Mensuration

Learing Objectives

In this chapter, you will learn

- To Differentiate between plane figures (triangles, quadrilaterals, pentagoes etc.) and solid figures.
- · To find perimeter and area of plane figures (quadrilaterals).
- · To find area of polygon.
- To find surface area and volume of some solids. (Cuboid, Cube, Cylinder) and use the concept in daily life.

9.1 Introduction

In earlier classes we have learnt about the perimeter and area of some closed plane figures. The perimeter of a figure is the total length of its boundary and the area of a figure is the region enclosed by it. We have already learnt about the perimeter and area of some plane figures such as triangles, rectangles, squares and circles etc. We have also learnt to find the area of pathways or borders of rectangular shapes.

In this chapter we will learn to find the area of simple quadrilaterals and some special types of quadrilaterals. We will also learn about surface area and volume of solids like cuboid, cube, cylinder etc.

9.2 Let us Recall

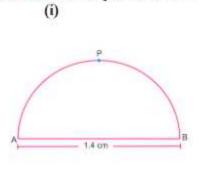
To review the previous knowledge let us recall the geometrical shapes and formulae (expressions) to find their perimeter and area.

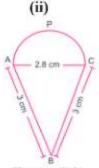
Shape	Diagram	Perimeter	Area
Triangle	a h	a + b + c	$\frac{1}{2} \times b \times h$
Rectangle	b a	a+b+a+b $= 2(a+b)$	a × b

Square	a	a + a + a + a = 4a	a × a
Parallelogram		a+b+a+b $= 2(a+b)$	a × h
Circle	•	2πr	πr^2

For circle we should use word circumference in place of perimeter.

Example 9.1 Find the perimeter of the following figures:





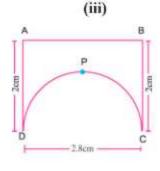


Figure 9.1

Sol. (i) We have, diameter of the semi circle = 1.4cm

$$\therefore Radius (r) of the semi circle = \frac{1.4}{2} = 0.7cm$$

So Perimeter (circumference) of the semi circle = $\pi r + 2r$

$$= \frac{22}{7} \times 0.7 + 2 \times 0.7 = 2.2 + 1.4 = 3.6 \text{ cm}$$

$$= AB + BC + \text{arc APC}$$

$$= 3 + 3 + \pi r$$

$$= 6 + \frac{22}{7} \times 1.4 [\therefore \text{Diameter} = 2.8 \text{cm}; \ r = \frac{2.8}{2} = 1.4 \text{cm}]$$

$$= 3 + 4.4 = 7.4 \text{ cm}$$

(iii) Perimeter of the figure
$$= AB + BC + arc CPD + AD$$

$$= 2.8 + 2 + \pi r + 2$$

$$= 6.8 + \frac{22}{7} \times 1.4 = 6.8 + 4.4 = 11.2cm$$

Example 9.2 The length and breadth of a rectangular field are in 3:2. If the area of the field is 294m², Find the cost of fencing the field at ₹8 per metre.

Sol. Let the length be 3x and breadth be 2x.

Given, Area of rectangle = 294 m²
i.e. length × breadth = 294
$$\Rightarrow 3x \times 2x = 294$$

$$\Rightarrow 6x^{2} = 294$$

$$\Rightarrow x^{2} = 49$$

$$\Rightarrow x^{2} = 7^{2}$$

$$\Rightarrow x = 7$$

$$\therefore length = 3x = 3 \times 7 = 21m$$
And breadth = $2x = 2 \times 7 = 14m$

Now, Perimeter of the rectangle = $2(\ell + b)$

$$= 2(21 + 14) = 2 \times 35 = 70m$$

∴ The cost of fencing the field = ₹(70×8) = ₹560

Example 9.3 A park is rectangular in shape having length 30 m and breadth 20 m. There are four square flower beds of size 2m × 2m each in the park as shown in figure 9.2 and rest has grass on it. Find



Figure 9.2

- (i) Perimeter of park
- (ii) Area of park
- (iii) Total Area of all four flower beds
- (iv) Area of park covered by grass

Sol. (i) Perimeter of park = 2 (length + breadth)
=
$$2 (30 + 20) m = 100 m$$

(ii) Area of park = length × breadth
=
$$(30 \times 20) \text{ m}^2 = 600 \text{ m}^2$$

(iii) Area of all four flower beds =
$$4 \times$$
 area of one flower bed = $4 \times (2 \times 2)$ m²
= $(4 \times 2 \times 2)$ m²
= 16 m²

(iv) Area of park covered by grass

= Total area of park – Area of all four flower beds
=
$$(600 - 16) \text{ m}^2 = 584 \text{ m}^2$$

Example 9.4 The shape of a garden is rectangular in the middle and semi circular at the ends as shown in figure 9.3. Find the area and perimeter of the garden.



Figure 9.3

Sol. The area of garden

= Area of rectangular part + 2 × Area of semi circle

= (Length × breadth) +
$$2 \times \frac{1}{2} \times \pi r^2$$

= (30 × 14) m² +
$$\left(2 \times \frac{1}{2} \times \frac{22}{7} \times 7 \times 7\right)$$
 m²

= (30 × 14) m² +
$$\left(2 \times \frac{1}{2} \times \frac{22}{7} \times 7 \times 7\right)$$
 m² So, radius $(r) = \frac{14}{2} = 7$ cm

$$= 420 \text{ m}^2 + 154 \text{ m}^2 = 574 \text{ m}^2$$

Perimeter of garden = length of boundry

= 2 × length of rectangular part + 2 × circumference of semi circle

=
$$(2 \times 30 + 2 \times \pi r) \text{ m} = \left(60 + 2 \times \frac{22}{7} \times 7\right) \text{ m}$$

$$= (60+44)$$
m $= 104$ m

Example 9.5. The area of a floor is 1080 m². Flooring tiles are in the shape of parallelogram shape whose base is 24 cm and corresponding height 10 cm are available. How many such tiles are required to cover the floor. (If required we can split the tiles to fill up corner.)

Sol. Area of floor = 1080
$$m^2$$
 = (1080 × 100 × 100) cm^2 = 108000000 cm^2

(Here note that area of floor is given in m2 unit where as the size of tile is given in cm2 unit. So to find the number of tiles, we have to convert them in same units.)

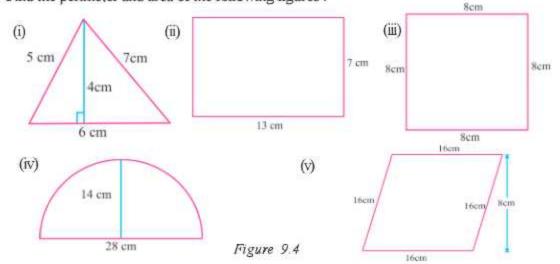
Area covered by one tile = $b \times h = (24 \times 10) \text{ cm}^2 = 240 \text{ cm}^2$

So Number of Tiles required
$$=$$
 $\frac{\text{Area of floor}}{\text{Area covered by one tile}}$

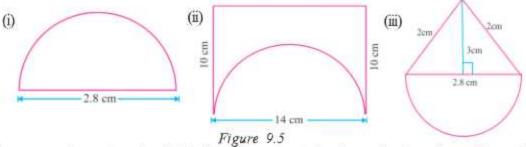
$$=\frac{10800000}{240}=45000$$



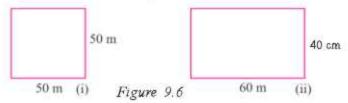
1. Find the perimeter and area of the following figures:



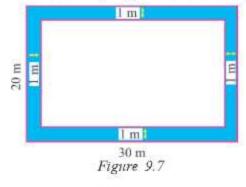
2. Find the area and perimeter of the following figures:



3. A square and a rectangular field with measurements (as shown in given figures) have the same perimeter. Which field has larger area and how much?



