# Mathematics

(Chapter – 8) (Introduction to Trigonometry)
(Class X)

## Exercise 8.1

**Question 1:**

In ΔABC right angled at B, AB = 24 cm, BC = 7 m. Determine

(i) sin A, cos A

(ii) sin C, cos C

**Answer 1:**

Applying Pythagoras theorem for ΔABC, we obtain

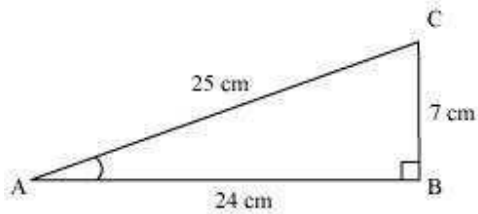
$AC^2 = AB^2 + BC^2$

$= (24\text{ cm})^2 + (7\text{ cm})^2$

$= (576 + 49)\text{ cm}^2$

$= 625\text{ cm}^2$

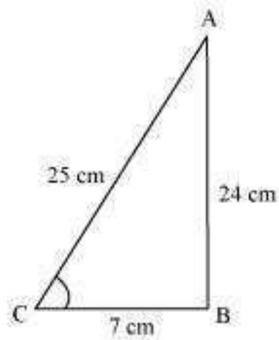
$\therefore$ AC = $\sqrt{625}$ cm = 25 cm




(i) sin A = $\frac{\text{Side opposite to } \angle A}{\text{Hypotenuse}} = \frac{BC}{AC} = \frac{7}{25}$

cos A = $\frac{\text{Side adjacent to } \angle A}{\text{Hypotenuse}} = \frac{AB}{AC} = \frac{24}{25}$

(ii)

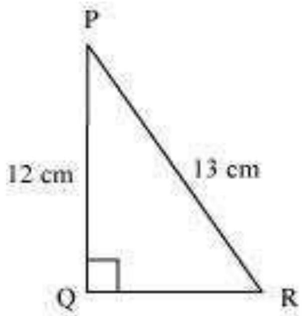


sin C = $\frac{\text{Side opposite to } \angle C}{\text{Hypotenuse}} = \frac{AB}{AC} = \frac{24}{25}$

cos C = $\frac{\text{Side adjacent to } \angle C}{\text{Hypotenuse}} = \frac{BC}{AC} = \frac{7}{25}$

**Question 2:**

In the given figure find tan P − cot R




**Answer 2:**

Applying Pythagoras theorem for ΔPQR, we obtain

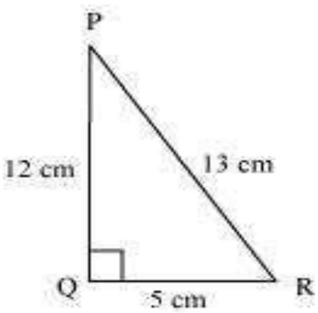
$PR^2 = PQ^2 + QR^2$

$(13\text{ cm})^2 = (12\text{ cm})^2 + QR^2$

$169\text{ cm}^2 = 144\text{ cm}^2 + QR^2$

$25\text{ cm}^2 = QR^2$

QR = 5 cm



$$\tan P = \frac{\text{Side opposite to } \angle P}{\text{Side adjacent to } \angle P} = \frac{QR}{PQ}$$

$$= \frac{5}{12}$$

$$\cot R = \frac{\text{Side adjacent to } \angle R}{\text{Side opposite to } \angle R} = \frac{QR}{PQ}$$

$$= \frac{5}{12}$$

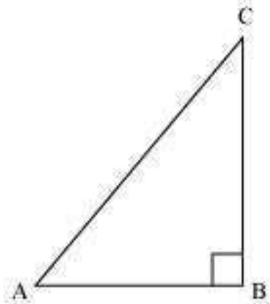
tan P − cot R = $\frac{5}{12} - \frac{5}{12} = 0$

**Question 3:**

If sin A = $\frac{3}{4}$, calculate cos A and tan A.

**Answer 3:**

Let ΔABC be a right-angled triangle, right-angled at point B.




Given that,

$$\sin A = \frac{3}{4}$$

$$\frac{BC}{AC} = \frac{3}{4}$$

Let BC be $3k$. Therefore, AC will be $4k$, where $k$ is a positive integer.

Applying Pythagoras theorem in ΔABC, we obtain

$AC^2 = AB^2 + BC^2$

$(4k)^2 = AB^2 + (3k)^2$

$16k^2 - 9k^2 = AB^2$

$7k^2 = AB^2$

AB = $\sqrt{7}k$

$$\cos A = \frac{\text{Side adjacent to } \angle A}{\text{Hypotenuse}}$$

$$= \frac{AB}{AC} = \frac{\sqrt{7}k}{4k} = \frac{\sqrt{7}}{4}$$

$$\tan A = \frac{\text{Side opposite to } \angle A}{\text{Side adjacent to } \angle A}$$

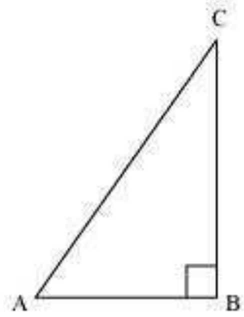
$$= \frac{BC}{AB} = \frac{3k}{\sqrt{7}k} = \frac{3}{\sqrt{7}}$$

**Question 4:**

Given 15 cot A = 8. Find sin A and sec A

**Answer 4:**

Consider a right-angled triangle, right-angled at B.




$$\cot A = \frac{\text{Side adjacent to } \angle A}{\text{Side opposite to } \angle A}$$

$$= \frac{AB}{BC}$$

It is given that,

cot A = $\frac{8}{15}$

$$\frac{AB}{BC} = \frac{8}{15}$$

Let AB be $8k$.Therefore, BC will be $15k$, where $k$ is a positive integer.

