^2-2x+3}$ ---(1)

$\Rightarrow \frac{dy}{dx} = \frac{-1}{(x^2-2x+3)^2} \times (2x-2) = \frac{-2(x-1)}{(x^2-2x+3)^2}$.

For tangents having slope 0

$\frac{dy}{dx} = 0$ ie $\frac{-2(x-1)}{(x^2-2x+3)^2} = 0$ ie $-2(x-1)=0$

ie $x = 1$.

When $x=1$; $y = \frac{1}{(1)^2-2(1)+3} = \frac{1}{1-2+3} = \frac{1}{2}$.

$\therefore$ Tangent to given curve at $(1, \frac{1}{2})$ with slope 0 will have equation $y-\frac{1}{2} = 0(x-1)$

ie $y-\frac{1}{2}=0 \Rightarrow 2y-1=0$

Q No. 13 Find the points on the curve $\frac{x^2}{9}+\frac{y^2}{16}=1$ at which tangents are (i) parallel to x-axis (ii) parallel to y-axis

Sol: Given curve is $\frac{x^2}{9}+\frac{y^2}{16}=1$ --(1)

Differentiating both sides w.r.t x

$\frac{2x}{9}+\frac{2y}{16}\cdot\frac{dy}{dx}=0 \Rightarrow \frac{dy}{dx} = \frac{-\frac{2x}{9}}{\frac{2y}{16}} = -\frac{16x}{9y}$.

(i) For tangent parallel to x-axis, we must have

$\frac{dy}{dx}=0$ ie $-\frac{16x}{9y} = 0$ ie $x=0$

and $x=0 \Rightarrow \frac{0^2}{9}+\frac{y^2}{16}=1 \Rightarrow y^2=16 \Rightarrow y=\pm 4$.

$\therefore$ Required points are $(0,4),(0,-4)$

(ii) For tangents parallel to y-axis we must have

$\frac{dx}{dy}=0$ ie $\frac{9y}{-16x}=0 \Rightarrow y=0$

and from (1) $y=0 \Rightarrow \frac{x^2}{9}+0=1 \Rightarrow x^2=9 \Rightarrow x=\pm 3$

$\therefore$ Required points are $(3,0),(-3,0)$

Q.No. 14 : Find the equations of tangents and Normal to the given Curve at indicated points..

(i) $y = x^4 - 6x^3 + 13x^2 - 10x + 5$ at $(0,5)$

Given Curve is $y = x^4 - 6x^3 + 13x^2 - 10x + 5$

$\Rightarrow \frac{dy}{dx} = 4x^3 - 18x^2 + 26x - 10$

$\therefore$ slope of tangent at $(0,5) = \left(\frac{dy}{dx}\right)_{x=0} = 0 - 0 + 0 - 10 = -10$

Hence eqn. of tangent at $(0,5)$ is

$y - 5 = -10(x - 0)$ or $10x + y - 5 = 0$

Again slope of Normal at $(0,5) = \frac{-1}{\text{Slope of tangent}} = \frac{-1}{-10} = \frac{1}{10}$.

$\therefore$ Eqn of Normal at $(0,5)$ is

$y - 5 = \frac{1}{10}(x - 0)$ ie $x - 10y + 50 = 0$

(ii) $y = x^4 - 6x^3 + 13x^2 - 10x + 5$ at $(1,3)$

Sol: Given Curve is $y = x^4 - 6x^3 + 13x^2 - 10x + 5$

$\frac{dy}{dx} = 4x^3 - 18x^2 + 26x - 10.$

$\therefore$ Slope of tangent at $(1,3) = \left(\frac{dy}{dx}\right)_{x=1} = 4(1)^3 - 18(1)^2 + 26(1) - 10 = 2$

$\therefore$ Eqn. of tangent at $(1,3)$ is $y - 3 = 2(x - 1)$

ie $2x - y + 1 = 0.$

Again slope of Normal at $(1,3) = \frac{-1}{\text{slope of tangent}} = -\frac{1}{2}.$

