SAMPLE QUESTION PAPER

BLUE PRINT

Time Allowed : 3 hours **Maximum Marks :** 80

S. No.	Chapter	VSA/Case based (1 mark)	SA-I (2 marks)	SA-II (3 marks)	LA (5 marks)	Total
1.	Relations and Functions	3(3)	–	1(3)	–	**4(6)**
2.	Inverse Trigonometric Functions	–	1(2)	–	–	**1(2)**
3.	Matrices	2(2)	–	–	–	**2(2)**
4.	Determinants	1(1)*	1(2)	–	1(5)*	**3(8)**
5.	Continuity and Differentiability	–	1(2)	2(6)	–	**3(8)**
6.	Application of Derivatives	1(4)	1(2)	1(3)	–	**3(9)**
7.	Integrals	2(2)#	2(4)#	1(3)	–	**5(9)**
8.	Application of Integrals	–	–	1(3)*	–	**1(3)**
9.	Differential Equations	1(1)*	1(2)	1(3)*	–	**3(6)**
10.	Vector Algebra	1(1)* + 1(4)	–	–	–	**2(5)**
11.	Three Dimensional Geometry	2(2)	1(2)*	–	1(5)*	**4(9)**
12.	Linear Programming	–	–	–	1(5)*	**1(5)**
13.	Probability	4(4)#	2(4)#	–	–	**6(8)**
	Total	**18(24)**	**10(20)**	**7(21)**	**3(15)**	**38(80)**

*It is a choice based question.
#Out of the two or more questions, one/two question(s) is/are choice based.

Subject Code : **041**

MATHEMATICS

Time allowed : 3 hours ***Maximum marks : 80***

General Instructions :

1. *This question paper contains two parts A and B. Each part is compulsory. Part-A carries 24 marks and Part-B carries 56 marks.*
2. *Part-A has Objective Type Questions and Part-B has Descriptive Type Questions.*
3. *Both Part-A and Part-B have internal choices.*

Part - A :

1. *It consists of two Sections-I and II.*
2. *Section-I comprises of 16 very short answer type questions.*
3. *Section-II contains 2 case study-based questions.*

Part - B :

1. *It consists of three Sections-III, IV and V.*
2. *Section-III comprises of 10 questions of 2 marks each.*
3. *Section-IV comprises of 7 questions of 3 marks each.*
4. *Section-V comprises of 3 questions of 5 marks each.*
5. *Internal choice is provided in 3 questions of Section-III, 2 questions of Section-IV and 3 questions of Section-V. You have to attempt only one of the alternatives in all such questions.*

PART - A

Section - I

1. Evaluate : $\int_{-1}^{0} \frac{dx}{2x+3}$

OR

Evaluate : $\int \frac{\sec^2 x}{2+\tan x} dx$

2. Let $A = \{1, 2, 3, 4\}$ and R be a relation in A given by $R = \{(1, 1), (2, 2), (3, 3), (4, 4), (1, 2), (2, 1), (3, 1)\}$. Then, show that R is reflexive.

3. Show that the matrix $A = \begin{bmatrix} 1 & -1 & 1 \\ 2 & 3 & 0 \\ 18 & 2 & 10 \end{bmatrix}$ is not invertible.

OR

Find the value of the determinant $\begin{vmatrix} x & -5x \\ 1 & x+10 \end{vmatrix}$.

4. Find the equation of a plane with intercepts 2, 3 and 4 on the X, Y and Z-axes respectively.

5. Find the degree of the differential equation $\frac{d^4y}{dx^4}+\sin\left(\frac{d^3y}{dx^3}\right)=0$.

OR

Find the order of the differential equation $x+\left(\frac{dy}{dx}\right)=\sqrt{1+\left(\frac{dy}{dx}\right)^2}$.

6. Let $A = \{1, 2, 3\}$ and $B = \{1, 2, 4\}$. Check whether $f = \{(1,1), (1,2), (2,1), (3, 4)\}$ from A to B is a function or not.

7. Let E and F be events associated with the sample space S of an experiment. Show that $P(S|F) = P(F|F) = 1$.

OR

Let A and B be two events associated with an experiment such that $P(A \cap B) = P(A)P(B)$. Show that $P(A|B) = P(A)$ and $P(B|A) = P(B)$.

8. If $A=\begin{bmatrix}0 & 1\\ 1 & 0\end{bmatrix}$, then find the value of A^2.

9. Find the angle between two vectors $\vec{a}$ and $\vec{b}$ with magnitude 1 and 2 respectively, such that $\vec{a}\cdot\vec{b}=1$.

OR

Find the projection of $\vec{a}=2\hat{i}-\hat{j}+\hat{k}$ on $\vec{b}=\hat{i}-2\hat{j}+\hat{k}$.

10. If $P(A) = \frac{3}{8}$, $P(B) = \frac{5}{8}$ and $P(A \cup B) = \frac{3}{4}$, then find $P(A\,|\,B)$.

11. Suppose you have two coins which appear identical in your pocket. You know that, one is fair and one is 2 headed. If you take one out, toss it and get a head, what is the probability that it was a fair coin?

12. Show that $f(x) = x^2 + 1$ is not one-one?

13. Suppose that five good fuses and two defective ones have been mixed up. To find the defective fuses, we test them one-by-one, at random and without replacement. What is the probability that we are lucky and find both of the defective fuses in the first two tests?

14. Evaluate : $\int_0^{2\pi}\frac{dx}{e^{\sin x}+1}$

15. What is the distance between the planes $2x + 2y - z + 2 = 0$ and $4x + 4y - 2z + 5 = 0$?

16. For matrix $A=\begin{bmatrix}-3 & 6 & 0\\ 4 & -5 & 8\\ 0 & -7 & -2\end{bmatrix}$, find $\frac{1}{2}(A-A')$ (where A' is the transpose of matrix A).

Section - II

Case study-based questions are compulsory. Attempt any 4 sub parts from each question. Each sub-part carries 1 mark.

17. If two vectors are represented by the two sides of a triangle taken in order, then their sum is represented by the third side of the triangle taken in opposite order.

(i) If $\vec{p},\vec{q},\vec{r}$ are the vectors represented by the sides of a triangle taken in orders, then $\vec{p}+\vec{q}=$

(a) $\vec{r}$ (b) $2\vec{r}$ (c) $-\vec{r}$ (d) None of these

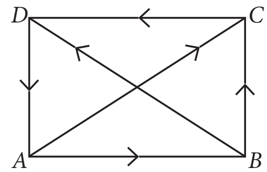
(ii) If $ABCD$ is a parallelogram and AC and BD are its diagonals, then $\overrightarrow{AC}+\overrightarrow{BD}=$

(a) $2\overrightarrow{DA}$ (b) $2\overrightarrow{AB}$ (c) $2\overrightarrow{BC}$ (d) $2\overrightarrow{BD}$

(iii) If $ABCD$ is a parallelogram, where $\overrightarrow{AB}=\vec{x}$ and $\overrightarrow{BC}=\vec{y}$, then $\overrightarrow{AC}-\overrightarrow{BD}=$