Applying Pythagoras theorem in ΔABC, we obtain

$AC^2 = AB^2 + BC^2 = (8k)^2 + (15k)^2$

$= 64k^2 + 225k^2$

$= 289k^2$

AC = 17$k$

$$\sin A = \frac{\text{Side opposite to } \angle A}{\text{Hypotenuse}} = \frac{BC}{AC}$$

$$= \frac{15k}{17k} = \frac{15}{17}$$

$$\sec A = \frac{\text{Hypotenuse}}{\text{Side adjacent to } \angle A}$$

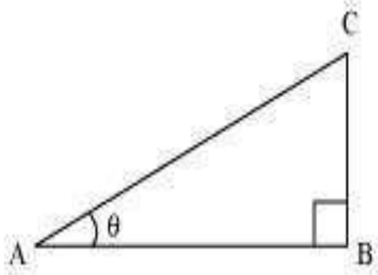
$$= \frac{AC}{AB} = \frac{17}{8}$$

**Question 5:**

Given $\sec\theta = \frac{13}{12}$, calculate all other trigonometric ratios.

**Answer 5:**

Consider a right-angle triangle ΔABC, right-angled at point B.




$$\sec\theta = \frac{\text{Hypotenuse}}{\text{Side adjacent to } \angle\theta}$$

$$\frac{13}{12} = \frac{AC}{AB}$$

If AC is 13$k$, AB will be 12$k$, where $k$ is a positive integer.

Applying Pythagoras theorem in ΔABC, we obtain

$(AC)^2 = (AB)^2 + (BC)^2$

$(13k)^2 = (12k)^2 + (BC)^2$

$169k^2 = 144k^2 + BC^2$

$25k^2 = BC^2$

BC = 5$k$

$$\sin\theta = \frac{\text{Side opposite to } \angle\theta}{\text{Hypotenuse}} = \frac{BC}{AC} = \frac{5k}{13k} = \frac{5}{13}$$

$$\cos\theta = \frac{\text{Side adjacent to } \angle\theta}{\text{Hypotenuse}} = \frac{\text{AB}}{\text{AC}} = \frac{12k}{13k} = \frac{12}{13}$$

$$\tan\theta = \frac{\text{Side opposite to } \angle\theta}{\text{Side adjacent to } \angle\theta} = \frac{\text{BC}}{\text{AB}} = \frac{5k}{12k} = \frac{5}{12}$$

$$\cot\theta = \frac{\text{Side adjacent to } \angle\theta}{\text{Side opposite to } \angle\theta} = \frac{\text{AB}}{\text{BC}} = \frac{12k}{5k} = \frac{12}{5}$$

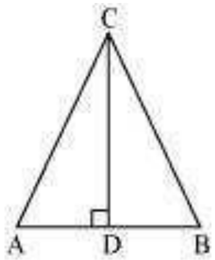
$$\operatorname{cosec}\theta = \frac{\text{Hypotenuse}}{\text{Side opposite to } \angle\theta} = \frac{\text{AC}}{\text{BC}} = \frac{13k}{5k} = \frac{13}{5}$$

**Question 6:**

If ∠A and ∠B are acute angles such that cos A = cos B, then show that ∠A = ∠B.

**Answer 6:**

Let us consider a triangle ABC in which CD ⊥ AB.

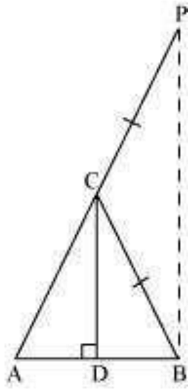



It is given that

cos A = cos B

$$\Rightarrow \frac{\text{AD}}{\text{AC}} = \frac{\text{BD}}{\text{BC}} \quad \text{.....................} (1)$$

We have to prove ∠A = ∠B. To prove this, let us extend AC to P such that BC = CP.




From equation (1), we obtain

$$\frac{AD}{BD}=\frac{AC}{BC}$$

$$\Rightarrow \frac{AD}{BD}=\frac{AC}{CP} \qquad \text{(By construction, we have BC = CP)} \qquad \ldots (2)$$

By using the converse of B.P.T,

CD||BP

⇒ ∠ACD = ∠CPB (Corresponding angles) … (3) And,

∠BCD = ∠CBP (Alternate interior angles) … (4)

By construction, we have BC = CP.

∴ ∠CBP **=** ∠CPB (Angle opposite to equal sides of a triangle) … (5)

From equations (3), (4), and (5), we obtain

∠ACD = ∠BCD … (6)

In ΔCAD and ΔCBD,

∠ACD = ∠BCD [Using equation (6)]

∠CDA = ∠CDB [Both 90°]

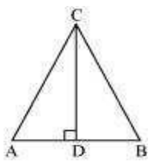
Therefore, the remaining angles should be equal.

∴ ∠CAD = ∠CBD

⇒∠A = ∠B

**Alternatively,**

Let us consider a triangle ABC in which CD ⊥ AB.




It is given that,

cos A = cos B

$$\Rightarrow \frac{AD}{AC} = \frac{BD}{BC}$$

$$\Rightarrow \frac{AD}{BD} = \frac{AC}{BC}$$

Let $\frac{AD}{BD} = \frac{AC}{BC} = k$

$\Rightarrow AD = k\ BD$ … (1)

And, $AC = k\ BC$ … (2)

Using Pythagoras theorem for triangles CAD and CBD, we obtain

$CD^2 = AC^2 - AD^2$ … (3)

And, $CD^2 = BC^2 - BD^2$ … (4)

From equations (3) and (4), we obtain

$AC^2 - AD^2 = BC^2 - BD^2$

$\Rightarrow (k\ BC)^2 - (k\ BD)^2 = BC^2 - BD^2$

$\Rightarrow k^2\ (BC^2 - BD^2) = BC^2 - BD^2$

$\Rightarrow k^2 = 1$

$\Rightarrow k = 1$

Putting this value in equation (2), we obtain

$AC = BC$

$\Rightarrow \angle A = \angle B$(Angles opposite to equal sides of a triangle)

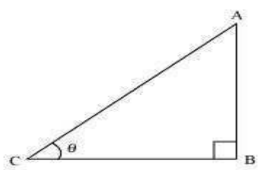
**Question 7:**

If $\cot\theta = \frac{7}{8}$, evaluate

(i) $\frac{(1+\sin\theta)(1-\sin\theta)}{(1+\cos\theta)(1-\cos\theta)}$ (ii) $\cot^2\theta$

**Answer 7:**

Let us consider a right triangle ABC, right-angled at point B.



$$\cot\theta = \frac{\text{Side adjacent to } \angle\theta}{\text{Side opposite to } \angle\theta} = \frac{BC}{AB}$$

$$= \frac{7}{8}$$

If BC is 7*k*, then AB will be 8*k*, where *k* is a positive integer.

