Chapter II

Numerical Applications

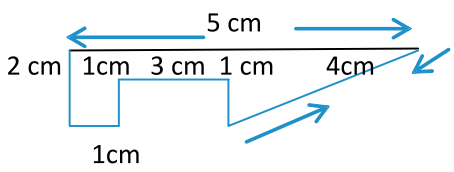
Learning Objectives:

After completion of this unit, Students will be able to:

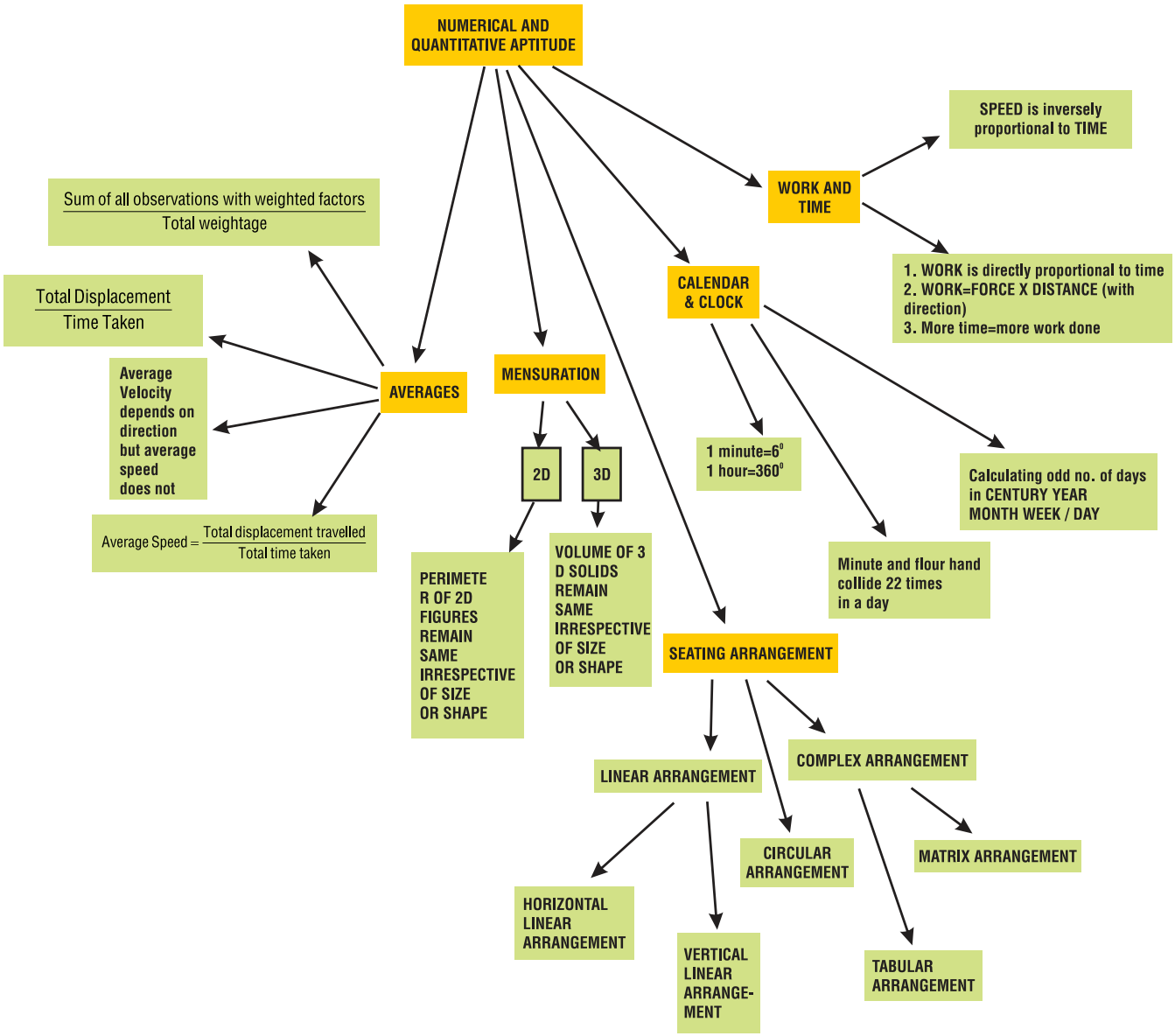
- Differentiate between average speed and average velocity in order to analyse the role of direction in speed.
- Establish the relationship between work and time in order to relate the efficiency and factors related to it.
- Determine the odd number of days in a month/year/century so as to decode it and find out the day of week for the same.
- Evaluate the angular value of a minute in order to compare the speed of the minute hand and hour hand.
- Derive the formulas for surface area and volume of combined solids in order to relate the same with real world problems.
- Compare the positive work done and negative work done in order to find workable solution for the posed problem.

Before You Start:

	You should know how to :	Juggle Your Brain
1.	Calculate mean (average) of given data Mean = $\frac{\sum_{i=1}^{n} fixi}{\sum_{i=1}^{n} fi}$	Find the mean marks if marks scored by 7 students are 38, 72, 91, 76, 25, 99, 65 respectively (out of 100)
2.	Determine the distance covered and the displacement of the body	An Artificial satellite moves in a circular orbit of radius 172 km and reaches to its initial position in 190 days. Find out the total distance travelled by the body in 1 revolution.
3.	Classify the given year as a leap year or a non-leap year.	Year 2002 was a _____ year.
4.	a) What is the measure of angle formed by clock hands in an hour? b) Convert days/hours/minutes/seconds accordingly.	a) In a clock minute hand points on 5 and hour hand points on 3, the region enclosed between them is known as _____. b) $7\frac{3}{8}$ Hours = 7 Hours __Minutes ___seconds.
5.	a) Differentiate between 2D and 3D figures	Given situation a) A right Δ with sides 3 cm, 4 cm and 5 cm is revolved about the side 4 cm.

	b) Find Perimeter/Area of combined 2D figures	I) Name the solid figure obtained. ii) Categorize it as 2D or 3D iii) Now If Δ was revolved about the side 5 cm. Draw figure and how it is different from the part (I) b) Find the perimeter of the given park. 
6.	Determine Speed $\text{Speed} = \frac{\text{Distance travelled}}{\text{Time taken}}$	A train travels 785 km in 3 hours 45 minutes find the Speed of the train. (Assuming that the train is moving at uniform speed).

Concept Map:



2.1 Introduction

Numerical and quantitative aptitude is a core concept that helps to deal with our day to day problems and find out workable solutions to those problems.

Numerical ability relates to number and its various operations. Quantitative Aptitude comprises of mathematical operations related to logic. It is a way or a language to express the logic along with the numerical approach. It is a skill rather than the concept which enhances analytical thinking to devise simple and more ways to find the solutions to the real-world situations. Also, it focuses on enhancing the problem-solving skill by finding simple and short approach through computation ability.

Numerical Skills:

CONTENT:

A Average

B Clock and Calendar

C Work, Time and Distance

D Mensuration

E Seating Arrangements

2.2 Average

Introductory Discussion:

Four football matches are played between two teams, team A and team B and their scores are:

SCORES TEAM A	10	5	4	6
SCORES TEAM B	5	2	8	5

a) What is the final score of team A and team B?

b) Which team scores better? Give reason.

$$Average\ (\text{Mean}) = \frac{Sum\ of\ all\ observations}{Total\ number\ of\ observations}$$

'Average or Mean' is a statistical measure, often known as a measure of central tendency of given data. It represents a calculated central value of the given data Average is also known as arithmetic mean or mean.