4. A park is of length 30 m and breadth 20 m. There is a path of one metre running inside along the perimeter of the park (fig. 9.7). The path has to be cemented. If 1 bag of cement is required to cement 4 m² area. How many bags of cement are required to construct the path.



5. Mr. Sandeep has a square plot as shown in figure 9.8. and he wants to construct a house in the middle of plot. A garden is developed around the house. Find the total cost of developing a garden around the house at the rate of ₹ 60 per m².

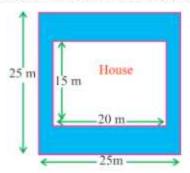


Figure 9.8

9.3 Area of a Quadrilateral

We know that a quadrilateral is a polygon having four sides. A quadrilateral can be split into two triangles by drawing one of its diagonal. It is known as triangulation. This "triangulation" helps us to find a formula to calculate the area of a quadrilateral. Study the figure 9.9.

Area of quadrilateral ABCD

= Area of ΔABD + Area of ΔCDB

$$= \left(\frac{1}{2} \times DB \times h_{_{1}}\right) + \left(\frac{1}{2} \times DB \times h_{_{2}}\right)$$

$$= \frac{1}{2} DB \times (h_1 + h_2) = \frac{1}{2} d(h_1 + h_2)$$

where d denotes the length of diagonal DB.

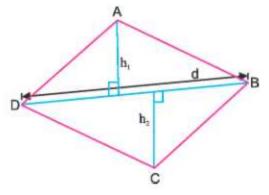


Figure 9.9

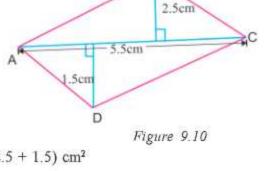
Example 9.6 Find the area of quadrilateral ABCD shown in figure 9.10.

Sol. In this case,

$$d = AC = 5.5 \text{ cm}, h_1 = 2.5 \text{ cm}$$

and
$$h_2 = 1.5$$
 cm

Using Area of quaderilateral
$$=\frac{1}{2} d(h_1 + h_2)$$



Area of quadrilateral ABCD =
$$\frac{1}{2} \times 5.5 \times (2.5 + 1.5) \text{ cm}^2$$

$$=\frac{1}{2} \times 5.5 \times 4 \text{ cm}^2 = 11 \text{ cm}^2$$

9.4. Area of Special Quadrilaterals :

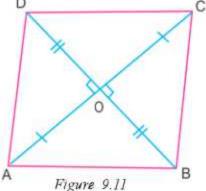
9.4.1 Area of Rhombus. We can use the same method of splitting a rhombus into two triangles (which is called triangulation) to find its area. We already know that rhombus is a parallelogram with

all sides of equal length and the diagonals of rhombus are perpendicular bisectors of each other.

Now Area of Rhombus ABCD

= Area of triangle ACD + Area of triangle ABC

$$\begin{split} &= \left(\frac{1}{2} \times AC \times OD\right) + \left(\frac{1}{2} \times AC \times OB\right) \\ &= \frac{1}{2} \times AC \times (OD + OB) = \frac{1}{2} \times AC \times BD \\ &= \frac{1}{2} \times d_1 \times d_2 \\ &= \frac{1}{2} \times (Product \ of \ diagonals) \end{split}$$



(where
$$d_1 = AC$$
 and $d_2 = BD$)

Note: We know that every square is a rhombus. So we can also calculate the area of a square in the form of diagonals. As the diagonals of a square are equal.

∴ Area of a square =
$$\frac{1}{2}$$
 × (product of diagonals) = $\frac{1}{2}$ d × d = $\frac{1}{2}$ d²

Example 9.7 Find the area of a rhombus whose diagonals are of lengths 20 cm and 8.2 cm.

Sol. Area of rhombus = $\frac{1}{2} \times d_1 \times d_2$ (where d_1 and d_2 are the diagonals of rhombus)

$$=\frac{1}{2} \times 20 \times 8.2 = 10 \times 8.2 = 82 \text{ cm}^2$$

Example 9.8 Find the area of a square whose diagonal is 12cm.

Sol. Given, diagonal (d) of a square = 12cm

Area of a square
$$= \frac{1}{2} \times \text{(product of diagonals)}$$
$$= \frac{1}{2} \times d^2 = \frac{1}{2} \times 12 \times 12 = 72 \text{ cm}^2$$

Example 9.9 Find the area of a rhombus whose base is 10 cm and height is 6.8 cm.

Sol. Given, base = 10cm and height = 6.8 cm

Area of a rhombus = base × height

= 10 × 6.8 = 68 cm²



Figure 9.12

Example 9.10 The area of a rhombus is 120cm² and one of its diagonal is 16cm. Find the length of the other diagonal.

Sol. We know,

Area of a rhombus=
$$\frac{1}{2}$$
 × (product of diagonals)

$$\Rightarrow 120 = \frac{1}{2} \times 16 \times d \Rightarrow d = \frac{120}{8} = 15 cm.$$

Example 9.11 Find the area of a rhombus whose side is 6cm and altitude is 4cm. If length of one diagonal is 8cm then find the length of the other diagonal.

Also Area of Rhombus
$$=\frac{1}{2} \times \text{(product of diagonals)}$$

i.e.
$$24 = \frac{1}{2} \times 8 \times d$$

$$\Rightarrow d = \frac{24}{4} = 6cm$$

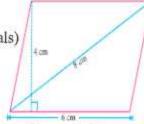


Figure 9.13

Note: See that by drawing a diagonal in a parallelogram (or rhombus) it is divided into two congurent triangles. So we can also find the area of a parallelogram (or rhombus) by finding the area of triangle on one side of its diagonal and then by doubling it.

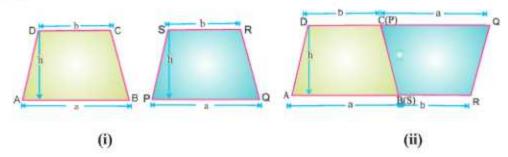
Now we shall find the area of a trapezium with the help of an activity.

Activity: Find area of trapezium with the help of an activity.

Material Required: Coloured paper, pair of scissors, scale, pencil, eraser, glue etc.

Previous Knowledge: Students know the area of parallelogram

Procedure:



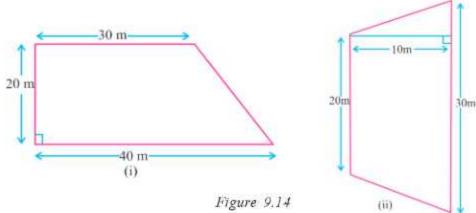
- On two pieces of coloured paper of different colours, draw two congruent trapezium as in fig (i) viz ABCD and PQRS.
- Flip one of the trapezium PQRS and place it along other trapezium as in fig (ii) to obtain parallelogram ARQD
- 3. Side of parallelogram ARQD is AR = a + b and height is h.

$$2 \times \text{Area of trapezium ABCD} = (a + b) \times h$$

Area of trapezium ABCD
$$=\frac{1}{2}(a+b) \times h$$

$$\therefore \text{ Area of trapezium ABCD } = \frac{1}{2} \times (\text{Sum of parallel sides}) \times \text{height}$$

Example 9.12 Find the area of trapezium shown in figure 9.14 (i) and 9.14 (ii).



