JEE (Main)-2025 (Online) Session-2

Memory Based Question with & Solutions

(Physics, Chemistry and Mathematics)

4th April 2025 (Shift-2)

Time: 3 hrs. **M.M.: 300**

IMPORTANT INSTRUCTIONS:

(1) The test is of 3 hours duration.

(2) This test paper consists of 75 questions. Each subject (PCM) has 25 questions. The maximum marks are 300.

(3) This question paper contains Three Parts. Part-A is Physics, Part-B is Chemistry and Part-C is Mathematics. Each part has only two sections: Section-A and Section-B.

(4) Section - A : Attempt all questions.

(5) Section - B : Attempt all questions.

(6) Section - A (01 - 20) contains 20 multiple choice questions which have only one correct answer. Each question carries +4 marks for correct answer and -1 mark for wrong answer.

(7) Section - B (21 – 25) contains 5 Numerical value based questions. The answer to each question should be rounded off to the nearest integer. Each question carries +4 marks for correct answer and -1 mark for wrong answer.

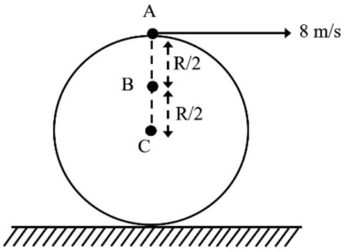
MEMORY BASED QUESTIONS JEE–MAIN EXAMINATION – APRIL, 2025

(Held On Friday 4th April, 2025) **TIME : 3 : 00 PM to 6 : 00 PM**

PHYSICS

SECTION-A

1. A disc is performing pure rolling, if speed of top point is 8 m/s. Find speed of point B.



(1) 3 m/s (2) 6 m/s

(3) 4 m/s (4) 2 m/s

Ans. **(2)**

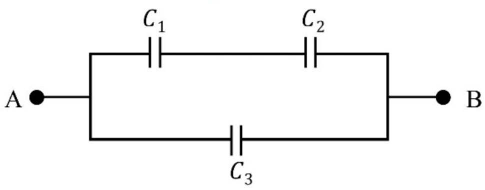
Sol. $v_A = \omega(2R)$

$v_B = \omega\left(\frac{3R}{2}\right)$

$\frac{8}{v_B} = \frac{4}{3}$

$v_B = 6 \text{ m/s}$

2. The equivalent capacitance between A and B is



(1) $\frac{C_1C_2+C_2C_3+C_3C_1}{C_1+C_2}$

(2) $\frac{C_1C_2+C_2C_3+C_3C_1}{C_2+C_3}$

(3) $C_1C_2 + C_2C_3 + C_3C_1$

(4) $C_1 + C_2 + C_3$

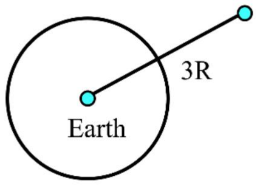
Ans. **(1)**

Sol. $C_{eq} = \frac{C_1C_2}{C_1+C_2} + C_3$

$C_{eq} = \frac{C_1C_2 + C_1C_3 + C_2C_3}{C_1 + C_2}$

3. A particle of mass m is at a distance 3R from the centre of Earth. Find minimum kinetic energy of particle to leave Earth's field

(R : Radius of Earth):



(1) 3mgR (2) $\frac{mgR}{3}$

(3) $\frac{3}{2}$mgR (4) $\frac{mgR}{2}$

Ans. **(2)**

Sol. $\frac{-GMm}{3R} + K.E. = 0$

$K.E. = \frac{mgR}{3}$

4. In a YDSE setup, the slits are separated by 1.5 mm and the distance between slits and screen is 2 m. On using light of wavelength 400 nm, it is observed that 20 maximas of double slit experiment lie inside the central maxima of single slit diffraction. The width of each slit is ________ μm

(1) 300 (2) 75

(3) 200 (4) 150

Ans. **(4)**

Sol. $\frac{20\lambda D}{d} = \frac{2\lambda D}{a}$

$\frac{20}{1.5\times10^{-3}} = \frac{2}{a}$

$a = \frac{1.5\times10^{-3}}{10}$

$a = 150\ \mu m$

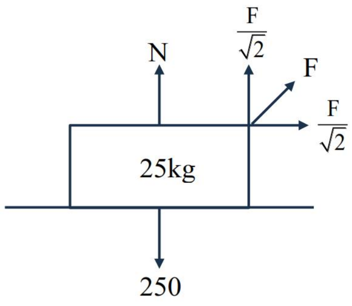
5. A block of mass 25 kg is pulled along a horizontal surface by a force at an angle of 45° with the horizontal. The friction coefficient between the block and the surface is 0.25. The block travels at a uniform velocity. The work done by the applied force during a displacement of 5 m of block is :-

(1) 300 J (2) 500 J

(3) 125 J (4) 250 J

Ans. **(4)**

Sol.



$$\frac{F}{\sqrt{2}} = \mu N$$

$$N = 250 - \frac{F}{\sqrt{2}}$$

$$\frac{F}{\sqrt{2}} = \mu\left(250 - \frac{F}{\sqrt{2}}\right)$$

$$\frac{F}{\sqrt{2}} = \frac{-F}{4\sqrt{2}} + \frac{250}{4}$$

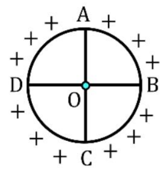
$$\frac{5F}{4\sqrt{2}} = \frac{250}{4}$$

$$\frac{F}{\sqrt{2}} = 50\text{ N}$$

$$W = \frac{F}{\sqrt{2}} \times 5$$

$$W = 50 \times 5 = 250\text{ J}$$

6. A metallic ring is uniformly charged as shown in figure. AC & BD are two mutually perpendicular diameters. Electric field due to arc AB at 'O' is E in magnitude. What would be the magnitude of Electric field at 'O' due to arc ABC ?