Mean is denoted by $\bar{x}$

More about Average:

1. Average lies between maximum and minimum value of given set of observations.

2. If value of each observation is increased or decreased by the same value *N* then the average will also be increased or decreased by the same value.

3. If value of each observation is multiplied or divided by the same value *N* then the mean will also be multiplied or divided by the same value.

Weighted Average

Example 1

The average monthly savings of a company was Rs. 12 Lakh for the first 3 months, Rs. 12.5 Lakh for the next 4 months and Rs. 31.2 Lakh during the next 5 months of a year. Total expenditure during the year is 78 Lakh. Find the average monthly earnings of the company.

Total Annual savings = 12 Lakh x 3 + 12.5 Lakh x 4 + 31.2 Lakh x 5

= (36 + 50 + 156) Lakh

= 242 Lakh

Total yearly expenditure = 78 Lakh

Total yearly income = 242 + 78 = 320 Lakh

Average monthly income = $\frac{320}{12} lakh = 26.67$ Lakh (approximately)

of company

In the above example we observe how the concept of average plays a vital role in the planning of the family budget to the planning of investment in the company. So, the need arises as to what percentage of the capital amount is to be invested in salaries, infrastructure etc. and on an average what amount is to be spent? So answer to these questions is weighted mean/average.

Weighted mean (average) is similar to arithmetic mean ($\bar{x}$), except that instead of all elements having equal weight in the case of arithmetic mean, some elements with high weight contribute more than the elements with low weight.

Weighted mean ($\bar{x}_w$) is the mean of a data set of 'n' elements when some elements carry more weight than other $\dfrac{\sum_{i=1}^{n} w_i . x_i}{\sum_{i=1}^{n} w_i}$

where

n= no of elements in data set

x_i = value of i^{th} element

w_i = weight of i^{th} element

$\bar{x}_w$= weighted mean

Note : If all the weights are equal then weighted mean = arithmetic mean

Example :2

A school follows the following criterion for grading a student for annual result:

ASSESSMENT	WEIGHTAGE
Homework	25%
Quiz	30%
Test	10%
Final exams	35%
	100%

In his math submissions, Joy scored 88 marks in homework, 71 marks in class quiz ,97 marks in tests conducted throughout the session and 90 marks in the final exams. What is Joy's annual result in Mathematics?

Weighted mean = $\bar{x}_w = \dfrac{\sum_{i=1}^{n} w_i x_i}{\sum_{i=1}^{n} w_i}$

$= \dfrac{25 \times 88 + 30 \times 71 + 10 \times 97 + 35 \times 90}{100}$

$= \dfrac{8450}{100}$ $\boxed{\bar{x}_w = 84.5\ \%}$

Example : 3

The average of 4 positive integers is 39. The highest integer is 73 and the lowest integer is 19. The difference between the remaining two integers in 18. Find the remaining integers.

AVERAGE (MEAN) = $\frac{Sum\ of\ all\ observations}{total\ number\ of\ observations}$

Let remaining two integers be x, x+18

$$39 = \frac{73 + X + X + 18 + 19}{4}$$

$46 = 2x$

X= 23
X+18 = 41

Hence, remaining integers are 23 and 41.

Example: 4

Five years ago, average age of a family of five was 32 years. The family includes Mrs. and Mr. Rai, their son, daughter and daughter in law. Recently, The daughter, Priya moved out of the house after marriage, at the age of 30 years. Calculate the family's present average age.

Total age of 5 family members 5 years ago = 5 x 32 = 160 years

Total age of 5 family members today = 160 + 25 = 185 years

Total age of 4 family members as Priya left = 185 – 30 = 155 years

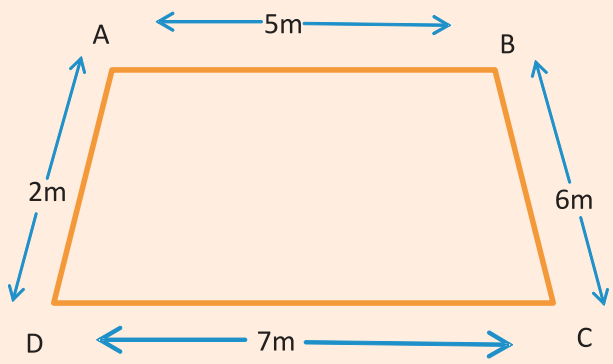
Present average age of Family = $\frac{155}{4}$ = 38.75 years

Differentiate Between Average Speed And Average Velocity:

AVERAGE SPEED	AVERAGE VELOCITY
Average Speed = $\frac{\text{Total distance travelled}}{\text{Total time taken}}$	Average Velocity = $\frac{\text{Total displacement of body}}{\text{Total time taken}}$

Example: 5

An insect starts from a point A and covers distance AB in 4 seconds. Then covers distance BC in 7 seconds, distance CD in 5seconds and distance DA in 2 seconds. Calculate the average velocity and average Speed.



Total Time Taken = 18 seconds

Total Distance Travelled = 20 m

Total Displacement = 0 m (as the final and initial position is same)

$$\text{Average Speed} = \frac{\text{Total distance travelled}}{\text{Total time taken}} = \frac{20}{18} = 1.11 \text{ m/s (approximately)}$$

$$\text{Average velocity} = \frac{\text{Total displacement}}{\text{Total time taken}} = \frac{0}{18} = 0 \text{ m/s}$$

Exercise: 1A

1. There are 44 boys and 36 girls in a class. The Average marks of boys are 40 and that of girls is 38. Find the average marks of the class?
2. The average of five numbers is 87. If one of the numbers is excluded, then the average gets decreased by 5. Find the excluded number.
3. A nursery is closed on Sunday. The average plants sold in the remaining six days of a week is 156 plants and the average sale from Monday to Friday is 124 plants. Find the number of plants sold on Saturday?
4. The average of five consecutive numbers is 125. Find the product of the number at the extreme positions.

5. Under MNREGA Schema 1000 new labourers are enrolled in Delhi. Earlier they were getting Rs 200 as daily wages, but now the authorities have increased the budget for them by 15 Lakh per month.

   a) Calculate the present monthly budget of the ministry for 1000 laborer

   b) Find the increase in daily income due to budget increase

   c) Find the new Average monthly income per labour.

6. The mean of 35 observations was found to be 98.6. But later, it was found that 72 was misread as 27. Find the correct mean.

7. The average annual PF contribution of certain number of defence officers is Rs28560 and that of other officers is Rs 22500. The number of defence officers is 22 times that of other officers. Then find the average savings of all the officers in total.

8. 5 years ago, the average age of the 3 children of Mr. Pandey was 8 years. A new baby is born in the family now. Find the present average age of the family.

9. In a restaurant 35 visitors can have lunch at a time. If the number of visitors increases by 7,then the expense of the restaurant on food increases by Rs. 42, while the average expenditure per head decreases by Rs. 1. Find the original expenditure of the restaurant.

10. The average of runs of Virat Kohli, the famous cricket player, of last 10 innings were 72. How many runs he must make in 11th inning to increase the average by 3 runs.

11. An ant is moving around a circular path of radius 3.5 cm and takes 3 seconds to complete 1 revolution. Find the average speed and average velocity?