$\therefore$ Eqn of Normal at $(1,3)$ is $y - 3 = -\frac{1}{2}(x - 1)$

ie $x + 2y - 7 = 0$

(iii) $y = x^3$ at $(1,1)$

Given Curve is $y = x^3 \Rightarrow \frac{dy}{dx} = 3x^2.$

$\therefore$ slope of tangent at $(1,1) = \left(\frac{dy}{dx}\right)_{(1,1)} = 3(1)^2 = 3$

$\therefore$ Eqn. of tangent at $(1,1)$: $y - 1 = 3(x - 1)$ ie $3x - y - 2 = 0$

Again slope of Normal at $(1,1) = -1/\text{slope of tangent} = -1/3.$

$\therefore$ Eqn. of reqd Normal is $y - 1 = -\frac{1}{3}(x - 1)$ ie $x + 3y - 4 = 0$

(iv) $y = x^2$ at $(0,0)$

Sol. Given Curve is $y = x^2$

$\therefore \frac{dy}{dx} = 2x$.

$\therefore$ Slope of tangent at $(0,0) = \left(\frac{dy}{dx}\right)_{(0,0)} = 2(0) = 0$

$\therefore$ Eqn of tangent at $(0,0)$ = $y - 0 = 0(x-0)$ ie $y = 0$ ie parallel to x-axis

$\therefore$ Nosmal to given cusve will be parallel to y-axis and its eqn is $x = 0$

[$\because$ line through $(0,0)$ and parallel to y-axis is $x = 0$]

(v) $x = \cos t$; $y = \sin t$ at $t = \pi/4$

Sol: Given Curve is $x = \cos t$; $y = \sin t$.

Now point on given curve corresponding to $t = \pi/4$ is $(\cos \pi/4, \sin \pi/4)$ or $\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$

Also slope of tangent at $t = \pi/4$

$$= \left(\frac{dy}{dx}\right)_{t=\pi/4} = \left(\frac{(dy/dt)}{(dx/dt)}\right)_{t=\pi/4} = \left(\frac{\cos t}{-\sin t}\right)_{t=\pi/4} = \frac{1/\sqrt{2}}{-\frac{1}{\sqrt{2}}} = -1$$

$\therefore$ Eqn. of tangent at $t = \pi/4$ is

$$y - \frac{1}{\sqrt{2}} = -1\left(x - \frac{1}{\sqrt{2}}\right) \text{ or } x + y - \sqrt{2} = 0$$

Again slope of Normal at $t = \pi/4 = \frac{-1}{\text{slope of tangent}} = +1$

$\therefore$ Eqn. of Normal is $y - \frac{1}{\sqrt{2}} = 1\left(x - \frac{1}{\sqrt{2}}\right)$ or $x - y = 0$

QNo 15 Find the eqn of the tangent line to the curve $y = x^2 - 2x + 7$ which is

(a) Parallel to $2x - y + 9 = 0$

(b) Perpendicular to the line $5y - 15x = 13$

Sol. : Given Curve is $y = x^2 - 2x + 7$.

$\Rightarrow \frac{dy}{dx} = 2x - 2$.

(a) Slope of given line $2x - y + 9 = 0$

is $-\left(\frac{2}{-1}\right) = 2$. $\left[\begin{array}{l} \text{Slope of } ax+by+c=0 \\ = -\frac{a}{b} = -\left(\frac{\text{Coeff of } x}{\text{Coeff of } y}\right) \end{array}\right.$

$\therefore$ For tangent(s) parallel to $2x - y + 9 = 0$

$2x - 2 = 2 \Rightarrow 2x = 4 \Rightarrow x = 2$.

When $x = 2$, $y = (2)^2 - 2(2) + 7 = 7$.

$\therefore$ Eqn of reqd. tangent at $(2,7)$ is

$y - 7 = 2(x-2)$ or $2x - y + 3 = 0$

(b) Slope of given line $5y - 15x = 13$ is $-\left(\frac{-15}{5}\right) = 3$.