Applying Pythagoras theorem in ΔABC, we obtain

$AC^2 = AB^2 + BC^2$

$= (8k)^2 + (7k)^2$

$= 64k^2 + 49k^2$

$= 113k^2$

AC = $\sqrt{113}k$

$$\sin\theta = \frac{\text{Side opposite to } \angle\theta}{\text{Hypotenuse}} = \frac{AB}{AC}$$

$$= \frac{8k}{\sqrt{113}k} = \frac{8}{\sqrt{113}}$$

$$\cos\theta = \frac{\text{Side adjacent to } \angle\theta}{\text{Hypotenuse}} = \frac{BC}{AC}$$

$$= \frac{7k}{\sqrt{113}k} = \frac{7}{\sqrt{113}}$$

(i) $\frac{(1+\sin\theta)(1-\sin\theta)}{(1+\cos\theta)(1-\cos\theta)} = \frac{(1-\sin^2\theta)}{(1-\cos^2\theta)}$

$$= \frac{1-\left(\frac{8}{\sqrt{113}}\right)^2}{1-\left(\frac{7}{\sqrt{113}}\right)^2} = \frac{1-\frac{64}{113}}{1-\frac{49}{113}}$$

$$= \frac{\frac{49}{113}}{\frac{64}{113}} = \frac{49}{64}$$

(ii) $\cot^2\theta = (\cot\theta)^2 = \left(\frac{7}{8}\right)^2 = \frac{49}{64}$

**Question 8:**

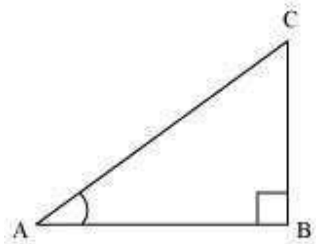
If 3 cot A = 4, Check whether $\frac{1-\tan^2 A}{1+\tan^2 A} = \cos^2 A - \sin^2 A$ or not.

**Answer 8:**

It is given that 3cot A = 4

Or, cot A = $\frac{4}{3}$

Consider a right triangle ABC, right-angled at point B.




$$\cot A = \frac{\text{Side adjacent to } \angle A}{\text{Side opposite to } \angle A}$$

$$\frac{AB}{BC} = \frac{4}{3}$$

If AB is 4*k*, then BC will be 3*k*, where *k* is a positive integer.

In ΔABC,

$(AC)^2 = (AB)^2 + (BC)^2$

$= (4k)^2 + (3k)^2$

$= 16k^2 + 9k^2$

$= 25k^2$

AC = 5*k*

$$\cos A = \frac{\text{Side adjacent to } \angle A}{\text{Hypotenuse}} = \frac{AB}{AC}$$

$$= \frac{4k}{5k} = \frac{4}{5}$$

$$\sin A = \frac{\text{Side opposite to } \angle A}{\text{Hypotenuse}} = \frac{BC}{AC}$$

$$= \frac{3k}{5k} = \frac{3}{5}$$

$$\tan A = \frac{\text{Side opposite to } \angle A}{\text{Hypotenuse}} = \frac{BC}{AB}$$

$$= \frac{3k}{4k} = \frac{3}{4}$$

$$\frac{1-\tan^2 A}{1+\tan^2 A}=\frac{1-\left(\frac{3}{4}\right)^2}{1+\left(\frac{3}{4}\right)^2}=\frac{1-\frac{9}{16}}{1+\frac{9}{16}}$$

$$=\frac{\frac{7}{16}}{\frac{25}{16}}=\frac{7}{25}$$

$\cos^2 A - \sin^2 A = \left(\frac{4}{5}\right)^2-\left(\frac{3}{5}\right)^2$

$$=\frac{16}{25}-\frac{9}{25}=\frac{7}{25}$$

$$\frac{1-\tan^2 A}{1+\tan^2 A}=\cos^2 A-\sin^2 A$$

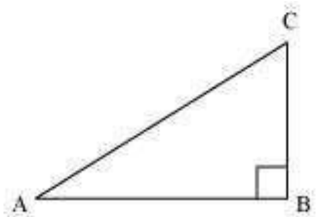
**Question 9:**

In ΔABC, right angled at B. If $\tan A=\frac{1}{\sqrt{3}}$, find the value of

(i) sin A cos C + cos A sin C

(ii) cos A cos C − sin A sin C

**Answer 9:**




$$\tan A=\frac{1}{\sqrt{3}}$$

$$\frac{BC}{AB}=\frac{1}{\sqrt{3}}$$

If BC is $k$, then AB will be $\sqrt{3}k$, where $k$ is a positive integer.

In ΔABC,

$AC^2 = AB^2 + BC^2$

$= \left(\sqrt{3}k\right)^2+(k)^2$

$= 3k^2 + k^2 = 4k^2$

$\therefore$ AC = $2k$

$$\sin A = \frac{\text{Side opposite to } \angle A}{\text{Hypotenuse}} = \frac{BC}{AC} = \frac{k}{2k} = \frac{1}{2}$$

$$\cos A = \frac{\text{Side adjacent to } \angle A}{\text{Hypotenuse}} = \frac{AB}{AC} = \frac{\sqrt{3}k}{2k} = \frac{\sqrt{3}}{2}$$

$$\sin C = \frac{\text{Side opposite to } \angle C}{\text{Hypotenuse}} = \frac{AB}{AC} = \frac{\sqrt{3}k}{2k} = \frac{\sqrt{3}}{2}$$

$$\cos C = \frac{\text{Side adjacent to } \angle C}{\text{Hypotenuse}} = \frac{BC}{AC} = \frac{k}{2k} = \frac{1}{2}$$

(i) sin A cos C + cos A sin C

$$= \left(\frac{1}{2}\right)\left(\frac{1}{2}\right) + \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{3}}{2}\right) = \frac{1}{4} + \frac{3}{4}$$

$$= \frac{4}{4} = 1$$

(ii) cos A cos C − sin A sin C

$$= \left(\frac{\sqrt{3}}{2}\right)\left(\frac{1}{2}\right) - \left(\frac{1}{2}\right)\left(\frac{\sqrt{3}}{2}\right) = \frac{\sqrt{3}}{4} - \frac{\sqrt{3}}{4} = 0$$

**Question 10:**

In ΔPQR, right angled at Q, PR + QR = 25 cm and PQ = 5 cm. Determine the values of sin P, cos P and tan P.

