CBSE Class 10 Mathematics Basic

Sample Paper - 10 (2020-21)

Maximum Marks: 80

Time Allowed: 3 hours

General Instructions:

i. This question paper contains two parts A and B.
ii. Both Part A and Part B have internal choices.

Part – A consists 20 questions

i. Questions 1-16 carry 1 mark each. Internal choice is provided in 5 questions.
ii. Questions 17-20 are based on the case study. Each case study has 5 case-based sub-parts. An examinee is to attempt any 4 out of 5 sub-parts.

Part – B consists 16 questions

i. Question No 21 to 26 are Very short answer type questions of 2 mark each,
ii. Question No 27 to 33 are Short Answer Type questions of 3 marks each
iii. Question No 34 to 36 are Long Answer Type questions of 5 marks each.
iv. Internal choice is provided in 2 questions of 2 marks, 2 questions of 3 marks and 1 question of 5 marks.

Part-A

1. State whether $\frac{129}{2^2 \times 5^7 \times 7^{17}}$ will have terminating decimal expansion or a non-terminating repeating decimal expansion.

OR

Find the HCF of the following polynomials: $x^8 - y^8; (x^4 - y^4)(x + y)$

2. State whether $x^2 - 6x + 4 = 0$ is a quadratic equation or not?
3. For what value of a the following pair of linear equation has infinitely many solution?
$ax - 3y = 1$

$-12x + ay = 2$

4. How many tangents, parallel to a secant can a circle have?
5. Does the sequence –1, –1, –1, –1, ... form an AP? Justify your answer.

OR

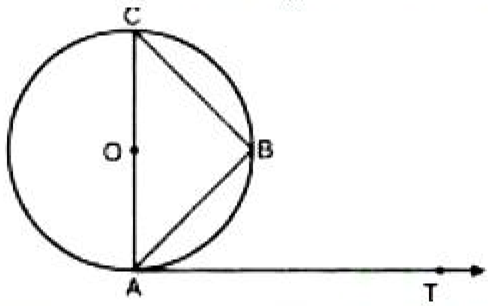
Find the 6^{th} term from the end of the A.P. 17,14,11,..., - 40

6. If $2x$, $x + 10$, $3x + 2$ are in A.P., find the value of x.
7. P & Q are centres of circles of radii 9 cm and 2 cm respectively. PQ = 17 cm. R is the centre of the circle of radius x cm which touches given circles externally. Given that angle PRQ is 90°. Write an equation in x and solve it.

OR

If one root of the quadratic equation $6x^2 - x - k = 0$ is $\frac{2}{3}$, then find the value of k.

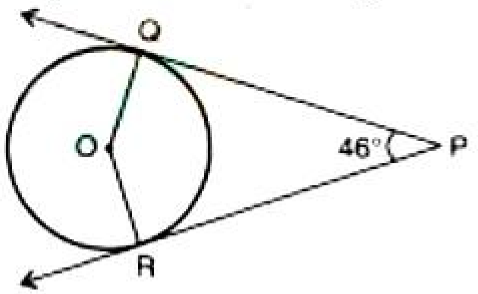
8. In the given figure, AB is a chord of the circle and AOC is its diameter such that $\angle ACB = 50^{\circ}$. If AT is the tangent to the circle at the point A, find $\angle BAT$



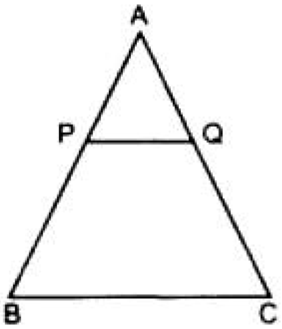
9. Distance between two parallel lines is 14 cm. Find the radius of the circle which will touch both the lines.

OR

If PQ and PR are two tangents to a circle with centre O. If $\angle QPR = 46^{\circ}$, find $\angle QOR$



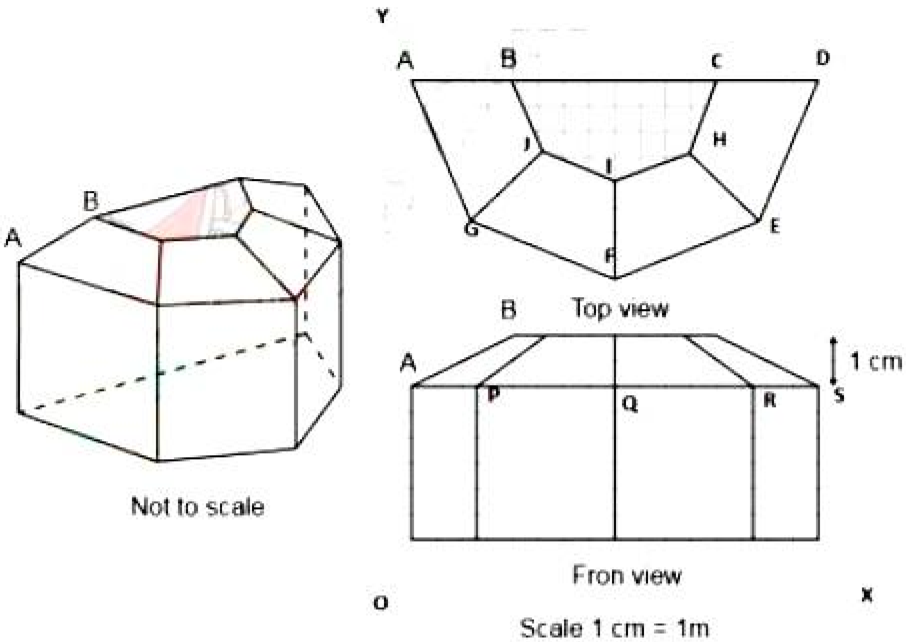
10. In the fig PQ || BC and AP: PB = 1:2. Find $\frac{ar(\Delta APQ)}{ar(\Delta ABC)}$.