(1) Zero (2) E/2

(3) 2E (4) $\sqrt{2}$E

Ans. **(4)**

Sol. $\frac{\sqrt{2}k\lambda}{R} = E_{AB}$

$$E'_{ABC} = \frac{2k\lambda}{R} = \sqrt{2}E$$

7. Displacement of a wave is expressed as $x(t) = 5\cos\left(628t + \frac{\pi}{2}\right)$ m. The wavelength of wave when its velocity is 300 m/s is :-

(1) 5 m (2) 1.5 m

(3) 3 m (4) 0.3 m

Ans. **(3)**

Sol. $f = \frac{628}{2\times\pi} = 100\text{ Hz}$

$$\lambda = \frac{v}{f} = \frac{300}{100} = 3\text{m}$$

8. Match the column :-

	Process		ΔQ
(a)	Adiabatic	(i)	$\frac{n\gamma R}{\gamma-1}\Delta T$
(b)	Isobaric	(ii)	0
(c)	Isochoric	(iii)	$nRT\ln\left(\frac{P_1}{P_2}\right)$
(d)	Isothermal	(iv)	$\frac{nR}{\gamma-1}\Delta T$

(1) (a)-(i), (b)-(ii), (c)-iii, (d)-(iv)

(2) (a)-(ii), (b)-(i), (c)-iv, (d)-(iii)

(3) (a)-(ii), (b)-(iv), (c)-iii, (d)-(i)

(4) (a)-(ii), (b)-(i), (c)-iv, (d)-(iii)

Ans. **(2)**

Sol. (1) Adiabatic, $\Delta Q = 0$

(2) Isobaric, $\Delta Q = 0\ \frac{n\gamma R}{\gamma - 1}\Delta T$

(3) Isochoric, $\frac{nR}{\gamma-1}\Delta T$

(4) Isothermal, $nRT\left(\frac{P_1}{P_2}\right)$

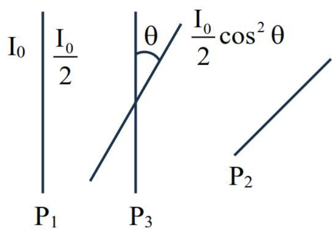
9. Two polarizers P_1 and P_2 are aligned in such a way that intensity is zero. P_3 polarizer is inserted between P_1 and P_2 such that final transmitted ray will have the maximum intensity. Find angle between P_1 and P_3.

(1) $\frac{\pi}{8}$ (2) $\frac{\pi}{3}$

(3) $\frac{\pi}{2}$ (4) $\frac{\pi}{4}$

Ans. **(4)**

Sol.



$\frac{I_0}{2}\cos^2\theta\sin^2\theta$

$\frac{I_0}{8}\sin^2 2\theta$

$2\theta = \frac{\pi}{2}$

$\theta = \frac{\pi}{4}$

10. A medium has relative permittivity $\frac{1}{0.085}$ and relative permeability is $\frac{10}{\pi}$. Find ratio of speed of light in vacuum to the medium.

(1) 1.27 (2) 3.14

(3) 2.28 (4) 6.12

Ans. **(4)**

Sol. $\mu = \frac{c}{v} = \frac{\frac{1}{\sqrt{\mu_0\varepsilon_0}}}{\frac{1}{\sqrt{\mu_m\varepsilon_m}}} = \sqrt{\mu_r\varepsilon_r}$

$\mu = \sqrt{\frac{1}{0.085}\times\frac{10}{\pi}}$

$\mu = 6.12$

11. Given below are two statements. One is labelled as Assertion (A) and the other is labelled as Reason (R)

Assertion (A): Plank's constant and linear momentum have same dimensions.

Reason (R): Bohr's angular momentum is integral multiple of $\frac{h}{2\pi}$.

In the light of the above statements, chose the correct answer from the options given below:

(1) Both (A) and (R) are true and (R) is the correct explanation of (A)

(2) Both (A) and (R) are true but (R) is NOT the correct explanation of (A)

(3) (A) is true but (R) is false

(4) (A) is false but (R) is true

Ans. **(4)**

Sol. $\vec{L} = \vec{r}\times\vec{P}$

$\vec{L} = \frac{nh}{2\pi}$

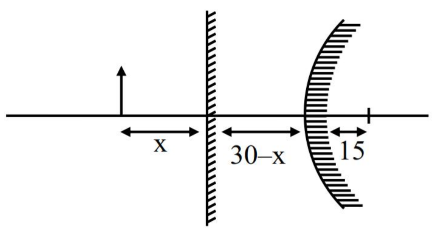
12. An object is placed at a distance of 30 cm in front of a convex mirror of focal length 30 cm. Now if a plane mirror is placed between convex mirror and the object such that the images from the two mirrors coincide. If the distance between the two mirrors is x cm then the value of 2x is:

(1) 15 (2) 30

(3) 7.5 (4) 10

Ans. **(3)**

Sol.



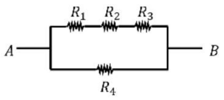
$30 - x + 15 = x$

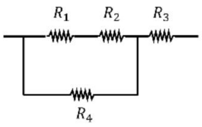
$45 = 2x$

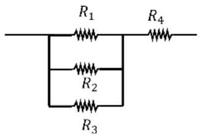
$x = \frac{45}{2}$

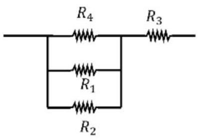
Distance $= 30 - \frac{45}{2} = \frac{15}{2}$

13. If resistor $R_1 = R_2 = R_3 = 5\Omega$ and $R_4 = 10\Omega$. Which circuit diagram is having equivalent resistance = 6Ω across A and B :-

(1) 

(2) 

(3) 

(4) 

Ans. **(1)**

Sol. $R_1 + R_2 + R_3 = 15$

$R_4 = 10$

$R_{eq} = \dfrac{15 \times 10}{25} = 6\Omega$

14. n identical bulbs each takes power P when connected with main supply. If these bulbs are connected in series with main supply, then power will be

(1) nP (2) $\dfrac{P}{n}$

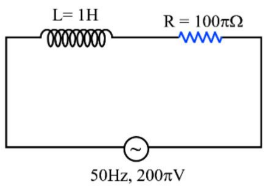
(3) $\dfrac{P}{n^2}$ (4) n^2P

Ans. **(2)**

Sol. $P = \dfrac{V^2}{R}$

$P_1 = \dfrac{V^2}{nR} = \dfrac{P}{n}$

15. An ac source of 100π volt is connected to the given circuit. Find maximum value of the current in the circuit.



(1) $\sqrt{3}\,A$ (2) $0.5\,A$

(3) $\dfrac{1}{\sqrt{2}}A$ (4) $2\,A$

Ans. **(4)**

Sol. $Z = \sqrt{R^2 + x_L^2}$

$Z = 100\pi\sqrt{2}$

$i_{rms} = \dfrac{v_{rms}}{Z} = \dfrac{200\pi}{100\pi\sqrt{2}} = \sqrt{2}$

$i_{peak} = 2\text{ A}$

16. Choose the incorrect statement from the below options

(1) In n-type semiconductor pentavalent impurities are present.

(2) If trivalent impurities are added to pure semiconductor, it becomes p-type semiconductor.

(3) In n-type semiconductor holes are majority charge carriers.

(4) In p-type semiconductor holes are majority charge carriers.