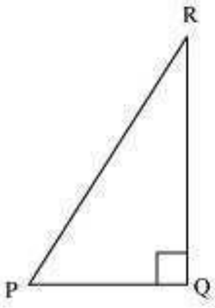
**Answer 10:**

Given that, PR + QR = 25

PQ = 5

Let PR be $x$.

Therefore, QR = $25 - x$




Applying Pythagoras theorem in ΔPQR, we obtain

$PR^2 = PQ^2 + QR^2$

$x^2 = (5)^2 + (25 - x)^2$

$x^2 = 25 + 625 + x^2 - 50x$

$50x = 650$

$x = 13$

Therefore, PR = 13 cm

QR = (25 − 13) cm = 12 cm

$$\sin P = \frac{\text{Side opposite to } \angle P}{\text{Hypotenuse}} = \frac{QR}{PR} = \frac{12}{13}$$

$$\cos P = \frac{\text{Side adjacent to } \angle P}{\text{Hypotenuse}} = \frac{PQ}{PR} = \frac{5}{13}$$

$$\tan P = \frac{\text{Side opposite to } \angle P}{\text{Side adjacent to } \angle P} = \frac{QR}{PQ} = \frac{12}{5}$$

**Question 11:**

State whether the following are true or false. Justify your answer.

(i) The value of tan A is always less than 1.

(ii) sec A = $\frac{12}{5}$ for some value of angle A.

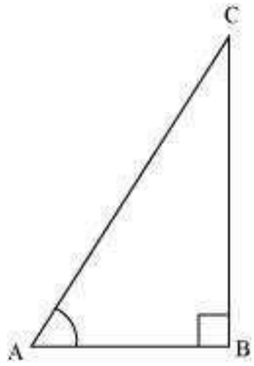
(iii) cos A is the abbreviation used for the cosecant of angle A.

(iv) cot A is the product of cot and A

(v) sin θ = $\frac{4}{3}$, for some angle θ

**Answer 11:**

(i) Consider a ΔABC, right-angled at B.




$$\tan A = \frac{\text{Side opposite to } \angle A}{\text{Side adjacent to } \angle A}$$

$$= \frac{12}{5}$$

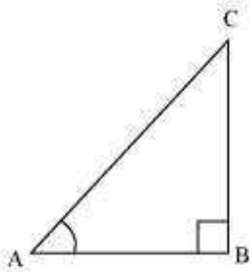
But $\frac{12}{5} > 1$

$\therefore \tan A > 1$

So, tan A < 1 is not always true.

Hence, the given statement is false.

(ii) $\sec A = \frac{12}{5}$




$$\frac{\text{Hypotenuse}}{\text{Side adjacent to } \angle A} = \frac{12}{5}$$

$$\frac{AC}{AB} = \frac{12}{5}$$

Let AC be $12k$, AB will be $5k$, where $k$ is a positive integer.

Applying Pythagoras theorem in ΔABC, we obtain

$AC^2 = AB^2 + BC^2$

$(12k)^2 = (5k)^2 + BC^2$

$144k^2 = 25k^2 + BC^2$

$BC^2 = 119k^2$

BC = 10.9$k$

It can be observed that for given two sides AC = 12$k$ and AB = 5$k$,

BC should be such that,

AC − AB < BC < AC + AB

$12k - 5k < BC < 12k + 5k$

$7k < BC < 17k$

However, BC = 10.9$k$. Clearly, such a triangle is possible and hence, such value of sec A is possible.

Hence, the given statement is true.

(iii) Abbreviation used for cosecant of angle A is cosec A. And cos A is the abbreviation used for cosine of angle A.

Hence, the given statement is false.

(iv) cot A is not the product of cot and A. It is the cotangent of ∠A.

Hence, the given statement is false.

(v) $\sin\theta = \frac{4}{3}$

We know that in a right-angled triangle,

$$\sin\theta = \frac{\text{Side opposite to } \angle\theta}{\text{Hypotenuse}}$$

In a right-angled triangle, hypotenuse is always greater than the remaining two sides.

Therefore, such value of sin θ is not possible.

Hence, the given statement is false

# Mathematics

(Chapter – 8) (Introduction to Trigonometry)
(Class X)

## Exercise 8.2

**Question 1:**

Evaluate the following

(i) sin60° cos30° + sin30° cos 60°

(ii) $2\tan^2 45° + \cos^2 30° - \sin^2 60°$

(iii) $\dfrac{\cos 45°}{\sec 30° + \operatorname{cosec} 30°}$

(iv) $\dfrac{\sin 30° + \tan 45° - \operatorname{cosec} 60°}{\sec 30° + \cos 60° + \cot 45°}$

(v) $\dfrac{5\cos^2 60° + 4\sec^2 30° - \tan^2 45°}{\sin^2 30° + \cos^2 30°}$

**Answer 1:**

(i) sin60° cos30° + sin30° cos 60°

$$= \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{3}}{2}\right) + \left(\frac{1}{2}\right)\left(\frac{1}{2}\right)$$

$$= \frac{3}{4} + \frac{1}{4} = \frac{4}{4} = 1$$

(ii) $2\tan^2 45° + \cos^2 30° - \sin^2 60°$

$$= 2(1)^2 + \left(\frac{\sqrt{3}}{2}\right)^2 - \left(\frac{\sqrt{3}}{2}\right)^2$$

$$= 2 + \frac{3}{4} - \frac{3}{4} = 2$$

(iii) $\dfrac{\cos 45°}{\sec 30° + \operatorname{cosec} 30°}$

$$= \frac{\frac{1}{\sqrt{2}}}{\frac{2}{\sqrt{3}}+2} = \frac{\frac{1}{\sqrt{2}}}{\frac{2+2\sqrt{3}}{\sqrt{3}}}$$

$$= \frac{\sqrt{3}}{\sqrt{2}\left(2+2\sqrt{3}\right)} = \frac{\sqrt{3}}{2\sqrt{2}+2\sqrt{6}}$$

