Long Answer Type Questions

Q.1. State and explain Charles's law.

Ans. It states that if the pressure remains constant, then the volume of a given mass of a gas increases or decreases by its volume at 0^{o}C for each 1^{o}C rise or fall in temperature.

Let V_0 be the volume of the given mass of a gas at 0^{o}C. according to Charles's law, its volume at 1^{o}C is

$$V_1 = V_0 + \frac{V_0}{273.15}$$

$$= V_0\left[1 + \frac{1}{273.15}\right]$$

Volume of the gas at 2^{o}C.

$$V_2 = V_0\left[1 + \frac{2}{273.15}\right]$$

∴ Volume of the gas at t^oC

$$V_t = V_0\left[1 + \frac{t}{273.15}\right]$$

$$= V_0\left(\frac{273.15+t}{273.15}\right)$$

If T_0 and T are temperatures on kelvin scale corresponding to 0^{o}C and t^oC, then

$$T_0 = 273.15 + 0 = 273.15$$

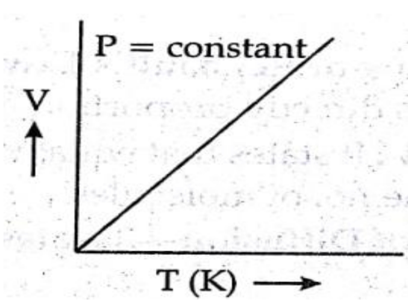
$$T = 273.15 + t$$

$$V_t = V_0\frac{T}{T_0}$$

$$\frac{V_t}{T} = \frac{V_0}{T_0}$$

$$\frac{V}{T} = \text{constant}$$

i.e., $V \propto T$



Q.2. State and derive the perfect or ideal gas equations.

Ans. This equation give the relation between pressure P, volume V and absolute temperature T of a gas.

$$PV = nRT$$

Derivation.

According to Boyle's law

$$V \propto \frac{1}{P} \qquad \ldots(1)$$

According to Charle's law

$$V \propto T \qquad \ldots(2)$$

Comparing (1) and (2), we get

$$\frac{PV}{T} = \text{constant}$$

As $PV = RT$

For n moles of gas $PV = nRT$

This is perfect or ideal gas equation.

Q.3. Determine the numerical values of R and k_B.

Ans. Numerical value R : Consider one mole of a gas at STP, then

$$R = \frac{P_0 V_0}{T_0}$$

Standard pressure

$$P_0 = 0.76 \text{ m of Hg column}$$

$$= 0.76 \times 13.6 \times 10^3 \times 9.8 \text{ N/m}^2$$

Standard temperature = $T_0 = 273.15$K

Volume of one mole of gas at

$$= 22.4 \times 10^{-3} \text{ m}^3$$

$$R = \frac{0.76\times13.6\times10^3\times9.8\times22.4\times10^{-3}}{273.15}$$

$$= 8.31 \text{ J mol}^{-1} \text{ K}^{-1}$$

In the C.G.S. system

$$R = \frac{8.31}{4.2} \text{ cal mol}^{-1} \,^{o}\text{C}^{-1}$$

$$= 1.98 \text{ cal mol}^{-1} \,^{o}\text{C}^{-1}$$

Numerical value of k_B:

We know that

$$k_B = \frac{R}{NA}$$

$$k_B = \frac{8.31 \text{ J mole}^{-1}\text{K}^{-1}}{6.02\times10^{23} \text{ mole}^{-1}}$$

$$=1.38 \times 10^{-23} \text{ J/K.}$$

Q.4. Determine γ from degree of freedom.

Ans. Suppose a polyatomic gas molecule has n degree of freedom.

Total energy associated with one gram molecule of the gas, i.e.,

$$\text{E} = n \times \frac{1}{2}\text{RT} \times 1$$

$$= \frac{n}{2}\text{RT}$$

As $$\text{C}_v = \frac{d}{d\text{T}}€$$

$$= \frac{d}{d\text{T}}\left(\frac{n}{2}\text{RT}\right)$$

$$= \frac{n}{2}\text{R}$$

$$\text{C}_p = \text{C}_v + \text{R}$$

$$\text{C}_p = \frac{n}{2}\text{R} + \text{R}$$

$$= \left(\frac{n}{2} + 1\right)\text{R}$$

$$\gamma = \frac{\text{C}_p}{\text{C}_V}$$

$$\gamma = \frac{\left(\frac{n}{2}+1\right)\text{R}}{\frac{n}{2}\text{R}}$$

∴ $$= \frac{2}{n}\left(\frac{n}{2} + 1\right)$$

$$\gamma = 1 + \frac{2}{n}$$

Q.5. State and explain Gay Lussac's law.