11. The sum of three numbers in AP is 21 and their product is 231. Find the numbers.
12. What is the value of $6\tan^2\theta - \frac{6}{\cos^2\theta}$?
13. Prove that: $\frac{1}{(cosec\theta - \cot\theta)}$ = (cosecθ + cotθ)
14. The largest sphere is carved out of a cube of side 10.5 cm. Find the volume of the sphere.
15. Find the value of x for which (x + 2), 2x, (2x + 3) are three consecutive terms of an AP.
16. The king, queen and jack of clubs are removed from a deck of 52 playing cards and the well shuffled. One card is selected from the remaining cards. Find the probability of getting a heart.
17. **SUN ROOM**

The diagrams show the plans for a sun room. It will be built onto the wall of a house. The four walls of the sunroom are square clear glass panels. The roof is made using

- Four clear glass panels, trapezium in shape, all the same size
- One tinted glass panel, half a regular octagon in shape



Scale 1 cm = 1m

i. Find the mid-point of the segment joining the points J (6, 17) and I (9, 16).[Refer to Top View]

a. $(\frac{33}{2}, \frac{15}{2})$

b. $(\frac{3}{2}, \frac{1}{2})$

c. $(\frac{15}{2}, \frac{33}{2})$

d. $(\frac{1}{2}, \frac{3}{2})$

ii. The distance of the point P from the y-axis is; [Refer to Top View]

a. 4

b. 15

c. 19

d. 25

iii. The distance between the points A and S is: [Refer to Front View]

a. 4

b. 8

c. 16

d. 20

iv. Find the coordinates of the point which divides the line segment joining the points A and B in the ratio 1:3 internally. [Refer to Front View]

a. (8.5, 2.0)

b. (2.0, 9.5)

c. (3.0, 7.5)

d. (2.0, 8.5)

v. If a point (x,y) is equidistant from the Q(9,8) and S(17,8), then [Refer to Front View]

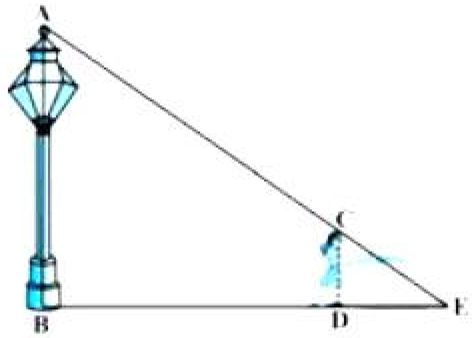
a. $x + y = 13$

b. $x - 13 = 0$

c. $y - 13 = 0$

d. $x - y = 13$

18. Some kindergarten students were playing near a lamp-post. They were so excited to see their shadows and trying to show that their shadow is the longest. The lamp was 3.6 m above the ground. One of the girl of height 90 cm was walking away from the base of a lamp-post at a speed of 1.2 m/s.



i. Which of the following line segment shows the length of the shadow?
   a. CE
   b. BE
   c. DE
   d. CD
ii. What would be the length of her shadow after 4 seconds?
   a. 1.2 m
   b. 1.6 m
   c. 2.3 m
   d. 1.4 m
iii. How far is the girl from the lamp-post?
   a. 4.8 m
   b. 1.2 m
   c. 12 m
   d. 6.4 m
iv. Triangle ABE and CDE are similar because:
   a. All sides are equal
   b. The shadow of a girl is equal to the height of the lamp-post
   c. Angle B and Angle E are common
   d. Both are related to the same length of the shadow
v. AB denotes the _______ and CD _______ after walking for 4 seconds away from the lamp-post.
   a. the girl, lamp-post
   b. the shadow, the girl
   c. the girl, the shadow
   d. lamp-post, the girl

19. The agewise participation of students of a school in the International Yoga day

Celebration that was held in Central City Ground Patna is shown in the following distribution. By Analysing the data given below answer the questions that follow:

Age(in years)	5 - 7	7 - 9	9 - 11	11 - 13	13 - 15	15 - 17	17 - 19
Number of students	x	15	18	30	50	48	x

Find the Following when the sum of frequencies is 181.

i. The mode of the data is:
   a. 17.81
   b. 11.81
   c. 18.41
   d. 14.81
ii. The value of missing frequency(x) is:
   a. 12
   b. 10
   c. 13
   d. 14
iii. The modal class is:
   a. 13 - 15
   b. 11 - 13
   c. 15 - 17
   d. 17 - 19
iv. The upper limit of the modal class is:
   a. 17
   b. 19
   c. 15
   d. 13

v. The construction of the cumulative frequency table is useful in determining the:

a. Mean

b. Median

c. Mode

d. All of the above

20. To promote cooperation, culture, creativity, sharing, self-confidence, and other social values, a student adventure camp was organized by the school for X-class students and their accommodation was planned in texts. The teacher divides the students into groups, each group of four students was given to prepare a conical tent of radius 7 m and canvas of area 551 m^2 in which 1 m^2 is used in stitching and wasting of canvas:

i. Curved surface of conical tent:

a. πrl

b. $\pi r^2 h$

c. $\frac{1}{3}\pi rl$

d. $2\pi r(r + l)$

ii. Height of the conical tent:

a. 23 m

b. 24 m

c. 25 m

d. 26 m

iii. Volume of tent:

a. 1234 m^3

b. 1232 m^3

c. $1332\ m^3$

d. $1343\ m^3$

iv. How much space is occupied by each student in the tent?

