PRACTICE PAPER 25

MATHEMATICS

Time allowed: 45 minutes **Maximum Marks: 200**

General Instructions: *As given in Practice Paper – 1.*

Section–A

Choose the correct option:

1. **For a matrix A, $AI = A$ and $AA^T = I$ is true**

(*a*) If A is a square matrix (*b*) If A is a non-singular matrix

(*c*) If A is a symmetric matrix (*d*) If A is any matrix

2. **Let** $\Delta = \begin{vmatrix} Ax & x^2 & 1 \\ By & y^2 & 1 \\ Cz & z^2 & 1 \end{vmatrix}$ **and** $\Delta_1 = \begin{vmatrix} A & B & C \\ x & y & z \\ zy & zx & xy \end{vmatrix}$ **then**

(*a*) $\Delta_1 = -\Delta$ (*b*) $\Delta \neq \Delta_1$ (*c*) $\Delta - \Delta_1 = 0$ (*d*) None of these

3. **If A and B are square matrix then $(AB)^{-1}$ is**

(*a*) $B^{-1}A^{-1}$ (*b*) $A^{-1}B^{-1}$ (*c*) B^{-1} (*d*) A^{-1}

4. **If $y = 2^x$ then $\dfrac{d^2y}{dx^2}$ is equal to**

(*a*) $2^x(\log_e 2)^2$ (*b*) $2^x \log_e 2$ (*c*) 2^x (*d*) None of these

5. **Slope of tangent to the curve $y = x^2 - x$ at the point where the line $y = 2$ cuts the curve in the first quadrant is**

(*a*) 2 (*b*) 3 (*c*) -3 (*d*) None of these

6. **The value of $\int \dfrac{dx}{\sqrt{5x^2 - 2x}}$ is**

(*a*) $\dfrac{1}{5}\log\left|x - \dfrac{1}{5} + \sqrt{x^2 - \dfrac{2x}{5}}\right| + C$ (*b*) $\dfrac{1}{\sqrt{5}}\log\left|\left(x - \dfrac{1}{5}\right) + \sqrt{x^2 - \dfrac{2x}{5}}\right| + C$

(*c*) $\dfrac{1}{2\sqrt{5}}\log\left|\left(x + \dfrac{1}{5}\right) + \sqrt{x^2 - \dfrac{x}{5}}\right| + C$ (*d*) None of these

7. **The value of $\int \dfrac{dx}{\sqrt{x^2 - 2x + 3}}$ equals**

(*a*) $\sin^{-1}\left(\dfrac{x-1}{\sqrt{2}}\right) + C$ (*b*) $\log\left|(x-1) + \sqrt{x^2 - 2x + 3}\right| + C$

(*c*) $\dfrac{1}{2}\log\left|(x-1) + \sqrt{x^2 - 2x + 3}\right| + C$ (*d*) $\log\left|(x-1) - \sqrt{x^2 - 2x + 3}\right| + C$

8. The value of $\int_{-1}^{1} \frac{x^3+|x|+1}{x^2+2|x|+1} dx$ is equal to

(a) log 2 (b) 2 log 2 (c) $\frac{1}{2}\log 2$ (d) 4 log 2

9. If f and g are continuous functions in [0, 1] satisfying $f(x) = f(a-x)$ and $g(x) + g(a-x) = a$, then $\int_0^a f(x) \cdot g(x)\, dx$ is equal to

(a) $\frac{a}{2}$ (b) $\frac{a}{2}\int_0^a f(x)dx$ (c) $\int_0^a f(x)dx$ (d) $a\int_0^a f(x)dx$

10. The area of the region bounded by the curve $y = \sin x$ between the ordinates $x = 0, x = \frac{\pi}{2}$ and the x-axis is

(a) 2 sq. units (b) 4 sq. units (c) 3 sq. units (d) 1 sq. unit

11. $y = ae^{mx} + be^{-mx}$ satisfies the differential equation

(a) $\frac{dy}{dx} + my = 0$ (b) $\frac{dy}{dx} - my = 0$ (c) $\frac{d^2y}{dx^2} - m^2y = 0$ (d) $\frac{d^2y}{dx^2} + m^2y = 0$

