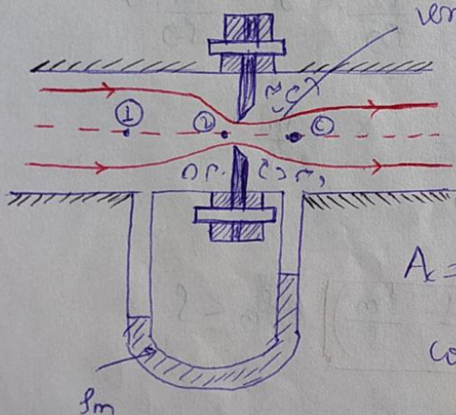
lecture 11
6/4/19

orificemeter:

Comparatively cheap device for comparatively measurement of discharge



$D_2 \in (0.4 - 0.85)\, D_1$

$A_c = C_c \cdot A_2$

↑ coefficient of contraction

$C_d \in 0.61 - 0.65$

Actual approach

Apply energy b/w ① and ©

$$\boxed{E_1 = E_c + h_L}$$

$$Z_1 + \frac{P_1}{\rho g} + \frac{V_1^2}{2g} = \frac{P_c}{\rho g} + \frac{V_2^2}{2g} + h_L + Z_c$$

$$\frac{V_c^2 - V_1^2}{2g} = \underbrace{\left[\frac{P_1}{\rho g} + Z_1\right] - \left[\frac{P_c}{\rho g} + Z_c\right]}_{h} - h_L$$

$$V_c^2 - V_1^2 = 2g\,(h - h_L)$$

Apply continuity

$$Q_{Act} = A_1V_1 = A_cV_c$$

$$Q_{act}^2\left[\frac{1}{A_c^2} - \frac{1}{A_1^2}\right] = 2g\,(h - h_L)$$

$$Q_{act}^2\left[\frac{A_1^2 - A_c^2}{A_c^2 \cdot A_1^2}\right] = 2g\,(h - h_L)$$

$$Q_{act} = \frac{A_1 A_c}{\sqrt{A_1^2 - A_c^2}}\sqrt{2g(h - h_L)}$$

$$[A_c = C_c A_2]$$

$$Q_{act} = \frac{C_c A_1 A_2}{\sqrt{A_1^2 - C_c^2 A_2^2}}\sqrt{2g(h-h_L)}$$

$$Q_{act} = \frac{A_1 A_2}{\sqrt{A_1^2 - A_2^2}}\sqrt{2gh} \times \frac{C_c\sqrt{A_1^2 - A_2^2}}{\sqrt{A_1^2 - C_c^2A_2^2}}\sqrt{\frac{h-h_L}{h}}$$

coefficient of orificemeter

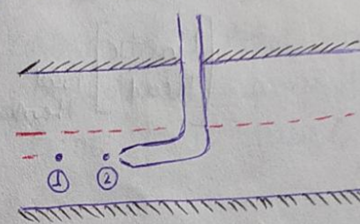
Pb - 16

$$\frac{\cancel{Q}}{\cancel{Q}} = \frac{0.98\,\frac{A_1 \cancel{A_2}}{\sqrt{A_1^2 - A_2^2}} \times \sqrt{2\cancel{g} h_v}}{0.61 \times \frac{A_1 \cancel{A_2}}{\sqrt{A_1^2 - A_2^2}} \times \sqrt{\cancel{2g} h_0}}$$

$$\boxed{\left(\frac{0.61}{0.98}\right)^2 = \frac{h_v}{h_0}}$$ Ans

PITOT TUBE

It is device basically used for the measurement of local velocity. It is 90° bend tube having a small opening.



