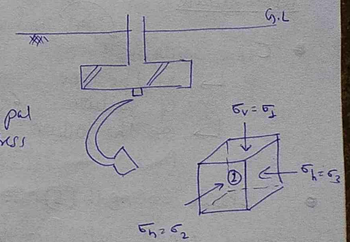
friction

lecture 9

13/11/19

Mohr's theory in Analysis of sand

considered the element just below the footing.



σ_1 = Major Principal stress

σ_2 = intermediate Principal stress

σ_3 = Minor Principal stress

$(\sigma_2 = \sigma_3)$

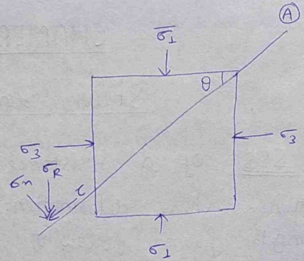
2D analysis

consider a Plane at an angle θ with major Principal Plane.

Normal stress

$\sigma_n = \sigma_1 \cos^2\theta + \sigma_3 \sin^2\theta$

$\sigma_n = \left(\frac{\sigma_1+\sigma_3}{2}\right) + \left(\frac{\sigma_1-\sigma_3}{2}\right)\cos 2\theta$



Shear stress

$\tau = \left(\frac{\sigma_1-\sigma_3}{2}\right) \times \sin 2\theta$

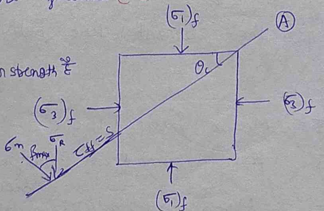
Resultant stress $\sigma_R = \sqrt{\sigma_n^2 + \tau^2}$

Angle of obliquity $\beta = \tan^{-1}\left(\frac{\tau}{\sigma_n}\right)$

⇒ the shear failure will occurs on that plane in which Resultant stress (σ_R) is most inclined with the normal of that plane. Such a plane is called critical plane (or) failure plane and the angle 'θ' is called critical angle of failure plane. (θ_c)

At failure

- τ_{ff} = Shear stress (यही shear strength है)
  - ↳ at the time of failure
  - ↳ on the failure point



- Shear strength (S) = to the shear stress produced on critical plane at limiting cond (Just before the failure)
- On the failure plane (σ_R) is most incline with normal of that plane. Hence β_{max} is called Maxm angle of obliquity and it is also termed as internal angle of friction (or) angle of Repose

(sand के लिए analyse हो रहा)

$\tau_{ff} = S$ = Shear strength

$\beta_{max} = \phi$ = internal angle of friction

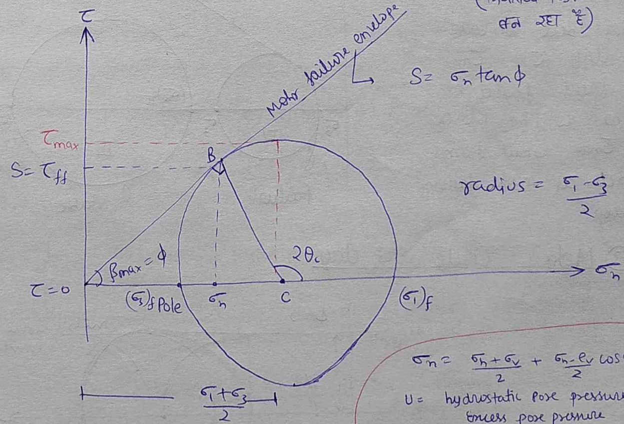
$\tan\beta_{max} = \frac{\tau_{ff}}{\sigma_n} = \frac{S}{\sigma_n}$

$S = \sigma_n \times \tan\beta_{max}$ then $\boxed{S = \sigma_n \tan\phi}$ ①

$\boxed{\phi = \beta_{max}}$

② Mohr's circle for sand

(triaxial test से circle बन रहा है)