12. The order and degree of differential equation $\left[1+\left(\frac{dy}{dx}\right)^2\right] = \frac{d^2y}{dx^2}$ are

(a) $2, \frac{3}{2}$ (b) 2, 3 (c) 2, 1 (d) 3, 4

13. The maximum value of $Z = x + 3y$ such that $2x + y \le 20$, $x + 2y \le 20$, $x \ge 0$, $y \ge 0$ is

(a) 10 (b) 30 (c) 60 (d) $\frac{80}{3}$

14. For the following distribution :

X	-4	-3	-2	-1	0
$P(X)$	0.1	0.2	0.3	0.2	0.2

$E(X)$ is equal to

(a) -1 (b) -3 (c) -2 (d) -1.8

15. If the probability that a person is not a swimmer is 0.3, then the probability that out of 5 persons 4 are swimmers is

(a) ${}^5C_4\,(0.7)^4\,(0.3)$ (b) ${}^5C_1\,(0.7)\,(0.3)^4$ (c) ${}^5C_4\,(0.7)\,(0.3)^4$ (d) $(0.7)^4\,(0.3)$

Section–B (B1)

16. The maximum number of equivalence relations on the set $A = \{1, 2, 3\}$ are

(a) 1 (b) 2 (c) 3 (d) 5

17. If $f : N \to S$ and given by $f(x) = 2x^2 - 4x + 7$ is an invertible function then $f^{-1}(x)$ is equal to

(a) $\sqrt{\frac{x+5}{2}} + 1, x \ge 5$ (b) $\sqrt{\frac{x-2}{5}} + 1$ (c) $\sqrt{\frac{x-5}{2}} + 1, x \ge 5$ (d) None of these

18. If $f : R \to R$ defined by $f(x) = \frac{3x+5}{2}$ is an invertible function, then $f^{-1}(1)$ is equal to

(a) -1 (b) 1 (c) 2 (d) ± 1

19. If * is a binary operation on $\mathbb{R}$ given by $a * b = a^b$ then 2 * 3 is equal to

(a) 8 (b) 9 (c) 2^6 (d) None of these

20. If $Y = \{n^2 : n \in N\} \subset N$. Consider $f : N \to Y$ as $f(x) = x^2$ then inverse of f is equal to

(a) $\sqrt{y}$ (b) y (c) $(y)^{3/2}$ (d) None of these

21. The value of $\tan\left(\frac{1}{2}\cos^{-1}\frac{2}{\sqrt{5}}\right)$ is

(*a*) $2+\sqrt{5}$ (*b*) $\sqrt{5}-2$ (*c*) $\frac{\sqrt{5}+2}{2}$ (*d*) $5+\sqrt{2}$

22. The principal value of $\sin^{-1}\frac{1}{2}$ is

(*a*) $\frac{\pi}{6}$ (*b*) $\frac{5\pi}{6}$ (*c*) $\frac{-\pi}{6}$ (*d*) Both (*a*) & (*b*)

23. The value of $\sin^{-1}\left[-\left(\frac{1}{2}\right)\right]+\cos^{-1}\left[-\left(\frac{1}{2}\right)\right]+\cot^{-1}(-\sqrt{3})+\operatorname{cosec}^{-1}(\sqrt{2})+\tan^{-1}(-1)+\sec^{-1}(\sqrt{2})$ equals

(*a*) $\frac{9\pi}{4}$ (*b*) $\frac{19\pi}{12}$ (*c*) $\frac{3\pi}{2}$ (*d*) $\frac{\pi}{2}$

24. The value of $\tan^{-1}\left(\frac{x}{y}\right)-\tan^{-1}\left(\frac{x-y}{x+y}\right)$ is equal to

(*a*) $\frac{\pi}{2}$ (*b*) $\frac{\pi}{3}$ (*c*) $\frac{\pi}{4}$ (*d*) $\frac{\pi}{4}$ or $\left(\frac{-3\pi}{4}\right)$

25. If A is a skew - symmetric matrix of order n, and C is a column matrix of order $n \times 1$, then $C'AC$ is

(*a*) An identity matrix of order n (*b*) A unit matrix of order one

(*c*) A zero matrix of order one (*d*) None of these

26. For the matrix $A=\begin{bmatrix}3 & 1\\ 7 & 5\end{bmatrix}$, value of x and y so that $A^2+xI=yA$ are

(*a*) (8, 8) (*b*) (– 8, 0) (*c*) (–8, –8) (*d*) None of these

27. If $x, y \in R$, then the determinant $\Delta=\begin{vmatrix}\cos x & -\sin x & 1\\ \sin x & \cos x & 1\\ \cos(x+y) & -\sin(x+y) & 0\end{vmatrix}$ lies in the interval