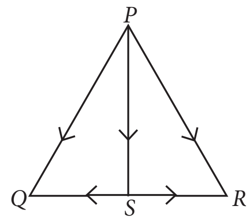
(a) $\vec{x}$ (b) $2\vec{x}$ (c) $\vec{y}$ (d) $2\vec{y}$

(iv) If $ABCD$ is a quadrilateral whose diagonals are $\overrightarrow{AC}$ and $\overrightarrow{BD}$, then $\overrightarrow{BA}+\overrightarrow{CD}=$



(a) $\overrightarrow{AC}+\overrightarrow{DB}$ (b) $\overrightarrow{AC}+\overrightarrow{BD}$ (c) $\overrightarrow{BC}+\overrightarrow{AD}$ (d) $\overrightarrow{BD}+\overrightarrow{CA}$

(v) If S is the mid point of side QR of ΔPQR, then $\overrightarrow{PQ}+\overrightarrow{PR}=$



(a) $2\overrightarrow{QS}$ (b) $2\overrightarrow{PS}$ (c) $2\overrightarrow{SR}$ (d) None of these

18. A can manufacturer designs a cylindrical can for a company making sanitizer and disinfector. The can is made to hold 5 litres of sanitizer or disinfector.

(i) If r cm be the radius and h cm be the height of the cylindrical can, then the surface area expressed as a function of r as

(a) $2\pi r^2$ (b) $2\pi r^2 + 5000$

(c) $2\pi r^2 + \dfrac{5000}{r}$ (d) $2\pi r^2 + \dfrac{10000}{r}$

(ii) The radius that will minimize the cost of the material to manufacture the can is

(a) $\sqrt[3]{\dfrac{500}{\pi}}$ cm (b) $\sqrt{\dfrac{500}{\pi}}$ cm (c) $\sqrt[3]{\dfrac{2500}{\pi}}$ cm (d) $\sqrt{\dfrac{2500}{\pi}}$ cm

(iii) The height that will minimize the cost of the material to manufacture the can is

(a) $\sqrt[3]{\dfrac{2500}{\pi}}$ cm (b) $2\sqrt[3]{\dfrac{2500}{\pi}}$ cm (c) $\sqrt{\dfrac{2500}{\pi}}$ (d) $2\sqrt{\dfrac{2500}{\pi}}$

(iv) If the cost of material used to manufacture the can is ₹ 100/m^2 and $\sqrt[3]{\dfrac{2500}{\pi}} \approx 9$, then minimum cost is approximately

(a) ₹ 16.7 (b) ₹ 18 (c) ₹ 19 (d) ₹ 20

(v) To minimize the cost of the material used to manufacture the can, we need to minimize the

(a) volume (b) curved surface area

(c) total surface area (d) surface area of the base

PART - B

Section - III

19. Separate the interval $\left[0, \dfrac{\pi}{2}\right]$ into sub-intervals in which the function $f(x) = \sin^4 x + \cos^4 x$ is increasing or decreasing.

20. If a discrete random variable X has the following probability distribution :

$X = x$	1	2	3	4	5	6	7
$P(X = x)$	k	$2k$	$2k$	$3k$	k^2	$2k^2$	$7k^2 + k$

Find the value of k.

OR

An urn contains 10 black balls and 5 white balls. Two balls are drawn from the urn one after the other without replacement. Find the probability that both drawn balls are black.

21. Evaluate : $\int_0^{\infty} \frac{dx}{(x^2+4)(x^2+9)}$

22. Solve for x : $\cos(\tan^{-1} x) = \sin\left(\cot^{-1}\frac{3}{4}\right)$.

23. Evaluate : $\int (e^{x \log a} + e^{a \log x} + e^{a \log a})\, dx$

OR

Evaluate : $\int \frac{dx}{x^{\frac{2}{3}}\sqrt{x^{\frac{2}{3}} - 4}}$

24. Find the solution of the differential equation $\frac{dy}{dx} = \cos(x + y)$.

25. If $A = \begin{bmatrix} 1 & 2 \\ 0 & 4 \end{bmatrix}$, then find A^{-1}.

26. A box contains N coins, of which m are fair and the rest are biased. The probability of getting head when a fair coin is tossed is $\frac{1}{2}$, while it is $\frac{2}{3}$ when a biased coin is tossed. A coin is drawn from the box at random and is tossed twice. The first time it shows head and the second time it shows tail. Find the probability that the coin drawn is fair.

27. Find the values of a and b such that the function f defined by $f(x) = \begin{cases} \frac{x-4}{|x-4|} + a, & \text{if } x < 4 \\ a + b, & \text{if } x = 4 \\ \frac{x-4}{|x-4|} + b, & \text{if } x > 4 \end{cases}$ is continuous at $x = 4$.

28. Find the cartesian equation of the plane passing through a point having position vector $2\hat{i} + 3\hat{j} + 4\hat{k}$ and perpendicular to the vector $2\hat{i} + \hat{j} - 2\hat{k}$.

OR

Find the distance of the point $(-1, -5, -10)$ from the point of intersection of the line $\vec{r} = 2\hat{i} - \hat{j} + 2\hat{k} + \lambda(3\hat{i} + 4\hat{j} + 12\hat{k})$ and the plane $\vec{r} \cdot (\hat{i} - \hat{j} + \hat{k}) = 5$.

Section - IV

29. Let N be the set of all natural numbers and let R be a relation in $N \times N$ defined by $(a, b)\, R(c, d) \Rightarrow ad = bc$ for all $(a, b), (c, d) \in N \times N$, show that R is an equivalence relation on $N \times N$.

30. Show that the surface area of a closed cuboid with square base and given volume is minimum, when it is a cube.

31. Solve : $(x+y)^2 \dfrac{dy}{dx} = a^2$

OR

Find one-parameter family of solution curve of the differential equation $\dfrac{dy}{dx}\cos^2 x = \tan x - y$.

32. If $\dfrac{x^2}{a^2} - \dfrac{y^2}{b^2} = 1$, then prove that $\dfrac{d^2y}{dx^2} = \dfrac{-b^4}{a^2y^3}$.

33. Evaluate : $\displaystyle\int \frac{1+x\cos x}{x(1-x^2e^{2\sin x})}dx$

34. Find the area enclosed by $y = 3x - 5$, $y = 0$, $x = 3$ and $x = 5$.

OR

Using integration, find the area of the region bounded by the lines $y - 1 = x$, the X-axis and the ordinates $x = -2$ and $x = 3$.

35. If $x = \dfrac{1-t^2}{1+t^2}$ and $y = \dfrac{2t}{1+t^2}$, then find $\dfrac{dy}{dx}$.

Section - V

36. The points $A(4, 5, 10)$, $B(2, 3, 4)$ and $C(1, 2, -1)$ are three vertices of a parallelogram $ABCD$. Find the vector and cartesian equations of the sides AB and BC and find coordinates of D.

OR

If the planes $x - cy - bz = 0$, $cx - y + az = 0$ and $bx + ay - z = 0$ pass through a straight line, then find the value of $a^2 + b^2 + c^2 + 2abc$.

37. Find the maximum value of $Z = x + y$ subject to $x + y \leq 10$, $3x - 2y \leq 15$, $x \leq 6$, $x \geq 0$, $y \geq 0$.