$\therefore$ Slope of any line which is perpendicular to given line will be $\left(-\frac{1}{3}\right)$

$\therefore$ For tangents perpendicular to given line

$\frac{dy}{dx} = -\frac{1}{3}$ ie $2x - 2 = -\frac{1}{3}$

$\Rightarrow 2x = 2 - \frac{1}{3}$ ie $x = \frac{5}{3\times 2} = \frac{5}{6}$

When $x = \frac{5}{6}$, $y = \left(\frac{5}{6}\right)^2 - 2\times\frac{5}{6} + 7 = \frac{25}{36} - \frac{10}{36} + 7 = \frac{217}{36}$

$\therefore$ The point on curve where tangent is perpendicular to given line is $\left(\frac{5}{6}, \frac{217}{36}\right)$ and

Eqn. of tangent is $y - \frac{217}{36} = -\frac{1}{3}\left(x - \frac{5}{6}\right)$

ie $y - \frac{217}{36} = -\frac{1}{3}x + \frac{5}{18}$

or $12x + 36y - 227 = 0$

Q.No.16. Show that the tangents to the curve $y=7x^3+11$ at the point where $x=2$ and $x=-2$ are parallel.

Sol Given Curve is $y=7x^3+11$

$\Rightarrow \frac{dy}{dx} = 21x^2$

$\therefore$ Slope of tangent at $x=2$ is $\left(\frac{dy}{dx}\right)_{x=2}$

ie $21(2)^2 = 21\times4 = 84.$

Slope of tangent at $x=-2$ is $\left(\frac{dy}{dx}\right)_{x=-2}$

ie $21(-2)^2 = 21\times4 = 84.$

As the slopes of tangent at $x=2$ and $x=-2$ are equal.

$\therefore$ Tangents at $x=2$ and $x=-2$ are parallel.

Q.No.17: Find the points on the curve $y=x^3$ at which the slope of the tangent is equal to the y-coordinate of the point.

Sol. The given curve is $y=x^3$

$\Rightarrow \frac{dy}{dx} = 3x^2$

According to given condition, we have

$\frac{dy}{dx} = y$ ie $3x^2=y.$

Now $y=x^3$

$\therefore 3x^2=x^3 \Rightarrow x^2(3-x)=0$

$\Rightarrow x=0$ or $x=3$

When $x=0$; $y=(0)^3=0$

When $x=3$; $y=(3)^3=27$

Hence the required are $(0,0)$ and $(3,27)$.

Q.No.18: For the curve $y=4x^3-2x^5$ find all the points at which the tangent passes through origin.

Sol: The given curve is $y=4x^3-2x^5$

Let (x_1, y_1) be the point at which tangent passes through the origin

Then $y_1 = 4x_1^3 - 2x_1^5$ ---(1)

Also. $\frac{dy}{dx} = 12x^2 - 10x^4$

$\Rightarrow \left(\frac{dy}{dx}\right)_{(x_1, y_1)} = 12x_1^2 - 10x_1^4$.

$\therefore$ Slope of tangent at (x_1, y_1) = $12x_1^2 - 10x_1^4$.

Hence the equation of tangent at (x_1, y_1) is

$$y - y_1 = (12x_1^2 - 10x_1^4)(x - x_1)$$

As it passes through origin $(0,0)$

$$\therefore \quad 0 - y_1 = (12x_1^2 - 10x_1^4)(0 - x_1)$$

or $\quad y_1 = 12x_1^3 - 10x_1^5 \quad \text{------- (2)}$

From (1) and (2)

$$4x_1^3 - 2x_1^5 = 12x_1^3 - 10x_1^5$$

$\Rightarrow \quad 8x_1^3 - 8x_1^5 = 0 \Rightarrow 8x_1^3(1 - x_1^2) = 0$

$\Rightarrow \quad 8x_1^3(1 - x_1)(1 + x_1) = 0.$

$\Rightarrow \quad x_1 = 0, 1, -1.$

When $x_1 = 0$; $y_1 = 4(0)^3 - 2(0)^5 = 0$

When $x_1 = 1$; $y_1 = 4(1)^3 - 2(1)^5 = 4 - 2 = 2$

When $x_1 = -1$; $y_1 = 4(-1)^3 - 2(-1)^5 = -4 + 2 = -2$

$\therefore$ The required points are $(0,0)$ $(1,2)$ and $(-1,-2)$.