a. $318\ m^3$

b. $813\ m^3$

c. $308\ m^3$

d. $391\ m^3$

v. The cost of canvas required for making the tent, if the canvas cost ₹ 70 per sq. m.

a. ₹ 40,000

b. ₹ 38570

c. ₹ 38575

d. ₹ 48470

Part-B

21. Given that HCF (306,1314) = 18. Find LCM (306,1314).

22. Find the distance of C(-4, -6) points from the origin.

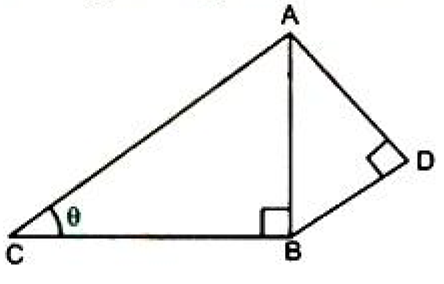
OR

Find the ratio in which the point $\left(\frac{-6}{5}, 4\right)$ divides the join of points (3, 5) and (-4, 5).

23. Find the zeroes of $100x^2 - 81$ and verify the relationship between the zeroes and their coefficients.

24. Draw a pair of tangents to a circle of radius 6 cm which are inclined to each other at 60°. Also write steps of construction.

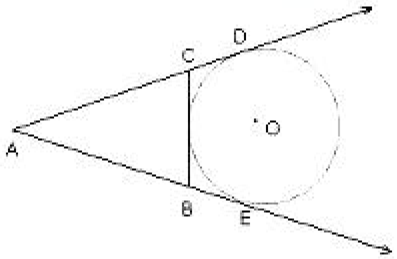
25. In the given figure AD = 4 cm, BD = 3 cm and CB = 12 cm, find cot θ.



OR

If x = 30°, verify that $\sin x = \sqrt{\frac{1-\cos 2x}{2}}$.

26. In the adjoining figure, if AD, AE and BC are tangents to the circle at D, E and F respectively, then prove that 2AD= AB + BC + CA.



27. Show that $3\sqrt{2}$ is an irrational number.

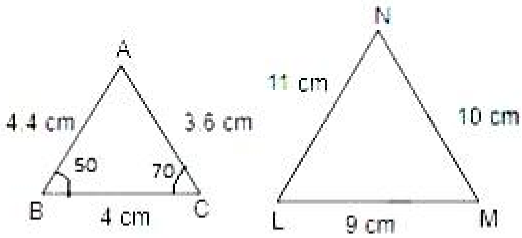
28. Solve the quadratic equation by factorization: $\frac{1}{2a+b+2x} = \frac{1}{2a} + \frac{1}{b} + \frac{1}{2x}$

OR

Solve for x : $9x^2 - 6ax + (a^2 - b^2) = 0$.

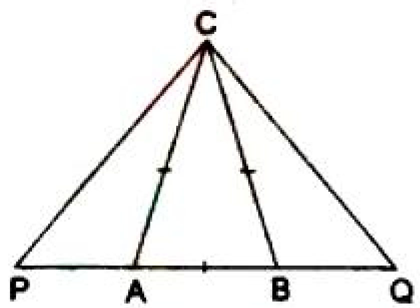
29. Find the zeroes of the quadratic polynomial $6x^2 - 3 - 7x$ and verify the relationship between the zeroes and the coefficients of the polynomial.

30. In the given figures, find $\angle MLN$



OR

In an isosceles, $\triangle ABC$ the base AB is produced both ways in P and Q such that $AP \times BQ = AC^2$ Prove that $\triangle ACP \sim \triangle BCQ$.



31. Cards marked with numbers 5 to 50 are placed in a box and mixed thoroughly. A card is drawn from the box at random. Find the probability that the number on the taken out card is

i. a prime number less than 10

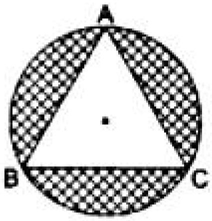
ii. a number which is a perfect square.

32. The horizontal distance between two towers is 60 m. The angle of elevation of the top of the taller tower as seen from the top of the shorter one is 30^o. If the height of the taller tower is 150 m, them find the height of the shorter tower.

33. Find the mean age of 100 residents of a colony from the following data.

Age in years (>or =)	**0**	**10**	**20**	**30**	**40**	**50**	**60**	**70**
No. of persons	**100**	**90**	**75**	**50**	**25**	**15**	**5**	**0**

34. On a circular table cover, of radius 42 cm, a design is formed by a girl leaving an equilateral triangle ABC in the middle, as shown in the figure. Find the covered area of the design. [Use $\sqrt{3}$ = 1.73 and $\pi = \frac{22}{7}$].



35. A part of monthly hostel charges in a college hostel are fixed and the remaining depends on the number of days one has taken food in the mess. When a student A takes food for 25 days, he has to pay ₹4500, whereas a student B who takes food for 30 days, has to pay ₹5200. Find the fixed charges and the cost of the food per day.

36. A person standing on the bank of a river observes that the angle of elevation of the top of a tree standing on the opposite bank is 60°. When he moves 30 metres away from the bank, he finds the angle of elevation to be 30°. Find the height of the tree and the width of the river. [Take $\sqrt{3}$ = 1.732.]

CBSE Class 10 Mathematics Basic

Sample Paper - 10 (2020-21)

Solution

Part-A

1. According to the question, The given number is $\frac{129}{2^2\times5^7\times7^{17}}$

   Clearly, none of $2, 5\ and\ 7$ is a factor of 129.

   So, the number will have a non terminating decimal expansion.