$$= \frac{\sqrt{3}\left(2\sqrt{6}-2\sqrt{2}\right)}{\left(2\sqrt{6}+2\sqrt{2}\right)\left(2\sqrt{6}-2\sqrt{2}\right)}$$

$$= \frac{2\sqrt{3}\left(\sqrt{6}-\sqrt{2}\right)}{\left(2\sqrt{6}\right)^2-\left(2\sqrt{2}\right)^2} = \frac{2\sqrt{3}\left(\sqrt{6}-\sqrt{2}\right)}{24-8} = \frac{2\sqrt{3}\left(\sqrt{6}-\sqrt{2}\right)}{16}$$

$$= \frac{\sqrt{18}-\sqrt{6}}{8} = \frac{3\sqrt{2}-\sqrt{6}}{8}$$

(iv) $\frac{\sin 30° + \tan 45° - \operatorname{cosec} 60°}{\sec 30° + \cos 60° + \cot 45°}$

$$= \frac{\frac{1}{2}+1-\frac{2}{\sqrt{3}}}{\frac{2}{\sqrt{3}}+\frac{1}{2}+1} = \frac{\frac{3}{2}-\frac{2}{\sqrt{3}}}{\frac{3}{2}+\frac{2}{\sqrt{3}}}$$

$$= \frac{\frac{3\sqrt{3}-4}{2\sqrt{3}}}{\frac{3\sqrt{3}+4}{2\sqrt{3}}} = \frac{\left(3\sqrt{3}-4\right)}{\left(3\sqrt{3}+4\right)}$$

$$= \frac{\left(3\sqrt{3}-4\right)\left(3\sqrt{3}-4\right)}{\left(3\sqrt{3}+4\right)\left(3\sqrt{3}-4\right)} = \frac{\left(3\sqrt{3}-4\right)^2}{\left(3\sqrt{3}\right)^2-\left(4\right)^2}$$

$$= \frac{27+16-24\sqrt{3}}{27-16} = \frac{43-24\sqrt{3}}{11}$$

(v) $\dfrac{5\cos^2 60° + 4\sec^2 30° - \tan^2 45°}{\sin^2 30° + \cos^2 30°}$

$$= \frac{5\left(\frac{1}{2}\right)^2 + 4\left(\frac{2}{\sqrt{3}}\right)^2 - (1)^2}{\left(\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2}$$

$$= \frac{5\left(\frac{1}{4}\right) + \left(\frac{16}{3}\right) - 1}{\frac{1}{4} + \frac{3}{4}}$$

$$= \frac{\frac{15 + 64 - 12}{12}}{\frac{4}{4}} = \frac{67}{12}$$

**Question 2:**

Choose the correct option and justify your choice.

(i) $\dfrac{2\tan 30°}{1 + \tan^2 30°} =$

(A). sin60° (B). cos60° (C). tan60° (D). sin30°

(ii) $\dfrac{1 - \tan^2 45°}{1 + \tan^2 45°} =$

(A). tan90° (B). 1 (C). sin45° (D). 0

(iii) sin2A = 2sinA is true when A =

(A). 0° (B). 30° (C). 45° (D). 60°

(iv) $\frac{2\tan 30°}{1-\tan^2 30°} =$

(A). cos60° (B). sin60° (C). tan60° (D). sin30°

**Answer 2:**

(i) $\frac{2\tan 30°}{1+\tan^2 30°}$

$$= \frac{2\left(\frac{1}{\sqrt{3}}\right)}{1+\left(\frac{1}{\sqrt{3}}\right)^2} = \frac{\frac{2}{\sqrt{3}}}{1+\frac{1}{3}} = \frac{\frac{2}{\sqrt{3}}}{\frac{4}{3}}$$

$$= \frac{6}{4\sqrt{3}} = \frac{\sqrt{3}}{2}$$

Out of the given alternatives, only $\sin 60° = \frac{\sqrt{3}}{2}$

Hence, (A) is correct.

(ii) $\frac{1-\tan^2 45°}{1+\tan^2 45°}$ $= \frac{1-(1)^2}{1+(1)^2} = \frac{1-1}{1+1} = \frac{0}{2} = 0$

Hence, (D) is correct.

(iii)Out of the given alternatives, only A = 0° is correct.

As sin 2A = sin 0° = 0

2 sinA = 2sin 0° = 2(0) = 0

Hence, (A) is correct.

(iv) $\frac{2\tan 30°}{1-\tan^2 30°}$

$$= \frac{2\left(\frac{1}{\sqrt{3}}\right)}{1-\left(\frac{1}{\sqrt{3}}\right)^2} = \frac{\frac{2}{\sqrt{3}}}{1-\frac{1}{3}} = \frac{\frac{2}{\sqrt{3}}}{\frac{2}{3}} = \sqrt{3}$$

Out of the given alternatives, only tan 60° $= \sqrt{3}$

Hence, (C) is correct.

**Question 3:**

If $\tan(A+B)=\sqrt{3}$ and $\tan(A-B)=\frac{1}{\sqrt{3}}$
0° < A + B ≤ 90°, A > B find A and B.

**Answer 3:**

$\tan(A+B)=\sqrt{3}$

$\tan(A+B)=\tan 60$

⇒A + B = 60 ………………..… (1)

$\tan(A-B)=\frac{1}{\sqrt{3}}$

⇒tan (A − B) = tan30

⇒A − B = 30 ……………….. (2)

On adding both equations, we obtain

2A = 90

⇒A = 45

From equation (1), we obtain

45 + B = 60

B = 15

Therefore, ∠A = 45° and ∠B = 15°

**Question 4:**

State whether the following are true or false. Justify your answer.

(i) sin (A + B) = sin A + sin B

(ii) The value of sinθ increases as θ increases

(iii) The value of cos θ increases as θ increases

(iv) sinθ = cos θ for all values of θ

(v) cot A is not defined for A = 0°

**Answer 4:**

(i) sin (A + B) = sin A + sin B Let A = 30° and B = 60°

sin (A + B) = sin (30° + 60°)

= sin 90° = 1

And sin A + sin B = sin 30° + sin 60°

$= \frac{1}{2} + \frac{\sqrt{3}}{2} = \frac{1+\sqrt{3}}{2}$

Clearly, sin (A + B) ≠ sin A + sin B

Hence, the given statement is false.