Point ① ⇒ Before Striking

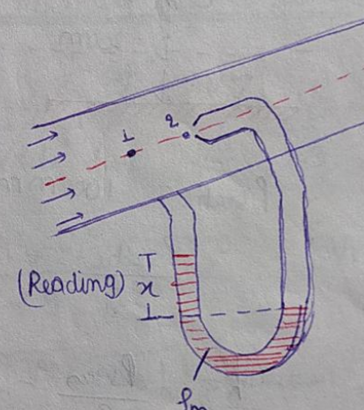
Point ② ⇒ stagnation Point $\{V_2 = 0\}$

Apply energy eqn

(theoritical) $E_1 = E_2$

(approach)

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\rho g} + \cancel{\frac{V_2^2}{2g}} + Z_2 \quad (V_2 = 0)$$



$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\rho g} + Z_2$$

$$\frac{V_1^2}{2g} = \left[\frac{P_2}{\rho g} + Z_2\right] - \left[\frac{P_1}{\rho g} + Z_1\right]$$

$$V_1 = \sqrt{2gh}$$

(theoritical)

$$\boxed{V_{1\,\text{actual}} = C_v \sqrt{2gh}}$$

C_v = coefficient of velocity

$C_v \in 0.97 - 0.99$

$h = x$ (Piezometer)

$h = x\left(\frac{\rho_m}{\rho} - 1\right)$ (U tube manometer) $\rho_m > \rho$

$h = x\left(1 - \frac{\rho_m}{\rho}\right)$ (inverted U tube manom.) $(\rho_m < \rho)$

$$\frac{P_1}{\rho g} + z_1 \Rightarrow \text{(Piezometric head) (static)}$$

$$\frac{P_2}{\rho g} + z_2 \Rightarrow \text{(Piezometric head) (Stagnation)}$$

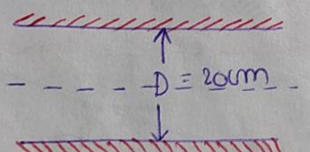
Inclined

$$\begin{bmatrix} \text{Kinematic} \\ \text{Head} \end{bmatrix}_{\substack{\text{local} \\ \text{theoretical}}} = \text{Stagnation Piezometric head} - \text{Static Piezometric head}$$

Horizontal ($Z_1 = Z_2$)

$$\begin{bmatrix} \text{Kinetic} \\ \text{Head} \end{bmatrix}_{\substack{\text{local} \\ \text{theoretical}}} = \text{Stagnation pressure head} - \text{static pressure head}$$

Pb - (36)



$P_{stag} = 200 \text{ KPas}$

$$\frac{P_{stag}}{\rho g} = \frac{200 \times \cancel{10^3}}{\cancel{10^3} \times 9.81} = 20.38 m$$

$P_{static} =$ 100 mm of Hg

$$\frac{P_{static}}{\rho g} = \frac{(13.6 \times 10^3)(9.81)(0.1) \text{ Pas}}{10^3 \times 9.81} = 1.36 m$$

$$h = \frac{P_{stag}}{\rho g} - \frac{P_{static}}{\rho g} = 20.38 - 1.38 = 19.02 M$$

$$\underset{\text{(actual)}}{V_{centerline}} = C_v \sqrt{2gh} = 0.97 \sqrt{2 \times 9.81 \times 19.02}$$

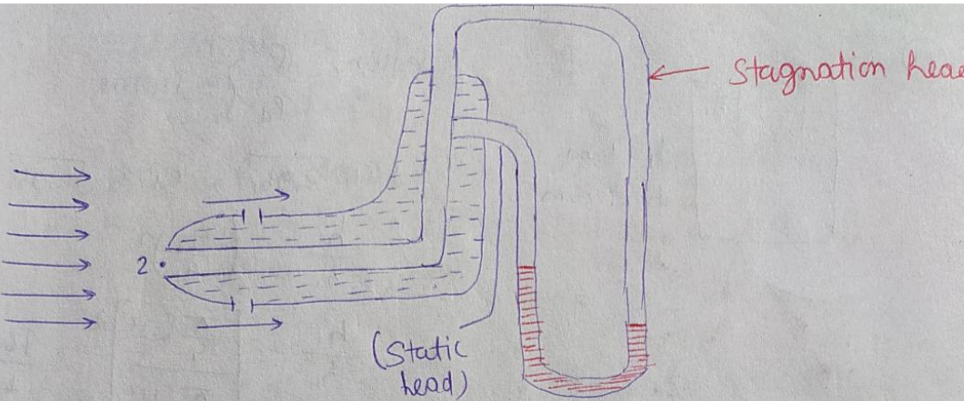
$$= 18.74 m/s$$

$$Q = A\bar{V} = \left[\text{Given } \bar{v} = \frac{V_{centerline \text{ (actual)}}}{2} \right]$$