Sol. Figure 9.14 (i) and (ii) are both of trapezium shape.

(i) Given parallel sides a = 30 m, b = 40 m and distance between them (h) = 20 m

Area of trapezium =
$$\frac{1}{2}$$
 (a + b) × h

∴ Area =
$$\frac{1}{2}$$
 (30 + 40) × 20 = $\frac{1}{2}$ × 70 × 20 = 700 m²

(ii) Given parallel sides a = 20 m, b = 30 m and distance (h) = 10 m

So Area of trapezium =
$$\frac{1}{2}$$
 (20 + 30) × 10 = 250 m².

Example 9.13. The area of a trapezium shaped field is 480 m². The distance between two parallel sides is 15 m. If one of the parallel side is 20 m. Find the other parallel side.

Sol. Here one of the parallel side of trapezium = 20 m.

Let another parallel side be b. Distance between parallel sides (h) = 15 m and Area of trapezium = 480 m^2

Now using, area of trapezium = $\frac{1}{2}$ × (sum of parallel sides) × h

We have,
$$480 = \frac{1}{2} \times (20 + b) \times 15$$

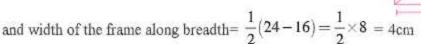
i.e.
$$20 + b = \frac{480 \times 2}{15} = 64$$

So
$$b = 64 - 20 = 44 \text{ m}$$

Hence other parallel side of trapezium = 44 m

Example 9.14. In given fig, the picture frame has outer dimensions of size 24cm × 28cm and inner dimensions of size 16cm × 20cm. Find the area of each section of the frame, if the width of each section is same.

Sol. Width of the frame along length $=\frac{1}{2}(28-20)$ $=\frac{1}{2}\times8=4$ cm



:. Each section of the frame in a trapezium with height 4cm.

Thus, Area of Section I $= \frac{1}{2} \times (16 + 24) \times 4$ $= \frac{1}{2} \times 40 \times 4 = 80 \text{cm}^2 = \text{Area of section III}$ Area of Section II $= \frac{1}{2} \times (20 + 28) \times 4 = \frac{1}{2} \times 48 \times 4 = 96 \text{cm}^2$ = Area of Section IV



1. Find the area of the quadrilaterals given below in figure 9.15.

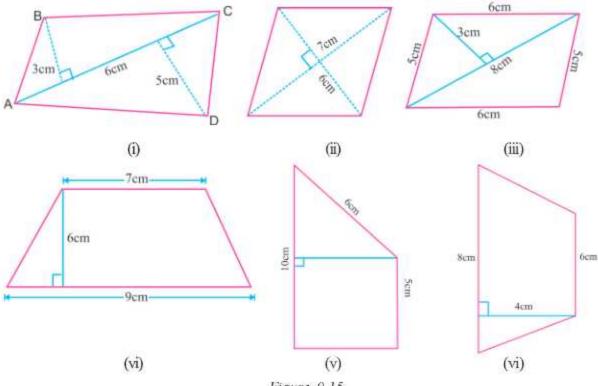


Figure 9.15

 The area of a rhombus is 320 cm². If length of its one diagonal is 16 cm. Find the length of the other diagonal.

 One diagonal of a quadrilateral field is 24 m and the altitudes dropped on it from the opposite vertices are 8 m and 13 m. Find the area of the field (fig. 9.16).

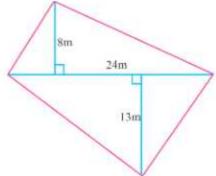


Figure 9.16

- 4. The diagonals of a rhombus are 7.5 cm and 12 cm. Find its area.
- 5. Find the area of a square whose length of diagonal is 10 cm.
- 6. Find the area of a rhombus with side 8 cm and altitude 4.8 cm.
- Find the area of rhombus whose side is 5 cm and whose altitude is 4.8 cm. If one of its diagonal is 8 cm. Find the length of other diagonal.
- Find the area of a trapezium shaped field if the parallel sides are of length 250 m and 160 m and the distance between them is 100 m.
- Find the other parallel side of trapezium if its area is 300 m². One parallel side is 15 m and distance between parallel sides is 15 m.
- Find the area of a trapezium whose parallel sides are 1m and 1.2m and perpendicular distance between them is 0.8 m.
- The floor of a building consists of 2400 tiles which are rhombus shaped having diagonals 45cm and 32cm in length. Find the total cost of polishing the floor, if the cost per m² is ₹4.

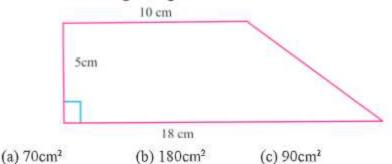
12. Multiple Choice Questions:

- (i) Find the area of a rhombus whose diagonals are 4cm and 6cm.
 - (a) 24cm²
- (b) 12cm²
- (c) 10cm2
- (d) 18cm²

- (ii) Find the area of a square whose diagonal is d.
 - (a) d²
- (b) $\frac{1}{2}$ d
- (c) 2d2
- (d) $\frac{1}{2} d^2$

(d) 120cm²

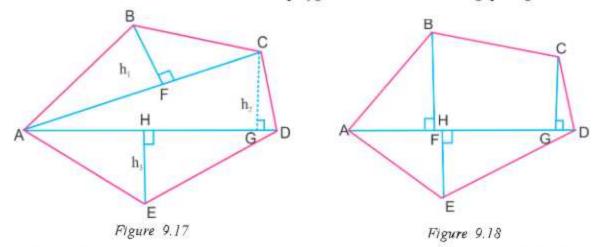
(iii) Find the area of the given figure:



9.5 Area of a Polygon

We know that a simple closed figure made up of only line segments is called a polygon. So far we have learnt about the area of three sided polygons i.e. triangles and four sided polygon i.e. quadrilaterals. Now we will try to find the area of a five sided polygon namely pentagon, six Sided polygon namely hexagon and so on.

By Joining the diagonals, we have divided a quadrilateral into triangles and found its area. Similar methods can be used to find the area of a polygon. Observe the following pentagon.



In fig. 9.17 by constructing two diagonals AC and AD the pentagon ABCDE is divided into three parts.

So Area of ABCDE = Area of \triangle ABC + Area of \triangle ACD + Area of \triangle AED

We can find area of ABCDE in another way also.

In fig. 9.18, by constructing one diagonal AD and three perpendiculars BF, CG and EH on it, Pentagon ABCDE is divided into four parts. So area of ABCDE = area of right angled triangle AFB + Area of triangle AFC + Area of Right angled triangle CGD + Area of triangle AED.

Example 9.15. Find the area of the Pentagon shown in figure 9.19

Sol. Here pentagon ABCDE is divided into right triangle AFB, trapezium BFHC, right triangle CHD and triangle AED.