OR

Find the maximum and minimum values of $Z = 7x_1 - 3x_2$ subject to $x_1 + 2x_2 \leq 2$, $2x_1 + 4x_2 \leq 8$, $x_1 \geq 0$, $x_2 \geq 0$.

38. Find a matrix X such that, $X\begin{bmatrix} 3 & 2 \\ 1 & -1 \end{bmatrix} = \begin{bmatrix} 4 & 1 \\ 2 & 3 \end{bmatrix}$.

OR

Find the inverse of the matrix $\begin{bmatrix} 1 & 0 & 1 \\ 0 & 2 & 3 \\ 1 & 2 & 1 \end{bmatrix}$, if exists. Also show that $|\text{adj } A| = |A|^2$.

SOLUTIONS

1. Let $I = \int_{-1}^{0} \frac{dx}{2x+3}$

$$= \left[\frac{\log(2x+3)}{2}\right]_{-1}^{0} = \left[\frac{\log 3}{2} - \frac{\log 1}{2}\right] = \frac{\log 3}{2}$$

OR

Let $I = \int \frac{\sec^2 x}{2+\tan x} dx$

Put $2 + \tan x = t \Rightarrow \sec^2 x\, dx = dt$

$\therefore \quad I = \int \frac{dt}{t} = \log|t| + C = \log|2+\tan x| + C$

2. Here, $A = \{1, 2, 3, 4\}$ and $R = \{(1, 1), (2, 2), (3, 3), (4, 4), (1, 2), (2, 1), (3, 1)\}$

Now, $(1, 1), (2, 2), (3, 3), (4, 4) \in R$;

$\therefore$ R is reflexive.

3. $A = \begin{bmatrix} 1 & -1 & 1 \\ 2 & 3 & 0 \\ 18 & 2 & 10 \end{bmatrix}$

$\therefore$ $|A| = 1(30 - 0) + 1(20 - 0) + 1(4 - 54)$

$= 30 + 20 - 50 = 0$

So, A^{-1} does not exist.

OR

We have, $\begin{vmatrix} x & -5x \\ 1 & x+10 \end{vmatrix} = x(x+10) + 5x$

$= x^2 + 10x + 5x = x^2 + 15x = x(x + 15)$

4. As the plane has intercepts 2, 3 and 4 on X, Y and Z axes respectively.

$\therefore$ The required equation of the plane is

$\frac{x}{2} + \frac{y}{3} + \frac{z}{4} = 1 \Rightarrow 6x + 4y + 3z = 12$

5. Since, the given differential equation cannot be expressed as a polynomial. So, its degree is not defined.

OR

We have, $x + \left(\frac{dy}{dx}\right) = \sqrt{1 + \left(\frac{dy}{dx}\right)^2}$

Highest order derivative is $\frac{dy}{dx}$. So, its order is 1.

6. Here, f is not a function from A to B as $f(1)$ is not unique.

7. We know that,

$$P(S|F) = \frac{P(S\cap F)}{P(F)} = \frac{P(F)}{P(F)} = 1$$

Also, $P(F|F) = \frac{P(F\cap F)}{P(F)} = \frac{P(F)}{P(F)} = 1$

Thus, $P(S \mid F) = P(F \mid F) = 1$

OR

Since, $P(A \cap B) = P(A)P(B)$, therefore, A and B are independent events.

$\therefore \quad P(A|B) = \frac{P(A\cap B)}{P(B)} = \frac{P(A)P(B)}{P(B)} = P(A)$

Similarly, $P(B|A) = P(B)$.

8. We have, $A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

$$A^2 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 0+1 & 0+0 \\ 0+0 & 1+0 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

9. We are given $|\vec{a}| = 1$, $|\vec{b}| = 2$ and $\vec{a}\cdot\vec{b} = 1$

Now, we have $\cos\theta = \frac{\vec{a}\cdot\vec{b}}{|\vec{a}||\vec{b}|} = \frac{1}{1\times 2} = \frac{1}{2}$

or $\quad \theta = \cos^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{3}$.

OR

We have, $\vec{a} = 2\hat{i} - \hat{j} + \hat{k}$ and $\vec{b} = \hat{i} - 2\hat{j} + \hat{k}$

Projection of $\vec{a}$ on $\vec{b}$ is $\frac{\vec{a}\cdot\vec{b}}{|\vec{b}|}$

$\vec{a}\cdot\vec{b} = (2\hat{i} - \hat{j} + \hat{k})\cdot(\hat{i} - 2\hat{j} + \hat{k}) = 2 + 2 + 1 = 5$

$|\vec{b}| = \sqrt{1^2 + (-2)^2 + 1^2} = \sqrt{1+4+1} = \sqrt{6}$

$\therefore$ Projection of $\vec{a}$ on $\vec{b}$ is $\frac{5}{\sqrt{6}}$.

10. $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$\therefore \quad P(A \cap B) = \frac{3}{8} + \frac{5}{8} - \frac{3}{4} = 1 - \frac{3}{4} = \frac{1}{4}$

$\therefore \quad P(A|B) = \frac{P(A\cap B)}{P(B)} = \frac{1/4}{5/8} = \frac{2}{5}$.

11. Let E_1 be the event that fair coin is drawn, E_2 be the event that 2 headed coin is drawn, and E be the event that tossed coin get a head.

$\therefore \quad P(E_1) = 1/2, P(E_2) = 1/2, P(E|E_1) = 1/2$ and $P(E|E_2) = 1$

Now, $P(E_1|E) = \frac{P(E_1)\cdot P(E|E_1)}{P(E_1)\cdot P(E|E_1) + P(E_2)\cdot P(E|E_2)}$

$$= \frac{\frac{1}{2}\cdot\frac{1}{2}}{\frac{1}{2}\cdot\frac{1}{2} + \frac{1}{2}\cdot 1} = \frac{\frac{1}{4}}{\frac{1}{4} + \frac{1}{2}} = \frac{\frac{1}{4}}{\frac{3}{4}} = \frac{1}{3}$$

12. Since $f(x) = f(-x) = x^2 + 1$ for all $x \in R$, therefore, f is not one-one.

13. Let D_1, D_2 be the events that we find a defective fuse in the first, second test respectively.

Required probability $= P(D_1 \cap D_2)$

$$= P(D_1)P(D_2 \mid D_1) = \frac{2}{7}\cdot\frac{1}{6} = \frac{1}{21}$$

14. Let $I = \int_0^{2\pi} \frac{dx}{e^{\sin x}+1}$ ...(i)

$$\Rightarrow \quad I = \int_0^{2\pi} \frac{dx}{e^{\sin(2\pi - x)}+1} \qquad \left(\because \int_0^a f(x)dx = \int_0^a f(a-x)dx\right)$$

$$\Rightarrow \quad I = \int_0^{2\pi} \frac{dx}{e^{-\sin x}+1} \quad \Rightarrow \quad I = \int_0^{2\pi} \frac{e^{\sin x}}{e^{\sin x}+1}dx \qquad \text{...(ii)}$$