$$Q = \frac{\pi}{4}(0.2)^2 \left[\frac{18.74}{2}\right) = 0.294 m^3/s$$

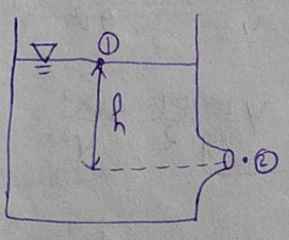
Ans

Pitot - Static tube :-



① The tube recording static pressure and stagnation pressure are combined into one instrument known as Pitot static tube
② A Pitot static tube built with standard proportion give accurate result. eg. Prandtl's Pitot static tube ($C_v \approx 1$)

Flow through orifice: Apply energy b/w ① and ②



Patm 0

$$\cancel{\frac{P_1}{\rho g}} + \cancel{\frac{V_1^2}{2g}} + z_1 = \cancel{\frac{P_2}{\rho g}} + \frac{V_2^2}{2g} + \cancel{z_2}$$

$$\frac{V_1^2 - V_2^2}{2g} + z_1 = 0 \quad [V_1 = 0]$$

$\boxed{V_2 = \sqrt{2gH}}$ (theoretical), $V_{2(actual)} = C_v\sqrt{2gh}$

C_v = coefficient of velocity

Point 1 = at free surface
point 2 = vena contracta
c/s area of orifice = a

apply continuity

$$A_1V_1 = A_2V_2$$

$$V_1 = \frac{A_2}{A_1} \cdot V_2 = 0 \; (A_1 >>> A_2) \; V_1 \approx 0$$

Discharge →

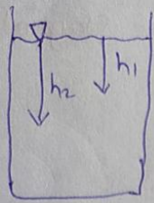
$$Q = A_2V_2$$

$$= C_c a \cdot C_v \cdot \sqrt{2gh} = \underbrace{C_c \cdot C_v}_{C_d} \cdot a\sqrt{2gh}$$

$$\boxed{Q = C_d \cdot A\sqrt{2gh}}$$

(a = area of orifice)

Pb-21



$d_1 = 1cm$
$d_2 = 2cm$

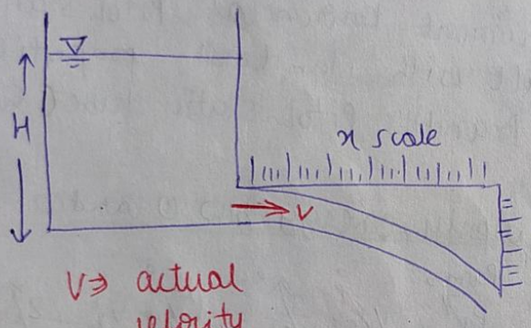
Given, $\left.\begin{matrix} Q \\ Q_a \end{matrix}\right\} = \text{same}$

$$C_d\, a_1 \sqrt{2gh_1} = C_d\, a_2 \sqrt{2gh_2}$$

$$\frac{h_1}{h_2} = \left(\frac{a_2}{a_1}\right)^2$$

$$\frac{h_1}{h_2} = \left[\frac{\frac{\pi}{4}(2)^2}{\frac{\pi}{4}(1)^2}\right]^2 = \frac{16}{1} \text{ Ans}$$

Experimental View:-



V ⇒ actual velocity

Distance travelled in time (t)

In x-dirn

$x = v.t$

In y-dirn

$y = \cancel{u}^{0} + \frac{1}{2}gt^2$

$y = \frac{1}{2}.gt^2 \quad \left\{ t = \frac{x}{v} \right\}$

$y = \frac{1}{2}.g.\frac{x^2}{v^2}$

$V = x\sqrt{\frac{g}{2y}}$

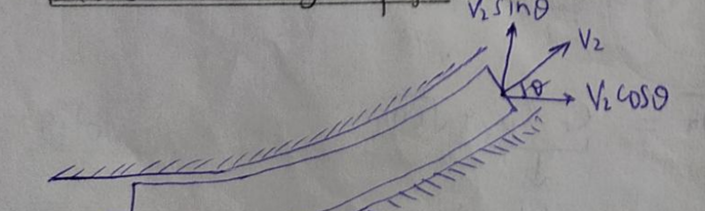
(or)