Ans. It states that if the volume remains constant, the pressure of a given mass of a gas increases or decreases by its pressure at 0°C for each 1°C rise or fall in temperature.

If P_0 and P_t are the pressure of a given mass of gas at 0°C and t°C respectively, then according to Gay Lussac's law

$$P_t = P_0\left(1 + \frac{t}{273.15}\right)$$

or $$P_t = P_0\left(\frac{273.15+t}{273.15}\right)$$

or $$P_t = P_0 \frac{T}{T_0}$$

where $T_0(\text{K}) = 273.15$

and $T(\text{K}) = 273.15 + t$

$$\frac{P_t}{P_0} = \frac{T}{T_0}$$

$$\frac{P}{T} = \text{constant}$$

or $$P \propto T$$

Q.6. Deduce Graham's law of diffusion from kinetic theory of gases using expression for pressure.

Ans. Let us consider two gases A and B diffusing into one another. Let ρ_1 and ρ_2 be their densities and v_1 and v_2 be their respective r.ms. velocities.

Pressure exerted by gas A,

$$P_1 = \frac{1}{3}\rho_1 v_1^2$$

and pressure exerted by gas B,

$$P_2 = \frac{1}{3}\rho_2 v_2^2$$

when steady state of diffusion is reached

$$P_1 = P_2$$

$$\frac{1}{3}\rho_1 v_1^2 = \frac{1}{3}\rho_2 v_2^2$$

$$\frac{v_1^2}{v_2^2} = \frac{\rho_2}{\rho_1}$$

or $$\frac{v_1}{v_2} = \sqrt{\frac{\rho_2}{\rho_1}}$$

If r_1 and r_2 be the rates of diffusion of gases A and respectively.

$$\frac{r_1}{r_2} = \frac{v_1}{v_2} = \sqrt{\frac{\rho_2}{\rho_1}}$$

Thus this law states that the rate of diffusion of a gas is inversely proportional to the square root of its density.

$$r \propto \frac{1}{\sqrt{\rho}}$$

Q.7. Derive Dalton's law of practical pressures on the basis of kinetic theory of gases.

Ans. Consider a mixture of gases occupying a volume V. Let m_1, m_2, m_3 be the molecular masses of the gases and n_1, n_2, n_3 be the number of their molecules, P_1, P_2, P_3 … ..the pressure exerted by individual gases and v_1, v_2, v_3 be the r.m.s velocities of the molecules of various gases.

According to kinetic theory

$$P_1 = \frac{1}{3}\frac{m_1 n_1}{V}v_1^2,$$

$$P_2 = \frac{1}{3}\frac{m_2 n_2}{V}v_2^2,$$

$$P_3 = \frac{1}{3}\frac{m_3 n_3}{V}v_3^2$$

Adding,

P = $P_1 + P_2 + P_3 = \frac{1}{3}\frac{m_1 n_1}{V}v_1^2 + \frac{1}{3}\frac{m_2 n_2}{V}v_2^2 + \frac{1}{3}\frac{m_3 n_3}{V}v_3^2$

As the temperature of all the gases in the mixture is same, so their average K.E. will be equal that is,

$\frac{1}{2}m_1 v_1^2 = \frac{1}{2}m_2 v_2^2 = \frac{1}{3}m_3 v_3^2 =\ldots.= \frac{1}{2}mv^2$ (say)

or $m_1 v_1^2 = m_2 v_2^2 = m_3 v_3^2 = \cdots .= mv^2$

Now,

P = $P_1 + P_2 + P_3 + \ldots. = \frac{1}{3V}(n_1 + n_2 + n_3 + \ldots.)mv^2 = \frac{1}{3}\frac{mn}{V}v^2$

here $n = n_1 + n_2 + n_3 + =$ total no. of molecules in the mixture.

But $\frac{1}{3}\frac{mn}{V}v^2$ = P = total pressure exerted by the mixture.

It proves Dalton's Law of particular pressure

∴ P = $P_1 + P_2 + P_3 + \ldots$...

Q.8. Explain why

(a) There is no atmospheric on moon.

(b) There is fall in temperature with altitude.

Ans. (a) The moon has small gravitational force and hence the escape velocity is small. As the moon is in the proximity of the Earth as seen from the Sun, the moon has the same amount of heat per unit area as that of the Earth. The air molecules have large range of speeds. Even though the rms speed of the air molecules is smaller than the escape velocity on the moon, a significant number molecules have speed greater than escape velocity and they escape. Now rest of the molecules arrange the speed distribution for the equilibrium temperature. Again a significant number of molecules escape as their speeds exceed escape speed. Hence, over a long time the moon has lost most of its atmosphere.