Adding (i) and (ii), we get

$$2I = \int_0^{2\pi} 1\cdot dx = 2\pi \quad \therefore \quad I = \pi$$

15. Given planes are $2x + 2y - z + 2 = 0$ and $4x + 4y - 2z + 5 = 0$

Let point $P(x_1, y_1, z_1)$ lie on plane $2x + 2y - z + 2 = 0$

$\Rightarrow$ $2x_1 + 2y_1 - z_1 = -2$

$$\therefore \quad d = \left|\frac{4x_1+4y_1-2z_1+5}{\sqrt{4^2+4^2+(-2)^2}}\right| = \left|\frac{2(2x_1+2y_1-z_1)+5}{\sqrt{16+16+4}}\right|$$

$$= \left|\frac{2(-2)+5}{\sqrt{36}}\right| = \frac{1}{6}\text{unit}$$

16. $A = \begin{bmatrix} -3 & 6 & 0 \\ 4 & -5 & 8 \\ 0 & -7 & -2 \end{bmatrix} \Rightarrow A' = \begin{bmatrix} -3 & 4 & 0 \\ 6 & -5 & -7 \\ 0 & 8 & -2 \end{bmatrix}$

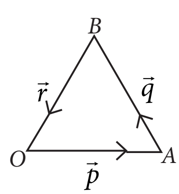
$$\therefore \quad \frac{1}{2}(A - A') = \frac{1}{2}\begin{bmatrix} 0 & 2 & 0 \\ -2 & 0 & 15 \\ 0 & -15 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 15/2 \\ 0 & -15/2 & 0 \end{bmatrix}$$

17. (i) (c) : Let OAB be a triangle such that

$OA = \vec{p}$, $AB = \vec{q}$, $OB = -\vec{r}$

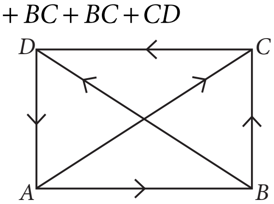
Now, $\vec{p} + \vec{q} = \overrightarrow{OA} + \overrightarrow{AB}$

$= \overrightarrow{OB} = -\vec{r}$



(ii) (c) : From triangle law of vector addition,

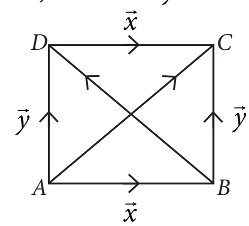
$\overrightarrow{AC} + \overrightarrow{BD} = \overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{BC} + \overrightarrow{CD}$



$= \overrightarrow{AB} + 2\overrightarrow{BC} + \overrightarrow{CD}$

$= \overrightarrow{AB} + 2\overrightarrow{BC} - \overrightarrow{AB} = 2\overrightarrow{BC}$ $\quad [\because \overrightarrow{AB} = -\overrightarrow{CD}]$

(iii) (b) : In ΔABC, $\overrightarrow{AC} = \vec{x} + \vec{y}$ ...(i)



and in ΔABD, $\vec{y} = \vec{x} + \overrightarrow{BD}$ ...(ii)

[By triangle law of addition]

Adding (i) and (ii), we have

$\overrightarrow{AC} + \vec{y} = 2\vec{x} + \vec{y} + \overrightarrow{BD}$

$\Rightarrow$ $\overrightarrow{AC} - \overrightarrow{BD} = 2\vec{x}$

(iv) (d) : In ΔABC, $\overrightarrow{BA} + \overrightarrow{AC} = \overrightarrow{BC}$ ...(i)

[By triangle law]

In ΔBCD, $\overrightarrow{BC} + \overrightarrow{CD} = \overrightarrow{BD}$ ...(ii)

From (i) and (ii), $\overrightarrow{BA} + \overrightarrow{AC} = \overrightarrow{BD} - \overrightarrow{CD}$

$\Rightarrow$ $\overrightarrow{BA} + \overrightarrow{CD} = \overrightarrow{BD} - \overrightarrow{AC} = \overrightarrow{BD} + \overrightarrow{CA}$

(v) (b) : Since S is the mid point of QR

So, $\overrightarrow{QS} = \overrightarrow{SR}$

Now $\overrightarrow{PQ} + \overrightarrow{PR} = (\overrightarrow{PS} + \overrightarrow{SQ}) + (\overrightarrow{PS} + \overrightarrow{SR})$ (By triangle law)

$= 2\overrightarrow{PS} + \overrightarrow{SQ} + \overrightarrow{SR} = 2\overrightarrow{PS}$ $\quad [\because \overrightarrow{SQ} = -\overrightarrow{QS}]$

18. (i) (d) : Given r cm is the radius and h cm is the height of required cylindrical can.

Given that, volume $= 5\ l = 5000\ \text{cm}^3$

$(\because 1\ \text{L} = 1000\ \text{cm}^3)$

$\Rightarrow$ $\pi r^2 h = 5000 \Rightarrow h = \frac{5000}{\pi r^2}$

Now, the surface area, as a function of r is given by

$$S(r) = 2\pi r^2 + 2\pi r h = 2\pi r^2 + 2\pi r\left(\frac{5000}{\pi r^2}\right)$$

$$= 2\pi r^2 + \frac{10000}{r}$$

(ii) (c) : Now, $S(r) = 2\pi r^2 + \frac{10000}{r}$

$\Rightarrow$ $S'(r) = 4\pi r - \frac{10000}{r^2}$

To find critical points, put $S'(r) = 0$

$$\Rightarrow \quad \frac{4\pi r^3 - 10000}{r^2} = 0$$

$$\Rightarrow \quad r^3 = \frac{10000}{4\pi} \Rightarrow r = \left(\frac{2500}{\pi}\right)^{1/3}$$

Also, $S''(r)\big|_{r=\sqrt[3]{\frac{2500}{\pi}}} = 4\pi + \frac{20000 \times 4\pi}{10000}$

$= 4\pi + 8\pi = 12\pi > 0$

Thus, the critical point is the point of minima.

(iii) (b) : The cost of material for the can is minimized when $r = \sqrt[3]{\frac{2500}{\pi}}$ cm and the height is

$$\frac{5000}{\pi\left(\sqrt[3]{\frac{2500}{\pi}}\right)^2} = 2\sqrt[3]{\frac{2500}{\pi}} \text{ cm.}$$

(iv) (a) : We have, minimum surface area $= \dfrac{2\pi r^3 + 10000}{r}$

$$= \frac{2\pi \cdot \frac{2500}{\pi} + 10000}{\sqrt[3]{\frac{2500}{\pi}}} = \frac{15000}{9} = 1666.67 \text{ cm}^2$$

Cost of 1 m^2 material = ₹100

$\therefore$ Cost of 1 cm^2 material = ₹ $\dfrac{1}{100}$

$\therefore$ Minimum cost = ₹ $\dfrac{1666.67}{100} = 16.7$

(v) (c) : To minimize the cost we need to minimize the total surface area.