V_{actual} $\quad V_{theo} = \sqrt{2gH}$

$$C_v = \frac{V_{actual}}{V_{theoretical}} = \frac{x\sqrt{\frac{\cancel{g}}{2y}}}{\sqrt{2\cancel{g}H}} = \frac{x}{2\sqrt{y.H}}$$

$$\boxed{C_v = \frac{x}{2\sqrt{y.H}}}$$

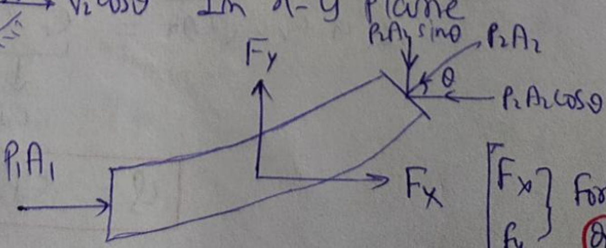
Flow through pipe bend



Force analysis

Fluid-system

In x-y plane



$\left[\begin{matrix} F_x \\ F_y \end{matrix}\right\}$ Force exerted on the fluid

Momentum eqn

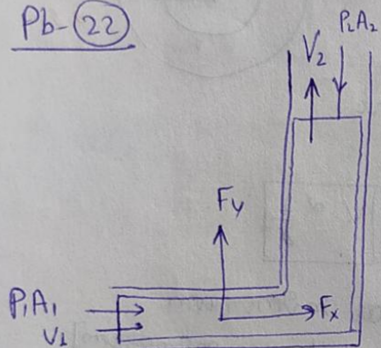
In x- dirn

$$P_1A_1 - P_2A_2\cos\theta + F_x = \underbrace{\dot{m}V_2\cos\theta}_{\text{Final momentum per sec}} - \overbrace{\dot{m}V_1}^{\text{Initial momentum per sec}}$$

In y- dirn.....

$$-P_2A_2\sin\theta + F_y = \dot{m}V_2\sin\theta - \dot{m}(0)$$

Pb-22



$D = 30$ cm

$V_1 = V_2 = 5$ m/s

In Y- dirn

$$-P_2A_2 + F_y = \dot{m}V_2 - \dot{m}(0)$$

$$F_y = \dot{m}(V_2) + P_2A_2$$

$$= \rho . A_2 . V_2^2 + P_2A_2$$

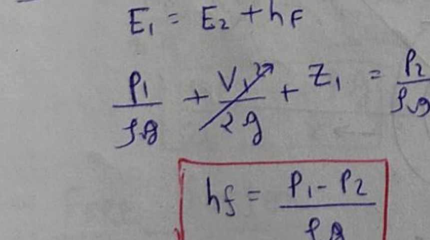
$$= ((10)^3 \times (5)^2 + (4000))\left(\frac{\pi}{4}(0.3)^2\right)$$

$$= 2.05 \text{ KN}$$

In general→

$\boxed{\text{Flow Physic}}$

case-1



$\dot{m}_1 = \dot{m}_2$

$Z_1 = Z_2$

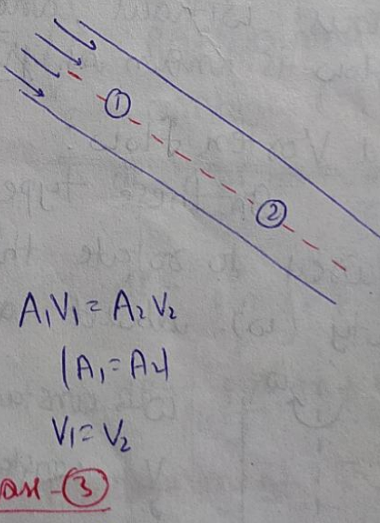
$E_1 > E_2$

$$E_1 = E_2 + h_F$$

$$\frac{P_1}{\rho g} + \cancel{\frac{V_1^2}{2g}} + Z_1 = \frac{P_2}{\rho g} + \cancel{\frac{V_2^2}{2g}} + Z_2 + h_f$$