OR

$P(x) = x^8 - y^8$

$= (x - y)(x + y)\left(x^2 + y^2\right)\left(x^4 + y^4\right)$ Using Identity $a^2 - b^2 = (a + b)(a - b)$

$Q(x) = \left(x^4 - y^4\right)(x + y)$

$= (x - y)(x + y)\left(x^2 + y^2\right)(x + y)$ Using Identity $a^2 - b^2 = (a + b)(a - b)$

$\therefore \text{HCF} = (x - y)(x + y)\left(x^2 + y^2\right) = x^4 - y^4$ Using Identity $a^2 - b^2 = (a + b)(a - b)$

2. Let $p(x) = x^2 - 6x + 4$

   It is of the form $ax^2 + bx + c = 0$, with $a = 1, b = -6, c = 4$

   Therefore, $x^2 - 6x + 4 = 0$ is a quadratic equation.

3. For infinitely many solutions,

   $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2} \Rightarrow \frac{a}{-12} = \frac{-3}{a} = \frac{1}{2}$

   $\Rightarrow \frac{a}{-12} = \frac{-3}{a}$ and $\frac{-3}{a} = \frac{1}{2}$

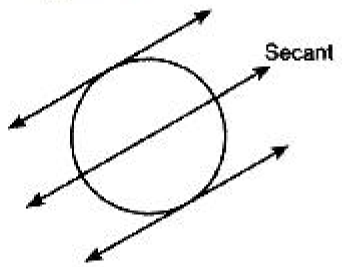
   $\Rightarrow a^2 = 36\ and\ a = -6$

   $\Rightarrow a = \pm6\ and\ a = -6$

   For $a = -6$, pair of given linear equations has infinitely many solutions.

4. A circle can have 2 tangents parallel to a secant.

   Diagram:



5. We have $a_1 = -1$, $a_2 = -1$, $a_3 = -1$ and $a_4 = -1$

$a_2 - a_1 = 0$

$a_3 - a_2 = 0$

$a_4 - a_3 = 0$

Clearly, the difference of successive terms is same, therefore given list of numbers form an AP.

OR

A.P. is 17,14,11,..., - 40

We have,

l = Last term = -40 , a = 17 and, d = Common difference= 14 - 17 = - 3

$\therefore$ 6th term from the end = l - (n -1)d

= l - (6-1) d

= -40 - 5 $\times$ (-3)

= -40 + 15

= -25

So, 6th term of given A.P. is -25.

6. If 2x, x + 10, 3x + 2 are in A.P., we have to find the value of x.

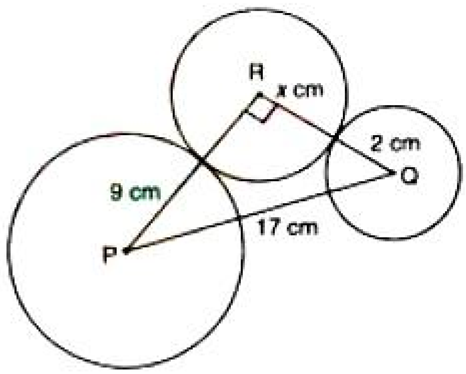
Since, 2x, x + 10, 3x + 2 are in A.P.

Therefore 2 (x + 10) = 2x + 3x + 2

$\Rightarrow$ 2x + 20 = 5x + 2

$\Rightarrow$ 3x = 18

$\Rightarrow$ x = 6

7. 

In right $\triangle PQR$, by Pythagoras theorem

$$PQ^2 = PR^2 + RQ^2$$

or, $17^2 = (x+9)^2 + (x+2)^2$

$289 = x^2 + 18x + 81 + x^2 + 4x + 4$

or, $x^2 + 11x - 102 = 0$

or, $x^2 + 17x - 6x - 102 = 0$

or, x(x + 17) - 6(x + 17) = 0

(x - 6)(x + 17) = 0

or, x = 6 or x =-17

or, x = 6 cm (x can't be negative)

OR

On substituting $x = \frac{2}{3}$, we get

$6 \cdot \left(\frac{2}{3}\right)^2 - \frac{2}{3} - k = 0$

$6 \times \frac{4}{9} - \frac{2}{3} - k = 0$

$k = 6 \times \frac{4}{9} - \frac{2}{3}$

$k = \frac{8}{3} - \frac{2}{3} = \frac{6}{3}$

$\therefore \quad k = 2$

8. $\because \angle ACB = 50°$

$\angle CBA = 90°$ (Angle in semi-circle)

$\therefore \angle OAB = 90° - 50°$

$= 40°$

$\angle BAT = 90° - \angle OAB$

$= 90° - 40°$

$= 50°$

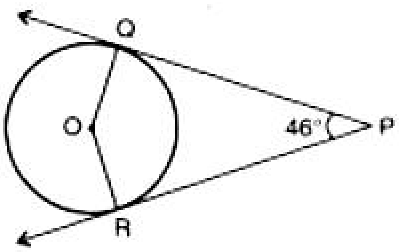
9. Circle touches both the parallel lines

Given, Distance between the parallel lines = 14 cm

We know that, Diameter of circle = Distance between the parallel lines

$\therefore$ Radius = $\frac{14}{2}$ = 7 cm

OR



Since, $OQ \perp OP \; and \; OR \perp RP$

$\angle QOR + \angle QPR + \angle PRQ + \angle QOR = 360^0$

or, $\angle QOR + 46^\circ = 180^\circ$

or, $\angle QOR = 180^\circ - 46^\circ = 134^\circ$

10. In ABC,

PQ || BC

$\therefore \quad \frac{AP}{AB} = \frac{AQ}{AC}$

Now in ΔAPQ and ΔABC,

$\frac{AP}{AB} = \frac{AQ}{AC}$ (As proved)

$\angle$A = $\angle$A (common angle)

$\triangle APQ \sim \triangle ABC$ (SAS similarity)

Since for similar triangles, the ratio of the areas is the square of the ratio of their corresponding sides. Therefore,

$\frac{ar(\Delta APQ)}{ar(\Delta ABC)} = \frac{AP^2}{AB^2} = \frac{AP^2}{(AP+PB)^2} = \frac{1^2}{3^2} = \frac{1}{9}$

11. Let the required numbers be (a-d), a and (a + d).............(1)

Then, according to question, (a - d) + a + (a + d) = 21

$\Rightarrow$ 3a = 21

$\Rightarrow$ a = 7.