Ans. **(3)**

Sol. Theoretical

17. If a charged particle is projected in a uniform magnetic field perpendicular to its velocity then the minimum time in which it returns to its original position :-

(1) $\dfrac{2\pi m}{qB}$ (2) $\dfrac{\pi m}{qB}$

(3) $\dfrac{qB}{2\pi m}$ (4) None

Ans. **(1)**

Sol. $t_{min} = T = \dfrac{2\pi m}{qB}$

SECTION – B

1. Dimensional formula of ratio of electric dipole moment and magnetic moment is $M^xL^yT^zA^w$ then the value of $(x - y)$ will be

Ans. **(1)**

Sol. $\dfrac{P}{M} = \dfrac{QL}{iA} = \dfrac{QL \cdot t}{QA}$

$= \left[M^0L^{-1}T^1A^0\right]$

CHEMISTRY

SECTION-A

1. Maximum IE and minimum IE of group-13 elements

(1) B, In (2) B, Tl

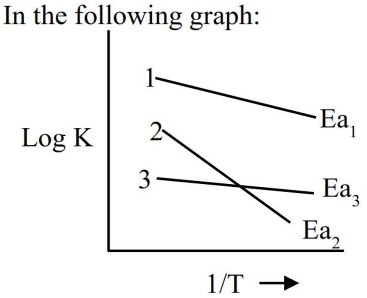
(3) Al, In (4) Al, Tl

Ans. **(1)**

Sol. IE order in group 13 elements

B > Tl > Ga > Al > In

2. In the following graph:



What is the correct order of increasing order of activation energy (Ea).

(1) $Ea_2 > Ea_1 > Ea_3$ (2) $Ea_1 > Ea_2 > Ea_3$

(3) $Ea_3 > Ea_2 > Ea_1$ (4) $Ea_2 > Ea_3 > Ea_1$

Ans. **(1)**

Sol. $k = Ae^{-E_a/RT}$

$$\log k = \log A - \frac{E_a}{2.303RT}$$

As negative slope increases activation energy increases.

3. Consider the following complex ions

(a) $Ni(CO)_4$ (b) $[Ni(CN)_6]^{2-}$

(c) $[FeF_6]^{3-}$ (d) $[CoF_6]^{3-}$

Which of the following order is correct for their unpaired electrons

(1) c > d > a = b (2) c > d > a > b

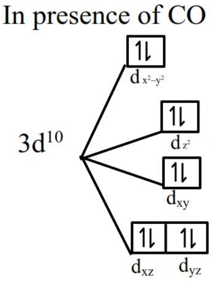
(3) a > b > c > d (4) c > a > d > b

Ans. **(1)**

(a) $Ni(CO)_4$

$Ni \Rightarrow 3d^8 4s^2$

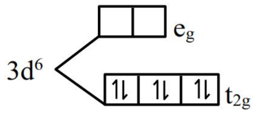
In presence of CO ligands Ni convert into $3d^{10}$



Unpaired $e^{\ominus}$(n) = 0

(b) $[Ni(CN)_6]^{2-}$

$Ni^{4\oplus} \Rightarrow 3d^6 4s^0$, $CN^{\ominus} \Rightarrow$ SFL

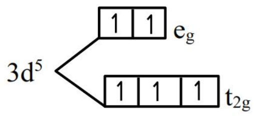


Unpaired $e^{\ominus}$(n) = 0

(c) $[FeF_6]^{3-}$

$Fe^{3\oplus} \Rightarrow 3d^5$

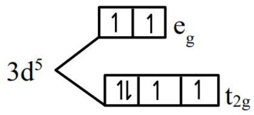
$F^{\ominus} \Rightarrow$ WFL



Unpaired $e^{\ominus}$(n) = 5

(d) $[CoF_6]^{3-}$

$Co^{3\oplus} \Rightarrow 3d^6$, $F^{\ominus} \Rightarrow$ WFL

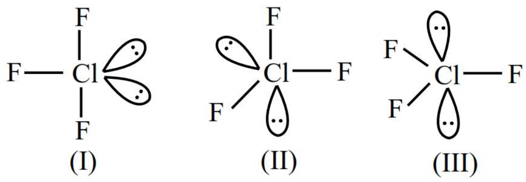


Unpaired $e^{\ominus}$(n) = 4

Order

c > d > a = b

4. Statement I: ClF_3 has 3 possible structures



Statement II: III is most stable structure due to least lp-bp repulsion.

(1) Statement I and Statement II both are correct.

(2) Statement I is correct but Statement II is incorrect.

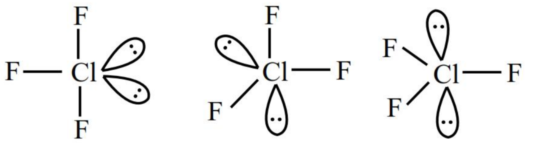
(3) Statement I is incorrect but Statement II is correct.

(4) Both Statements are incorrect.

Ans. **(2)**

Sol. **Statement-I**

According to arrangement of lone pairs ClF_3 has 3 possible structures.



(I) (II) (III)

Statement-2

According to Bent's rule structure (I) is the most stable structure of ClF_3

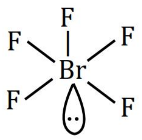
5. A compound [A = $MCl_4 \cdot 3NH_3$] reacts with $AgNO_3$ and gives x mole of AgCl. Now x is the number of lone pair of e^- on BrF_5. Then calculate G.I. of compound (A).

(1) 0 (2) 1

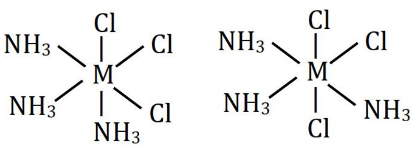
(3) 2 (4) 3

Ans. **(2)**

Sol.



x = no. of lone pair = 1



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6. Incorrect order of I.E. is

(1) $Mn^+ < Mn^{2+}$ (2) $Mn^{+2} < Fe^{+2}$

(3) $Fe^{+2} < Fe^{+3}$ (4) $Mn^+ < Cr^+$

Ans. **(2)**

Sol. (1) $Mn^+ < Mn^{2+}$ (correct)

(d^6) (d^5)

1509 3260

(2) $Mn^{+2} < Fe^{+2}$ (Incorrect)

(d^5) (d^6)

3260 2963

(3) $Fe^{+2} < Fe^{+3}$ (correct)

(d^6) (d^5)

(4) $Mn^+ < Cr^+$ (correct)

(d^6) (d^5)

1509 1592

7. Given below are two statements.

Statement I: The formula of cryoscopic constant is given as $K_f = \frac{MRT_f^2}{1000 \times \Delta H_{fusion}}$.

Statement II: K_f of water is greater than benzene.

In light of the above statements choose the most appropriate option.

(1) Statement I and Statement II both are correct.

(2) Statement I is correct but Statement II is incorrect.