$$\boxed{h_f = \frac{P_1 - P_2}{\rho g}}$$

case-2 Inclined

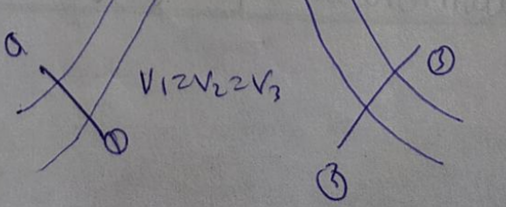


$A_1V_1 = A_2V_2$

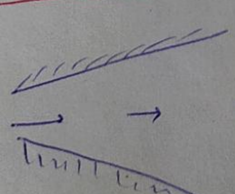
$(A_1 = A_2)$

$V_1 = V_2$

case-3



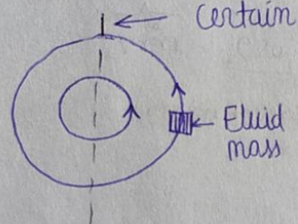
case-4



$\boxed{A\uparrow}$, $\boxed{V\downarrow}$

$\boxed{P\uparrow}$

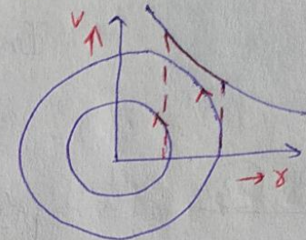
Vortex Flow :-



When a certain mass of fluid is rotated about the particular axis due to some natural effect or external agency, such a flow are known as vortex flows.

Type of vortex flow :-

- Free vortex flow
- forced vortex flow



Free Vortex flow :

$\vec{T}_{external} = 0$

$\frac{d\vec{J}}{dt} = 0$

$\frac{d}{dt}(m.v.r) = 0$

Inte. it

$V.r = \text{Constant}$

$V.r = C$

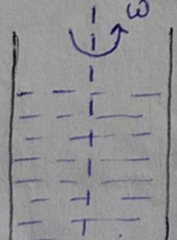
$\boxed{V = \frac{C}{r}}$ or $\boxed{V \propto \frac{1}{r}}$

$\vec{J}$ = angular momentum
[Moment of linear momentum]

⟹ When the fluid mass is rotating about a particular axis without any external torque, then such a fluid flow is known as free vortex Flow

Forced Vortex Flow :

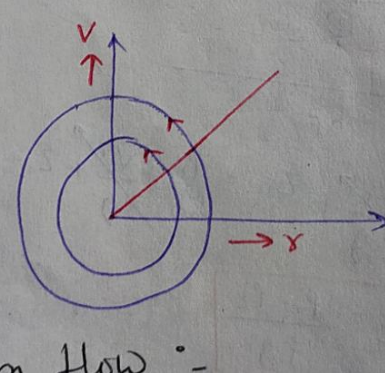
In these type of vortex flow, ~~sum~~ some external torque is required to rotate the fluid mass at a constant angular velocity (ω) about a particular axis



$\omega \to$ constant

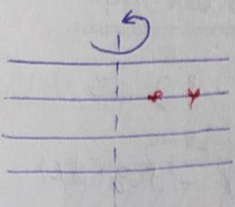
$\frac{V}{r} \to$ constant

$\boxed{V = \omega r}$ or $\boxed{V \propto r}$

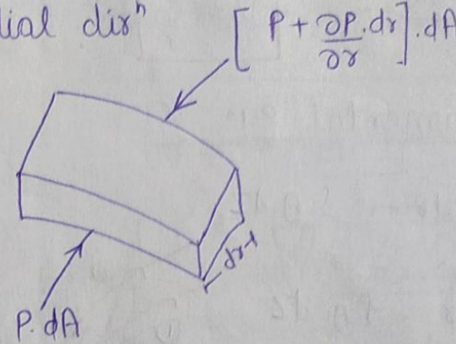


Fundamental eqn of Vortex Flow :-

- Fundamental -



In Radial dir^n



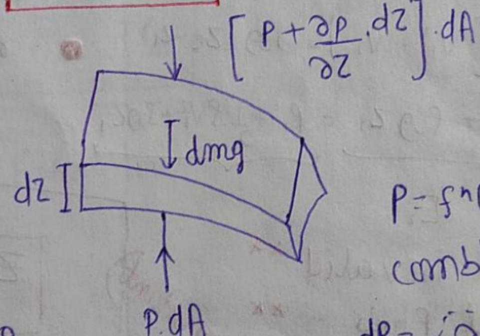
$$\left[\cancel{P} + \frac{\partial P}{\partial r}.dr\right].dA - \cancel{P}\,dA = dm.\frac{V^2}{r}$$