Q.No. 19 : Find the points on the curve $x^2 + y^2 - 2x - 3 = 0$ at which tangents are parallel to x-axis.

Sol: Given Curve is $x^2 + y^2 - 2x - 3 = 0$

Differentiating both sides w.r.t x., we get

$$2x + 2y \cdot \frac{dy}{dx} - 2 = 0$$

$\Rightarrow \quad 2y\frac{dy}{dx} = 2 - 2x = 2(1 - x)$

$\Rightarrow \quad \frac{dy}{dx} = \frac{2(1-x)}{2y} = \frac{1-x}{y} \quad \text{---- (1)}$

For tangents parallel to x-axis, we must have

$$\frac{dy}{dx} = 0 \Rightarrow \frac{1-x}{y} = 0.$$

$\Rightarrow 1-x=0 \,;\, y \neq 0$ ie $x=1, y\neq 0.$

$\therefore$ From given Curve $(1)^2 + y^2 + 2\times 1 - 3 = 0$

ie $y^2 - 4 = 0 \Rightarrow y = \pm 2$

Hence the required points are $(1,2)$ and $(1,-2)$.

QNo 20: Find the equation of normal at point (am^2, am^3) for curve $ay^2 = x^3$.

Sol: Given Curve is $ay^2 = x^3$.

$$\Rightarrow a\cdot 2y\cdot\frac{dy}{dx} = 3x^2 \Rightarrow \frac{dy}{dx} = \frac{3x^2}{2ay}.$$

$\therefore$ Slope of tangent at (am^2, am^3)

$$= \left(\frac{dy}{dx}\right)_{(am^2, am^3)} = \frac{3(am^2)^2}{2a\cdot am^3} = \frac{3a^2m^4}{2a^2m^3} = \frac{3}{2}m.$$

$\therefore$ Slope of Normal at given point $= -\frac{2}{3m}$

Hence eqn. of Normal at given point is

$$y - am^3 = -\frac{2}{3m}(x - am^2)$$

ie $3my - 3am^4 = -2x + 2am^2$.

ie $2x + 3my - 3am^4 - 2am^2 = 0$

QNo 21: Find the equation of Normal to curve $y = x^3 + 2x + 6$ which are parallel to line $x + 14y + 4 = 0$

Sol: given line is $x + 14y + 4 = 0$

$\therefore$ Slope of given line $= -\frac{1}{14}$.

$\therefore$ Slope of Normal which is parallel to given curve $= -\frac{1}{14}$ ---- (1)

Now Given Curve is $y = x^3 + 2x + 6$

Let (x_1, y_1) be the point where the Normal is parallel to given line

Now $\frac{dy}{dx} = 3x^2 + 2$.

Slope of Tangent at $(x_1, y_1) = \left(\frac{dy}{dx}\right)_{(x_1, y_1)} = 3x_1^2 + 2$

$\therefore$ slope of Normal at $(x_1, y_1) = \frac{-1}{\text{Slope of tangent}} = \frac{-1}{3x_1^2+2}$ ... (2)

From (1) and (2)

$$\frac{-1}{3x_1^2+2} = -\frac{1}{14}.$$

$\Rightarrow$ $3x_1^2 + 2 = 14 \Rightarrow 3x_1^2 = 12 \Rightarrow x_1^2 = 4 \Rightarrow x_1 = \pm 2.$

Now As (x_1, y_1) lies on given Curve

$\therefore$ $y_1 = x_1^3 + 2x_1 + 6$

When $x_1 = 2$; $y_1 = (2)^3 + 2(2) + 6 = 18$

When $x_1 = -2$; $y_1 = (-2)^3 + 2(-2) + 6 = -6$

$\therefore$ There are two points $(2, 18)$ and $(-2, -6)$ where Normal is parallel to given line.

$\therefore$ Equations of required Normal are

$y - 18 = -\frac{1}{14}(x-2)$ and $y + 6 = -\frac{1}{14}(x+2)$

ie $14y - 252 = -x + 2$ and $14y + 84 = -x - 2$

or $x + 14y - 254 = 0$ and $x + 14y + 86 = 0$

QNo 22: Find the equations of the tangent and Normal to the parabola $y^2 = 4ax$ at the point $(at^2, 2at)$

Sol: Given Curve is $y^2 = 4ax$.