$S = \sigma_n \tan\phi$

radius = $\frac{\sigma_1 - \sigma_3}{2}$

$\sigma_n = \frac{\sigma_h+\sigma_v}{2} + \frac{\sigma_h-\sigma_v}{2}\cos 2\alpha$

U = hydrostatic pore pressure + excess pore pressure

$\sigma_n' = \sigma_n - U$

$\tau = c + \bar{\sigma}_n \tan\phi$

In ΔOBC

$\angle O + \angle B + \angle C = 180$

$\beta_{max} + 90 + 180 - 2\theta_c = 180$

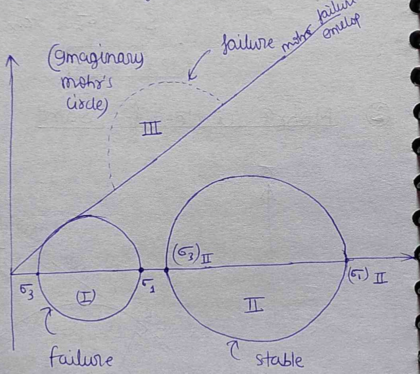
$\frac{\beta_{max}}{2} + 45 - \theta_c = 0$

$\theta_c = 45° + \frac{\beta_{max}}{2} = 45° + \frac{\phi}{2}$

$\boxed{\theta_c = 45° + \frac{\phi}{2}}$ ★②

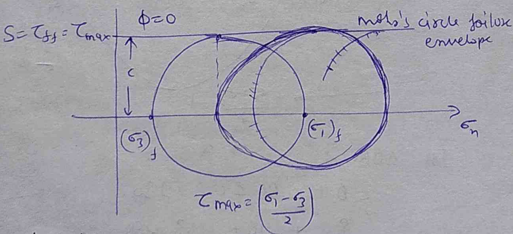
with major principal axis

Note ① Failure will occur on that plane in which shear stress lie on (and) above the Mohr's failure envelop whereas. If the point is located below the failure envelop that such cond represents the stable cond



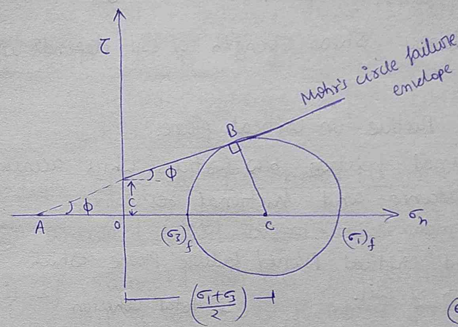
Note ② In case of sand and silt τ_{ff} is less than τ_{max} but in case of clay $\tau_{ff} = \tau_{max}$. In clay's failure plane will be plane of τ_{max} which forms Horizontal mohr failure envelop

③ Mohr's circle for clay



④ Mohr's circle for silt

In Δ ABC,



In ΔABC $\left(\tan\phi = \frac{c}{AO}\right)$

$$\sin\phi = \frac{BC}{AC} = \frac{\left(\frac{\sigma_1-\sigma_3}{2}\right)}{AO+OC}$$

$$\sin\phi = \frac{\left(\frac{\sigma_1-\sigma_3}{2}\right)}{c\cot\phi + \left(\frac{\sigma_1+\sigma_3}{2}\right)}$$

$$\left(\frac{\sigma_1-\sigma_3}{2}\right) = c\cot\phi\sin\phi + \left(\frac{\sigma_1+\sigma_3}{2}\right)\sin\phi$$

$$(\sigma_1-\sigma_3) = 2c\cot\phi\sin\phi + (\sigma_1+\sigma_3)\sin\phi$$

$$\sigma_1(1-\sin\phi) = \sigma_3(1+\sin\phi) + 2c\cos\phi$$

$$\sigma_1 = \sigma_3\left(\frac{1+\sin\phi}{1-\sin\phi}\right) + 2c\frac{\cos\phi}{1-\sin\phi}$$