19. Here $f(x) = \sin^4 x + \cos^4 x$ ...(i)

On differentiating (i) w.r.t. x, we get

$f'(x) = 4\sin^3 x \cos x - 4\cos^3 x \sin x = 4\sin x \cos x(\sin^2 x - \cos^2 x) = -2\sin 2x \cos 2x = -\sin 4x$

Put $f'(x) = 0 \Rightarrow x = \dfrac{\pi}{4}$

Intervals	**Sign of $f'(x)$**	**Conclusion**
$\left(0, \frac{\pi}{4}\right)$	–ve as $0 < 4x < \pi$	f is decreasing in $\left(0, \frac{\pi}{4}\right)$
$\left(\frac{\pi}{4}, \frac{\pi}{2}\right)$	+ve as $\pi < 4x < 2\pi$	f is increasing in $\left(\frac{\pi}{4}, \frac{\pi}{2}\right)$

20. Since $\sum_{x=1}^{7} P(X = x) = 1$

$\therefore$ $k + 2k + 2k + 3k + k^2 + 2k^2 + 7k^2 + k = 1$

$\Rightarrow$ $10k^2 + 9k - 1 = 0 \Rightarrow (10k - 1)(k + 1) = 0$

$\Rightarrow$ $k = \dfrac{1}{10}$ or $k = -1$

Since probability cannot be negative.

$\therefore$ $k = \dfrac{1}{10}$

OR

Let E and F respectively denote the events that first and second ball drawn is black. We have to find $P(E \cap F)$ or $P(EF)$.

Now, $P(E) = P(\text{black ball in first draw}) = \dfrac{10}{15}$

and $P(F | E) = \dfrac{9}{14}$

By multiplication rule of probability, we have

$$P(E \cap F) = P(E) \cdot P(F|E) = \frac{10}{15} \times \frac{9}{14} = \frac{3}{7}$$

21. Let $I = \displaystyle\int_0^\infty \frac{dx}{(x^2+4)(x^2+9)}$

$$= \frac{1}{5}\left(\int_0^\infty \frac{1}{x^2+4}dx - \int_0^\infty \frac{1}{(x^2+9)}dx\right)$$

$$= \frac{1}{5}\left[\left[\frac{1}{2}\tan^{-1}\frac{x}{2}\right]_0^\infty - \left[\frac{1}{3}\tan^{-1}\frac{x}{3}\right]_0^\infty\right]$$

$$= \frac{1}{5}\left[\left(\frac{1}{2}\cdot\frac{\pi}{2} - 0\right) - \left(\frac{1}{3}\cdot\frac{\pi}{2} - 0\right)\right] = \frac{1}{5}\left(\frac{\pi}{4} - \frac{\pi}{6}\right) = \frac{\pi}{60}$$

22. Let $\cot^{-1}\dfrac{3}{4} = \theta \Rightarrow \cot\theta = \dfrac{3}{4}$

$\therefore$ $\operatorname{cosec}\theta = \sqrt{1+\cot^2\theta} = \sqrt{1+\dfrac{9}{16}} = \dfrac{5}{4}$

$\therefore$ $\sin\theta = \dfrac{4}{5} \Rightarrow \theta = \sin^{-1}\dfrac{4}{5}$

So, $\sin\left(\cot^{-1}\dfrac{3}{4}\right) = \sin\left(\sin^{-1}\dfrac{4}{5}\right) = \dfrac{4}{5}$

Let $\tan^{-1} x = \phi$. Then, $\tan\phi = x$

$\therefore$ $\sec\phi = \sqrt{1+\tan^2\phi} = \sqrt{1+x^2}$

$\therefore$ $\cos\phi = \dfrac{1}{\sqrt{1+x^2}}$

So, $\cos(\tan^{-1} x) = \cos\phi = \dfrac{1}{\sqrt{1+x^2}}$

Thus, $\dfrac{1}{\sqrt{1+x^2}} = \dfrac{4}{5} \Rightarrow \dfrac{1}{1+x^2} = \dfrac{16}{25} \Rightarrow 16x^2 = 9$

$\Rightarrow$ $x^2 = \dfrac{9}{16} \Rightarrow x = \pm\dfrac{3}{4}$

23. Let $I = \int(e^{x\log a} + e^{a\log x} + e^{a\log a})\,dx$

$= \int(e^{\log a^x} + e^{\log x^a} + e^{\log a^a})\,dx$

$= \int(a^x + x^a + a^a)\,dx$ $\quad [\because e^{\log\lambda} = \lambda]$

$= \int a^x dx + \int x^a dx + \int a^a dx = \dfrac{a^x}{\log a} + \dfrac{x^{a+1}}{a+1} + a^a x + c$

OR

Let $I = \displaystyle\int \frac{dx}{x^{\frac{2}{3}}\sqrt{x^{\frac{2}{3}} - 4}} = \int \frac{x^{\frac{-2}{3}}}{\sqrt{x^{\frac{2}{3}} - 2^2}}dx$

Put $x^{\frac{1}{3}} = t \Rightarrow x^{\frac{-2}{3}}\,dx = 3dt$

$\therefore$ $I = 3\displaystyle\int \frac{dt}{\sqrt{t^2 - 2^2}} = 3\log\left|t + \sqrt{t^2 - 4}\right| + c$

$= 3\log\left|x^{\frac{1}{3}} + \sqrt{x^{\frac{2}{3}} - 4}\right| + c$

24. We have $\dfrac{dy}{dx}=\cos(x+y)$

Put $x+y=u \Rightarrow \dfrac{dy}{dx}=\dfrac{du}{dx}-1$

$$\frac{du}{dx}-1=\cos u \Rightarrow \int\frac{du}{1+\cos u}=\int dx+c$$

$$\Rightarrow \int\frac{1}{2}\sec^2\left(\frac{u}{2}\right)du=x+c \Rightarrow \tan\left(\frac{u}{2}\right)=x+c$$

$$\Rightarrow \tan\left(\frac{x+y}{2}\right)=x+c$$

25. $|A|=\begin{vmatrix}1 & 2\\ 0 & 4\end{vmatrix}=4-0=4\neq 0 \quad \therefore \quad A^{-1}$ exists.

$$\therefore \text{ adj } A=\begin{bmatrix}4 & -2\\ 0 & 1\end{bmatrix}$$

$$\Rightarrow A^{-1}=\frac{1}{|A|}(\text{adj } A)=\frac{1}{4}\begin{bmatrix}4 & -2\\ 0 & 1\end{bmatrix}=\begin{bmatrix}1 & -1/2\\ 0 & 1/4\end{bmatrix}$$

26. Let E be the event that the coin tossed twice shows first head and then tail and F be the event that the coin drawn is fair.

$$P(F/E)=\frac{P(F)\cdot P(E/F)}{P(F)\cdot P(E/F)+P(\overline{F})P(E/\overline{F})}$$

$$=\frac{\frac{m}{N}\cdot\frac{1}{2}\cdot\frac{1}{2}}{\frac{m}{N}\cdot\frac{1}{2}\cdot\frac{1}{2}+\frac{N-m}{N}\cdot\frac{2}{3}\cdot\frac{1}{3}}=\frac{m/4}{m/4+2(N-m)/9}$$

$$=\frac{9m}{m+8N}.$$

27. We have, L.H.L. (at $x = 4$)

$$=\lim_{x\to 4^-} f(x)=\lim_{x\to 4^-}\frac{x-4}{|x-4|}+a=-1+a$$

R.H.L. (at $x = 4$)

$$=\lim_{x\to 4^+} f(x)=\lim_{x\to 4^+}\frac{x-4}{|x-4|}+b=1+b$$

and $f(4) = a + b$

Since $f(x)$ is continuous at $x = 4$.