(3) Statement I is incorrect but Statement II is correct.

(4) Both Statements are incorrect.

Ans. **(2)**

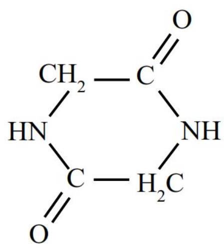
Sol. $k_f = \frac{MRT_f^2}{1000 \times \Delta H_{fus}}$

$k_f \propto M$

Molar mass $\Rightarrow$ Water < Benzene

$(k_f)_{water} < (k_f)_{benzene}$

8. x is a peptide which is hydrolysed to 2 amino acids y and z. y when react with HNO_2 gives lactic acid. z when heated gives cyclic structure as below:



y and z respectively are

(1) Alanine and Lysine

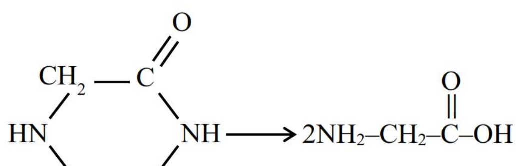
(2) Alanine and Glycine

(3) Glycine and Alanine

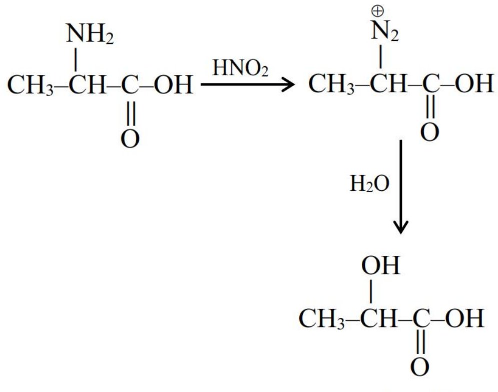
(4) Valine and Glycine

Ans. **(3)**

Sol.



(Z)



Lactic acid

9. Statement I: Alcohol is prepared from alkyl halide in presence of aq. KOH by elimination.

Statement II: Alkenes are prepared from alkyl halide with alc. KOH by β-elimination.

(1) Statement I and Statement II both are correct.

(2) Statement I is correct but Statement II is incorrect.

(3) Statement I is incorrect but Statement II is correct.

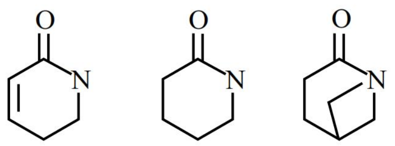
(4) Both Statement are incorrect.

Ans. **(3)**

Sol. Statement I is incorrect as aq. KOH gives mostly substitution reaction and gives substitution product.

Statement II is correct as alc. KOH gives β-elimination reaction.

10. Write Basic Strength order of following compounds



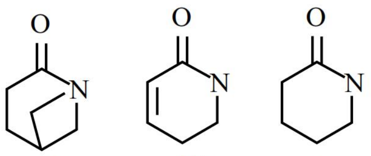
(1) (2) (3)

(1) 3,1,2 (2) 2,3,1

(3) 1,3,2 (4) 2,1,3

Ans. **(1)**

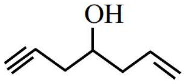
Sol. Basic Strength order



(3) (1) (2)

Cross conjugation

11. Find the IUPAC name of the given compound



(1) Hept-2-ene-6-yne-4-ol

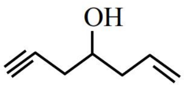
(2) Hept-6-ene-2-yne-4-ol

(3) Hept-1-ene-6-yne-4-ol

(4) Hept-6-ene-1-yne-4-ol

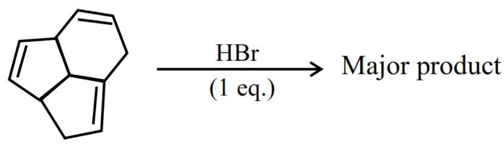
Ans. **(3)**

Sol.

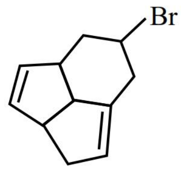


Hept-1-ene-6-yne-4-ol

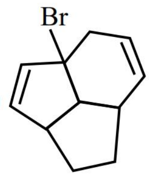
12. Find the major product



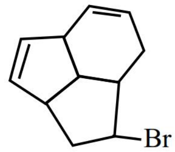
(1)



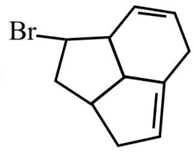
(2)



(3)

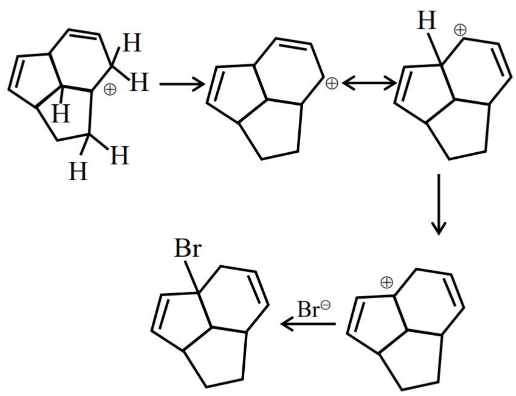


(4)

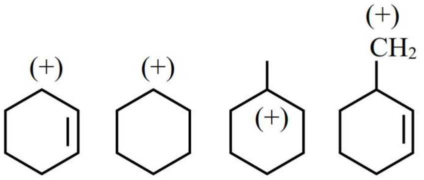


Ans. **(2)**

Sol.



13. Arrange the following carbocation in decreasing order of their stability.



(A) (B) (C) (D)

(1) (A) > (B) > (C) > (D)

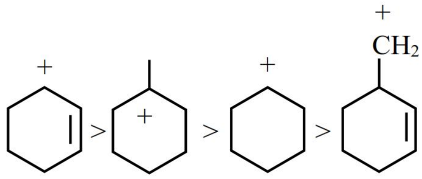
(2) (A) > (C) > (B) > (D)

(3) (C) > (A) > (B) > (D)

(4) (B) > (C) > (A) > (D)

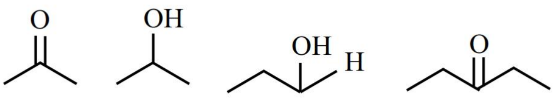
Ans. **(2)**

Sol. Carbocation stability order



(A) (Resonance) (C) α-H=7 (B) α-H=4 (D) α-H=1

14. Which of the following gives yellow precipitate with I_2/NaOH

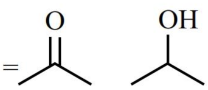


(a) (b) (c) (d)

(1) a & b (2) b & c

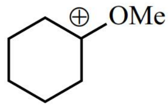
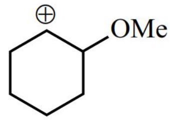
(3) a & c (4) b & d