$\therefore$ $2y\frac{dy}{dx} = 4a \Rightarrow \frac{dy}{dx} = \frac{4a}{2y}.$

$\therefore$ slope of tangent at given point $= \left(\frac{dy}{dx}\right)_{(at^2, 2at)}$

$= \frac{2a}{2at} = \frac{1}{t}$

and $\therefore$ Slope of Normal at given point $= -t$.

$\therefore$ Eqn. of tangent at $(at^2, 2at)$ is

$$y - 2at = \frac{1}{t}(x - at^2) \quad \text{or} \quad x - ty + at^2 = 0$$

and Eqn of Normal at $(at^2, 2at)$ is

$$y - 2at = -t(x - at^2) \quad \text{or.} \quad tx + y - 2at - at^3 = 0$$

QNo 23: Prove that the curves $x = y^2$ and $xy = k$ cut at right angles if $8k^2 = 1$.

Sol: Given Curve are $x = y^2 \text{ ---- } (1)$

$xy = k \text{ -- } (2)$

Let $A(x_1, y_1)$ be the point of intersection of (1) and (2)

$\Rightarrow x_1 = y_1^2$ and $x_1 y_1 = k$

$\Rightarrow \frac{k}{y_1} = y_1^2$ or $k = y_1^3 \Rightarrow y_1 = (k)^{1/3}$

$\therefore$ from (1) we get $x_1 = (k^{1/3})^2 = (k)^{2/3}$

$\therefore$ (1) and (2) intersect at $(k^{2/3}, k^{1/3})$

Now. From (1) $\frac{dy}{dx} = \frac{1}{2y}$.

$\therefore$ slope of tangent at $(k^{2/3}, k^{1/3}) = \frac{1}{2k^{1/3}}$

From (2) $y = \frac{k}{x} \Rightarrow \frac{dy}{dx} = -\frac{k}{x^2}$

$\therefore$ slope of tangent at $(k^{2/3}, k^{1/3}) = -\frac{k}{(k^{2/3})^2} = -\frac{1}{k^{1/3}}$

Now Two Curves will cut at right angle if tangents are perpendicular

i.e if product of slopes $= -1$

ie. if $\left(\frac{1}{2k^{1/3}}\right)\left(-\frac{1}{k^{1/3}}\right) = -1.$

ie if $\frac{-1}{2k^{2/3}} = -1$ ie $1 = 2k^{2/3}$

$\Rightarrow (1)^3 = \left(2k^{2/3}\right)^3$

$\Rightarrow 1 = 8k^2$ ie $8k^2 = 1.$

QNo 24: Find the equations of tangent and Normal to the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ at the point (x_0, y_0)

Sol: Given Curve is $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ --- (1)

Differentiating both sides

$$\frac{2x}{a^2} - \frac{2y}{b^2} \cdot \frac{dy}{dx} = 0 \Rightarrow \frac{dy}{dx} = \frac{b^2 x}{a^2 y}$$

$\therefore$ Slope of tangent at $(x_0, y_0) = \left(\frac{dy}{dx}\right)_{(x_0, y_0)}$

$$= \frac{b^2 x_0}{a^2 y_0}.$$

Hence eqn. of tangent at (x_0, y_0) is

$$y - y_0 = \frac{b^2 x_0}{a^2 y_0}(x - x_0)$$

or $a^2 y_0 y - a^2 y_0^2 = b^2 x_0 x - b^2 x_0^2$

or. $\frac{y_0 y}{b^2} - \frac{y_0^2}{b^2} = \frac{x_0 x}{a^2} - \frac{x_0^2}{a^2}$ [Dividing by $a^2 b^2$]

or. $\frac{x_0 x}{a^2} - \frac{y_0 y}{b^2} = \frac{x_0^2}{a^2} - \frac{y_0^2}{b^2}$

or $\frac{x_0 x}{a^2} - \frac{y_0 y}{b^2} = 1$ $\left[\because \frac{x_0^2}{a^2} - \frac{y_0^2}{b^2} = 1 \text{ as } (x_0, y_0) \text{ lies on (1)}\right]$