(ii) The value of sin θ increases as θ increases in the interval of 0° < θ < 90° as sin 0° = 0

$\sin 30° = \frac{1}{2} = 0.5$

$\sin 45° = \frac{1}{\sqrt{2}} = 0.707$

$\sin 60° = \frac{\sqrt{3}}{2} = 0.866$

sin 90° = 1

Hence, the given statement is true.

(iii) cos 0° = 1

$\cos 30° = \frac{\sqrt{3}}{2} = 0.866$

$\cos 45° = \frac{1}{\sqrt{2}} = 0.707$

$\cos 60° = \frac{1}{2} = 0.5$

cos90° = 0

It can be observed that the value of cos θ does not increase in the interval of 0°<θ<90°.

Hence, the given statement is false.

(iv) sin θ = cos θ for all values of θ.

This is true when θ = 45°

As $\sin 45° = \frac{1}{\sqrt{2}}$

$\cos 45° = \frac{1}{\sqrt{2}}$

It is not true for all other values of θ.

As $\sin 30° = \frac{1}{2}$ and $\cos 30° = \frac{\sqrt{3}}{2}$

Hence, the given statement is false.

(v) cot A is not defined for A = 0°

As $\cot A = \frac{\cos A}{\sin A}$,

$\cot 0° = \frac{\cos 0°}{\sin 0°} = \frac{1}{0}$ = undefined

Hence, the given statement is true.

# Mathematics

(Chapter – 8) (Introduction to Trigonometry)
(Class X)

## EXERCISE 8.3

**Question 1:**

Express the trigonometric ratios sin A, sec A and tan A in terms of cot A.

**Answer 1:**

We know that,

$$\text{cosec}^2 A = 1 + \cot^2 A$$

$$\frac{1}{\text{cosec}^2 A} = \frac{1}{1 + \cot^2 A}$$

$$\sin^2 A = \frac{1}{1 + \cot^2 A}$$

$$\sin A = \pm \frac{1}{\sqrt{1 + \cot^2 A}}$$

Therefore, $\sin A = \frac{1}{\sqrt{1 + \cot^2 A}}$

We know that, $\tan A = \frac{\sin A}{\cos A}$

However, $\cot A = \frac{\cos A}{\sin A}$

Therefore, $\tan A = \frac{1}{\cot A}$

Also, $\sec^2 A = 1 + \tan^2 A$

$$= 1 + \frac{1}{\cot^2 A}$$

$$= \frac{\cot^2 A + 1}{\cot^2 A}$$

$$\sec A = \frac{\sqrt{\cot^2 A + 1}}{\cot A}$$

**Question 2:**

Write all the other trigonometric ratios of ∠A in terms of sec A.

**Answer 2:**

We know that,

$$\cos A = \frac{1}{\sec A}$$

Also, $\sin^2 A + \cos^2 A = 1$

$\sin^2 A = 1 - \cos^2 A$

$$\sin A = \sqrt{1 - \left(\frac{1}{\sec A}\right)^2}$$

$$= \sqrt{\frac{\sec^2 A - 1}{\sec^2 A}} = \frac{\sqrt{\sec^2 A - 1}}{\sec A}$$

$\tan^2 A + 1 = \sec^2 A$

$\tan^2 A = \sec^2 A - 1$

$$\tan A = \sqrt{\sec^2 A - 1}$$

$$\cot A = \frac{\cos A}{\sin A} = \frac{\frac{1}{\sec A}}{\frac{\sqrt{\sec^2 A - 1}}{\sec A}}$$

$$= \frac{1}{\sqrt{\sec^2 A - 1}}$$

$$\operatorname{cosec} A = \frac{1}{\sin A} = \frac{\sec A}{\sqrt{\sec^2 A - 1}}$$

**Question 3:**

Choose the correct option. Justify your choice.

(i) $9 \sec^2 A - 9 \tan^2 A =$

(A) 1 (B) 9 (C) 8 (D) 0

(ii) $(1 + \tan \theta + \sec \theta)(1 + \cot \theta - \operatorname{cosec} \theta)$

(A) 0 (B)1 (C) 2 (D) −1

(iii) $(\sec A + \tan A)(1 - \sin A) =$

(A) secA (B) sinA (C) cosecA (D) cosA

(iv) $\frac{1 + \tan^2 A}{1 + \cot^2 A}$

(A) $\sec^2 A$ (B) −1 (C) $\cot^2 A$ (D) $\tan^2 A$

**Answer 3:**

(i) $9 \sec^2 A - 9 \tan^2 A$

$= 9 (\sec^2 A - \tan^2 A)$

$= 9 (1)$ [As $\sec^2 A - \tan^2 A = 1$]

$= 9$

Hence, alternative (B) is correct.

(ii) (1 + tan θ + sec θ) (1 + cot θ − cosec θ)

$$=\left(1+\frac{\sin\theta}{\cos\theta}+\frac{1}{\cos\theta}\right)\left(1+\frac{\cos\theta}{\sin\theta}-\frac{1}{\sin\theta}\right)$$

$$=\left(\frac{\cos\theta+\sin\theta+1}{\cos\theta}\right)\left(\frac{\sin\theta+\cos\ \theta-1}{\sin\theta}\right)$$

$$=\frac{(\sin\theta+\cos\ \theta)^2-(1)^2}{\sin\theta\cos\theta}$$

$$=\frac{\sin^2\theta+\cos^2\theta+2\sin\theta\ \cos\ \theta-1}{\sin\theta\cos\ \theta}$$

$$=\frac{1+2\sin\theta\cos\ \theta-1}{\sin\theta\cos\ \theta}$$

$$=\frac{2\sin\theta\ \cos\ \theta}{\sin\ \theta\cos\ \theta}=2$$

Hence, alternative (C) is correct.

(iii) (secA + tanA) (1 − sinA)

$$=\left(\frac{1}{\cos A}+\frac{\sin A}{\cos A}\right)(1-\sin A)$$

$$=\left(\frac{1+\sin A}{\cos A}\right)(1-\sin A)$$

$$=\frac{1-\sin^2 A}{\cos A}=\frac{\cos^2 A}{\cos A}$$

= cosA

Hence, alternative (D) is correct.