So Area of Pentagon ABCDE

= Area of right \triangle AFB + Area of trapezium BFHC + Area of right \triangle CHD +

Area of right A AED

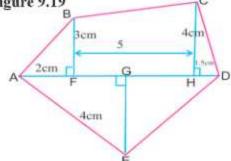


Figure 9.19

$$= \left(\frac{1}{2} \times AF \times BF\right) + \left(\frac{1}{2} (BF + CH) \times FH\right) + \left(\frac{1}{2} \times HD \times CH\right) + \left(\frac{1}{2} \times AD \times EG\right)$$

$$= \left(\frac{1}{2} \times 2 \times 3\right) + \left[\frac{1}{2} (3 + 4) \times 5\right] + \left(\frac{1}{2} \times 1.5 \times 4\right) + \left(\frac{1}{2} \times 8.5 \times 4\right)$$

$$= 3 \text{ cm}^2 + 17.5 \text{ cm}^2 + 3 \text{ cm}^2 + 17 \text{ cm}^2 = 40.5 \text{ cm}^3$$

Example 9.16 Find the area of a Pentagon shown in figure 9.20.

Sol. We can find the area of Pentagon ABCDE by two different methods.

Method 1:

From A draw perpendicular AF on DC as shown in figure 9.21.

AF divides the pentagon into two congurent trapeziums. You can verify it by paper folding.

Now Area of trapezium AFDE = $\frac{1}{2}$ (AF + ED) × DF

$$= \frac{1}{2} (30 + 15) \times 7.5 \text{ cm}^2 \left[DF = \frac{1}{2} DC \right]$$
$$= 168.75 \text{ cm}^2$$

So Area of Pentagon ABCDE = 2 × Area of trapezium AFDE $= 337.5 \text{ cm}^2$

Method 2:

We can also split this pentagon into two parts as shown in figure 9, 22 Now Area of Pentagon ABCDE = Area of ΔAEB + Area of square EBCD

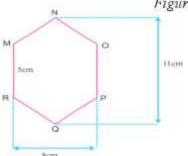
$$= \left(\frac{1}{2} \times EB \times AF\right) + (DC \times CB)$$

$$= \left(\frac{1}{2} \times 15 \times 15\right) \text{ cm}^2 + (15 \times 15) \text{ cm}^2$$

$$= 112.5 \text{ cm}^2 + 225 \text{ cm}^2 = 337.5 \text{ cm}^2$$

Example 9.17 Find the area of hexagon MNOPQR shown in figure 9.23

Where
$$NM = NO = QP = QR$$



Sol. We can find its area by two different methods.

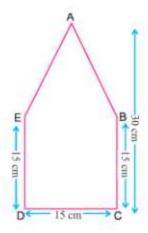


Figure 9.20 30 cm

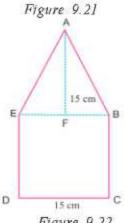


Figure 9.22



Method 1:

NQ divides the hexagon into two congurent trapeziums as shown in figure 9.24 You can verify it by paper folding.

Now Area of trapezium MNQR

$$= \frac{1}{2} (5 + 11) \times 4 \text{ cm}^2 = 32 \text{ cm}^2$$

So Area of hexagon MNOPQR

$$= 2 \times 32 \text{ cm}^2 = 64 \text{ cm}^2$$

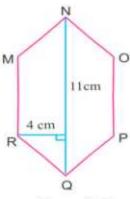


Figure 9.24

Method 2:

We can also split the hexagon as shown in figure 9.25. Here ΔMNO and ΔRQP are congurent triangles with perpendicular 3 cm.

Area of hexagon MNOPQR

= 2 Area of ΔMNO + Area of Rectangle MOPR

$$= 2 \times \left(\frac{1}{2} \times 8 \times 3\right) \text{ cm}^2 + (8 \times 5) \text{ cm}^2 = 2 \times \frac{1}{2} \times 8 \times 3 \text{ cm}^2 + 40 \text{ cm}^2$$

 $= 64 \text{ cm}^2$

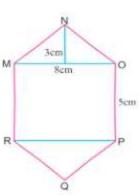


Figure 9.25

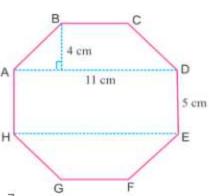
Example 9.18 Find the area of regular octagon as shown in figure 9.26

Sol. Area of octagon ABCDEFGH= Area of trapezium ABCD + Area of rectangle ADEH+ Area of trapezium EFGH

$$= \left[\frac{1}{2}(11+5)\times 4\right] \text{ cm}^2 + (11\times 5) \text{ cm}^2 + \left[\frac{1}{2}(11+5)\times 4\right] \text{ cm}^2 \qquad Figure 9.26$$

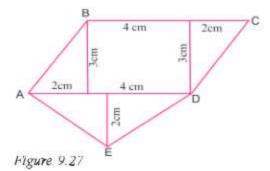
(Here trapezium ABCD and EFGH are congurent.)

$$= 32 \text{ cm}^2 + 55 \text{ cm}^2 + 32 \text{ cm}^2 = 119 \text{ cm}^2$$



Exercise 9.3

 Find the area of pentagon ABCD shown in figure 9.27.



There is a pentagonal shaped park as shown in figure 9.28. Jyoti and Kavita divided it in two different ways. Find the area of park using both ways.

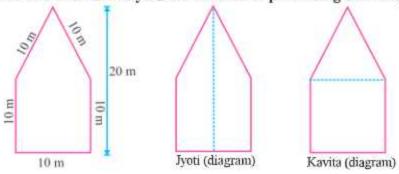
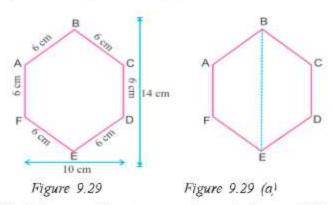
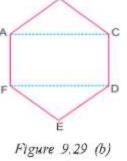


Figure 9.28

 Find the area of hexagon shown in figure 9.29 by two different ways as shown in figure 9.29 (a) and 9.29 (b). Where AB = BC = CD = DE = EF = FA



4. Find the area of octagon as shown in figure 9.30.



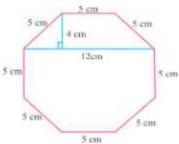


Figure 9.30

5. Find the area of Hexagon shown in the figure 9.31

where	
MP = 9 cm	MD = 7 cm
MC = 6 cm	MB = 4 cm
MA = 2cm	AN = 2.5 cm
OC = 3 cm	QD = 2 cm
RB = 2.5 cm	

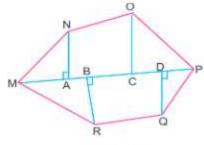
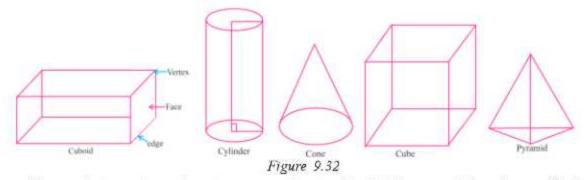


Figure 9.31

9.6 Solid Shapes

In your earlier classes you have studied that two dimensional figures can be identified as the faces of three dimensional shapes. Observe the solids which we have discussed so far in figure 9.32.



Observe that some shapes have two or more than two identical (congruent) faces. In case of **cuboid** all six faces are rectangular and opposite faces are identical. So there are three pairs of identical faces.

In case of a cube, all six faces are square and all are identical whereas cylinder has one curved surface and two circular faces which are identical and parallel to each other.

Observe in cylinder, the line segment joining the centre of circular faces is perpendicular to the base (fig. 9.32). Such cylinders are known as **right circular cylinders**. We are going to study only this type of cylinder though there are other types of cylinders as well.