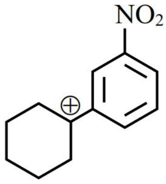
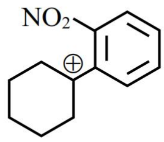
Ans. **(1)**

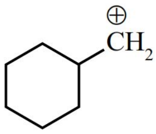
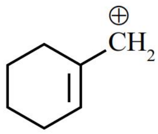
Sol. Iodoform test (positive) = 

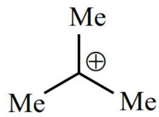
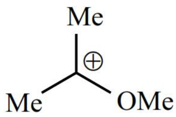
(a) (b)

15. In which of the following first cation is more stable the second?

(a)  and 

(b)  and 

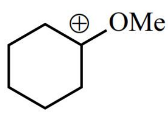
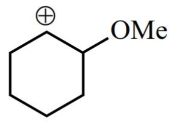
(c)  and 

(d)  and 

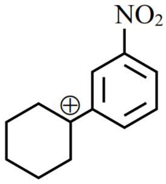
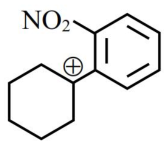
(1) a & b (2) b & c

(3) a & c (4) b & d

Ans. **(1)**

Sol. (a)  and 

Due to back bonding 1st is more stable than II

(b)  and 

Due to distant electron withdrawing group 1st is more stable than II

SECTION-B

16. Total number of electrons in chromium (z = 24) for which the value of azimuthal quantum number (ℓ) is 1 and 2.

Ans. **(17)**

Sol.

Cr(z=24)	$1s^2$	$2s^2$	$2p^6$	$3s^2$	$3p^6$	$3d^5$	$4s^1$
n =	1	2	2	3	3	3	4
ℓ =	0	0	1	0	1	2	0

Total electrons having $\ell = 1$ and $\ell = 2$

$\Rightarrow 6 + 6 + 5 = 17$

17. A sample of limestone having moss of 150 Kg which is 75% pure. Find the mass of calcium oxide if the sample is heated till decomposition.

Ans. **(63)**

Sol. $CaCO_3 \longrightarrow CaO + CO_2$

150 kg

Pure $CaCO_3$ is 150 kg $\times \frac{75}{100}$

$= 150 \times 10^3 \times \frac{75}{100} = 1125 \times 10^2 \text{gm}$

Mole of $CaCO_3 = \frac{1125 \times 10^2}{100} = 1125 \text{mole}$

Mole of CaO = 1125 mole

Mass of CaO = 1125 × 56

= 63000 gm

= 63 kg

18. Consider the following zero order reaction:

A → Products

Half-life of the reaction is 1 hr if initial concentration of the reactant is 2 mol/L. Find the half-life of the reaction in minutes if the initial concentration of the reaction is 0.5 mol/L.

Ans. **(15)**

Sol. $t_{1/2} \propto [A_0]$ for zero order

$$\frac{(t_{1/2})}{(t_{1/2})_2} = \frac{[A_0]_1}{[A_0]_2}$$

$$\frac{1}{(t_{1/2})_2} = \frac{2}{0.5}$$

$t_{1/2} = 0.25$ hrs.

19. If x mg of $Mg(OH)_2$ is added in 1L of solution to make a solution with pH = 10, then find the value of x.

[Given : MW of $Mg(OH)_2$ = 58 g/mol]

Assume $Mg(OH)_2$ dissociates completely in water.

Ans. **(3)**

Sol. $Mg(OH)_2 \longrightarrow Mg^{+2} + 2OH^{\ominus}$

pH = 10

$[H^+] = 10^{-10}$

$[OH^{\ominus}] = 10^{-4}$

$[Mg(OH)_2] = \frac{10^{-4}}{2}$

$\Rightarrow$ mole of $Mg(OH)_2 = \frac{10^{-4} \times 1}{2}$

Mass of $Mg(OH)_2 = \frac{10^{-4}}{2} \times 58$

$= 29 \times 10^{-4}$ gm

= 2.9 mg ≈ 3 mg

MATHEMATICS

1. $\cot^{-1}\left(\frac{7}{4}\right)+\cot^{-1}\left(\frac{19}{4}\right)+\cot^{-1}\left(\frac{39}{4}\right)+\cdots\infty$

is equal to

(1) $\cot^{-1}(2)$ (2) $\cot^{-1}\left(\frac{1}{2}\right)$

(3) $\cot^{-1}\left(\frac{1}{3}\right)$ (4) $\cot^{-1}(3)$

Ans. (2)

Sol. $\tan^{-1}\left(\frac{4}{7}\right)+\tan^{-1}\left(\frac{4}{19}\right)+\tan^{-1}\left(\frac{4}{39}\right)+.....\infty$

$$\Rightarrow\sum_{r=1}^{\infty}\tan^{-1}\left(\frac{4}{(2r)^2+3}\right)=\sum_{r=1}^{\infty}\tan^{-1}\left(\frac{1}{r^2+3/4}\right)$$

$$\Rightarrow\sum_{r=1}^{\infty}\tan^{-1}\left(\frac{1}{1+\left(r^2-\frac{1}{4}\right)}\right)$$

$$\Rightarrow\sum_{r=1}^{\infty}\tan^{-1}\left(\frac{\left(r+\frac{1}{2}\right)-\left(r-\frac{1}{2}\right)}{1\left(r+\frac{1}{2}\right)\left(r-\frac{1}{2}\right)}\right)$$

$$\Rightarrow\sum_{r=1}^{n}\left(\tan^{-1}\right)\left(r+\frac{1}{2}\right)-\tan^{-1}\left(r-\frac{1}{2}\right)$$

$$\Rightarrow\tan^{-1}\left(\frac{3}{2}\right)-\tan^{-1}\left(\frac{1}{2}\right)$$

$$+\tan^{-1}\left(\frac{5}{2}\right)-\tan^{-1}\left(\frac{3}{2}\right)$$

$$+\tan^{-1}\left(\frac{5}{2}\right)-\tan^{-1}\left(\frac{3}{2}\right)+\tan^{-1}\left(n+\frac{1}{2}\right)-\tan^{-1}\left(n-\frac{1}{2}\right)$$

$$+........\infty$$

$$\Rightarrow\tan^{-1}(\infty)-\tan^{-1}\left(\frac{1}{2}\right)$$

$$=\frac{\pi}{2}-\tan^{-1}\left(\frac{1}{2}\right)$$

$$=\cot^{-1}\left(\frac{1}{2}\right)$$

2. $\sum_{k=1}^{n}\left(\alpha^k+\frac{1}{\alpha^k}\right)^2=20,$

α is one of the root of $x^2+x+1=0$, then n is equal to

Ans. (11)

Sol.