12. Sara walks 7.2 km in one and half hour and 3.5 km in 2 hours in the same direction.

    What is the Sara's average speed for the whole journey?

2.3 Clock And Calendar

Introductory Discussion

Lalita Babar is three times winner of Mumbai Marathon. She won Bronze medal in 3000 m steeplechase at the 2014 Asian Games and broke the National record with the time of 9 hours 35 minutes and 37 seconds.

Time To Work:

1. Write the length of steeple chase race in decimeters.

2. Convert the time taken in seconds.

For thousands of years people measured time by observing the passage of day and night, the stars and the change of seasons. But it is very important to measure the time accurately. This lead to the inventions of tools for measuring time such as Sundial, hourglass, water clock etc. Many devices were invented over the centuries to measure the time accurately, and finally in due course of time Clock was invented. The most accurate clock in the world is The Cesium Fountain Atomic Clock developed at NIST Laboratories in Colorado, USA.

Units of Time :

Time is measured in seconds, minutes, hours, days and so on. Question arises as to how these units were derived.

The units of time we use are based on the sun, the moon and the earth rotation and revolution.

Common Units Related To Time :

1 Minute = 60 seconds
1 hour = 60 minutes = 3600 seconds
1 day = 24 hours =1440 min =86400 seconds
1 week =7 days
1 year = 12 months = $365\frac{1}{4} days$
1 Decade = 10 years
1 Century = 100 years
1 Millennium = 1000 years

Also,

1 millisecond = $\frac{1}{1000}$ = 10^{-3} second

1 microsecond = $\frac{1}{1000000}$ second = 10^{-6} second

Example : 6

What would be the time four and a half hour before 2.15 pm?

= 2:15 pm - $4\frac{1}{2}$ hours

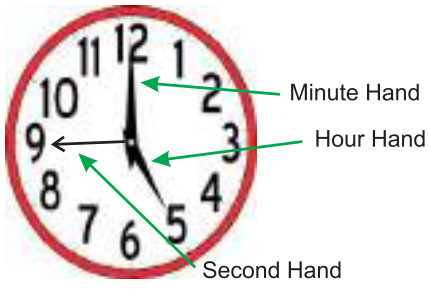
= 2:15 pm - (4 hours + 30 minutes)

= 10:15 am - 30 minutes

= 9:45 am

Structure of Clock

A clock is device provided with three hands as an hour hand, minute hand and seconds hand.



"HOROLOGY"
Study and measurement of time and making of clocks

A clock hand makes an angle of 360^0 in one revolution

Angle of clock hand in one revolution = 360^0

Therefore,

Angular value of an hour hand in one hour = $\frac{360^0}{12} = 30^0$

$\Rightarrow$ Speed of Hour hand = 30^0 per hour

Angular value of minute hand in one minute $= \frac{360^0}{60} = 6^0$

$\Rightarrow$ Speed of minute hand = 6^0 per minute

Angular value of a second hand in one second $= \frac{360^0}{60} = 6^0$

$\Rightarrow$ Speed of second hand = 6^0 per second

Collisions of Hands of A Clock

Two hands of clock collide when angle between them is 0°

Let's find how many times in a day the minute and hour hands of a clock coincide with each other?

But how?

Number of hours in a day = 24

So, hour hand completes two revolutions in a day

first collision is at : 12:00 am

Second collision is at 1: 05 am

Therefore, minute and hour hands must collide 24 times a day but Hands will not collide between 11'o Clock to 12'o clock. This will happen 2 times a day. Hence, 24 - 2 = 22 times. Answer: 22 times a day.

Time of Collision

As Hour and Minute hands collide 22 times in 24 hours.

Time of one collusion = $\frac{24}{12}$ *hour*

Therefore, Time of one collision = $\frac{12}{11}$ *hour*

$= \frac{12 \times 60}{11}$ minutes

$= \frac{720}{11}$ minutes

$= 65\frac{5}{11} = 65.45$ minutes (approx.)

So, if 1st collision is at 12'O clock, then next collision will be after $65\frac{5}{11}$ minutes after 12'o clock

Stretch Your Brain:

Complete the following table on the basis of the above information:

FREQUENCY	Time in fractions	Exact time	Time that Appears on clock (approx.)
1st	12:00:00	12:00:00	12:00
2nd	$1:05:\frac{5}{11}$	1:05:27	1:05
3rd	$2:10:\frac{10}{11}$	2:10:54	2:10
4th			
5th	$4:21:\frac{9}{11}$	4:21:16	4:20
6th			
7th			
8th			
9th			
10th			
11th	$11:59:\frac{11}{11}$	12:00:00	12:00:00

Investigation 1: Calculate the speed of seconds hand?

Investigation 2: Find out the difference in the speed of minute hand and hour hand ?

Relation Between Time and Angle Between the Hands of Clock:

Angle in 12 hours = 360^0

Angle in 1 hour = 30^0

Angle in 1 minute = $\frac{30^0}{60^0} = \left(\frac{1}{2}\right)^0$

Hour hand = $\left(30H + \frac{m}{2}\right)^0$

But 1 minute = 6^0

So, Angle between hands of clock(A) = $30H + \frac{m}{2} - 6m$

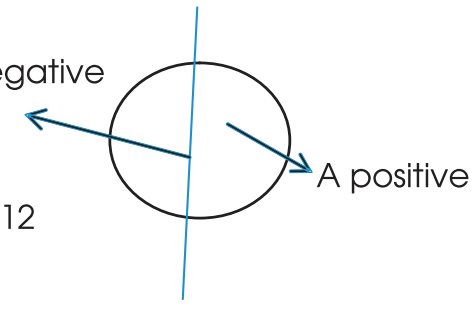
H means hour shown by hour hand

m means minutes shown by minute hand.

2 11

$T = \frac{2}{11}(H \times 30° \pm A)$ or $A = 30° \times H - \frac{11}{2} \times t$ (t = time in minutes indicated by minute hand)

T = time at which angle is formed
H = running hour
A = angle between hands
A is positiveif hands collide between 12 to 6
And negative if hands collide between 6 to 12



Note: Value of A is always taken as positive, negative sign is ignored.

Example 7

At what time between 4'o clock and 5'o clock, will the hands of a clock make an angle of 20°

H = 4 A = 20°

$T = \frac{2}{11}[4x30^{\circ} \pm A]$

$= \frac{2}{11}[4x30^{\circ} + 20^{\circ}]$

A is (positive because the hands of clock lies in between 12 to 6 first half of clock)

$= \frac{2}{11}[120^{\circ} + 20^{\circ}]$

$= \frac{2}{11}[140^{0}]$

$= \frac{280^{0}}{11} = 25\frac{5}{11}$ = 25 minutes $\frac{5}{11}$ seconds

= 25 minutes 27 seconds

Time = 4:25:27

Example 8

At what time between 3'o clock and 4'o clock will the hands of a clock collide?

At 3'o clock hour hand is at 3 and minute hand is at 12. i.e. the minute hand is 15 min. space apart from hour hand.

Hence the hand will collide, if minute hand covers 15 minutes over the hour hand.

As, Time of one collision = $\frac{12}{11}$ hours

Therefore 15 minutes will be covered in $\left(\frac{12}{11} \times 15\right) = \frac{180}{11} = 16\frac{4}{11}$ minutes

So, the hands will collide at $16\frac{4}{11}$ minutes past 3.