$$\because \cos\phi = \sqrt{1-\sin^2\phi} = \sqrt{(1-\sin\phi)(1+\sin\phi)}$$

$$\therefore \frac{\cos\phi}{1-\sin\phi} = \frac{\sqrt{(1-\sin\phi)(1+\sin\phi)}}{(1-\sin\phi)} = \sqrt{\frac{1+\sin\phi}{1-\sin\phi}}$$

$$\therefore \sigma_1 = \sigma_3\left(\frac{1+\sin\phi}{1-\sin\phi}\right) + 2c\sqrt{\frac{1+\sin\phi}{1-\sin\phi}}$$

$$\tan^2\left(45+\frac{\phi}{2}\right) = \frac{1+\sin\phi}{1-\sin\phi}$$

★★★★★ ③

$$\boxed{\sigma_1 = \sigma_3\tan^2\left(45+\frac{\phi}{2}\right) + 2c\tan\left(45+\frac{\phi}{2}\right)}$$

At Plastic equilibrium
(failure होते के Just Pahle की cond)

⑤ Mohr's Coulomb's theory of Shear Strength

- A/c to this theory Shear strength of soil depends upon the

① cohesion b/w the particle on critical plane.

② Angle of internal friction b/w particles which account for interlocking resistance and frictional resistance (dense sand have greater friction angle)

③ Normal stress and critical plane / failure plane which increases with ↑ depth of soil.

★★★★

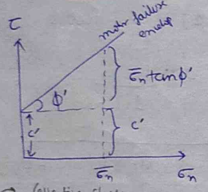
$$S = C' + \bar{\sigma}_n \tan\phi'$$ ④

(empirical formula)

C → total cohesion

φ → total friction angle

σ_n → total normal stress on critical plane



$\sigma_n = \bar{\sigma}_n$ → (effective stress liye hai)

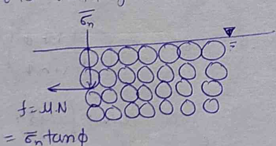
Note the above theory ($S = C + \sigma_n \tan\phi$) is found to be invalid for the cond when water table is present. Hence, the above theory is modified and improv by terzaghi which is called as Modified mohr's coulomb theory

Pahle ($S = C + \sigma_n \tan\phi$) eqn था It is valid for all the cond's

C' = effective cohesion.

φ' = effective friction angle

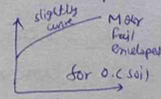
$\bar{\sigma}_n$ = effective normal stress on critical plane $= (\sigma_n - u)$



⑥ Limitation's in Mohr coulomb theory

① Mohr failure envelop is approximately to a straight line which found invalid for over-consolidated soil

Practical result shows that Mohr failure envelop in OC soil is found to be slightly curved.



② The analysis is 2D in which (σ_1 and σ_3) is considered whereas actual stress cond in soil is 3D in which ($\sigma_2 = \sigma_3$).

(lab की बात है)

⑦ Type of Shear Parameter (C and φ)

Ⓐ undrained / total shear parameter (C_u, ϕ_u) and (C, φ)

1) If saturated soil mass is subjected to shear loading and soil is undrained it means pore pressure will developed then in such case total stress should be used

Note If fast loading is done on saturated clay sample then in short term cond undrained shear parameter should be determine.

Ⓑ drained / effective shear parameter (C_d, ϕ_d), (C', ϕ')

(पानी निकल जायेगा)

1) If loading rate is slow and expulsion of pore water complete during loading then pore pressure will become zero. in such case effective stress should be used

Note ① loading in sand on slow rate for short term and long term cond both whereas in clay for long term cond under slow rating rate drained shear parameter should be determined.

Note② C and φ is not fundamental properties of soil these r depend on the type of test, water content, and drainage cond under which testing is done.

→ test must represent the ~~stable~~ field cond. the field cond could be rapid Ⓞⓡ slow cond, granual Ⓞⓡ claye soil, ~~rapid Ⓞⓡ slo~~ drain Ⓞⓡ undrain cond.