$\therefore \quad -1 + a = a + b \Rightarrow b = -1$

and $1 + b = a + b \quad \Rightarrow \quad a = 1$

Thus, $f(x)$ is continuous at $x = 4$ if $a = 1$ and $b = -1$.

28. Here, $(x_1, y_1, z_1) = (2, 3, 4)$, $a = 2$, $b = 1$, $c = -2$

Cartesian equation is

$a(x - x_1) + b(y - y_1) + c(z - z_1) = 0$

$\Rightarrow \quad 2(x - 2) + 1(y - 3) -2(z - 4) = 0$

$\Rightarrow \quad 2x - 4 + y - 3 - 2z + 8 = 0$

$\Rightarrow \quad 2x + y - 2z = -1$

OR

Any point on line

$\vec{r}=2\hat{i}-\hat{j}+2\hat{k}+\lambda(3\hat{i}+4\hat{j}+12\hat{k})$

is $(2 + 3\lambda, -1 + 4\lambda, 2 + 12\lambda)$ ...(i)

If it lies on the plane $\vec{r}\cdot(\hat{i}-\hat{j}+\hat{k})=5$,

then, $1(2 + 3\lambda) - 1(-1 + 4\lambda) + 1(2 + 12\lambda) = 5$

$\Rightarrow \quad 2 + 3\lambda + 1 - 4\lambda + 2 + 12\lambda = 5$

$\Rightarrow \quad 11\lambda = 0 \ \Rightarrow \ \lambda = 0$

Putting $\lambda = 0$ in (i), we get $(2, -1, 2)$

Required distance $=\sqrt{(2+1)^2+(-1+5)^2+(2+10)^2}$

$=\sqrt{9+16+144}=13$ units

29. Given $(a, b)\, R(c, d)$

$\Rightarrow \quad ad = bc \ \forall\ (a, b), (c, d) \in N \times N$

Reflexive : Let $(a, b) \in N \times N$ when $a \in N$, $b \in N$

$(a, b)\, R\, (a, b) \Rightarrow ab = ba$, which is true

So, R is reflexive.

Symmetric : Let $(a, b), (c, d) \in N \times N$

$(a, b)\, R\, (c, d) \Rightarrow ad = bc$

$\Rightarrow \quad cb = da \Rightarrow (c, d)\, R(a, b)$

So R is symmetric.

Transitive : Let $(a, b), (c, d), (e, f) \in N \times N$

$(a, b)\, R(c, d) \Rightarrow ad = bc$

$(c, d)\, R\, (e, f) \ \Rightarrow cf = de$

$\Rightarrow \quad adcf = bcde \Rightarrow af = be \Rightarrow (a, b)\, R\, (e, f)$

So R is transitive.

Since R is reflexive, symmetric and transitive, so R is an equivalence relation.

30. Let V be the volume of a closed cuboid with length x, breadth x and height y. Let S be the surface area of cuboid. Then

$x^2y = V$ and $S = 2(x^2 + xy + xy) = 2(x^2 + 2xy)$

$$\therefore \quad S=2\left[x^2+2x\cdot\frac{V}{x^2}\right]=2\left[x^2+\frac{2V}{x}\right]$$

$$\therefore \quad \frac{dS}{dx}=2\left[2x-\frac{2V}{x^2}\right]$$

$$\therefore \quad \frac{dS}{dx}=0 \Rightarrow x^3=V=x^2y \Rightarrow x=y$$

$$\text{Now, } \frac{d^2S}{dx^2}=2\left[2+\frac{4V}{x^3}\right]>0$$

$\therefore \quad x = y$ will give minimum surface area

and $x = y$, means all the sides are equal.

$\therefore \quad$ Cube will have minimum surface area.

31. Given equation is $(x+y)^2\dfrac{dy}{dx}=a^2$

Put $x+y=u \ \Rightarrow \ 1+\dfrac{dy}{dx}=\dfrac{du}{dx}$

$$\therefore \quad u^2\left(\frac{du}{dx}-1\right)=a^2 \ \Rightarrow \ u^2\frac{du}{dx}=u^2+a^2$$

$$\Rightarrow \int \frac{u^2}{u^2+a^2}du = \int dx \Rightarrow \int du - \int \frac{a^2 du}{u^2+a^2} = x + C$$

$$\Rightarrow u - a^2 \times \frac{1}{a}\tan^{-1}\frac{u}{a} = x + C$$

$$\Rightarrow x + y - a\tan^{-1}\left(\frac{x+y}{a}\right) = x + C$$

$$\Rightarrow y - a\tan^{-1}\left(\frac{x+y}{a}\right) = C$$

$$\Rightarrow y = a\tan^{-1}\left(\frac{x+y}{a}\right) + C$$

OR

We have, $\frac{dy}{dx}\cos^2 x = \tan x - y$

$$\Rightarrow \frac{dy}{dx} + \left(\frac{1}{\cos^2 x}\right)y = \frac{\tan x}{\cos^2 x} \quad ...(i)$$

This is a linear differential equation of the form

$\frac{dy}{dx} + Py = Q$, where $P = \frac{1}{\cos^2 x}$, $Q = \frac{\tan x}{\cos^2 x}$

$\text{I.F.} = e^{\int \sec^2 x\, dx} = e^{\tan x}$

$\therefore$ The solution is given by

$$y \cdot (e^{\tan x}) = \int \tan x \cdot \sec^2 x \cdot e^{\tan x} dx + c \quad ...(ii)$$

Let $I = \int \tan x \cdot \sec^2 x \cdot e^{\tan x} dx, \quad \cos x \neq 0$

Put $\tan x = t \Rightarrow \sec^2 x\, dx$

$\therefore \; I = \int te^t dt = te^t - e^t$

$\Rightarrow \; I = \tan x \cdot e^{\tan x} - e^{\tan x}$

Putting this value in (ii), we get

$ye^{\tan x} = (\tan x - 1)e^{\tan x} + c$

$\Rightarrow y = (\tan x - 1) + ce^{-\tan x}$

32. Given $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 \quad ...(i)$

Let $x = a\sec\theta$ and $y = b\tan\theta$

$$\Rightarrow \frac{dx}{d\theta} = a\sec\theta\tan\theta \text{ and } \frac{dy}{d\theta} = b\sec^2\theta$$

$$\therefore \frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta}$$

$$\Rightarrow \frac{dy}{dx} = \frac{b\sec^2\theta}{a\sec\theta\tan\theta} \Rightarrow \frac{dy}{dx} = \frac{b\sec\theta}{a\tan\theta}$$

$$\Rightarrow \frac{dy}{dx} = \frac{b\,\text{cosec}\,\theta}{a}$$

Differentiating with respect to x, we get

$$\frac{d^2y}{dx^2} = \frac{-b\,\text{cosec}\,\theta\cot\theta}{a} \times \frac{d\theta}{dx}$$

$$\Rightarrow \frac{d^2y}{dx^2} = \frac{-b\,\text{cosec}\,\theta\cot\theta}{a\,(a\sec\theta\tan\theta)} = \frac{-b}{a^2\tan^3\theta}$$

$$= \frac{-b \times b^3}{a^2 \times y^3} \quad \left\{\text{From (i), } \tan\theta = \frac{y}{b}\right\}$$

$$\therefore \frac{d^2y}{dx^2} = \frac{-b^4}{a^2y^3}$$

33. We have, $I = \int \frac{1 + x\cos x}{x(1 - x^2e^{2\sin x})}dx$

Put $xe^{\sin x} = t \Rightarrow e^{\sin x}(1 + x\cos x)dx = dt$

$$\therefore \; I = \int \frac{dt}{t(1-t^2)} = \int \frac{dt}{t(1+t)(1-t)}$$

Let $\frac{1}{t(1+t)(1-t)} = \frac{A}{t} + \frac{B}{1+t} + \frac{C}{1-t}$

$\Rightarrow 1 = A(1 - t^2) + B(t - t^2) + C(t + t^2)$

Equating the coefficients of t^2, t and constant terms, we get $A + B - C = 0$, $B + C = 0$ and $A = 1$