9.7 Surface Area of Cuboid, Cube and Cylinder

To find the total surface area of cube, cuboid and cylinder, we will find the area of each face. The surface area of a solid is the sum of the area of its faces. To clarify further, we take each shape one by one.

9.7.1 Cuboid

We know the cuboid has three pairs of identical faces.

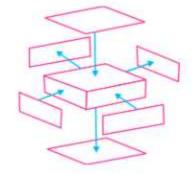


Figure 9.33

Now cut open a cuboidal box and lay it flat (figure 9.33), we can see a net as shown in (figure 9.34).

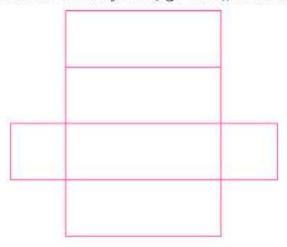


Figure 9.34



Activity

Find the formula of total surface area of the cuboid by activity.

Required Material: Chart Paper, Geometry Box, Coloured sketch or Pencils.

Procedure :

- Take a chart paper and draw a net of cuboid as shown.
- 2. Cut the net from the chart paper.
- Complete net is divided into 6 rectangles.
- Here, Fig I and Fig V, Fig III and Fig IV, Fig II and Fig VI are congruent rectangles.
- When we fold this net from partitions, we shall get a cuboid of dimensions.

$$5cm \times 4cm \times 3cm i.e. (I \times b \times h)$$

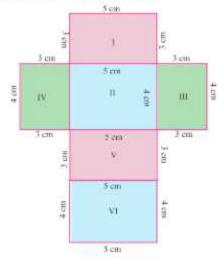


Figure 9.35

Observation:

Total Surface of Area of a cuboid = Area of all six rectangles

= Area of Rectangle (I+II+III+IV+V+VI)

= 2× Area of Rectangle (I + II + III)

 $=2\times(5\times3+5\times4+3\times4)$

If L = 5cm, b = 4cm and h = 3cm

Then Total Surface Area of a cuboid $= 2 \times (l \times h + l \times b + h \times b)$

= 2 (1b + bh + h1)

Note: The side walls (the faces excluding the top and bottom) make the lateral surface area of the cuboid. For example the total area of all the four walls of the cuboidal room in which you are sitting is the lateral surface area of the room. If I, b and h are length, breadth and height of the room respectively, then lateral surface area of room

$$= (I \times h) + (b \times h) + (I \times h) + (b \times h)$$

$$= 2 I h + 2bh$$

$$= 2h(I + b)$$

This is also known as Area of four walls.

VIVA VOCE

Q 1. What is the base of a cuboid?

Ans: Rectangle

Q.2. What is lateral surface area of a cuboid?

Ans: $2(l+b) \times h$

Q 3. What is total surface area of a cuboid?

Ans: 2(/b + bh + h/)

Example 9.19 Find the total surface area and lateral surface area of a room having length 6 m, breadth 5 m and height 4m.

Sol. Given, I = 6m, b = 5m and h = 4m

Total surface area of room= 2(length × breadth + breadth × height + height × length) m2

$$= 2(6 \times 5 + 5 \times 4 + 4 \times 5) \text{ m}^2 = 140 \text{ m}^2$$

Lateral surface of room $= 2 \times (length + breadth) \times height$

$$= 2 \times (6 + 5) \times 4 = 2 \times 11 \times 4$$

$$= 88 \text{ m}^2$$

9.7.2 Cube

We know that if length, breadth and height of a cuboid are equal then it is known as a cube. All faces of a cube are square in shape. All faces are of equal area.

Lateral surface Area = $4 \times$ Area of one face = $4l^2$

And Total surface area of cube $= 6 \times \text{Area of one face}$

= 6 P

where I is the length of each side of the cube.

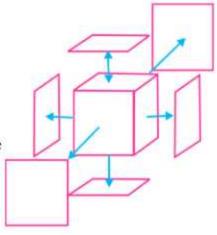


Figure 9.36

Example 9.20 Find the total surface area and lateral surface area of cube having side 10 cm.

Sol. We know that total surface area of cube = 6P, where l is the length of side.

Given, side (
$$l$$
) of a cube = 10cm

So Total surface area =
$$6 \times (10 \times 10)$$
 cm² = 600 cm²

Now lateral surface area of cube=
$$4P = 4 \times (10)^2$$

$$= 400 \text{ cm}^2$$

Example 9.21 Find the side of a cube whose surface area is 600cm².

Sol. Give, Surface area of a cube = 600cm²

$$\Rightarrow 6\ell^2 = 600 \Rightarrow \ell^2 = \frac{600}{6} = 100$$

$$\Rightarrow \ell^2 = 10^2 \Rightarrow \ell = 10$$

Thus, Side of the cube = 10cm.

Example 9.22 Vasu is painting the walls and ceiling of a cuboidal hall with dimensions 15m×10m×7m. From each can of paint, 100m² of area can be painted. How many cans of paint will he need to paint the room?

Sol. Dimensions of the hall are $15m \times 10m \times 7m$

Since, he is painting walls and ceiling

Surface area of the hall = Area of four walls + Area of top
=
$$2 (\ell+b) h + \ell b$$

= $2 (15+10) \times 7 + 15 \times 10$
= $2 \times 25 \times 7 + 150 = 350 + 150 = 500 \text{m}^2$

⇒ Surface Area of the hall = 500m²

Given, Area to be painted with 1 can = 100m²

$$\therefore \qquad \text{Number of Cans} \qquad = \frac{500}{100} = 5$$

Thus, 5 cans of paint will be needed to paint the room.

Example 9.23 What will happen to the surface area of the cube if its edge is doubled?

Sol. Let the edge of the cube be I

then, Surface Area of the cube = 6P

Now, If the edge is doubled, i.e. new edge = 2l

∴ Surface area 6 (2ℓ)² = 4 (6ℓ²) = 4 × (Original surface area)

Thus, If edge of a cube is doubled then the total surface area will be four times the original surface area.

Example 9.24 Three cubes each of side 4cm are joined end to end. Find the surface area of the cuboid so formed.

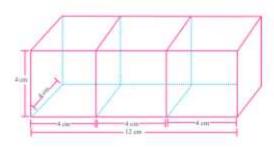
Sol. The cuboid formed by joining three cubes is shown, whose dimensions are

$$l = 4 + 4 + 4 = 12$$
cm, $b = 4$ cm,

h = 4cm

.. Total Surface area of the cuboid

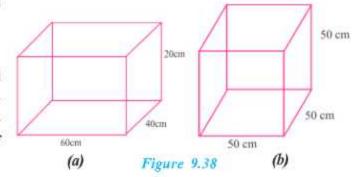
$$= 2 (lb + bh + hl)$$



$$= 2 (12 \times 4 + 4 \times 4 + 4 \times 12)$$

$$= 2 (48 + 16 + 48) = 224 \text{cm}^2$$

Example 9.25 There are two cuboidal boxes as shown in the adjoining figure 9.38. Which box requires the lesser amount of material to make?



Sol. To find the amount of material to make the box (a) and (b), we have to find the total surface area of the boxes

Total surface area of box (a) = 2 (
$$\ell$$
b × bh + he)
= 2 ($60 \times 40 + 40 \times 20 + 20 \times 60$) cm²
= 8800 cm²

Total surface area of box (b) = $6l^2 = 6 \times 50 \times 50 = 15000 \text{ cm}^2$

So box (a) requires lesser amount of material to make.