(*a*) $[-\sqrt{2}, \sqrt{2}]$ (*b*) $[-1, 1]$ (*c*) $[-\sqrt{2}, 1]$ (*d*) $[-1, -\sqrt{2}]$

28. If $A=\begin{bmatrix}1 & -1 & 1\\ 2 & -1 & 0\\ 1 & 0 & 0\end{bmatrix}$ then the value of A^{-1} is

(*a*) A^3 (*b*) A^2 (*c*) A (*d*) 1

29. If $f(x)$ is everywhere differentiable, then the values of a and b if $f(x)=\begin{cases}x^2+3x+a, & \text{for } x \le 1\\ bx+2, & \text{for } x>1\end{cases}$ is

(*a*) $a=3, b=5$ (*b*) $a=0, b=5$ (*c*) $a=0, b=3$ (*d*) $a=3, b=3$

30. If $x\sqrt{1+y}+y\sqrt{1+x}=0$ and $x \neq y$, then the value of $\frac{dy}{dx}$ is

(*a*) $\frac{-1}{(1+x)^2}$ (*b*) $\frac{1}{(1+x)^2}$ (*c*) $\frac{1}{(1-x)^2}$ (*d*) $\frac{-1}{(1-x)^2}$

31. If $x=a\sec^3\theta$ and $y=a\tan^3\theta$, then $\frac{dy}{dx}$ at $\theta=\frac{\pi}{3}$ is

(*a*) $\frac{2}{\sqrt{3}}$ (*b*) $\frac{1}{\sqrt{3}}$ (*c*) $\sqrt{3}$ (*d*) $\frac{\sqrt{3}}{2}$

32. The function $f(x)=\frac{4-x^2}{4x-x^3}$ is

(*a*) discontinuous at only one point (*b*) discontinuous at exactly two points

(*c*) discontinuous at exactly three points (*d*) none of these

33. The values of a for which the function $f(x) = \sin x - ax + b$ increases on R are

(a) $(-\infty, \infty)$ (b) $[-1, 1]$ (c) $(-\infty, -1)$ (d) none of these

34. The value of $\int \frac{\cos^2 x - \cos 2x}{1 - \cos x} dx$ equals

(a) $x + \sin x + C$ (b) $x - 2\sin x + C$ (c) $-x + 2\cos x + C$ (d) $x - 2\cos x + C$

35. If $x = \int_0^y \frac{dt}{\sqrt{1+9t^2}}$ and $\frac{d^2y}{dx^2} = ay$, then a is equal to

(a) 3 (b) 6 (c) 9 (d) 1

36. The integral of $\int \frac{x}{\sqrt{x}+1} dx$ is equal to

(a) $2\left[\frac{x\sqrt{x}}{3} - \frac{x}{2} + \sqrt{x} - \log\left|(\sqrt{x}+1)\right|\right] + C$ (b) $\frac{x\sqrt{x}}{3} + \frac{x}{2} - \sqrt{x} + \log(\sqrt{x}+1) + C$

(c) $\sqrt{x} - \log(\sqrt{x}+1) + C$ (d) None of these

37. The area of the region bounded by the circle $x^2 + y^2 = 1$ is

(a) 2π sq. units (b) π sq. units (c) 3π sq. units (d) 4π sq. units

38. The general solution of differential equation $\frac{dy}{dx} = e^{\frac{x^2}{2}} + xy$ is

(a) $y = Ce^{\frac{-x^2}{2}}$ (b) $y = Ce^{\frac{x^2}{2}}$ (c) $y = (x + C)e^{\frac{x^2}{2}}$ (d) $y = (C - x)e^{\frac{x^2}{2}}$

39. The general solution of $\frac{dy}{dx} + y\tan x = \sec x$ is

(a) $y\sec x = \tan x + C$ (b) $y\tan x = \sec x + C$

(c) $\tan x = y\tan x + C$ (d) $x\sec x = \tan y + C$

40. If the points P, Q, R, S have position vector $\vec{p}, \vec{q}, \vec{r}, \vec{s}$ respectively such that $\vec{p} - \vec{q} = 2(\vec{r} - \vec{s})$, then