$x^2+x+1=0$ with roots ω, ω_2

Let $\alpha=\omega$

$$\sum_{k=1}^{n}\left(\omega^k+\frac{1}{\omega^k}\right)^2=\left(\omega+\frac{1}{\omega}\right)^2+\left(\omega^2+\frac{1}{\omega^2}\right)^2+\left(\omega^3+\frac{1}{\omega^3}\right)^2+.......$$

$=(-1)^2+(-1)^2+(2)^2+......$

$=(1+1+4)+(1+1+4)+(1+1+4)+1+1$

$=20$

So, $(9+2)=11$ terms

So, $n=11$

3. $\int\frac{\left(\sqrt{1+x^2}+x\right)^{10}}{\left(\sqrt{1+x^2}-x\right)^{9}}dx=\frac{1}{m}\left[\left(\sqrt{1+x^2}+x\right)^n\left(n\sqrt{1+x^2}-x\right)\right]+c,$

where c is the constant of integration and $m,n\in\mathbb{N}$, then $m+n$ is equal to

Ans. (379)

Sol. $I=\int\left(\sqrt{1+x^2}+x\right)^{19}dx$

Let $\sqrt{1+x^2}+x=t\Rightarrow\sqrt{1+x^2}=t-x$

$$\Rightarrow 1+x^2=t^2+x^2-2tx$$

$$\Rightarrow 1=t^2-2tx\Rightarrow x=\frac{t^2-1}{2t}$$

$$\Rightarrow x=\frac{1}{2}\left(t-\frac{1}{t}\right)$$

$$\Rightarrow dx=\frac{1}{2}\left(1+\frac{1}{t^2}\right)dt$$

$$I=\int t^{19}\left(\frac{1}{2}\left(1+\frac{1}{t^2}\right)\right)dt$$

$$I=\frac{1}{2}\int\left(t^{19}+t^{17}\right)dt$$

$$=\frac{1}{2}\frac{t^{20}}{20}+\frac{1}{2}\frac{t^{18}}{18}+c$$

$$=\frac{t^{19}}{4}\left(\frac{t}{10}+\frac{1}{t\cdot 9}\right)+c$$

$$=\frac{t^{19}}{360}\left(9t+\frac{10}{t}\right)+c$$

$$=\frac{t^{19}}{360}\left(9\left(x+\sqrt{1+x^2}\right)+10\left(\sqrt{1+x^2}-x\right)\right)+c$$

$$=\frac{t^{19}}{360}\left(19\sqrt{1+x^2}-x\right)$$

$\Rightarrow$ n = 19

$\Rightarrow$ m = 360

So, m + n = 379

4. Let the mean & variance of observation 2, 3, 3, 4, 5, 7, a, b is 4 and 2, then mean deviation about mode of the observation is

Ans. **(1)**

Sol. $\bar{n}=\frac{2+3+3+4+5+7+a+b}{8}=4$

$\Rightarrow$ a + b = 8 ...(1)

Also,

$$\sigma^2=\frac{4+9+9+16+25+49+a^2+b^2}{8}-(4)^2=2$$

$\Rightarrow a^2+b^2+112=18\times 8$

$\Rightarrow a^2+b^2=32$ ...(2)

From (1) & (2) $\Rightarrow$ a = b = 4

Now, mode z = 4

$$M.D.=\frac{2+1+1+0+1+3+0+0}{8}=1$$

5. If $1^2\cdot{}^{15}C_1+2^2\cdot{}^{15}C_2+3^2\cdot{}^{15}C_3+\cdots+15^2\cdot{}^{15}C_{15}=2^m\cdot 3^n\cdot 5^k$, then $m+n+k$ is equal to

(1) 19 (2) 20
(3) 21 (4) 18

Ans. **(1)**

Sol. $\sum_{r=1}^{15} r^2.\ {}^{15}C_r=\sum r.15\ {}^{14}C_{r-1}$

$\Rightarrow 15\left(\sum\{(r-1)+1\}^{14}C_{r-1}\right)$

$\Rightarrow 15\left(\sum(r-1).\frac{14}{(r-1)}\ {}^{13}C_{r-2}+\sum{}^{14}C_{r-1}\right)$

$\Rightarrow 15(14.\ 2^{13}+2^{14})$

$\Rightarrow 15\times 2^{13}(16)$

$\Rightarrow 3\times 5\times 2^{17}$

$=2^m\times 3^n.\ 5^k$ $\Rightarrow$ m = 17, n = 1, k = 1

m + n + k = 19.

6. Let domain of

$$f(x)=\log_4\log_7(8-\log_2(x^2+2x+2))$$

is (α,β) & domain of

$g(x)=\sin^{-1}\left(\frac{7x+10}{x-2}\right)$ is $[\gamma,\ \delta]$,

then find the value of $\alpha^2+\beta^2+\gamma^2+\delta^2$.

Ans. **(261)**

Sol. For domain of f(x)

$\log_7\left(8-\log_2\left(x^2+2x+2\right)\right)>0$

$\Rightarrow 8-\log_2\left(x^2+2x+2\right)>1$

$\Rightarrow 7>\log_2\left(x^2+2x+2\right)$

$\Rightarrow x^2+2x+2<128$

$\Rightarrow (x+1)^2-127<0$

$x\in(\alpha,\beta)$

$\alpha+\beta=-2;\alpha$

$x^2+2x-126=0$

$\alpha^2+\beta^2=126-2\alpha+126-2\beta$

$=252-2(\alpha+\beta)$

= 256

and $8-\log_2\left(x^2+2x+2\right)>0$

$\Rightarrow 256>x^{2+2}+2x+2$

$\Rightarrow (x+1)^2-255<0$

this inequality gives us domain as a superset of $x\in(\alpha,\beta)$ so, ultimately domain for f(x) is $x\in(\alpha,\beta)$

For domain of g(x)

$$-1 \le \frac{7x+10}{x-2} \le 1 \Rightarrow x \in [-2,-1]$$

$$\gamma+\delta=-3, \gamma=-2$$

$$\delta=-1$$

$$\Rightarrow \gamma^2+\delta^2=5$$

So, $\alpha^2+\beta^2+\gamma^2+\delta^2=261$

7. If the sum of first 20 terms of series

$\frac{4.1}{4+3.1^2+1^4}+\frac{4.2}{4+3.2^2+2^4}+\frac{4.3}{4+3.3^2+3^4}+\frac{4.4}{4+3.4^2+4^4}+\cdots$ is $\frac{m}{n}$, where m, n are co-primes, then $m+n$ is equal to