9.7.3 Right Circular Cylinder

The cylinder has two congruent circular faces that are parallel to each other. The line segment

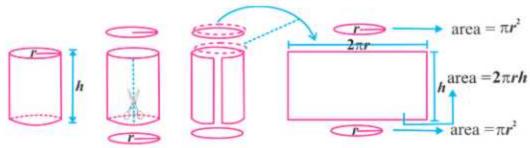


Figure 9.39

joining the centre of circular faces is perpendicular to the base in most of the cylinders, these are right circular cylinders. For example, water pipes, tube lights, round pillars etc.

To find the surface area of the cylinder, cut a cylinder as shown in figure 9.39.

Curved surface area of a cylinder is the area of its curved part, part of cylinder is formed from a rectangle having length $2\pi r$ and width h. So curved surface area of a cylinder is $2\pi r$ h.



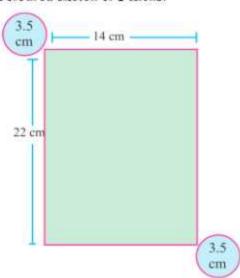
Activity

To find the total surface area of a cylinder by an activity.

Material Required: Chart paper, Geometry box, Coloured sketch or Pencils.

- Procedure:
- Take a chart paper and draw a net of closed cylinder as shown in which we have a rectangle of size 22cm × 14cm and two congruent circles of radius 3.5cm each.
- 2. Cut the net from the chart paper.
- Now, when rectangle is rolled along side14cm, there is formation of an open cylinder and height of cylinder will be 14cm.
- Close the open cylinder from top and bottom with two congruent circles of radius 3.5cm (whose

circumference =
$$2\pi r = 2 \times \frac{22}{7} \times 3.5 = 22$$
cm)



Thus, A closed cylinder is formed by suitable transformation.

Observation:

Total Surface of Area of a cylinder = Area of the given net a closed cylinder

= Area of Rectangle + 2 × Area of circle

 $= 22cm \times 14cm + 2 \times \pi r^2$

 $= (2\pi r) \times h + 2\pi r^2$

 $=2\pi r (h+r)$

By Step 4, $2\pi r = 22$ and in step 3, h = 14

Thus, Total Surface area of a closed cylinder $= 2\pi r (h + r)$

Note: The word 'Lateral surface Area' is used for solids with plain surface like cube, cuboid etc. Where the word 'Curved surface Area' is used for solids with curved surfaces. e.g. cylinder, cone, sphere.

VIVA VOCE

Q 1. What is the base of a right circular cylinder?

Ans: Circle.

Q.2. What is the area of base of cylinder?

Ans: πr²

Q 3. What is the curved surface area of a cylinder?

Ans: 2πrh

Example 9.26 Find the total and curved surface area of cylinder shown in figure 11.40.

Sol. Radius (r) of base of cylinder is 2 m and height (h) is 5 m.

So total surface area of cylinder = $2\pi r (h + r)$

$$2\pi \times 2 (5+2) \text{ m}^2 = 2\pi \times 2 \times 7 \text{ m}^2$$

$$= 2 \times \frac{22}{7} \times 2 \times 7 \text{ m}^2 = 88 \text{ m}^2$$



Figure 9.40

and Curved surface area =
$$2\pi rh = 2\pi \times 2 \times 5 = 20\pi = 20 \times \frac{22}{7}m^2 = \frac{440}{7}m^2$$

Example 9.27. Find the curved area of a cylinder whose circumference of the base is 22cm and height is 7cm.

Sol. Curved surface area of cylinder = $2\pi rh$

=
$$(2\pi r) \times h$$
 = (Circumference of the base) × h

$$= 22 \times 7 = 154$$
cm²

Example 9.28. The curved surface area of a hollow cylinder is 4224cm². It is cut along its height and formed a rectangular sheet of width 33cm. Find the perimeter of the sheet.

Sol.

Circumference of the base =
$$\frac{4224}{33}$$
 = 128 cm

33 cm

Figure 9.41

Since, circumference of the base = length of the rectangle = 128cm

$$\therefore$$
 Perimeter of the rectangular sheet = 2 ($l + b$)

$$= 2 (128 + 33) = 2 \times 161 = 322$$
cm

Example 9.29. A road roller takes 750 complete revolutions to move once over to level a road. Find the area of the road if the diameter of a road roller is 84cm and length 1m.

Sol. Diameter of the road roller = 84cm

$$\therefore \text{ radius (r) of the road roller} = \frac{84}{2} = 42 \text{cm}$$

and length (h) =
$$1m = 100cm$$

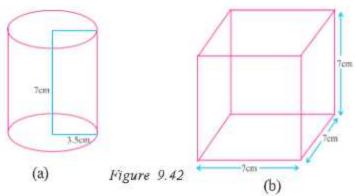
Area of road leveled in 750 revolutions = 750 × curved surface area of roller

$$= 750 \times 2\pi rh$$

$$=750\times2\times\frac{22}{7}\times42\times100$$

$$= \frac{19800000}{100 \times 100} \ m^{\text{2}} \quad = 1980 \ m^{2}$$

Example 9.30 Find the curved surface Area (lateral surface area) of the figures 9.42 (a) and (b).



Sol. Figure 9.42 (a) is cylinderical in shape. So curved surface area of figure 9.42 (a)

=
$$2\pi \text{rh cm}^2$$

= $2 \times \frac{22}{7} \times 3.5 \times 7 \text{ cm}^2$
= 154 cm^2

Figure 9.42 (b) is cubical in shape, so curved surface area of fig. 9.42 (b) is

$$=4P=4\times7\times7\text{ cm}^2$$

 $= 196 \text{ cm}^2$

Example 9.31 A company packs its milk powder in cylindrical container whose base has a diameter 14 cm and height 20 cm. Company places a label around the surface of the container (as shown in figure 9.43). If the label is placed 1 cm from top and bottom. What is the area of label?

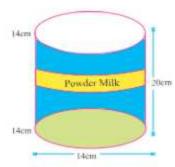


Figure 9.43

Sol. We have to find the area of the label. As label is placed around the cylinderical container having radius 7 cm and height 20 cm. Since the label is placed 1 cm from top and bottom. So height of label is 18 cm.