(iv) $$\frac{1+\tan^2 A}{1+\cot^2 A}=\frac{1+\frac{\sin^2 A}{\cos^2 A}}{1+\frac{\cos^2 A}{\sin^2 A}}=\frac{\frac{\cos^2 A+\sin^2 A}{\cos^2 A}}{\frac{\sin^2 A+\cos^2 A}{\sin^2 A}}=\frac{\frac{1}{\cos^2 A}}{\frac{1}{\sin^2 A}}$$

$$=\frac{\sin^2 A}{\cos^2 A}=\tan^2 A$$

Hence, alternative (D) is correct.

**Question 4:**

Prove the following identities, where the angles involved are acute angles for which the expressions are defined.

**Answer 4:**

(i) $(\text{cosec}\theta - \cot\theta)^2 = \dfrac{1-\cos\theta}{1+\cos\theta}$

$$\text{L.H.S.} = (\text{cosec}\,\theta - \cot\theta)^2$$

$$= \left(\frac{1}{\sin\theta} - \frac{\cos\theta}{\sin\theta}\right)^2$$

$$= \frac{(1-\cos\theta)^2}{(\sin\theta)^2} = \frac{(1-\cos\theta)^2}{\sin^2\theta}$$

$$= \frac{(1-\cos\theta)^2}{1-\cos^2\theta} = \frac{(1-\cos\theta)^2}{(1-\cos\theta)(1+\cos\theta)} = \frac{1-\cos\theta}{1+\cos\theta}$$

$$= \text{R.H.S.}$$

(ii) $\dfrac{\cos A}{1+\sin A} + \dfrac{1+\sin A}{\cos A} = 2\sec A$

$$\text{L.H.S.} = \frac{\cos A}{1+\sin A} + \frac{1+\sin A}{\cos A}$$

$$= \frac{\cos^2 A + (1+\sin A)^2}{(1+\sin A)(\cos A)}$$

$$= \frac{\cos^2 A + 1 + \sin^2 A + 2\sin A}{(1+\sin A)(\cos A)}$$

$$= \frac{\sin^2 A + \cos^2 A + 1 + 2\sin A}{(1+\sin A)(\cos A)}$$

$$= \frac{1+1+2\sin A}{(1+\sin A)(\cos A)} = \frac{2+2\sin A}{(1+\sin A)(\cos A)}$$

$$= \frac{2(1+\sin A)}{(1+\sin A)(\cos A)} = \frac{2}{\cos A} = 2\sec A$$

$$= \text{R.H.S.}$$

(iii) $\dfrac{\tan\theta}{1-\cot\theta} + \dfrac{\cot\theta}{1-\tan\theta} = 1 + \sec\theta\,\text{cosec}\theta$

$$\text{L.H.S.} = \frac{\tan\theta}{1-\cot\theta} + \frac{\cot\theta}{1-\tan\theta}$$

$$= \frac{\frac{\sin\theta}{\cos\theta}}{1-\frac{\cos\theta}{\sin\theta}} + \frac{\frac{\cos\theta}{\sin\theta}}{1-\frac{\sin\theta}{\cos\theta}}$$

$$= \frac{\frac{\sin\theta}{\cos\theta}}{\frac{\sin\theta-\cos\theta}{\sin\theta}} + \frac{\frac{\cos\theta}{\sin\theta}}{\frac{\cos\theta-\sin\theta}{\cos\theta}}$$

$$= \frac{\sin^2\theta}{\cos\theta(\sin\theta-\cos\theta)} + \frac{\cos^2\theta}{\sin\theta(\sin\theta-\cos\theta)}$$

$$= \frac{1}{(\sin\theta-\cos\theta)}\left[\frac{\sin^2\theta}{\cos\theta} - \frac{\cos^2\theta}{\sin\theta}\right]$$

$$= \left(\frac{1}{\sin\theta-\cos\theta}\right)\left[\frac{\sin^3\theta-\cos^3\theta}{\sin\theta\cos\theta}\right]$$

$$= \left(\frac{1}{\sin\theta-\cos\theta}\right)\left[\frac{(\sin\theta-\cos\theta)(\sin^2\theta+\cos^2\theta+\sin\theta\cos\theta)}{\sin\theta\cos\theta}\right]$$

$$= \frac{(1+\sin\theta\ \cos\theta)}{(\sin\theta\ \cos\theta)}$$

= secθ cosec θ + 1 = R.H.S.

(iv) $\frac{1+\sec A}{\sec A} = \frac{\sin^2 A}{1-\cos A}$

$$\text{L.H.S.} = \frac{1+\sec A}{\sec A} = \frac{1+\frac{1}{\cos A}}{\frac{1}{\cos A}}$$

$$= \frac{\frac{\cos A+1}{\cos A}}{\frac{1}{\cos A}} = (\cos A+1)$$

$$= \frac{(1-\cos A)(1+\cos A)}{(1-\cos A)}$$

$$= \frac{1-\cos^2 A}{1-\cos A} = \frac{\sin^2 A}{1-\cos A}$$ = R.H.S

(v) $\frac{\cos A-\sin A+1}{\cos A+\sin A-1}=\operatorname{cosec}A+\cot A$

Using the identity $\operatorname{cosec}^2 A = 1 + \cot^2 A$

L.H.S = $\frac{\cos A-\sin A+1}{\cos A+\sin A-1}$

$$=\frac{\frac{\cos A}{\sin A}-\frac{\sin A}{\sin A}+\frac{1}{\sin A}}{\frac{\cos A}{\sin A}+\frac{\sin A}{\sin A}+\frac{1}{\sin A}}$$

$$=\frac{\cot A-1+\operatorname{cosec}A}{\cot A+1-\operatorname{cosec}A}$$

$$=\frac{\{(\cot A)-(1-\operatorname{cosec}A)\}\{(\cot A)-(1-\operatorname{cosec}A)\}}{\{(\cot A)+(1-\operatorname{cosec}A)\}\{(\cot A)-(1-\operatorname{cosec}A)\}}$$

$$=\frac{(\cot A-1+\operatorname{cosec}A)^2}{(\cot A)^2-(1-\operatorname{cosec}A)^2}$$