(1) 420 (2) 421

(3) 422 (4) 423

Ans. **(2)**

Sol. $s=\sum_{r=1}^{n}\frac{4r}{4+3r^2+r^4}$

$$2\sum_{r=1}^{n}\frac{2r}{(r^2+2)^2-r^2}$$

$$2\sum_{r=1}^{n}\frac{(r^2+2+r)-(r^2+2-r)}{(r^2+2+r)(r^2+2-r)}$$

$$2\sum_{r=1}^{n}\left(\frac{1}{r^2+2-r}-\frac{1}{r^2+2+r}\right)$$

$$s_n=2\begin{bmatrix}\frac{1}{2}-\frac{1}{4}\\ \frac{1}{4}-\frac{1}{8}\\ \frac{1}{n^2+2-n}-\frac{1}{n^2+2+n}\end{bmatrix}$$

$$s_n=2\left[\frac{1}{2}-\frac{1}{n^2+2+n}\right]$$

$$s_n=\frac{n^2+n}{n^2+n+2}$$

$$s_{20}=\frac{400+20}{400+20+2}=\frac{210}{211}=\frac{m}{n}$$

m + n = 210 + 211

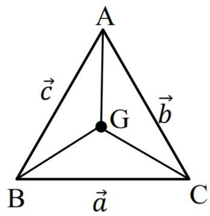
= 421.

8. Let the three sides of a triangle ABC is given by vectors $2\hat{\imath}-\hat{\jmath}+\hat{k}, \hat{\imath}-3\hat{\jmath}-5\hat{k}$ and $3\hat{\imath}-4\hat{\jmath}-4\hat{k}$, let G be the centroid of triangle ABC, then

$6\left(|\overrightarrow{AG}|^2+|\overrightarrow{BG}|^2+|\overrightarrow{CG}|^2\right)$ is equal to

Ans. **(164)**

Sol.



Let ℓ_1, ℓ_2, ℓ_3 be medians then

$$\ell_1^2+\ell_2^2+\ell_3^2=\frac{3}{4}(a^2+b^2+c^2)$$

$$=\frac{3}{4}(6+35+41)=\frac{3\times 41}{2}=\frac{123}{2}$$

Now, $|\overrightarrow{AG}|^2+|\overrightarrow{BG}|^2+|\overrightarrow{CG}|^2=\frac{4}{9}(\ell_1^2+\ell_2^2+\ell_3^2)$

$$=\frac{4}{9}\times\frac{123}{2}$$

So, $6\left(|\overrightarrow{AG}|^2+|\overrightarrow{BG}|^2+|\overrightarrow{CG}|^2\right)=\frac{4}{9}\times\frac{123}{2}\times 6$

= 164

9. Consider two sets A & B containing three numbers in A.P. Let the sum and the product of the elements of A be 36 and p respectively and the sum and the product of B 36 and q respectively. Let d and D be the common difference of A.P's in A & B respectively such that $D=d+3, d>0$. If $\frac{p+q}{p-q}=\frac{19}{5}$, then $p-q$ is equal to

Ans. **(540)**

Sol. A = {12 – d, 12, 12 + d}

B = {12 – 0, 12, 12 + D}

p = 12 (144 – d^2)

q = 12(144 – D^2)

$$\frac{p+q}{p-q}=\frac{19}{5} \Rightarrow \frac{p}{q}=\frac{12}{7}$$

$$\frac{144-d^2}{144-(d+3)^2}=\frac{12}{7}$$

$\Rightarrow$ $5d^2 + 72d - 612 = 0$

$5d^2 - 30d + 102d - 612 = 0$

$(5d + 106)(d - 6) = 0$

$\Rightarrow$ $d = 6$

$p = 12(144 - 36) = 12 \times 108$

$q = 12(144 - 81) = 12 \times 63$

$p - q = 12(108 - 63)$

$= 12 \times 45$

$p - q = 540.$

10. Let $f(x)$ and $g(x)$ satisfies the functional equation $2g(x) + 3g\left(\frac{1}{x}\right) = x$ and

$2f(x) + 3f\left(\frac{1}{x}\right) = x^2 + 5$. If $\alpha = \int_1^2 f(x)dx$ and $\beta = \int_1^2 g(x)dx$, then $(9\alpha + \beta)$ is equal to

(1) $\frac{27 + 6\,ln2}{10}$ (2) $\frac{27 - 6\,ln2}{10}$

(3) $\frac{3}{5}ln2$ (4) $\frac{3}{5}ln2 + \frac{7}{30}$

Ans. **(1)**

Sol.
$$\left.\begin{array}{l}\left(2g(x)+3g\left(\frac{1}{x}\right)=x\right)2\\ \left(2f(x)+3f\left(\frac{1}{x}\right)=x^2+5\right)2\\ \left(2f\left(\frac{1}{x}\right)+3f(x)=\frac{1}{x^2}+5\right)3\\ \left(2g\left(\frac{1}{x}\right)+3g(x)=\frac{1}{x}\right)3\end{array}\right\}$$

$$4g(x)+6g\left(\frac{1}{x}\right)=2x \qquad \ldots(i)$$

$$6g\left(\frac{1}{x}\right)+9g(x)-\frac{3}{x} \qquad \ldots(ii)$$

Solving (i) and (ii) we get

$$-5g(x)=2x-\frac{3}{x}$$

$$\Rightarrow g(x)=\frac{1}{5}\left(\frac{3}{x}-2x\right)$$

$$4f(x)+6f\left(\frac{1}{x}\right)=2x^2+10 \qquad \ldots(iii)$$

$$9f(x)+6f\left(\frac{1}{x}\right)=\frac{3}{x^2}+15 \qquad \ldots(iv)$$

Solving (iii) and (iv) we get

$$-5f(x)=2x^2-\frac{3}{x^2}-5$$

$$\Rightarrow f(x)=\frac{1}{5}\left(\frac{3}{x^2}+5-2x^2\right)$$

$$\beta=\frac{1}{5}\left[3\ell nx-\frac{2x^2}{2}\right]_1^2$$

$$=\frac{1}{5}\left[(3\ell n2-4)-(0-1)\right]$$

$$=\frac{1}{5}(3\ell n2-3)$$

$$\alpha=\frac{1}{5}\left[\left(-\frac{3}{x}+5x-\frac{2x^3}{3}\right)\right]_1^2$$

$$\alpha=\frac{1}{5}\left[\left(\frac{-3}{2}+10-\frac{16}{3}\right)-\left(-3+5-\frac{2}{3}\right)\right]$$

$$\alpha=\frac{1}{5}\left[\frac{19}{6}-\frac{4}{3}\right]$$

$$9\alpha=\frac{9}{5}\left[\frac{11}{6}\right]=\frac{33}{10}$$

So, $9\alpha+\beta=\frac{6\ell n2+27}{10}$

11. Let the matrix $A=\begin{bmatrix}1&0&0\\1&0&1\\0&1&0\end{bmatrix}$ satisfy

$A^n = A^{n-2} + A^2 - I$ for $n \ge 3$,

then the sum of all the elements of A^{50} is equal to

Ans. **(53)**

Sol. $A^n - A^{n-2} = A^2 - I$

$A^{50} - A^{48} = A^2 - I$

$A^{48} - A^{46} = A^2 - I$

$\vdots$

$A^4 - A^2 = A^2 - I$

$A^{50} - A^2 = 24(A^2 - I)$

$A^{50} = 25A^2 - 24I$

$$A^2=\begin{pmatrix}1&0&0\\1&0&1\\0&1&0\end{pmatrix}\begin{pmatrix}1&0&0\\1&0&1\\0&1&0\end{pmatrix}=\begin{pmatrix}1&0&0\\1&1&0\\1&0&1\end{pmatrix}$$