Example 9

Find the angle between the hands of clock when it struck 15 minutes past 7?

$$A = 30° \times H - \frac{11}{2} T$$

$$= 30^0 \times 7 - \frac{11}{2} \times 15$$

$$= 210^0 - 82.5^0$$

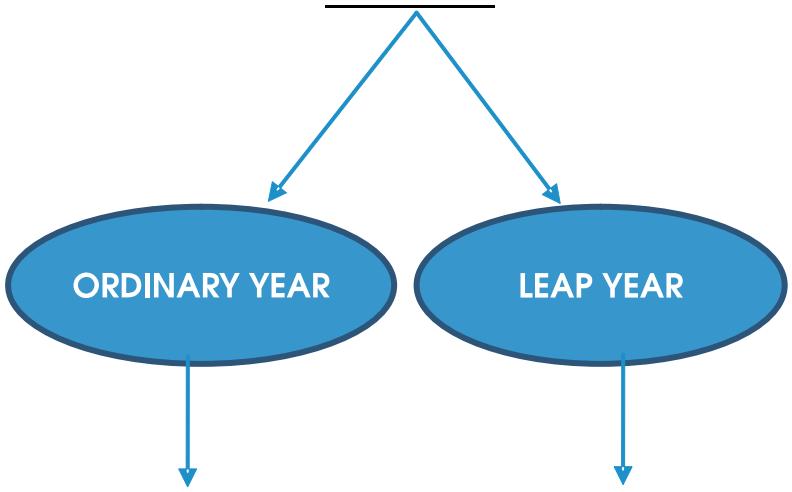
$$= 127.5^0$$

Calendar

Clock apprises us about the time span for seconds, minutes, day etc. whereas calendar depicts the days, weeks and months of a specific year.

Structure of Calendar

Calendar



No. of Days	365	366
No. of weeks	52 Weeks and 1 Day	52 weeks and 2 Days
No. of odd days	1	2

ODD DAY* The extra day(s) left after exact number of weeks is considered as an Odd day.

Calculating Number of Odd Days in a Month
Juggle Your Brain

MONTH	NO. OF ODD DAYS
JAN	3
FEB	0/1
MARCH	3
APR	2
MAY	
JUNE	
JULY	
AUG	
SEP	
OCT	
NOV	
DEC	

January has 31 days
= 4 weeks + 3 days
January has 3 odd days

Complete the table

$\frac{30}{7}$ = 4 weeks and 2 days

February has either
28 or 29 days
=4 weeks + 0 day
or 4 weeks + 1 day
Februray has eighter 0 or 1 odd days

Decoding for Day of Week

Coding and Decoding is one of most important parameters of day to day life. As nowadays most of the thing we use is based on cryptogamy (coding of data) Example our mobile phones passwords, debit cards, computers etc. uses this coding methodology and so the need arises for decoding of the same.

Decoding the day when date is given is done on the basis of odd number. of days in the month/year/century. If we consider that the week begins with a Monday and ends on a Sunday, then decoding is done as :

Code	Day
0	Sunday
1	Monday
2	Tuesday
3	Wednesday
4	Thursday
5	Friday
6	Saturday

Example :10

If 30th March 2020 was Monday. What would be the day after 61 days?
Reference date : 30 March 2020
Reference day : Monday
Duration : 61 days
61 days : 8 weeks and 5 days

$\frac{61}{7}$ = 8 weeks and 5 days

So the day after the 8th week would be the same i.e. 56TH Day and it Will be MONDAY. 57th day will be TUESDAY.
Therefore, the 61st day will be a Saturday.

Leap Year:

A Leap year occurs every four years, To check whether the given year is a leap year or not, divide the year by 4 and if the remainder comes as 0, then the year is a leap year

FUN FACT
A number is divisible by 4 if the last two digits are divisible by 4

Calculating Number Of Odd Days In A Century :

ILLUSTRATION: What day of the week was on 31st DECEMBER 100 A.D.?

STEP 1	Write the given year as multiple of 400	Total number of years 100
STEP 2	Find the no. of ordinary years and leap years in a century.	Number of leap years in 100 years = 24 years because 100/4 = 25 but 100 is divisible by 4 so (25– 1 = 24) So, ❖ Total leap year = 25-1 = 24 ❖ Ordinary years = 100-24 =76
STEP 3	Write the total no. of odd days in a century	Number of Odd days in a century = 76x1 + 24x2 = 76+ 48 = 124 days
STEP 4	Dividing the total no. of odd days by7 remainder indicates the last day of the year.	= 124/7 = 17 weeks and 5 odd days = Dec 31st = Friday

Number Of Odd Days In A Century

Stretch Your Mind:

Complete the table

Century	No. of Odd Day		Last Day of Century
100	5		Friday
200	5+5=10=7x1+3=	3	Wednesday
300	5x3=15=7x2+1=	1	Monday
400	5x4=20 multiple of 4	0	Sunday
500		5	Friday
600			
700		1	
800		0	
1000	5+5=10=7x1+3	3	
1400			
1600			
1700			
2000			
2100			

Exercise 1(b)

1. If today is a Tuesday, what will be the day on 7706th day?
2. Find the total number of days from 26th January 2008 to 15 May 2008?
3. If the second day of April month is a Friday, then find the last day of the next month?
4. Workout for the day of week on the given date:

   (1) 15th August 1947

   (2) 22nd November 2025

   (3) 21st September 2080

   (4) 18thOctober 2100

5. A local train from Mumbai leaves every 40 minutes from the station. When inquired by a passenger, the help desk executive informed that the train had already left 10 minutes ago. If this information was given at 10:15 a.m.

   a) At what time did the train leave the station?

   b) At what time will the next train leave the station?

   c) For how long will that man has to wait for the next train?

   d) Find the angle formed between the minute hand and the hour hand when the passenger will board the next train?

6. Pranil works in an electronics goods shop and the shop has offered 30% discount on MRP on all the goods in the month of October. On top of it, the shop gives successive discount of 10% if the person uses e-payment mode. Pranil remembers that the maximum sale occurred after 17th October but before 21st October while his colleague remembers that the maximum sale happened after 19th October but before 24th October. Owner listens to both of them and concludes a date which is precisely common. What is the date as per your point of view?

7. a) In a day, how many times is a straight angle formed between minute and hour hands?

   b) In a day, how many times is a right angle formed between minute and hour hands?

8. Find the angle between the minute hand and the hour hand at 7: 40 pm?

9. By 20 minutes past 5, how many degrees has the hour hand has turned through?

10. At what time between 3'o clock and 4'o clock are the hands of a clock three degrees apart?

11. Indian government has announced a complete lockdown for entire country from the Midnight of 25th March 2020 till the midnight of 14th April 2020 due to pandemic disease Covid-19 outbreak. But keeping in view the grim situation, the lockdown was further extended till 3rd May 2020.

a) Calculate the total number of days for which the lockdown lasted.

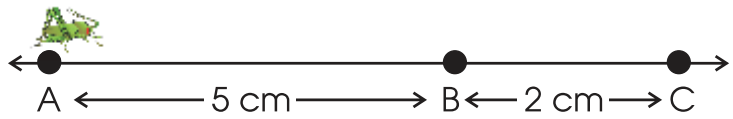
b) If 25th March 2020 is Wednesday, then find which day of the week will fall on 3rd May 2020.