Hence area of label = $(2 \times \pi \times \text{radius} \times \text{height}) \text{ cm}^2$

$$=$$
 $\left(2 \times \frac{22}{7} \times 7 \times 18\right)$ cm² = 792 cm²



1.	Find the lateral and	ind the lateral and total surface area of the cuboid having dimensions:					
	(i) 6cm × 5cm × 4c	m (ii)15m	\times 12m \times 8m	(iii) $8m \times 10m \times 8m$			
2.	Find the lateral and	total surface area of	the cubes having e	edge:			
	(i) 8cm	(ii) 12m		(iii) 15mm			
3.	Find the side of a c	ube whose surface a	rea is 2400 cm².				
4.	a communicación de servicio de la companya de la c	Neetu painted the outside of a cabinet of measure $3m \times 2m \times 1.5 m$. How much surface area she covered if she painted all cabinet except bottom.					
5.	.0.	Ashima painted her room of measure $15m \times 12m \times 7m$. How much surface area did he cover if he painted all except the floor?					
6.		Manu wants her room to be painted. If the measures of her room is $20m \times 12m \times 15m$ then find the cost of painting the room except the floor at \ref{floor} 6 per m^2 ?					
7.		A suitcase with measurement 80 cm × 48 cm × 24 cm is to be covered with a cloth. How many metres of cloth of width 96 cm is required to cover the suitcase?					
8.	What will happen to	What will happen to the surface area of a cube if its edge is (i) tripled (ii) halved.					
9.	Three cubes each o so formed.	Three cubes each of side 5cm are joined end to end. Find the surface area of the cuboid so formed.					
10.	Find the curved and total surface area of a cylinder whose dimensions are						
	(i) $r = 7$ cm, $h = 20$ 6	(i) $r = 7$ cm, $h = 20$ cm (ii) $r = 14$ cm, $h = 15$ m (iii) diameter = 7cm, $h = 12$ cm					
11.	Find the curved sur height is 12cm	Find the curved surface area of a cylinder whose circumference of the base is 77cm and height is 12cm					
12.	Find the radius of c	Find the radius of cylinder whose curved surface area is 1056cm ² and height 12cm.					
13.				surface area is 968 cm ² .			
14.	A cylindrical pipe of surface area?	A cylindrical pipe open from both sides has radius 21 cm and height 50 cm. What is its surface area?					
15.		A road roller takes 950 complete revolutions to move once over to level a road. Find the area of road leveled if the diameter of road roller is 84 cm and length is 1 m.					
16.	A closed cylindrical tank of radius 7 m and height 3m is made from a sheet of matel. What is cost of tank if rate of matel sheet is 20 Rs per m ² .						
17.	Multiple Chocie Q	uestions :					
(i	i) Lateral surface a	rea of cube is:					
	(a) 6 <i>l</i> ²	(b) 5l ²	(c) 4l ²	(d) $2l^2$			
(i	Curved surface area of cylinder is:						
	(a) 2πrh	(b) πrh	(c) 2πr	(d) πr ² h			
(i	ii) If the edge of a c	ube is doubled then	what will happen t	o the surface area?			
	(a) 2 times	(b) 4 times	(c) 3 times	(d) Half			

9.8 Volume of Cuboid, Cube and Cylinder

Amount of space occupied by a three dimensional object is called its volume. We can compare the volume of objects surrounding us. Volume of a room is greater than the volume of a almirah kept inside it. Similarly volume of almirah is greater than a box kept inside it.

Remember to find the area of a region we use square units. Here we will use cubic units to find the volume of a solid. For finding the area we divide the region into square units. Similarly to find the volume of a solid we need to divide it into cubical units.

Observe that the volume of each of the adjoining solid is 8 cubic units. (Figure 9.44)

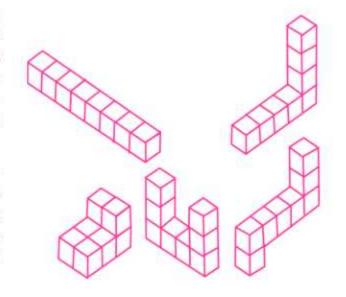


Figure 9.44

We can say that the volume of a solid is measured by counting the number of unit cubes it contains. Cubic units which are generally use to measure volume are

Now we find some expressions to find volume of a cuboid, cube and cylinder.

9.8.1 Cuboid

Take 36 cubes of equal size (i.e. length of each cube is same). Arrange them to form a cuboid. You can arrange them in many ways as shown in the following table:

Table

Cuboid

length

breadth

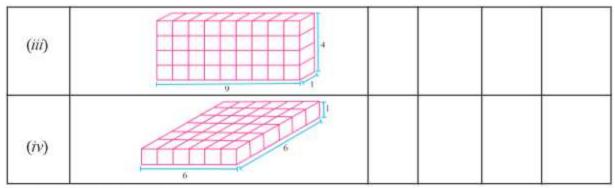
height $1 \times b \times h = V$ 12

12

3

1 $12 \times 3 \times 1 = 36$ (ii)

195



What do we observe?

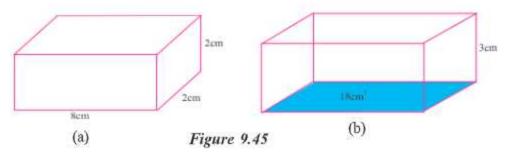
As we used 36 cubes to form these cuboids, so volume of each cuboid is 36 cubic units. We can see that volume of each cuboid is equal to product of length, breadth and height of the cuboid.

So Volume of Cuboid =
$$I \times b \times h$$

As $I \times b$ is the area of base, so we can also say that

Volume of Cuboid = area of base × height

Example 9.32 Find the volume of cuboid shown in figure 9.45. (a) and 9.45 (b)



Sol. In figure 9.45 (a) length, bredath and height of cuboid is 8 cm, 2 cm and 2 cm respectively.

So Volume of Cuboid =
$$l \times b \times h = (8 \times 2 \times 2) \text{ cm}^3$$

= 32 cm³

In figure 9.45 (b), Area of base of Cuboid is 18 cm² and height is 3 cm.

9.8.2 Cube

As we know that cube is a special case of cuboid, where l = b = h.

Hence Volume of Cube = $1 \times 1 \times 1 = 1^3$ i.e. (side)³

Example 9.33. Find the volume of cube having side (i) 5 cm (ii) 2.5 cm.

Sol. We know Volume of cube = (side)3

(ii) Side =
$$2.5 \text{cm}$$

Example 9.34 Find the height of a cuboid whose base area is 180 cm² and volume is 900 cm³.

Sol. Given, Volume of Cuboid = 900 cm³ and its base area = 180 cm²; Let h cm be the height of cuboid. We know that

Volume of Cuboid = Base Area × height

i.e.
$$h = \frac{900}{180} = 5 \text{ cm}$$

Example 9.35 A cuboid is of dimensions 60 cm × 54 cm × 30 cm. How many small cubes of side 6 cm can be placed in the given cuboid?

Sol. Volume of cuboid =
$$(60 \times 54 \times 30)$$
 cm³ = 97200 cm³
Volume of a small cube = $(6 \times 6 \times 6)$ cm³ = 216 cm³

$$\therefore$$
 Number of small cubes that can be placed in the cuboid = $\frac{\text{Volume of Cuboid}}{\text{Volume of Cube}} = \frac{97200}{216} = 450$

9.8.3 Cylinder

In case of cuboid we have seen that volume of cuboid = $l \times b \times h$

Also we have seen that volume of cuboid = Area of base × height

Can we find the volume of a cylinder in the same way?

Like cuboid, top and bottom of a cylinder are congruent and parallel to each other. Its curved surface is also perpendicular to the base, just like cuboid.

So the Volume of Cylinder = Area of base × height
=
$$\pi r^2 \times h = \pi r^2 h$$

where r is the radius of circular face and h is the height of cylinder.