$$\frac{\partial P}{\partial r}.dr.dA = dm.\frac{V^2}{r}$$

$$\frac{\partial P}{\partial r}.dr.\cancel{dA} = \rho\,[dA/dr].\frac{V^2}{r}$$

$$\boxed{\frac{\partial P}{\partial r} = \frac{\rho.V^2}{r}}$$

In Z dir^n



$$P.\cancel{dA} - \left[\cancel{P} + \frac{\partial P}{\partial z}.dz\right].dA - dmg = 0$$

$$-\frac{\partial P}{\partial z}.dz.dA = dm.g$$

$$\frac{\partial P}{\partial z}.dz.dA = -\rho.d\forall.g$$

$$\frac{\partial P}{\partial z}\,dz\,\cancel{dA} = -\rho\,(dA/dz)\,g$$

$$\boxed{\frac{\partial P}{\partial z} = -\rho g}$$

$P = f^n(r)$ and also $f^n(z)$

combine $P = f^n(r, z)$

$$dP = \frac{\partial P}{\partial r}.dr + \frac{\partial P}{\partial z}.dz$$

$$dP = \frac{\rho V^2}{r}.dr + \rho.g.dz$$

$$dP = \frac{\rho V^2}{r}.dr + \rho.g.dz$$

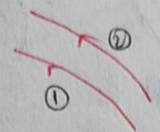
"(Fundamental eq^n of vortex flow)"

- Free Vortex

Free Vortex :- $\boxed{v = \frac{c}{r}}$

By Fundamental eqn

$$dP = \rho \frac{v^2}{r} . dr - \rho . g . dz$$



$$dP = \frac{\rho c^2}{r^3} dr - \rho g . dz$$

int. it

$$\int_1^2 dP = \rho c^2 \int_1^2 \frac{dr}{r^3} - \rho g \int_1^2 dz$$

$$P_2 - P_1 = \frac{\rho c^2 [r^{-2}]_1^2}{(-2)} - \rho g (Z_2 - Z_1)$$

$$P_2 - P_1 = \frac{1}{2} \times \rho \left(\frac{c^2}{r^2}\right)_2^1 - \rho g (Z_2 - Z_1)$$

$$P_2 - P_1 = \frac{1}{2} \rho [V_1^2 - V_2^2] - \rho g (Z_2 - Z_1)$$

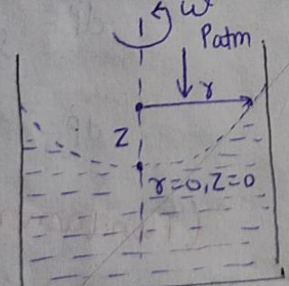
$$\underbrace{P_2 + \frac{1}{2} \rho V_2^2 + \rho g Z_2}_{} = \underbrace{P_1 + \frac{1}{2} \rho V_1^2 + \rho g Z_1}_{}$$

$$E_1 = E_2$$

- Bernaullis is valid **
- Irrotational Flow **

(Important)

- Eqn of free surface in forced vortex



By Fundamental eqn

$$dP = \frac{\rho V^2}{r} . dr - \rho g . dz$$

$[v = \omega r]$

[dP=0 since the pressur is same at all the Points on the free surface]

$$0 = \rho \omega^2 . r . dr - \rho g . dz$$

$$dz = \frac{\omega^2 . r . dr}{g}$$

Forced Vortex

By fundamental eqn

$$dP = \rho \frac{v^2}{r} . dr - \rho g . dz$$

$$dP = \rho (\omega^2 r) dr - \rho g (dz)$$

Int it

$$\int_1^2 dP = \rho \omega^2 \int_1^2 r . dr - \rho g \int_1^2 dz$$

$$P_2 - P_1 = \frac{1}{2} . \rho \omega^2 [r^2]_1^2 - \rho g (Z_2 - Z_1)$$

$$P_2 - P_1 = \frac{1}{2} (\rho) (V_2^2 - V_1^2) - \rho g (Z_2 - Z_1)$$

$$P_2 - \frac{1}{2} \rho V_2^2 + \rho g Z_2 = P_1 - \frac{1}{2} \rho V_1^2 + \rho g Z_1$$