8) Type of shear test

1) triaxial test
2) unconfined compression test
3) Vane shear test
4) Direct shear test
5) Torsion balance test
6) Ring shear test

② Field Method
A) Vane shear test
B) SPT
C) CPT

③ Empirical method

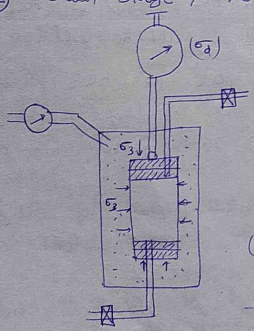
$$\frac{C_u}{\bar{\sigma}_n} = 0.11 + 0.0037 I_p$$

$\bar{\sigma}_n$ = Effect. stress
C_u = Undrained cohesion
I_p = Plasticity index

1) Triaxial shear test

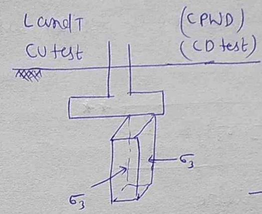
A cylindrical soil specimen is prepared for a saturated soil mass which is enclosed inside the impermeable rubber membrane. The length of sample usually 2 times of it's dia.

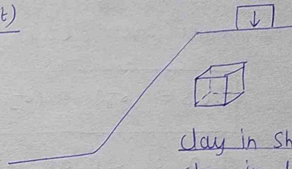
There are two stages of soil loading
1) confining Pressure stage / Cell Pressure stage
2) Shear stage / Deviator stage



triaxial shear test

Type of triaxial test	Stage I confining stage	Stage II shear stage	suitable cond.
(a) Unconsolidated undrained quick test / UU test (5-7 min)	× (closed)	×	clay in short term
(S) consolidated drained slow test / CD test	✓ (open)	✓	clay in long term (and) sand in short term and long term
(R) Consolidated undrained CU test	✓	×	sudden draw down, G.W. table and unloading cond.





clay in short term UU
clay in long term CD

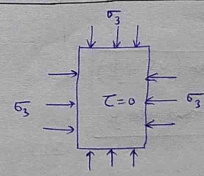
Note: unconsolidated drained test is not performed practically because confining pressure acts from a very long time and if soil is unconsolidated in long time then it can't be drained in small period of shear loading.

9) Process to determine c and ϕ

test is performed in two stages.

1) confining pressure stage / cell pressure stage / consolidation stage (stage)

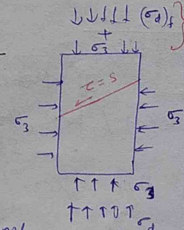
⇒ All around confining Pressure (σ_c and σ_3) is applied using external water pressure. If test is unconsolidated then drainage valve will be close and if test is consolidated then valve should be open



Stage I

Stage ② Shear stage / Deviator stage

Confined Pressure is kept constant and additional axial stress is applied called as deviator stress (σ_d). (σ_d) increases gradually untill the soil fails in shear. ② If test is undrained then drainage valve should be closed and if test is drained then valve should be opened



Stage II

Note ① the deviator stress at failure is termed as confined compressive stress (C.C.S)

$$C.C.S = (\sigma_d)_f = (\sigma_1 - \sigma_3)_f = \frac{P}{A_f}$$ ★★

A_f = area of c/s at failure

⇒ For σ_1 and σ_3 at failure a mohr's circle is drawn and test is repeated on another sample of same soil with changed value of minor principle stress (σ_3) and corresponding (σ_d) and (σ_1) is noted.

Mohr circle for each case is drawn and common tangent to all the Mohr's circle is termed as Mohr's failure envelope (C and ϕ can be determined)

$$\sigma_1 = \sigma_3 \tan^2\left(45 + \frac{\phi}{2}\right) + 2c \tan\left(45 + \frac{\phi}{2}\right)$$