Example 9.36 Find the volume of cylinder shown in figure 9.46 (a) and 9.46 (b)

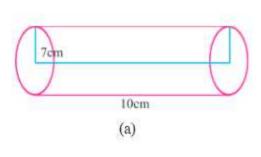
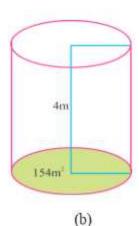


Figure 9.46



Sol. In figure 9.46 (a), radius (r) of base = 7 cm and height (h) of cylinder = 10 cm

So Volume of cylinder =
$$\pi r^2 h = \left(\frac{22}{7} \times 7 \times 7 \times 10\right) \text{ cm}^3$$

= 1540 cm³

In figure 9.46 (b), Area of base of cylinder = 154 m² and height of cylinder = 4 m,

So Volume of cylinder = Area of base × height
=
$$(154 \times 4) \text{ m}^3 = 616 \text{ m}^3$$

9.9 Volume and Capacity

Volume refers to the amount of space occupied by an object and capacity refer to the quantity that a container holds. For example if a water bottle holds 1000 cm³ of water, then the capacity of the water bottle is 1000 cm³.

Capacity is also measured in terms of litres. The relation between litre and cm³ is $1 \text{ m}\ell = 1 \text{ cm}^3$; $1L = 1000 \text{ cm}^3$.

Now 1 m³ =
$$(100 \times 100 \times 100)$$
 cm³ = 1000000 cm³ = 1000 L

Example 9.37 A milk tank is in the form of a cylinder whose radius is 1.5 m and length is 7 m. Find the quantity of milk in litres that can be stored in the tank?

Sol. Volume of milk tank = $\pi r^2 h$

$$= \left(\frac{22}{7} \times 1.5 \times 1.5 \times 7\right) \text{ m}^{3}$$

$$= 49.5 \text{ m}^{3} = 49.5 \times 1000I \qquad [\therefore 1\text{m}^{3} = 1000I]$$

$$= 49500 I$$

Example 9.38 A rectangular piece of paper 11 cm × 4 cm is folded without overlapping to make a cylinder of height 4 cm. Find the volume of cylinder.

Sol. Length of the paper becomes the circumfrence of the base of the cylinder and width becomes the height of cylinder.

Let radius of cylinder = r cm and height = 4 cm

Now perimeter of the base of cylinder = circumference of circle

$$\Rightarrow 2\pi r = 11$$

$$2 \times \frac{22}{7} \times r = 11 \Rightarrow r = \frac{7}{4} \text{ cm}$$

Volume of cylinder $V = \pi r^2 h$

$$= \left(\frac{22}{7} \times \frac{7}{4} \times \frac{7}{4} \times 4\right) \text{ cm}^3 = 38.5 \text{ cm}^3$$



1. Find the volume of a cuboid having dimension							
	(i) 4	lm, 3m, 5m	(ii)12cm, 8cm, 10cm	(iii) 1.5cm, 2	m, 3.4m		
2.	Find the volume of a cube having edge:						
	(i) 6	icm	(ii) 12cm	(iii) 1.5m			
3.	Fine	d the volume o	f cuboid whose area of	base is 24 cm ²	and height is 3	cm.	
4.		By doubling the side of cube, how many times (a) its surface area becomes (b) its volume becomes.					
5.	Fino	find the height of a cuboid whose volume is 275 cm ³ and base area is 25 cm ² .					
6.		A godown is in the form of a cuboid of measure 60 m \times 32 m \times 30 m. How many cuboidal boxes can be stored in it if the volume of one box is 8 m ³ .					
7.	Find the volume of a cylinder whose:						
	(i) r	= 7cm, h = 12	cm (ii) $r = 3.5$ cm, h	i = 15cm	iii) r = 14m h =	10m	
8.		Find the height of the cylinder whose volume is 1.54 m³ and whose diameter of the base is 140 cm.					
9.	Fino	Find the volume of a cylinder having base area 1.54 m ² and height 3.5 m.					
10.		a rectangular paper of width 14 cm is rolled along its width and a cylinder of radius 20 cm s formed. Find the volume of the cylinder.					
11.		Water is pouring into a cuboidal reservoir at the rate of 60 litres per minutes. If the volume of reservoir is 108 m³, find the number of hours it will take to fill up reservoir.					
12.	Mulitple Choice Questions :						
	(i)	(i) Find the area of base of a cylinder.					
		(a) $\pi r^2 h$	(b) πr ²	(c) 2πr	h (d) 2πr	
	(ii)	Find the volum	ne of a cuboid having dir	mension 4m × 2	$5m \times 2m$		
		(a) 20m³	(b) 40m³	(c) 30n	n³ (d) 200m³	
	(iii)	(iii) If edge of cube is doubled then what will happen to its volume?					
		(a) Double	(b) 4 times	(c) 8 ti	mes (d) 6 times	
	(iv)	11 =	cm³				
		(a) 1000	(b) 100	(c) 10	(d) <i>l</i>	
	(v) The volume of a cube with edge 1.1 is						
		(a) 13.31	(b) 1.331	(c) 133	.1 (d) 1331	



Learning Outcomes

After completion of the chapter, the students are now able to:

- Differentiate between plane and solid figures.
- Find perimeter and area of the plane figures (quadrilateral)
- Find area of a polygon.
- Find surface area and volume of some solids (cuboid, cube, cylinder) and use the concept in daily life.

Answers

Exercise 9.1

- 1. (i) P-18 cm, A-12 cm²
- (ii) P-40 cm, A-91 cm²
- (iii) P-32 cm, A-64 cm²
- (iv) P-72 cm, A-308 cm² (v) P-64cm A-128cm²

- 2. (i) A-3.08 cm², P-7.2 cm
- (ii) A-63 cm², 46 cm (iii) A-7.28 cm², P-8.4 cm
- square, 100 m²
- 4. 24

5. ₹ 19500

Exercise 9.2

- (i) 24 cm²
- (ii) 21 cm²
- (iii) 24 cm²
- (iv) 48 cm² (v) 37.5 cm² (vi) 28 cm²

- 2. 40 cm
- 3. 252 m²
- 4. 45 cm²
- 5. 50 cm²
- 6. 38.4 m² 7. 24 cm²;6cm

- 20500m²
- 9. 25m
- 10. 0.88m²
- 11. 691.20

- 12. (i) (b)
- (ii) (d)
- (iii) (a)

Exercise 9.3

- 1. 24 cm²
- 2, 150 m²
- 3. 100 cm²
- 4. 128 cm²
- 5. 31.75 cm²

Exercise 9.4

- 1. (i) 88 cm², 148cm²
- (ii) 432cm², 792m²
- (iii) 288m²; 448m²

- 2. (i) 256 cm²; 384cm²
- (ii) 576cm²; 864m²
- (iii) 900mm²; 1350mm²

- 3. 20 cm
- 4. 21m2
- 5. 558m²
- 6. ₹7200
- 7. 144cm

- 8. (i) 9 times (ii) One fourth
- 9. 350cm²
- 10. (i) 880cm²; 1188cm²
- (ii) 1320cm²; 2552cm²
- (iii) 264cm²; 341cm²

- 11. 924cm 12. 14cm 13. 15cm 14. 6600cm² 15. 2508m²
- 16. ₹ 8800 17. (i) (c), (ii) (a), (iii) (b)

Exercise 9.5

- 1. (i) 60 m³ (ii) 960cm³ (iii) 10.4m³
- 2. (i) 216cm³ (ii) 1728cm³ (iii) 3.375m³
- 3. 72 cm³ 4. (a) 4 times (b) 8 times
- 5. 11 cm 6. 7200 7. (i) 1848 cm³ (ii) 577.5cm³ (iii) 6160m³
- 8. lm 9. 5.39m³ 10. 17600cm³ 11. 30h
- 12. (i) (b) (ii) (a) (iii) (c) (iv) (a) (v) (b)