And, (a - d) $\times$ a $\times$ (a + d) = 231 $\Rightarrow$ a $(a^2 - d^2)$ = 231

$\Rightarrow 7(49 - d^2) = 231$ [$\because$a = 7]

$\Rightarrow 7d^2 = 343 - 231 = 112$

$\Rightarrow d^2 = 16$

$\Rightarrow$ d = $\pm$4.

Thus, a = 7 and d = $\pm$4. Now substitute these values of a and d in above equation (1).

Therefore, the required numbers are(3, 7, 11) or (11, 7, 3).

12. We have

$6\tan^2\theta - \frac{6}{\cos^2\theta} = 6\tan^2\theta - 6\sec^2\theta = -6\left(\sec^2\theta - \tan^2\theta\right)$

We know that, $\sec^2\theta - \tan^2\theta = 1$

Therefore, $6\tan^2\theta - \frac{6}{\cos^2\theta} = -6$

13. We have,

L.H.S. = $\frac{1}{(cosec\theta - \cot\theta)} = \frac{1}{(cosec\theta - \cot\theta)} \times \frac{(cosec\theta + \cot\theta)}{(cosec\theta + \cot\theta)}$

$= \frac{(cosec\theta + \cot\theta)}{(cosec^2\theta - \cot^2\theta)} = cosec\theta + \cot\theta$ = R.H.S. [$\because$ $\text{cosec}^2\theta - \cot^2\theta = 1$]

$\therefore$ L.H.S. = R.H.S.

14. Side of cube = 10.5 cm

The largest sphere is carved out from this cube

Then, the diameter of sphere = 10.5 cm

$\therefore$ radius of sphere = $\frac{10.5}{2}$ cm

$\therefore$ Volume of sphere = $\frac{4}{3}\pi r^3$

= $\frac{4}{3} \times \frac{22}{7} \times \frac{10.5}{2} \times \frac{10.5}{2} \times \frac{10.5}{2}$

= 606.375 cm^3

15. If T_1, T_2, T_3 are consecutive terms of an AP, then

$T_2 - T_1 = T_3 - T_2 \text{ or } 2T_2 = T_1 + T_3$

$\therefore$ x + 2, 2x, 2x + 3 are in AP, if

$2(2x) = x + 2 + 2x + 3$

$\Rightarrow$ 4x = 3x + 5 $\Rightarrow$ x = 5

16. After removing king, queen and jack of clubs from a deck of 52 playing cards there are 49 cards left in the deck. Out of these 49 cards one card can be chosen in 49 ways.

$\therefore$ Total number of elementary events = 49

There are 13 heart cards in the deck containing 49 cards out of which one heart card can be chose in 13 ways.

$\therefore$ Favourable number of elementary events = 13

Hence, P (Getting a heart) = $\frac{13}{49}$

17. i. (c) $(\frac{15}{2}, \frac{33}{2})$
   ii. (a) 4
   iii. (c) 16
   iv. (d) (2.0, 8.5)
   v. (b) x - 13 = 0

18. i. (c) DE
   ii. (b) 1.6 m

iii. (a) 4.8 m

iv. (c) Angle B and Angle E are common

v. (d) lamp-post, the girl

19. Sum of the frequencies = 181

$\Rightarrow$ x + 15 + 18 + 30 + 50 + 48 + x = 181

$\Rightarrow$ 2x + 161 = 181

$\Rightarrow$ x = 10

Thus, the missing frequencies are 10 and 10.

Clearly, the modal class is 13 - 15, as it has the maximum frequency.

$\therefore l = 13$, h = 2, f_1 = 50, f_0 = 30, f_2 = 48

Mode, $M_o = l + \left\{h \times \frac{(f_1 - f_0)}{(2f_1 - f_0 - f_2)}\right\}$

$= 13 + 2\left\{\frac{50-30}{2(50)-30-48}\right\}$

$= 13 + 2 \times \frac{20}{22}$

= 13 + 1.81 = 14.81

i. (d) 14.81

ii. (b) 10

iii. (a) 13 - 15

iv. (c) 15

v. (b) Median

20. i. (a) πrl

ii. (b) 24 m

iii. (b) 1232 m^3

iv. (c) 308 m^3

v. (b) ₹ 38570

Part-B

21. $HCF(306, 1314) = 18$

$LCM(306, 1314) = ?$

Let, a = 306

b = 1314

LCM (a, b) $\times$ HCF (a,b) = a $\times$ b

or, LCM (a, b) $\times$ 18 = 306 $\times 1314$

or LCM(a,b) = $\frac{306\times1314}{18}$ =22338

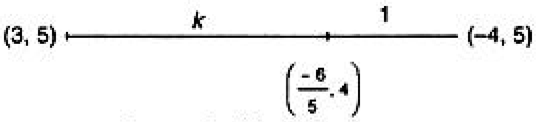
Therefore, $LCM(306, 1314) = 22338$

22. The given point is C(-4, -6) and let O(0,0) be the origin

Then, $CO = \sqrt{(-4-0)^2+(-6-0)^2} = \sqrt{16+36} = \sqrt{52} = 2\sqrt{13}\ units$