Solving these equations, we get

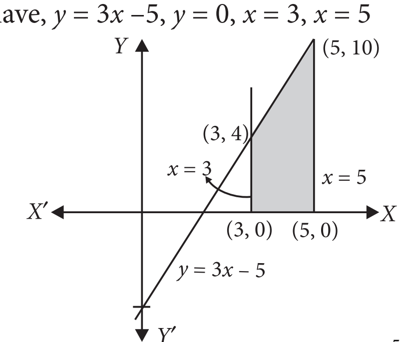
$A = 1, B = \frac{-1}{2}$ and $C = \frac{1}{2}$

$$\therefore \; I = \int \frac{dt}{t(1-t^2)} = \int \frac{dt}{t} - \frac{1}{2}\int \frac{dt}{1+t} + \frac{1}{2}\int \frac{dt}{1-t}$$

$$= \log t - \frac{1}{2}\log|1+t| - \frac{1}{2}\log|1-t| + c$$

$$= \log|xe^{\sin x}| - \frac{1}{2}\log|1 - x^2e^{2\sin x}| + c$$

34. We have, $y = 3x - 5$, $y = 0$, $x = 3$, $x = 5$



$$\text{Required area} = \int_3^5 (3x-5)\,dx = \left[\frac{3x^2}{2} - 5x\right]_3^5$$

$$= \frac{3}{2}(5^2 - 3^2) - 5(5 - 3)$$

$$= \frac{3}{2}(25 - 9) - 5 \times 2 = 24 - 10 = 14 \text{ sq. units}$$

OR

The line $y - 1 = x$ meets the X-axis at the point where $y = 0$, *i.e.*, where $0 - 1 = x$, *i.e.*, at the point $(-1, 0)$. It meets Y-axis at the point where $x = 0$, *i.e.*, where $y - 1 = 0$, *i.e.*, at the point $(0, 1)$.

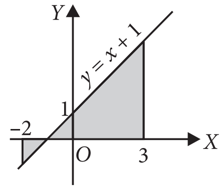
Required area (shown shaded) is given by

$$= \left|\int_{-2}^{-1}(x+1)\,dx\right| + \int_{-1}^{3}(x+1)\,dx$$

$$=\left|\left[\frac{(x+1)^2}{2}\right]_{-2}^{-1}\right|+\left[\frac{(x+1)^2}{2}\right]_{-1}^{3}$$

$$=\left|\frac{1}{2}(0-1)\right|+\frac{1}{2}(4^2-0)$$

$$=\frac{1}{2}+8=\frac{17}{2}\text{ sq. units}$$



35. We have $x=\dfrac{1-t^2}{1+t^2}$ and $y=\dfrac{2t}{1+t^2}$

Putting $t = \tan\theta$ in both the equations, we get

$$x=\frac{1-\tan^2\theta}{1+\tan^2\theta}=\cos 2\theta \quad \text{...(i)}$$

and $$y=\frac{2\tan\theta}{1+\tan^2\theta}=\sin 2\theta \quad \text{...(ii)}$$

Differentiating (i) and (ii), we get

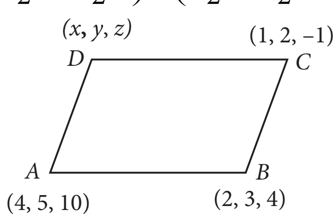
$$\frac{dx}{d\theta}=-2\sin 2\theta \text{ and } \frac{dy}{d\theta}=2\cos 2\theta$$

Therefore, $$\frac{dy}{dx}=\frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}}=-\frac{\cos 2\theta}{\sin 2\theta}=-\frac{x}{y}$$

36. In a parallelogram, diagonals bisect each other.

$\therefore$ Mid point of BD = Mid point of AC

$$\Rightarrow \left(\frac{x+2}{2},\frac{y+3}{2},\frac{z+4}{2}\right)=\left(\frac{4+1}{2},\frac{5+2}{2},\frac{10-1}{2}\right)$$



$\Rightarrow$ $x + 2 = 5$, $y + 3 = 7$, $z + 4 = 9$

$\Rightarrow$ $x = 3$, $y = 4$, $z = 5$

So coordinates of D are (3, 4, 5).

Cartesian equation of side AB is

$$\frac{x-4}{2-4}=\frac{y-5}{3-5}=\frac{z-10}{4-10}$$

$$\Rightarrow \frac{x-4}{-2}=\frac{y-5}{-2}=\frac{z-10}{-6} \Rightarrow \frac{x-4}{1}=\frac{y-5}{1}=\frac{z-10}{3}$$

and vector equation of side AB is

$$\vec{r}=4\hat{i}+5\hat{j}+10\hat{k}+\lambda(\hat{i}+\hat{j}+3\hat{k})$$

Again, cartesian equation of side BC is

$$\frac{x-2}{1-2}=\frac{y-3}{2-3}=\frac{z-4}{-1-4}$$

$$\Rightarrow \frac{x-2}{-1}=\frac{y-3}{-1}=\frac{z-4}{-5} \Rightarrow \frac{x-2}{1}=\frac{y-3}{1}=\frac{z-4}{5}$$

and vector equation of side BC is

$$\vec{r}=2\hat{i}+3\hat{j}+4\hat{k}+\lambda(\hat{i}+\hat{j}+5\hat{k})$$

OR

Given planes are

$x - cy - bz = 0$ ...(i)

$cx - y + az = 0$ ...(ii)

$bx + ay - z = 0$ ...(iii)

The d.r.'s of normal to plane (i), (ii) and (iii) are $(1, -c, -b)$, $(c, -1, a)$ and $(b, a, -1)$ respectively.

All planes pass through same line, then the line is perpendicular to each of the three normals.