- Bernallie invalid
- rotational Flow

→ Int. it

$$\boxed{Z = \frac{\omega^2 . r^2}{2g} + C}$$ Parabolic

Applying boundary condn

At $r=0, Z=0$ $\quad C=0$

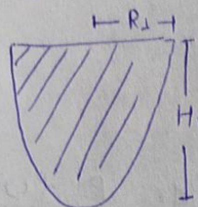
$$\boxed{Z = \frac{\omega^2 . r^2}{2g}}$$

[Shape → Paraboloid]

*****

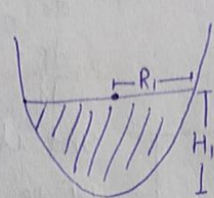
Volume of Paraboloid = $\dfrac{\text{Volume of cylinder of same height}}{2}$

Case-1



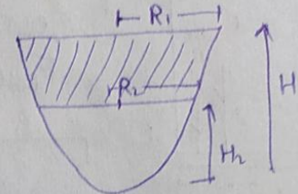
$$\left[V = \frac{\pi R_1^2 . H_1}{2} \right]$$

Case-2



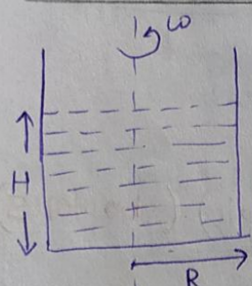
$$\left[V = \frac{\pi R_1^2 H_1}{2} \right]$$

Case-3

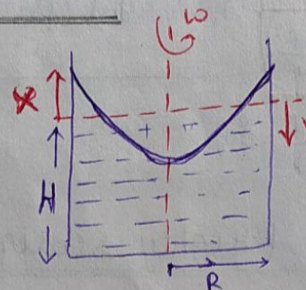


$$V = \left[\frac{\pi (R_1^2 . H_1) - \pi R_2^2 . H_2}{2} \right]$$

Conservation of Volume =



Initial cond^n



Final cond^n

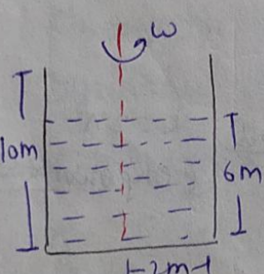
$$V_{initial} = V_{final}$$

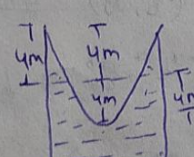
$$\pi R^2 H = \pi R^2 (H+x) - \frac{\pi R^2 (x+y)}{2}$$

$$H = H + x - \frac{x}{2} - \frac{y}{2}$$

$$\frac{x}{2} = \frac{y}{2} = \boxed{x = y}$$

Pb - (1)





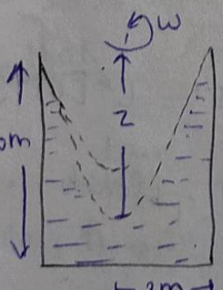
Determine $\omega_{max} = ?$
So that V_{olm} spillout = 0

Sol^n $$Z = \frac{\omega^2 . R^2}{2g}$$

$$4 + 4 = \frac{\omega^2 (2)^2}{2g}$$

$$\omega = 6.26 \text{ rad/s Ans}$$

Pb - (2)



$$Z = \frac{\omega^2 R^2}{2g}$$

$$= \frac{(6.4)^2 (2)^2}{2g}$$

$$= 8.35 m$$

$$V_{final} = \pi (2)^2 \times 10 - \frac{\pi (2)^2 (8.35)}{2}$$

$$= 73.2 \ m^3$$

$$V_{initial} = \pi (2)^2 (6)$$

$$= 75.4 m^3$$

$$V_{spillout} = V_{initial} - V_{final}$$