Again slope of Normal at (x_0, y_0)

$$= \frac{-1}{\text{slope of tangent}} = -\frac{a^2 y_0}{b^2 x_0}$$

Hence equation of Normal at (x_0, y_0) is

$$y - y_0 = -\frac{a^2 y_0}{b^2 x_0}(x - x_0)$$

or $$\frac{b^2(y - y_0)}{y_0} = -\frac{a^2(x - x_0)}{x_0}$$

or $$\frac{b^2 y}{y_0} - b^2 = -\frac{a^2 x}{x_0} + a^2$$

or $$\frac{a^2 x}{x_0} + \frac{b^2 y}{y_0} = a^2 + b^2$$

QNo25: Find the equation of the tangent to the curve $y = \sqrt{3x-2}$ which is parallel to line $4x - 2y + 5 = 0$

Sol: Given Curve is $y = \sqrt{3x-2}$ ---- (1)

Let (x_1, y_1) be the point on (1) where tangent is parallel to line $4x - 2y + 5 = 0$

$$\therefore \left(\frac{dy}{dx}\right)_{(x_1, y_1)} = -\left(\frac{4}{-2}\right) \qquad \left[\because \text{slope of } 4x - 2y + 5 = -\frac{4}{-2}\right.$$

ie $$\frac{d}{dx}\left(\sqrt{3x-2}\right)_{(x_1, y_1)} = 2.$$

$$\Rightarrow \left[\frac{1}{2\sqrt{3x-2}} \cdot 3\right]_{(x_1, y_1)} = 2 \quad \text{or} \quad \frac{3}{2\sqrt{3x_1 - 2}} = 2$$

$$\Rightarrow \sqrt{3x_1 - 2} = \frac{3}{4} \quad \text{or} \quad 3x_1 - 2 = \frac{9}{16}$$

$$\Rightarrow 3x_1 = \frac{9}{16} + 2 \quad \text{or} \quad 3x_1 = \frac{41}{16} \quad \text{or} \quad x_1 = \frac{41}{48}$$

Also Since (x_1, y_1) lies on (1)

$$\therefore y_1 = \sqrt{3\left(\frac{41}{48}\right) - 2} = \sqrt{\frac{123 - 96}{48}} = \sqrt{\frac{27}{48}} = \sqrt{\frac{9}{16}} = \frac{3}{4}$$

$\therefore$ Reqd equation of Reqd tangent is

$$y - \frac{3}{4} = 2\left(x - \frac{41}{48}\right)$$

or $$48x - 24y - 23 = 0$$

Q No 26 : The slope the normal to the curve $y = 2x^2 + 3\sin x$ at $x = 0$ is

(A) 3 (B) $\frac{1}{3}$ (C) -3 (D) $-\frac{1}{3}$

Sol : Given Curve is $y = 2x^2 + 3\sin x$

$\therefore \frac{dy}{dx} = 4x + 3\cos x.$

$\therefore$ Slope of tangent at $x = 0$ is $\left(\frac{dy}{dx}\right)_{x=0} = 4(0) + 3\cos 0^\circ = 3$

$\therefore$ slope of Normal $= \frac{-1}{\text{Slope of tangent}} = -\frac{1}{3}$

$\therefore$ (D) is Right option.

Q No 27 : The line $y = x + 1$ is tangent to curve $y^2 = 4x$ at the point

(A) (1, 2) (B) (2, 1) (C) (1, −2) (D) (−1, 2)

Sol : Given Curve is $y^2 = 4x$ ---(1)

Given line is $y = x + 1$ ---(2)

Slope of (2) $x - y + 1 = 0$ is $-\left(\frac{1}{-1}\right) = 1.$

From (1) $2y\left(\frac{dy}{dx}\right) = 4 \Rightarrow \frac{dy}{dx} = \frac{4}{2y} = \frac{2}{y}.$

$\therefore$ According to question $\frac{2}{y} = 1$ or $y = 2.$

$\therefore$ from (1) $(2)^2 = 4x \Rightarrow x = 1.$

$\therefore$ The reqd point is (1, 2)

$\therefore$ (A) is the correct option.

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