12. India is 9 hours and 30 minutes ahead of Ottawa ON, Canada. What is time in Canada when it is 1:25 am in India?

2.4 Time, Work And Distance

Introductory Problem

An insect moves from point A to point C and returns to point B taking a total time of 6 seconds.



a) What is the total distance travelled by the insert?

b) Calculate the speed of insect?

Relation Of Speed , Distance And Time

Speed is inversely proportional to time (if distance is constant) i.e. When time increases, speed decreases and vice versa.

EXAMPLE: 11

Speeds of two cars, A and B is in the ratio

$$\frac{Speed\ of\ car\ A}{Speed\ of\ car\ B} = \frac{x}{y}$$

$$\frac{Time\ taken\ by\ car\ A}{Time\ taken\ by\ car\ B} = \frac{x}{y}$$

$$\frac{Time\ taken\ by\ car\ B}{Time\ taken\ by\ car\ A} = \frac{x}{y}$$

Time taken by Car A: Car B= y : x

Work And Time

The physical or mental effort directed towards doing something refers to the work. In simple language work is an outcome of an effort applied for a particular time.

Work is directly proportional to time. i.e. more time given for work will increase the work. If amount of work i.e. a task remains constant, then two things play a vital role in the condition

1. Number of persons working
2. Number of hours

If In a task, the number of persons increase then the time decrease and vice versa

Relation Of Time And Work Done

Ravi does a work (W) for *n* days

then work done by Ravi in 1 day = $1/n^{th}$ of W = W/n

Nitish does same work (W) for *m* days

Then, work done by Nitish in 1 day = $1/m^{th}$ of W = W/m

Ravi and Nitish started working together and finished the work in *p* days.

Work finished by both in 1 day = $1/p^{th}$ of W

Therefore, $\frac{W}{n}+\frac{W}{m}=\frac{W}{p}$

So result can be generalized as $\frac{1}{n}+\frac{1}{m}=\frac{1}{p}$ ..... (1)

Hence total no. of days taken by both of them:

$$p=\frac{mn}{m+n} \quad \ldots\ldots \ (2)$$

If more than two persons work together for same task than result becomes:

$$\frac{1}{n_1}+\frac{1}{n_2}+\ldots+\frac{1}{n_a}=\frac{1}{p}$$

(n_a represents number of days taken by a^{th} person)

Example 12:

Reena completes her canvas painting in 4 days. Mihir finish the same canvas painting in 5 days. If they both work together find out the number of days taken by them to finish it.

No. of days taken together = $\frac{4\times5}{4+5} = \frac{20}{9}$ days

= $2\frac{2}{9}$ days

Negative Work:

If two objects or persons are working against each other due to which a delay or incompletion in task occurs is referred as negative work.

Example : 13

Pipe A can fill a tank in 20 hours. But due to a leakage it is taking thrice the time to fill the tank. If the tank is full, how long will it take to drain out the tank, If the pipe A is closed?

Let Initial Time taken by pipe A = x hours

Tank filled in 1 hour = $\frac{1}{x}$ $\qquad x = 20$ hours

After leakage time taken by pipe = 3 x

$\Rightarrow$ Water filled in tank after leakage (in 1 hour) = $\frac{1}{3x}$

Let the tank get emptied in p hours

Water flows out in 1 hour = 1/p

Amount of water flows in 1 hour = water left in tank due to leakage

$$\frac{1}{p} = \frac{1}{x} - \frac{1}{3x}$$

$$\frac{1}{p} = \frac{2}{3x}$$

$$p = \frac{3x}{2} \text{ hours} = \frac{3}{2}\times 20 = 30 \text{ hours}$$

$\Rightarrow$ Tank is drained out in 30 hours

Exercise: 1C

1. Navya is twice as efficient as Nitti. If they take10 days to finish a certain job together. How much time will they take individually to finish the same job?

2. A piece of work is finished in 30 days by A. Since C is thrice as good as A andA is twice as good as B. If A, B, C work together, then how many days will they take to finish the work?

3. X can do a piece of work in 60 days, whereas Y can do the same work in 40 days. Both started the work together, but X left after 10 days before the completion of work. Find how many days will it take to complete the work?

4. A train is running at 7/11 of its own speed due to fog and reached a place in 44 hours. What was the original time taken by the train if it runs at its own speed?

5. a) The signal poles on a railroad are placed 100 m apart, how many poles will be passed by a train in 8hours if the speed of the train is 45 km/h.

   b) If 7201 poles are to be installed within two stations at equal distance covering distance of 360km, Find out the distance between two consecutive poles.

6. 100 persons begin to work together on a project which was expected to be completed in 40 days. But after few days 40 persons left. As a result, the project got delayed by 10 days. How many days after the commencement of the project did the 40 persons left?

7. The efficiency of x,y,z are in ratio of 3:2:6 to finish a task. If they work together, they can finish it in 2 hours; find the time taken by them if they do the task individually?

8. A is three times as efficient as B. Also, A takes 30 days less than B for doing a piece of work. Find the time taken by them if they work a) individually b) together?

9. Machine P is 40% more efficient than Machine Q. Machine P can make 100 bags alone in 30 hours. Find the time taken to complete the order if both the machines work together?

10. A Corporate firm is running in loss, they had categorized their employees into two teams, Team A and Team B. The firm has funds to pay Team A for 21 days only. If they work with Team B then the firm can pay for 28 days only. Management has decided that they will allow both the teams to work for as long as they are having enough funds to pay them. Find out for how many days both the teams could work together?

2.5 Mensuration

Mensuration is a branch of Mathematics which deals with area, perimeter, volume of geometric figures

Introductory Discussion

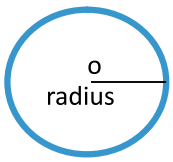
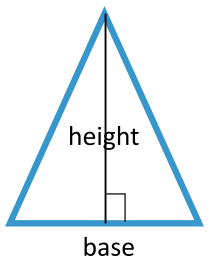
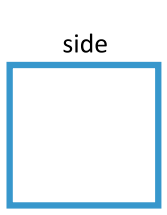
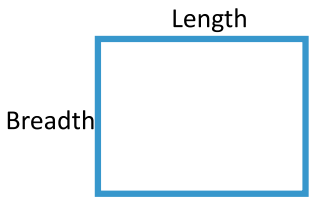
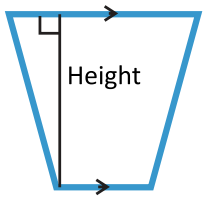
A restaurant sells paranthas in three different sizes. Paranthas are circular in shape. The small parantha is 15 cm in diameter and costs Rs 49. The medium sized parantha is 30 cm in diameter and costs Rs 79. The king sized parantha of diameter 35 cm costs Rs 99. Which of the paranthas is worth purchasing?

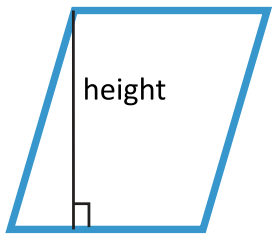
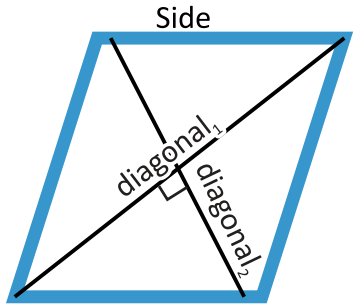
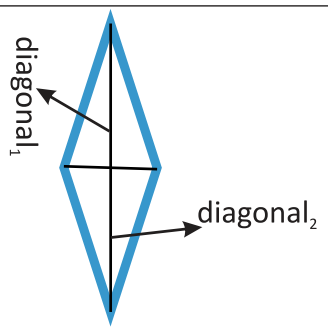
Concept of 2D And 3D Shapes:

A two dimensional figures is a Nane figure having length and breadth . It has no thickness. Example: Square, triangle, circle etc.