So, $25A^2 - 24I = \begin{pmatrix} 25 & 0 & 0 \\ 25 & 25 & 0 \\ 25 & 0 & 25 \end{pmatrix} - \begin{pmatrix} 24 & 0 & 0 \\ 0 & 24 & 0 \\ 0 & 0 & 24 \end{pmatrix}$

$$= \begin{pmatrix} 1 & 0 & 0 \\ 25 & 1 & 0 \\ 25 & 0 & 1 \end{pmatrix}$$

Sum of all elements = 1 + 25 + 25 + 1 + 1 = 53.

12. Let the values of P for which shortest distance between the lines

$\frac{x+1}{3} = \frac{y}{4} = \frac{z}{5}$ &

$\vec{r} = (p\hat{i} + 2\hat{j} + \hat{k}) + \lambda(2\hat{i} + 3\hat{j} + 4\hat{k})$ is $\frac{1}{\sqrt{6}}$ be $a, b (a < b)$, then the length of the latus rectum of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is equal to

Ans. **(2/3)**

Sol. $\frac{x+1}{3} = \frac{y}{4} = \frac{z}{5}$

$\frac{x-p}{2} = \frac{y-2}{3} = \frac{z-1}{4}$

So, from the above two equations we have

$\vec{a}_1 = (-1, 0, 0)$

$\vec{a}_2 = (p, 2, 1)$

$$\vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 4 & 5 \\ 2 & 3 & 4 \end{vmatrix}$$

$= \hat{i} - 2\hat{j} + \hat{k}$

$|\vec{b}_1 \times \vec{b}_2| = \sqrt{6}$

$$S.D. = \begin{vmatrix} p+1 & 2 & 1 \\ 3 & 4 & 5 \\ 2 & 3 & 4 \end{vmatrix} \div \sqrt{6}$$

$$= \frac{(p+1)(1) - 2(2) + (1)}{\sqrt{6}} = \pm\frac{1}{\sqrt{6}}$$

$\Rightarrow p - 2 = \pm 1$

$\Rightarrow p = 3, 1$

$\Rightarrow a = 1$ and $b = 3$

So, the ellipse is $\frac{x^2}{1} + \frac{y^2}{9} = 1$

$\Rightarrow$ length of latus rectum $= \frac{2a^2}{b} = \frac{2}{3}$

13. Let $A = \{-3, -2, -1, 0, 1, 2, 3\}$ and $xRy \Rightarrow 2x - y \in \{0, 1\}$. If l is number of elements in given relation, m and n are minimum number of elements to be added to make it reflexive and symmetric, respectively. Then $l + m + n$ is equal to

Ans. **(17)**

Sol. For $2x - y = 0 \Rightarrow y = 2x$

$x = -1, 0, 1$

Ordered pairs $\Rightarrow (-1, -2), (0, 0), (1, 2)$

For $2x - y = 1 \Rightarrow y = 2x - 1$

$x = -1, 0, 1, 2$

ordered pairs $\Rightarrow (-1, -3), (0, -1), (1, 1), (2, 3)$

$l = 7$

$m = 5$

$n = 5$ $\Rightarrow l + m + n = 17.$

14. Let the sum of the focal distance of the point P(4,3) on the hyperbola $H: \frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ be $8\sqrt{\frac{5}{3}}$, then if the length of the latus rectum is l then value of $9l^2$ is

Ans. **(40)**

Sol. $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

Passes (4, 3)

$\frac{16}{a^2} - \frac{9}{b^2} = 1$

$\frac{16}{a^2} - \frac{9 \times 3}{2a^2} = 1$

$16 - \frac{27}{2} = a^2$

$a^2 = \frac{5}{2}$

$b^2 = \frac{2}{3} \times \frac{5}{2}$

$= \frac{5}{3}$

$\ell = \frac{2b^2}{a}$

$$=\frac{2\times\frac{5}{3}}{\sqrt{\frac{5}{2}}}$$

$$=\frac{10}{3}\times\sqrt{\frac{2}{5}}$$

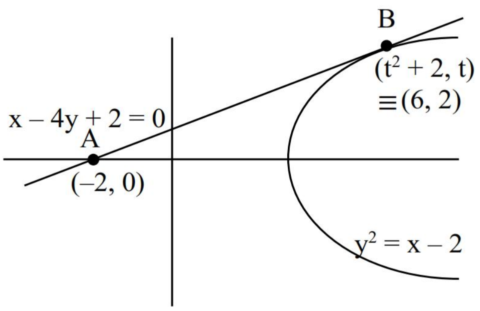
$$\ell^2=\frac{100}{9}\times\frac{2}{5}$$

$$=\frac{40}{9}$$

$$\left\{\begin{array}{l}(ex_1-a)+(ex_4+a)=8\sqrt{\frac{5}{3}}\\ 2e\times4=8\frac{\sqrt{5}}{\sqrt{3}}\\ e=\sqrt{\frac{5}{3}}\\ b^2=a^2\left(\frac{5}{3}-1\right)\\ b^2=\frac{2}{3}a^2\end{array}\right\}$$

15. A line passing through the point A(-2, 0) touches the parabola $P: y^2 = x - 2$ at the point B in the first quadrant. The area of the region bounded by the line AB, parabola P and the x-axis is equal to

Ans. $\frac{8}{3}$



Sol.

$$m_T=\frac{1}{2t}$$

$$\Rightarrow\frac{t}{t^2+4}=\frac{1}{2t}$$

$$\Rightarrow 2t^2=t^2+4$$

$$\Rightarrow t^2=4$$

So, $A=\int_0^2\left((y^2+2)-(4y-2)\right)dxy$

$$=\left.\frac{(y-2)^3}{3}\right|_0^2=\frac{8}{3}$$