OR

Let the ratio be k : 1



Then $\frac{-6}{5} = \frac{3-4k}{k+1}$

$\Rightarrow$ -6k - 6 = 15 - 20k

$\Rightarrow$ 14k = 15 + 6

$\Rightarrow k = \frac{21}{14} = \frac{3}{2}$

$\therefore$ required ratio is 3 : 2

23. The given polynomial is

p(x) = $100x^2$- 81

For zeroes, p(x) = 0

$\Rightarrow 100x^2 - 81 = 0$

$\Rightarrow (10x)^2 - (9)^2 = 0$

$\Rightarrow$ (10x - 9) (10x + 9) = 0

$\Rightarrow$ 10x - 9 = 0 or 10x + 9 = 0

$\Rightarrow x = \frac{9}{10}$ or $x = \frac{-9}{10}$

Compare $100x^2$- 81 with $ax^2 + bx + c$

Now a = 100, b = 0, c = -81

$\frac{-b}{a} = \frac{0}{100} = 0$ ..... (i)

Sum of zeroes = $\frac{9}{10} + \left(\frac{-9}{10}\right)$ = 0 .... (ii)

From (i) and (ii), we get

Sum of zeroes = $\frac{-b}{a}$

Also, $\frac{c}{a} = \frac{-81}{100}$ ...... (iii)

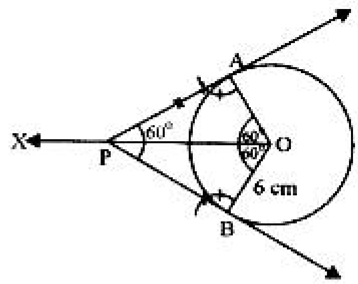
Product of zeroes = $\frac{9}{10} \times \left(\frac{-9}{10}\right) = \frac{-81}{100}$ .... (iv)

From (iii) and (iv), we get

Product of zeroes = $\frac{c}{a}$

24. Steps of construction:
    i. Draw a circle with centre O and radius = 6 cm.
    ii. Take a ray OP.
    iii. Construct $\angle POA$ and $\angle POB$ = 60° such that $\angle AOB = 120°$.
    iv. Construct $\angle OAP = \angle OBP = 90°$ with radii OA and OB respectively such that their arms AP and BP intersect ray OP at P, then $\angle APB = 60°$

    Such that tangents PA and PB are inclined at 60° with each other.



25. In right $\triangle$ABD, $\angle D = 90°$

$AB^2 = AD^2 + BD^2$

$\Rightarrow AB^2 = 4^2 + 3^2$

$\Rightarrow$ AB = ± 5 cm

$\Rightarrow AB = 5cm$ (Neglecting negative value)

In right $\triangle$ABC , cot θ = $\frac{\text{BC}}{\text{AB}} = \frac{12}{5}$

OR

We have,

x = 30°

$\Rightarrow 2x = 60°$

$\therefore \quad \sqrt{\frac{1-\cos 2x}{2}} = \sqrt{\frac{1-\cos 60°}{2}} = \sqrt{\frac{1-\frac{1}{2}}{2}} = \sqrt{\frac{1}{4}} = \frac{1}{2}$ and $\sin x = \sin 30° = \frac{1}{2}$

$\therefore$ sin x $= \sqrt{\frac{1-\cos 2x}{2}}$. Hence verified.

26. CD = CF, BE = BF

$\Rightarrow$ CD + BE = CF + BF = BC

Now AD = AC + CD = AC + CF

AE = AB + BE = AB + BF

$\therefore$ AD + AE = AB + AC + BC

$\Rightarrow$ 2AD = AB + BC + AC

27. Let us preassume that $3\sqrt{2}$ is a rational number.

In that case, $3\sqrt{2}$ can be writtin as $\frac{p}{q}$, where p and q are co-prime integers and q is not zero.

So, $\frac{p}{q} = \frac{3\sqrt{2}}{1}$

$\Rightarrow \frac{p}{3q} = \frac{\sqrt{2}}{1}$

Since, p is an integer and 3q is also an integer where 3q is not zero.

So, $\frac{p}{3q}$ is a rational number but the equalient number $\sqrt{2}$ should also be a rational number.

But this contradicts the fact that $\sqrt{2}$ is an irrational number.

so, this assumption is wrong and $3\sqrt{2}$ is an irrational number.

28. Consider $\frac{1}{2a+b+2x} = \frac{1}{2a} + \frac{1}{b} + \frac{1}{2x}$

$\Rightarrow \frac{1}{2a+b+2x} - \frac{1}{2x} = \frac{1}{2a} + \frac{1}{b}$

$\Rightarrow$ 2ab (2x - 2a – b – 2x) = (2a + b)2x (2a + b + 2x)

$\Rightarrow$ 2ab (-2a - b) = 2(2a+ b) (2ax + bx + $2x^2$)

$\Rightarrow$ -ab = 2ax + bx + $2x^2$

$\Rightarrow$ $2x^2$ + 2ax + bx + ab = 0

$\Rightarrow$ 2x (x + a) + b (x + a) = 0

$\Rightarrow$ (2x + b) (x + a) = 0

$\Rightarrow \mathbf{x = -a, -\frac{b}{2}}$

Hence the roots are $-a, -\frac{b}{2}$.