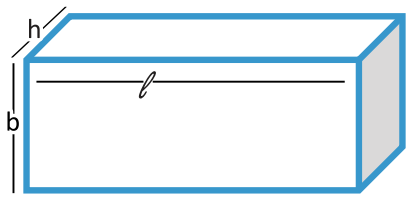
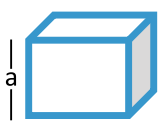
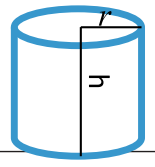
A three dimensional solid has three dimensions length, breadth and height. Example: Cuboid, Cube cylinder, cone etc

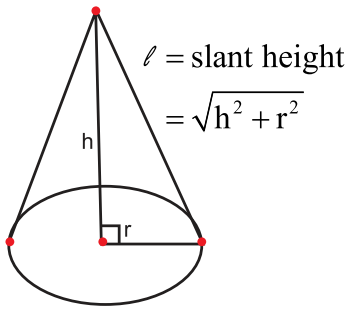
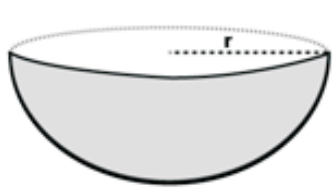
Let's recall formulas for some 2D Shapes

NAME	SHAPE	PERIMETER	AREA
CIRCLE		$2\pi r$	πr^2
TRIANGLE		Sum of all sides	If base *b* and corresponding height *h* are given, then area= $\frac{1}{2}\times base\times corresponding\ height$ If three sides a, b, c are given then $s=\frac{a+b+c}{2}$ and Area= $\sqrt{s(s-a)(s-b)(s-c)}$
SQUARE		4 X side	Side X Side
RECTANGLE		2 (length + breadth)	Length X breadth
TRAPEZIUM		Sum of all sides	$\frac{1}{2}\times height\times$ $(sum\ of\ parallel\ sides)$

PARALLELO-GRAM		2 x sum of adjacent sides)	Base X corresponding height
RHOMBUS		4 X side	$\frac{1}{2} \times diagonal_1 \times diagonal_2$
KITE		Sum of all sides	$\frac{1}{2} \times diagonal_1 \times diagonal_2$

Let's Recall Formulas for some 3D figures

NAME	FIGURE	LATERAL CURVED SURFACE AREA	TOTAL SURFACE AREA	VOLUME
CUBOID		2hx(l + b)	2(lb + bh +hl)	l X b X h
CUBE		$4a^2$	$6a^2$	a^3
CYLINDER		$2\pi rh$	$2\pi r(r+h)$	$\pi r^2 h$

CONE		$\pi r \ell$	$\pi r(r + l)$	$\frac{1}{3}\pi r^2 h$
SPHERE		$4\pi r^2$	$4\pi r^2$	$\frac{4}{3}\pi r^3$
HEMISPHERE		$2\pi r^2$	$3\pi r^2$	$\frac{2}{3}\pi r^3$

FUN FACT

Conversion of Solids

If a solid is melted to form a new solid, then the volume will remain same in both cases.

Length of Longest Rod :

Length of longest rod that can be fitted in a cuboid $=\sqrt{L^2+B^2+H^2}$

Where L = length, B= breadth, H =height

Length of longest rod that can be fitted in a cube $=\sqrt{3}a$

Where a = side of a cube

Exercise: 1 D

1. A cylindrical tank of base diameter 16 m and height 6 m requires a non-porous lining on its circular base and curved walls. The lining of the base costs Rs .14. 20 per m^2 and on the curved walls it costs Rs 16.30 per square meter

   a) Find the area of the base.

   b) Find the total area of lining.

   c) Determine the total cost of lining.

2. The area of a rectangular field is 104000 m^2. This rectangular area has been drawn on a map to the scale of 1cm to 100 m. The length is shown as 7. 50 cm on the map.

   a) Find the actual breadth of the rectangular field.

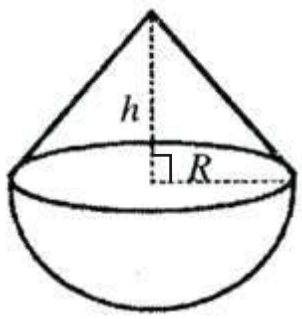
   b) Find the perimeter of rectangular field represented on map.

3. A rope by which a mare is tied at a centre of the circular field is decreased from 23m to 11 m.

   a) What is the decrease in area more will be able to graze now?

   b) Find the percentage decrease in area grazed? due to decreased length of rope.

4. A cone is surmounted on a hemisphere as shown here. The radius of the solid formed is 6 cm and its height is 21 cm. Find:



i) Curved surface area of the solid

ii) Volume of the solid.

5. Find the area of the greatest circle which can be inscribed in a square whose perimeter is 180 cm?

6. The length and breadth of rectangle are increased by 30% and decreased by 20 % respectively. Determine the percentage of increase or decrease in its area.

7. Dimensions of the cuboidal hall are 10 m x 12 m x 15 m. Find the length of the longest rod that can be fitted into the hall.

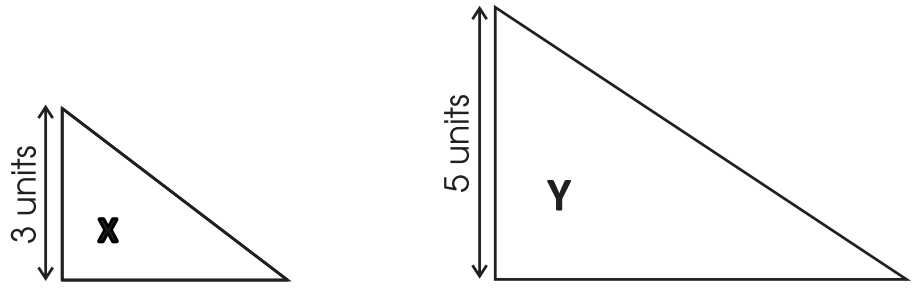
8. A rectangular hall is to be carpeted with a vinyl flooring sheet 25 cm wide, available at the rate of Rs. 22.60 per metre square. If the area of the hall is 117 m^2, then

a) Find the length of vinyl flooring sheet required.

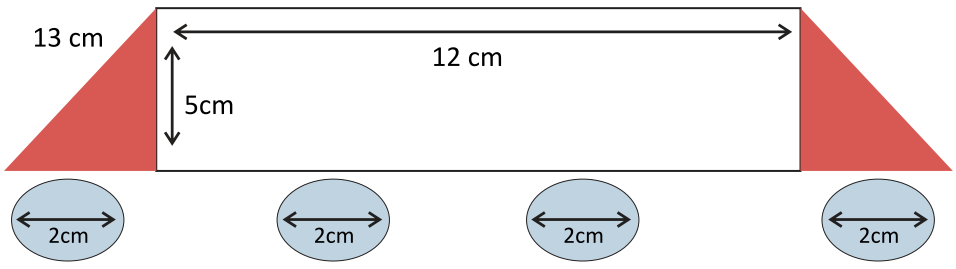
b) Calculate the cost of vinyl sheet.