$$=\frac{\cot^2 A+1+\operatorname{cosec}^2 A-2\cot A-2\operatorname{cosec}A+2\cot A\operatorname{cosec}A}{\cot^2 A-(1+\operatorname{cosec}^2 A-2\operatorname{cosec}A)}$$

$$=\frac{2\operatorname{cosec}^2 A+2\cot A\operatorname{cosec}A-2\cot A-2\operatorname{cosec}A}{\cot^2 A-1-\operatorname{cosec}^2 A+2\operatorname{cosec}A}$$

$$=\frac{2\operatorname{cosec}A(\operatorname{cosec}A+\cot A)-2(\cot A+\operatorname{cosec}A)}{\cot^2 A-\operatorname{cosec}^2 A-1+2\operatorname{cosec}A}$$

$$=\frac{(\operatorname{cosec}A+\cot A)(2\operatorname{cosec}A-2)}{-1-1+2\operatorname{cosec}A}$$

$$=\frac{(\operatorname{cosec}A+\cot A)(2\operatorname{cosec}A-2)}{(2\operatorname{cosec}A-2)}$$

= cosec A + cot A

= R.H.S

(vi) $\sqrt{\frac{1+\sin A}{1-\sin A}}=\sec A+\tan A$

$$\text{L.H.S.} = \sqrt{\frac{1+\sin A}{1-\sin A}}$$

$$= \sqrt{\frac{(1+\sin A)(1+\sin A)}{(1-\sin A)(1+\sin A)}}$$

$$= \frac{(1+\sin A)}{\sqrt{1-\sin^2 A}} \qquad = \frac{1+\sin A}{\sqrt{\cos^2 A}}$$

$$= \frac{1+\sin A}{\cos A} \qquad = \sec A + \tan A$$

$$= \text{R.H.S.}$$

(vii) $\dfrac{\sin\theta - 2\sin^3\theta}{2\cos^3\theta - \cos\theta} = \tan\theta$

$$\text{L.H.S.} = \frac{\sin\theta - 2\sin^3\theta}{2\cos^3\theta - \cos\theta}$$

$$= \frac{\sin\theta\left(1-2\sin^2\theta\right)}{\cos\theta\left(2\cos^2\theta - 1\right)}$$

$$= \frac{\sin\theta \times \left(1-2\sin^2\theta\right)}{\cos\theta \times \left\{2\left(1-\sin^2\theta\right)-1\right\}}$$

$$= \frac{\sin\theta \times \left(1-2\sin^2\theta\right)}{\cos\theta \times \left(1-2\sin^2\theta\right)}$$

$$= \tan\theta = \text{R.H.S}$$

(viii) $(\sin A + \operatorname{cosec} A)^2 + (\cos A + \sec A)^2 = 7 + \tan^2 A + \cot^2 A$

$$\text{L.H.S} = (\sin A + \operatorname{cosec} A)^2 + (\cos A + \sec A)^2$$

$$= \sin^2 A + \operatorname{cosec}^2 A + 2\sin A \operatorname{cosec} A + \cos^2 A + \sec^2 A + 2\cos A \sec A$$

$$= \left(\sin^2 A + \cos^2 A\right) + \left(\operatorname{cosec}^2 A + \sec^2 A\right) + 2\sin A\left(\frac{1}{\sin A}\right) + 2\cos A\left(\frac{1}{\cos A}\right)$$

$$= (1) + \left(1 + \cot^2 A + 1 + \tan^2 A\right) + (2) + (2)$$

$$= 7 + \tan^2 A + \cot^2 A$$

$$= \text{R.H.S}$$

(ix) $\left(\text{cosec}A-\sin A\right)\left(\sec A-\cos A\right)=\dfrac{1}{\tan A+\cot A}$

$$\begin{aligned}
\text{L.H.S} &= \left(\text{cosec}A-\sin A\right)\left(\sec A-\cos A\right)\\
&= \left(\frac{1}{\sin A}-\sin A\right)\left(\frac{1}{\cos A}-\cos A\right)\\
&= \left(\frac{1-\sin^2 A}{\sin A}\right)\left(\frac{1-\cos^2 A}{\cos A}\right)\\
&= \frac{\left(\cos^2 A\right)\left(\sin^2 A\right)}{\sin A\cos A}\\
&= \sin A\cos A
\end{aligned}$$

$$\begin{aligned}
\text{R.H.S} &= \frac{1}{\tan A+\cot A}\\
&= \frac{1}{\dfrac{\sin A}{\cos A}+\dfrac{\cos A}{\sin A}}=\frac{1}{\dfrac{\sin^2 A+\cos^2 A}{\sin A\cos A}}\\
&= \frac{\sin A\cos A}{\sin^2 A+\cos^2 A}=\sin A\cos A
\end{aligned}$$

Hence, L.H.S = R.H.S

(x) $\left(\dfrac{1+\tan^2 A}{1+\cot^2 A}\right)=\left(\dfrac{1-\tan A}{1-\cot A}\right)^2=\tan^2 A$

$$\begin{aligned}
\frac{1+\tan^2 A}{1+\cot^2 A} &= \frac{1+\dfrac{\sin^2 A}{\cos^2 A}}{1+\dfrac{\cos^2 A}{\sin^2 A}}=\frac{\dfrac{\cos^2 A+\sin^2 A}{\cos^2 A}}{\dfrac{\sin^2 A+\cos^2 A}{\sin^2 A}}\\
&= \frac{\dfrac{1}{\cos^2 A}}{\dfrac{1}{\sin^2 A}}=\frac{\sin^2 A}{\cos^2 A}\\
&= \tan^2 A
\end{aligned}$$

$$\left(\frac{1-\tan A}{1-\cot A}\right)^2 = \frac{1+\tan^2 A - 2\tan A}{1+\cot^2 A - 2\cot A}$$

$$= \frac{\sec^2 A - 2\tan A}{\operatorname{cosec}^2 A - 2\cot A}$$

$$= \frac{\frac{1}{\cos^2 A} - \frac{2\sin A}{\cos A}}{\frac{1}{\sin^2 A} - \frac{2\cos A}{\sin A}} = \frac{\frac{1-2\sin A\cos A}{\cos^2 A}}{\frac{1-2\sin A\cos A}{\sin^2 A}}$$

$$= \frac{\sin^2 A}{\cos^2 A} = \tan^2 A$$