OR

Given equation, $9x^2 - 6ax + a^2 - b^2 = 0$

$\therefore \quad x = \frac{6a \pm \sqrt{(-6a)^2 - 4\times 9\times (a^2 - b^2)}}{2\times 9}$

or, $x = \frac{6a \pm \sqrt{36a^2 - 36a^2 + 36b^2}}{18}$

or, $x = \frac{6a+6b}{18}, x = \frac{6a-6b}{18}$

or, $x = \frac{6(a+b)}{18}, x = \frac{6(a-b)}{18}$

or, $x = \frac{a+b}{3}, x = \frac{a-b}{3}$

29. The given polynomial is

p(x) = $6x^2 - 7x - 3$

Factorize the above quadratic polynomial, we have

$6x^2 - 7x - 3 = 6x^2 - 9x + 2x - 3$

= 3x(2x - 3) + 1(2x - 3)

= (3x + 1)(2x - 3)

For p(x) = 0, either 3x + 1 = 0 or 2x - 3 = 0

$\Rightarrow \quad x = \frac{-1}{3}$ or $x = \frac{3}{2}$

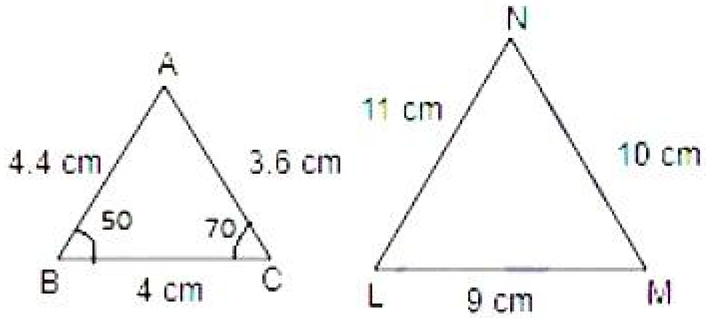
Verification: we have a = 6, b = -7, c = -3

Sum of zeroes = $\frac{-1}{3} + \frac{3}{2} = \frac{7}{6}$

Also, $\frac{-b}{a} = \frac{-(-7)}{6} = \frac{7}{6}$

Now, product of zeroes = $\left(-\frac{1}{3}\right) \times \frac{3}{2} = \frac{-1}{2}$

Also, $\frac{c}{a} = \frac{-3}{6} = \frac{-1}{2}$

30. 

In △ABC and △NLM,

$\frac{MN}{BC} = \frac{10}{4}$ = 2.5

$\frac{LM}{AC} = \frac{9}{3.6}$ = 2.5

$\frac{NL}{AB} = \frac{11}{4.4}$ = 2.5

We know that, if all the sides of two triangles are proportional then the two triangles are similar.

i.e. $\triangle ABC \sim \triangle LMN$

⇒ ∠A = ∠L (Corresponding Part of Similar Triangles) ----(1)

∠B = ∠M = 50^o (CPST)

∠C = ∠N = 70^o (CPST)

In △ABC,

∠A +∠B +∠C = 180^o (Angle sum property)

∠A + 50^o + 70^o = 180°

$\angle A = 180^o - 120^o$

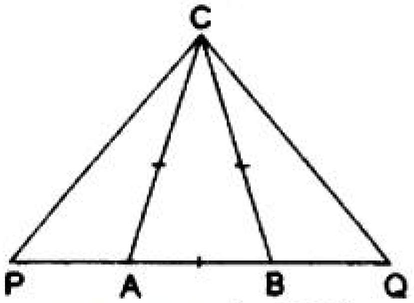
$\angle A = 60^o$

Hence $\angle M = 50^o$; $\angle L = 60^o$; $\angle N = 70^o$

$\Rightarrow \angle A = \angle MLN = 60^o$ [from (1)]

$\Rightarrow \angle MLN = 60^o$

OR



It is given that $\triangle ABC$ is an isosceles triangle, therefore we have

CA = CB

$\Rightarrow \angle CAB = \angle CBA$

$\Rightarrow 180^0 - \angle CAB = 180^0 - \angle CBA$

$\Rightarrow \angle CAP = \angle CBQ$ (Angles opposite to equal sides of a triangle are equal)

Also, we have

$\text{AP} \times \text{BQ} = AC^2$

$\Longrightarrow \frac{AP}{AC} = \frac{AC}{BQ}$

$\Longrightarrow \frac{AP}{AC} = \frac{BC}{BQ}$ [$\because$ AC = BC]

Thus, by SAS similarity theorem, we obtain

$\triangle \text{APC} \sim \triangle \text{BCQ}$

31. According to question we are given that Cards marked with numbers 5 to 50 are placed in a box and mixed thoroughly. A card is drawn from the box at random. Therefore All possible outcomes are 5, 6, 7, 8 .... 50.

Number of all possible outcomes = 46

i. Out of the given numbers, the prime numbers less than 10 are 5 and 7.

   Suppose E_1 be the event of getting a prime number less than 10.

   Then, number of favorable outcomes = 2

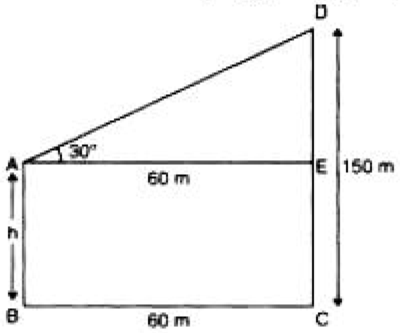
   Therefore, P(getting a prime number less than 10) = P(E) = $\frac{2}{46} = \frac{1}{23}$

ii. Out of the given numbers, the perfect squares are 9, 16, 25, 36 and 49.