(a) PQ and RS bisect each other (b) PQ and RS trisect each other

(c) PS and QR trisect each other (d) QS and PR bisect each other

41. The two vectors $\hat{j} + \hat{k}$ and $3\hat{i} - \hat{j} + 4\hat{k}$ represents the two sides AB and AC, respectively of ΔABC. The length of the median through A is

(a) $\sqrt{34}/2$ (b) $\sqrt{21}$ (c) $\sqrt{48}/2$ (d) 4

42. If $|\vec{a}| = 8, |\vec{b}| = 3$ and $|\vec{a} \times \vec{b}| = 12$, then value of $\vec{a} \cdot \vec{b}$ is

(a) $\sqrt{3}$ (b) $12\sqrt{3}$ (c) $6\sqrt{3}$ (d) None of these

43. Vector in the direction of $\vec{a} = 3\hat{i} + 4\hat{j} - 5\hat{k}$ that has magnitude $\frac{\sqrt{2}}{5}$ units is

(a) $\frac{3\hat{i} + 4\hat{j} - 5\hat{k}}{25}$ (b) $\frac{3\hat{i} - 4\hat{j} + 5\hat{k}}{25}$ (c) $\frac{-3\hat{i} + 4\hat{j} + 5\hat{k}}{25}$ (d) $\frac{3\hat{i} - 4\hat{j} - 5\hat{k}}{25}$

44. If the plane passes through the points (2, 0, 0), (0, 3, 0) and (0, 0, 4), the equation of plane is

(a) $6x + 4y + 3z = 12$ (b) $6x - 4y + 3z = 12$ (c) $4x + 6y + 3z = 12$ (d) none of these

45. The direction cosines of the vector $(2\hat{i} + 2\hat{j} - \hat{k})$ are

(a) $\frac{2}{3}, \frac{1}{3}, \frac{-1}{3}$ (b) $\frac{2}{3}, \frac{2}{3}, \frac{-1}{3}$ (c) $\frac{-1}{3}, \frac{-1}{3}, \frac{2}{3}$ (d) none of these

46. The vector equation of the line through the points (3, 4, – 7) and (1, – 1, 6) is

(a) $\vec{r} = 3\hat{i} + 4\hat{j} - 7\hat{k} + \lambda(3\hat{i} + 7\hat{j} + 2\hat{k})$ (b) $\vec{r} = (3\hat{i} + 4\hat{j} - 7\hat{k}) + \lambda(-2\hat{i} - 5\hat{j} + 13\hat{k})$

(c) $\vec{r} = 3\hat{i} + 4\hat{j} - 7\hat{k} + \lambda(3\hat{i} - 7\hat{j} + 5\hat{k})$ (d) $\vec{r} = (3\hat{i} + 4\hat{j} - 7\hat{k}) + \lambda(2\hat{i} + 5\hat{j} - 13\hat{k})$

47. **The cartesian equation of the plane $\vec{r}.(\hat{i}+\hat{j}-\hat{k})=2$ is**

(*a*) $x+y+z=0$ (*b*) $x+y-z=2$ (*c*) $x-y-z=7$ (*d*) $x-y+z=0$

48. **The probability of guessing correctly atleast 8 out of 10 answers on a true false type examination is**

(*a*) $\frac{7}{64}$ (*b*) $\frac{7}{128}$ (*c*) $\frac{145}{1024}$ (*d*) $\frac{17}{41}$

49. **In a college, 30% students fail in Physics, 25% fail in Mathematics and 10% fail in both. One student is chosen at random. The probability that she fails in Physics, if she has failed in Mathematics is**

(*a*) $\frac{3}{10}$ (*b*) $\frac{2}{5}$ (*c*) $\frac{7}{90}$ (*d*) $\frac{1}{3}$

50. **10% of the bulbs produced in a factory are of red colour and 2% are red and defective. If one bulb is picked up at random, the probability of its being defective, if it is red is**

(*a*) $\frac{1}{7}$ (*b*) $\frac{20}{27}$ (*c*) $\frac{1}{5}$ (*d*) $\frac{2}{5}$