At 300 K,

$V_{rms} = \sqrt{\frac{3kT}{m}} = \sqrt{\frac{3\times1.38\times10^{-23}\times300}{7.3\times10^{-26}}}$ = 1.7 km/s

V_{esc} for moon = 4.6 km/s

(b) As the molecules move higher their potential energy increases and hence kinetic energy decreases and hence temperature reduces. At greater height more volume is available and gas expands and hence some cooling takes place.

Q.9. A box of 1.00 m^3 is filled with nitrogen at 1.50 atm at 300 K. the box has a hole of an area 0.010 mm^2. How much time is required for the pressure to reduce by 0.10 atm, if the pressure outside is 1 atm.

Ans. V_{ix} = speed of molecule inside the box along x direction

n_1 = number of molecules per unit volume.

In time Δt, particles moving along the wall will collide if they are within ($V_{ix}\Delta t$) distance. Let a = area of the wall.

No, of particles colliding in time $\Delta t = \frac{1}{2} n_i (V_{ix}\Delta t) a$ (factor of 1/2 due to motion towards wall).

In general, gas Is In equilibrium as the wall is very large as compared to hole.

$\therefore \quad V_{1x}^2 + V_{1y}^2 + V_{1z}^2 = v_{rms}^2$

$\therefore \quad v_{1x}^2 = \frac{V_{rms}^2}{3}$

$\frac{1}{2} m V_{rms}^2 = \frac{3}{2} kT \Rightarrow V_{rms}^2 = \frac{3kT}{m}$

$\therefore V_{1x}^2 = \frac{kT}{m}$

$\therefore$ No. of particles colliding in time $\Delta t = \frac{1}{2} n_1 \sqrt{\frac{kT}{m}} \Delta t\, a$. if particles collide along hole, they move out. Similarly outer particles colliding along hole will move in.

$\therefore$ Net particle flow in time $\Delta t = \frac{1}{2}(n_1 - n_2)\sqrt{\frac{kT}{m}} \Delta t\, a$ as temperature is same in and out.

pV = uRT ⇨ μ = $\frac{PV}{RT}$

$n = \frac{\mu N_A}{V} = \frac{P N_A}{RT}$

After some time τ pressure changes to p_1' inside

$\therefore n_1' = \frac{P_1' N_A}{RT}$

$n_1 V - n_1' V$ = no. of particle gone out

$= \frac{1}{2}(n_1 - n_2)\sqrt{\frac{kT}{m}} \tau a$

$\therefore \frac{P_1 N_A}{RT} V - \frac{P_1' N_A}{RT} V = \frac{1}{2}(P_1 - P_2)\frac{N_A}{RT}\sqrt{\frac{kT}{m}}a$

$\therefore \tau = 2\left(\frac{P_1 - P_1'}{P_1 - P_2}\right)\frac{V}{a}\sqrt{\frac{m}{kT}}$

$= 2\left(\frac{1.5-1.4}{1.5-1.0}\right)\frac{5\times1.00}{0.01\times10^{-6}}\sqrt{\frac{46.7\times10^{-27}}{1.38\times10^{-23}\times300}}$

Q.10. Consider a rectangular block of wood mobbing with a velocity v_0 in a gas at temperature T and mass density ρ. Assume the velocity is along x-axis and the area of cross-section of the block perpendicular to v_0 is A. show that the drag force on the block is 4ρ$Av_0\sqrt{\frac{kT}{m}}$, where m is the mass of the gas molecule.

Ans. n = no. of molecules per unit volume

v_{rms} = rms speed o gas molecules

When block is moving with speed v_0, relative speed of molecules w.r.t. front face = $v + v_0$

Coming heat on, momentum transferred to block per collision

$= 2m(v + v_0)$, where m = mass of molecule.

No. of collission in time $\Delta t = \Delta t \frac{1}{2}(v + v_0)n\Delta t A$,

where A = area of cross section of block and factor of 1/2 appears due to particles moving towards block.

∴ Momentum transferred in time $\Delta t = m(v + v_0)^2 nA\Delta t$ from front surface.

Similarly momentum transferred in time $\Delta t = m(v + v_0)^2 nA\Delta t$ from back surface.

∴ Net force (drag force) $mnA[(v + v_0) - (v - v_0)^2]$ from fron = $mnA(vv_0) = (4mnAv)v_0$

$= (4\rho Av)v_0$

We also have $\frac{1}{2}mv^2 = \frac{1}{2}kT$ (∵ v is the velocity along x-axis)

Therefore, $v = \sqrt{\frac{kT}{m}}$.

Thus, drag = $4\rho A\sqrt{\frac{kT}{m}}v_0$.