Suppose E_2 be the event of getting a perfect square.

Then, number of favorable outcomes = 5

Therefore, P(getting a perfect square) = P(E) = $\frac{5}{46}$

32. 

Let AB and CD be two towers.

Let the height of the shorter tower AB =h

In $\triangle$DEA

tan 30^{o} = $\frac{DE}{EA} = \frac{1}{\sqrt{3}}$

DE=$\frac{60}{\sqrt{3}} = 20 \times \sqrt{3} = 20 \times 1.73 = 34.6m$

So h = CE = 150 - 34.6 = 115.4 m

33. First we convert the cumulative frequency distribution into an ordinary frequency distribution:

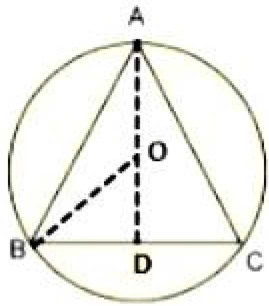
Age (in years)	No. of persons
0 – 10	100-90 = 10
10-20	90 – 75 = 15
20-30	75 – 50 = 25
30-40	50 – 25 = 25
40-50	25 – 15 = 10
50-60	15 – 5 = 10
60-70	5 – 0 = 5

Class-interval	Mid-point(x_i)	Frequency(f_i)	$U_i = (x_i-35)/5$	fiui
0-10	5	10	-6	-60
10-20	15	15	-4	-60
20-30	25	25	-2	-50
30-40	35	25	0	0

40-50	45	10	2	20
50-60	55	10	4	40
60-70	65	5	6	30
Total		100		-80

Here a = 35, $\sum f_i = 100,\ \sum f_i u_i = -80, h = 5$

$\therefore\ \overline{x} = a\ + \frac{\sum_{i\ =\ 1}^{7} f_i u_i}{\sum_{i\ \ =1}^{7} f_i}\ \times h{=}35 + \frac{-80}{100} \times 5 = 35 - 4\ \ {=}31$

34. 

Let O be the centre of the circumcircle.

Join OB and draw $AD \perp BC$

Then, OB = 42 cm

and $\angle OBD = 30^\circ$

In $\triangle OBD$,

$\sin 30^\circ = \frac{OD}{OB}$

$\Rightarrow \frac{1}{2} = \frac{\text{OD}}{42}$

$\Rightarrow$ OD = 21 cm

Now, $BD^2 = OB^2 - OD^2$

$= 42^2 - 21^2$

= (42 + 21) (42 - 21)

$= 63 \times 21$

$\Rightarrow BD = \sqrt{63 \times 21}$

$= \sqrt{3 \times 21 \times 21}$

$= 21\sqrt{3}$ cm

$\Rightarrow BC = 2 \times 21\sqrt{3}$

$= 42\sqrt{3}\text{cm}$

Now, area of the shaded region

= Area of the circle - Area of an equilateral $\triangle$ ABC

$= \frac{22}{7} \times 42 \times 42 - \frac{\sqrt{3}}{4} \times 42\sqrt{3} \times 42\sqrt{3}$

= (5544 - 2291.50) cm^2

= 3252.5 cm^2

35. Let the fixed charges be ₹x and the cost of food per day be ₹y.

Then as per the question,

$x + 25y = 4500$........(i)

$x + 30y = 5200$........(ii)

Subtracting (i) from (ii), we get

$5y = 700$

$\Rightarrow y = 140$

Substituting $y = 140$ in (i), we get

$\Rightarrow x + 25 \times 140 = 4500$

$\Rightarrow x = 1000$

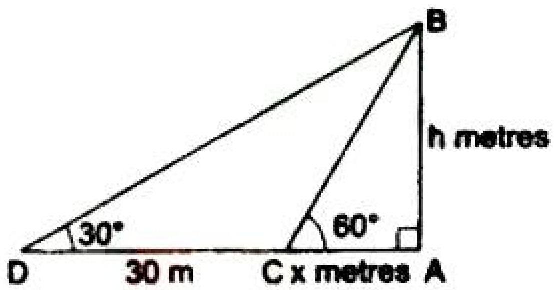
Therefore, the fixed charges is ₹1000 and the cost of the food per day is ₹140.

36. Let AB be the tree of height h m and AC be the river.

Let C be the position of a man standing on the opposite bank of the river . After moving 30 m away from point C. Let new position of man be D i.e CD = 30m

Then, $\angle ACB = 60^\circ$, $\angle ADB = 30^\circ$, $\angle DAB = 90^\circ$ and CD = 30m.

Let AB = h metres and AC = x metres.



From right ΔCAB, we have

$\frac{AC}{AB} = \cot 60^\circ = \frac{1}{\sqrt{3}}$

$\Rightarrow \frac{x}{h} = \frac{1}{\sqrt{3}} \Rightarrow x = \frac{h}{\sqrt{3}}$ ...(i)

From right ΔDAB, we have

$\frac{AD}{AB} = \cot 30^\circ = \sqrt{3}$

$\Rightarrow \frac{x+30}{h} = \sqrt{3} \Rightarrow x = \sqrt{3}h - 30$....(ii)

Equating the values of x from (i) and (ii), we get

$\frac{h}{\sqrt{3}} = \sqrt{3}h - 30 \Rightarrow h = 3h - 30\sqrt{3}$

$\Rightarrow 2h = 30\sqrt{3} \Rightarrow h = 15\sqrt{3} = 15 \times 1.732 = 25.98$

Putting $h = 15\sqrt{3}$ in (i), we get $x = \frac{15\sqrt{3}}{\sqrt{3}} = 15.$

Hence, the height of the tree is 25.98m and the width of the river is 15 metres.