9. Area of circle is numerically equal to twice of its circumference. Find the diameter of the circle.

10. The following isosceles right-angled triangles represent the monthly expenditures of two law firms X and Y. If the monthly expenditure of firm X is Rs. 63000.Find the monthly expenditure of firm Y.



11. Shakuntala designed a toy car. Help her to find the area of the given below.



(Both the triangles are identical)

12. A circular park120 m in diameter has a path of 10m wide all around outside, which is to be gravelled. The MCD received tenders from three companies as follows:

COMPANY	A	B	C
COST OF GRAVELLING (in Rs)	120 per m^2	140 per m^2	10000 per $100m^2$

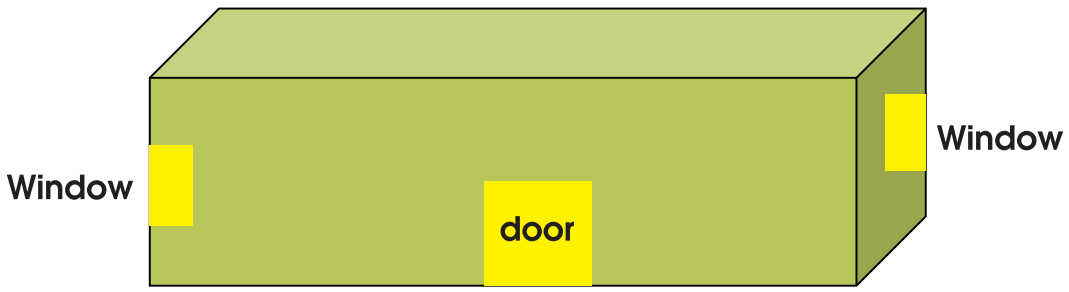
a) Find the area of path.

b) Which company will get the tender? (Hint: MCD will be selecting the tender of the company which has quoted the least)

c) Calculate the total amount paid by MCD to the selected company.

13. A swimming pool 36 m wide and 27 m long is 3m deep on the shallow end and 4 m deep at the deeper end. Find the capacity of the pool (in litres).

14. Nishant owns a house of dimensions 10 feet x 8 feet x 9 feet. His house has two entry doors on the opposite sides, of dimension 7 feet by 3 feet each and two windows on opposite walls of dimensions 3 ft x 2 ft each. He wants wall papering to be done in his house. If the wallpaper that he selects is 2 ft wide and the cost is Rs. 25/ ft. Find the total cost paid by Nishant, if the labour charges Rs. 10/ sq feet.



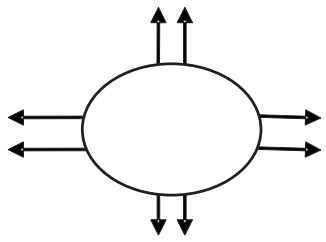
15. A cubical gold block of dimensions of 9cm x 11cm x 12 cm is melted and recasted into spherical balls of radius 3 mm. Find the number of spherical balls formed.

2.6 Seating Arrangements

Introductory Discussion

Eight students A, B, C, D, E, F, G and H participated in a game in which they are seated on a circle., While A,B,C,D are seated on odd number. of seats respectively, E,F,G and H are seated on even number respectively.

Try all the ways of arranging their seating and answer the following questions accordingly.

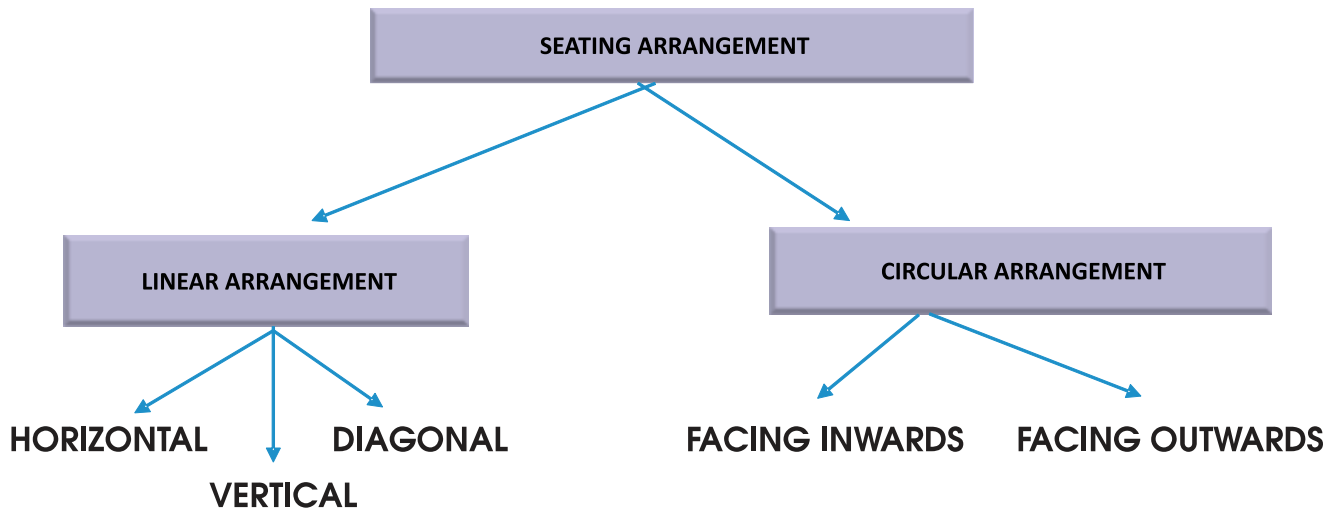


a) Name the students seating on 4th and 8th position

b) Name the student who are adjacent to G

The concept of arrangement plays a very important role in making our life simpler and easier. In this topic we will be dealing with the seating arrangements done under certain condition(s).

Arrangement is a plan which specifies how a group of objects/ persons are placed/allocated their positions in a particular situation/framework. It represents the patterns which may be linear or circular.



Linear Arrangement

Logical arrangement of any objects/persons; either horizontally or vertically or diagonally is referred as linear arrangement.

Series connection of resistors is an example of horizontal linear arrangement.
Students sitting in classroom or examination hall are an example of vertical linear arrangement.

Circular Arrangement :

If objects/persons are arranged in a circular manner than the arrangement is known as a circular arrangement.

A. FACING INWARDS (TOWARDS CENTRE): Objects or people in an arrangement, facing towards the center of circle. Example - Round table conference.

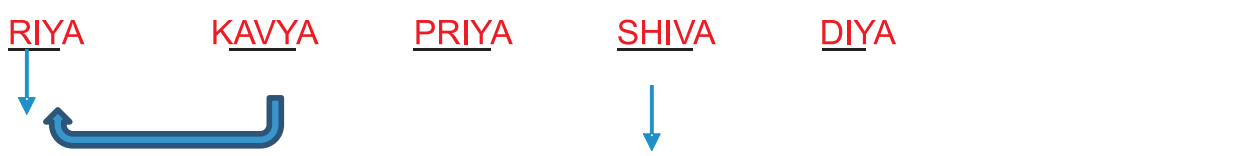
B. FACING OUTWARDS(AWAY FROM CENTRE):Every object or person in an arrangement, facing outwards. Example: Musical Chair

Seating Arrangement In A Photograph:

When a photographer is clicking a photograph, the right and the left references change i.e. the viewer's right hand side becomes photographer's left hand side and vice-versa.