The d.r's. of line from planes (i) and (ii) are

$$\begin{vmatrix}-c & -b\\ -1 & a\end{vmatrix}, -\begin{vmatrix}1 & -b\\ c & a\end{vmatrix}, \begin{vmatrix}1 & -c\\ c & -1\end{vmatrix}$$

i.e., $-ac - b$, $-a - bc$, $-1 + c^2$ ...(iv)

The d.r.'s of line from planes (ii) and (iii) are

$$\begin{vmatrix}-1 & a\\ a & -1\end{vmatrix}, -\begin{vmatrix}c & a\\ b & -1\end{vmatrix}, \begin{vmatrix}c & -1\\ b & a\end{vmatrix}$$

i.e., $1 - a^2$, $c + ab$, $ac + b$ ...(v)

The d.r.'s in (iv) and (v) are in proportion, then

$$\frac{-ac-b}{1-a^2}=\frac{-a-bc}{c+ab}=\frac{-1+c^2}{ac+b}$$

$$\Rightarrow \frac{-ac-b}{1-a^2}=\frac{-a-bc}{c+ab}$$

$\Rightarrow$ $ac^2 + bc + a^2bc + ab^2 = a + bc - a^3 - a^2bc$

$\Rightarrow$ $c^2 + abc + b^2 = 1 - a^2 - abc$

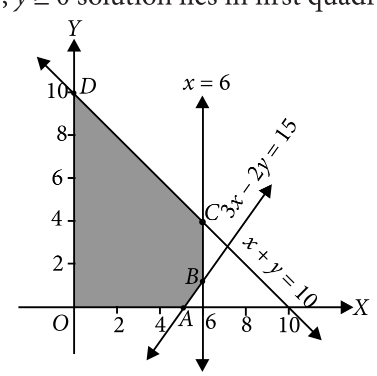
$\Rightarrow$ $a^2 + b^2 + c^2 + 2abc = 1$

37. Converting inequations into equations and drawing the corresponding lines, we have

$x + y = 10$, $3x - 2y = 15$, $x = 6$

i.e., $\dfrac{x}{10}+\dfrac{y}{10}=1$, $\dfrac{x}{5}+\left(\dfrac{y}{-7.5}\right)=1$, $x=6$

Also, $x \ge 0$, $y \ge 0$ solution lies in first quadrant



B is the point of intersection of the lines $x = 6$ and $3x - 2y = 15$ *i.e.*, $B \equiv \left(6, \dfrac{3}{2}\right)$.

C is the point of intersection of the lines $x = 6$ and $x + y = 10$ *i.e.*, $C \equiv (6, 4)$.

We have points $A(5, 0)$, $B\left(6, \dfrac{3}{2}\right)$, $C(6, 4)$ and $D(0, 10)$.

Now $Z = x + y$

$\therefore \quad Z(O) = 0 + 0 = 0$

$Z(A) = 5 + 0 = 5$

$Z(B) = 6 + \dfrac{3}{2} = 7.5$

$Z(C) = 6 + 4 = 10$

$Z(D) = 0 + 10 = 10$

$\therefore \quad Z$ has maximum value 10 at two points $C(6, 4)$ and $D(0, 10)$.

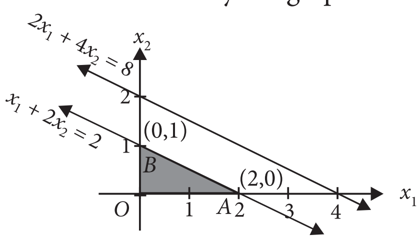
OR

Converting inequations into equations and drawing the corresponding lines, we have

$x_1 + 2x_2 = 2$; $2x_1 + 4x_2 = 8$

i.e $\dfrac{x_1}{2} + \dfrac{x_2}{1} = 1$, $\dfrac{x_1}{4} + \dfrac{x_2}{2} = 1$

The constraints are shown by the graph.



Also, $x_1 \geq 0$, $x_2 \geq 0$ solution lies in first quadrant.

We have points $A(2, 0)$ and $B(0, 1)$.

Now, $Z = 7x_1 - 3x_2$

$\therefore \quad Z(O) = 7(0) - 3(0) = 0$

$Z(A) = 7(2) - 3(0) = 14$

$Z(B) = 7(0) - 3(1) = -3$

$\therefore \quad Z$ has maximum value 14 at point $(2, 0)$ and minimum value -3 at point $(0, 1)$.

38. We have, $X\begin{bmatrix} 3 & 2 \\ 1 & -1 \end{bmatrix} = \begin{bmatrix} 4 & 1 \\ 2 & 3 \end{bmatrix}$

Let $A = \begin{bmatrix} 3 & 2 \\ 1 & -1 \end{bmatrix}$ and $B = \begin{bmatrix} 4 & 1 \\ 2 & 3 \end{bmatrix}$

$\therefore \quad |A| = -3 - 2 = -5 \neq 0$, so A^{-1} exists.

Now, the given matrix equation is $XA = B$

$\therefore \quad X(AA^{-1}) = BA^{-1} \Rightarrow X = BA^{-1}$ ...(i)

Now, cofactors of matrix A are given by $A_{11} = -1$, $A_{12} = -1$, $A_{21} = -2$, $A_{22} = 3$

$\therefore \quad \text{adj } A = \begin{bmatrix} -1 & -1 \\ -2 & 3 \end{bmatrix}^1 = \begin{bmatrix} -1 & -2 \\ -1 & 3 \end{bmatrix}$

$\therefore \quad A^{-1} = \dfrac{1}{|A|}(\text{adj } A) = -\dfrac{1}{5}\begin{bmatrix} -1 & -2 \\ -1 & 3 \end{bmatrix}$

Now, from (i), $X = BA^{-1}$

$\Rightarrow X = \left(-\dfrac{1}{5}\right)\begin{bmatrix} 4 & 1 \\ 2 & 3 \end{bmatrix}\begin{bmatrix} -1 & -2 \\ -1 & 3 \end{bmatrix}$

$\Rightarrow X = \left(-\dfrac{1}{5}\right)\begin{bmatrix} -4-1 & -8+3 \\ -2-3 & -4+9 \end{bmatrix}$

$\Rightarrow X = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$

OR

Let $A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 2 & 3 \\ 1 & 2 & 1 \end{bmatrix}$

$\therefore \quad |A| = 1(2 - 6) - 0 + 1(0 - 2) = -4 - 2 = -6 \neq 0$

So, A^{-1} exists.

Let A_{ij} be the co-factor of a_{ij} in $|A|$. Then

$A_{11} = -4$, $A_{12} = 3$, $A_{13} = -2$

$A_{21} = 2$, $A_{22} = 0$, $A_{23} = -2$

$A_{31} = -2$, $A_{32} = -3$, $A_{33} = 2$

$\therefore \text{ adj } A = \begin{bmatrix} -4 & 2 & -2 \\ 3 & 0 & -3 \\ -2 & -2 & 2 \end{bmatrix}$

Hence, $A^{-1} = \dfrac{1}{|A|}(\text{adj } A) = \dfrac{-1}{6}\begin{bmatrix} -4 & 2 & -2 \\ 3 & 0 & -3 \\ -2 & -2 & 2 \end{bmatrix}$

$= \dfrac{1}{6}\begin{bmatrix} 4 & -2 & 2 \\ -3 & 0 & 3 \\ 2 & 2 & -2 \end{bmatrix}$

Now, $|\text{adj } A| = -4(0 - 6) - 2(6 - 6) - 2(-6 - 0)$

$= 24 - 0 + 12 = 36 = 6^2 = (-6)^2$

$= |A|^2$

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