Example : 14

Five children are sitting in a row. Shiva is sitting next to Priya but not Diya. Kavya is sitting next to Riya who is sitting on extreme left and Diya is not sitting next to Kavya. Who are adjacent neighbours of Shiva.

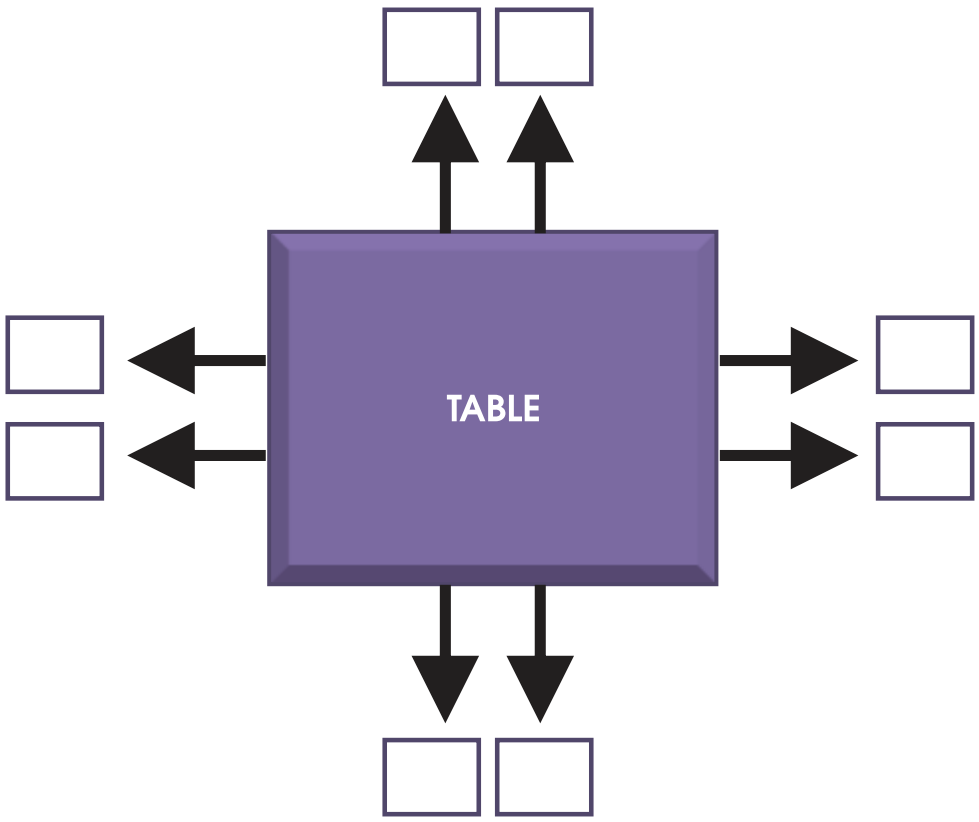


Hence Priya and Diya are adjacent neighbors of Shiva.

Exercise 1E

1. Six students P,Q,R,S,T and U are sitting in two rows, three are in each row. T is not at the end of any row. S is second to the left of U. R is the neighbour of T,Q is the neighbour of U. Arrange the members of two rows. Who is sitting diagonally opposite to Q?

2. Group of 8 persons A,B,C,D,E,F,G and H are seated around a square table while facing towards each other ,two persons are seated on each side. There are three ladies in a group and they are not allowed to sit next to each other. F is seated between H and B, also C is seated between E and B. D is a lady member who is seated second to the left of F. A is a lady member seated opposite to B. B is a male member. A lady member must be seated in between B and E.



Answer the following questions on the basis of the above information:

i) Identify the lady members.

ii) Name the members who is seated immediate left to B

iii) Name the members who are adjacent to D

3. Riya is known for her good photography skill and has won many prizes. On the occasion of family function five cousins approached her for clicking their photograph. Riya had them all sit on a bench. Siya is seated left to Rani and to the right of Bubbly. Maria is seated to the right of Rani, Reet is between Rani and Maria.

   a) Who is seated in the middle

   b) Who is seated immediate right to Reet.

4. In an exhibition seven different electronic companies are displaying their new Air Conditioner Model on seven different tables - LG, LLOYD, VOLTAS, HITACHI, DECCAN, SAMSUNG, and WHIRLPOOL. All the models are facing towards east. LG is next to the right of WHIRLPOOL, WHIRLPOOL is placed

fourth to the right of VOLTAS. HITACHI is placed between LLOYD and SAMSUNG. VOLTAS which is placed third to the left of LLOYD, is at one end.

a) Write the arrangement of companies of AC from the left to the right.

b) Which AC brand lies between HITACHI and WHIRLPOOL.

c) Name the air conditioner that is placed at the extreme right.

5. Three Doctors Dr. Aggarwal, Dr.Chabbra and Dr. Roy are available in LIFE hospital of New Delhi. Dr. Aggarwal is available in hospital between 1 pm and 5 pm on Monday, Wednesday and Sunday. Dr. Chabbra is available in hospital between 11 am and 3 pm on Tuesday, Wednesday, Friday and Sunday. Dr. Roy is available between 10 am and 1 pm on Tuesday and Thursday and also available on Friday, Saturday and Sunday between 3 pm and 5 pm.

a) On which day are all three doctors available in the hospital.

b) Ravi wants to consult two doctors Dr. Chabbra and Dr. Roy on the same day. Workout suitable day and time for him to visit the hospital.

6. Five friends are standing in a line and facing towards the wall wearing Red, Grey, Yellow, Violet and Black Shoes. The persons wearing Yellow and Red Shoes are not standing at the end position. The person in the middle position is wearing Black shoes and the person with Red Shoes is not in his left. The person wearing violet shoes is standing on extreme right.

a) Who is on 4th position from right?

b) Who is standing on the extreme left position?

7. Six persons P, Q ,R,S,T and U are sitting in a circle with their faces towards the centre. S is on the immediate left of T, P is on the left of S and U is immediate neighbor of T. R is sitting second to the right of U..

a) Name the immediate neighbours of Q.

b) Who is between S and R?

Summary

1. AVERAGE (MEAN) = $\frac{Sum\ of\ all\ observations}{total\ number\ of\ observations}$

2. Weighted Average is the mean of a set of numbers in which some elements of the given data carry more weightage than others.

$$\bar{x}_w = \frac{\sum_{i=1}^{n} w_i x_i}{\sum_{i=1}^{n} w_i}$$

3. Average speed is independent of direction and average velocity is dependent of direction.

4. Average speed cannot be zero but average velocity can be zero.

5. **Calendar** is collection of days, weeks and months in a tabular form for a specific year.

6. **Odd Days** are the number of days left after exact number of weeks.

7. A century which is a leap year also has zero number of odd days.

8. Angular value of a minute hand is 6^0.

9. Speed of an hour hand = ½ degree per minute

10. Speed of a minute hand is 6^0 per minute

11. Two hands of clock minute hand and an hour hand collides each other after every $65\frac{5}{11}$ minutes.

12. Time and angle formed between minute and hour hand is related as

    i) $t = \frac{2}{11}(H \times 30^0 + A)$

    ii) $A = 30 \times H\frac{11}{2} \times t$ (t = time in minutes indicated by minute hand)

Note: Value of A is always taken as positive, negative sign is ignored.

13. Volume of a solid = Area of base X height of object