Chapter 3. Trigonometric Functions

Question-1

Find the radian measure corresponding to the following degree measures:

a) 15^{o}
b) $-37^{0}\ 30'$
c) 240^{0}
d) 530^{0}

Solution:

a) $1^{o} = \frac{\pi}{180}$ radian

$15^{o} = \frac{\pi}{180} \times 15^{o} = \frac{\pi}{12}$ radian

b) $1^{o} = \frac{\pi}{180}$ radian

$-37^{o}30' = \frac{\pi}{180} \times -37.5^{o} = \frac{-5\pi}{24}$ radian

c) $1^{o} = \frac{\pi}{180}$ radian

$240^{o} = \frac{\pi}{180} \times 240 = \frac{4\pi}{3}$ radian

d) $1^{o} = \frac{\pi}{180}$ radian

$530^{0} = \frac{\pi}{180} \times 530 = \frac{53}{18}\pi$ radian

Question-2

Find the degree measure corresponding to the following radian measures:

a) $\frac{3}{4}$
b) -4
c) $\frac{5\pi}{3}$
d) $\frac{7\pi}{6}$

Solution:

a)1 radian $= \frac{180^{o}}{\pi}$

$\frac{3}{4}$ radian $= \frac{180^{o}}{\pi} \times \frac{3}{4} = \frac{135^{o}}{\pi} = \frac{135^{o}}{22} \times 7 = 42.95 = 42^{o}57'$

b)1 radian $= \frac{180^{o}}{\pi}$

-4radian $= \frac{180^{o}}{\pi} \times -4 = \frac{-720}{22} \times 7 = -32.72 \times 7 = -229^{o}5'24''$

c)1 radian $= \frac{180^{o}}{\pi}$

$\frac{5\pi}{3}$ radian $= \frac{180^{o}}{\pi} \times \frac{5\pi}{3} = 300^{0}$

d) 1 radian = $\frac{180^o}{\pi}$

$\frac{7\pi}{6}$ radian = $\frac{180}{\pi} \times \frac{7\pi}{6}$ = 210^o

Question-3

A wheel makes 360 revolutions in one minute. Through how many radians does it turn is one second?

Solution:

One complete revolution = 2π360 revolutions = 360 × 2π= 720π radians / minute = 720π /60 radians/sec = 12π radians/sec.

Question-4

Find the degree measure of the angle subtended at the centre of a circle of diameter 200cm by an arc of length 22cm.

Solution:

Radius (r) = (200/2)cm = 100cm

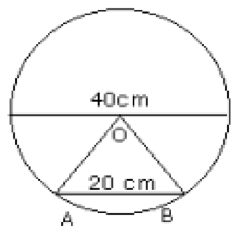
Length of an arc (*l*) = 22cm

θ = *l*/r = 22/100radians = $\frac{22}{100} \times \frac{180}{\pi}$ = $12^o36'$

Question-5

In a circle of diameter 40cm. The length of a chord is 20cm. Find the length of minor arc corresponding to the chord.

Solution:



Question-6

If, in two circles, arcs of the same length subtend angles of 60^o and 75^o at the centre, find the ratio of their radii.

Solution:

Let the radii be r_1 and r_2. Let the angles subtend by the arcs in two circles be θ_1 and θ_2.

$l = \theta r$ where θ is the angle, l the length of an arc and r the radius of the circle.

$\theta_1 = 60^o = 60 \times \frac{\pi}{180} = \frac{\pi}{3}$ radian

$\theta_2 = 75^o = 75 \times \frac{\pi}{180} = \frac{5\pi}{12}$ radian

$\frac{\pi}{3} r_1 = \frac{5\pi}{12} r_2$

$r_1 / r_2 = \frac{\frac{5}{12}}{\frac{1}{3}} = \frac{5}{4}$

Therefore the required ratio is 5:4.

Question-7

Find the angle in radian through which a pendulum swings if its length is 75cm and the tip described an arcs of length

(i) 10cm

(ii) 15cm

(iii) 21cm

Solution:

(i) Length of the pendulum (r) = 75cm

Length of an arc (l) = 10cm

$\theta = l/r = 10/75$ radians $= 2/15$ radians

(ii) Length of the pendulum (r) = 75cm

Length of an arc (l) = 15cm

$\theta = l/r = 15/75$ radians $= 1/5$ radians

(iii) Length of the pendulum (r) = 75cm

Length of an arc (l) = 21cm

$\theta = l/r = 21/75$ radians $= 7/25$ radians

Question-8

Find the values of the other five trigonometric functions in the following problem: cosθ = -1/2, is quadrant III

Solution:

cosθ and secθ are negative; tan θ and cot θ are positive.

$\cos\theta = -\frac{1}{2}$

$\sin^2\theta = 1 - \frac{1}{4} = \frac{3}{4}$

$\therefore \sin\theta = -\frac{\sqrt{3}}{2}$

$\therefore \tan\theta = \dfrac{\frac{-\sqrt{3}}{2}}{\frac{-1}{2}} = \sqrt{3}$

$\therefore \text{cosec}\,\theta = -\dfrac{1}{\frac{\sqrt{3}}{2}} = -\frac{2}{\sqrt{3}}$

$\sec\theta = \dfrac{1}{\frac{-1}{2}} = -2$

$\cot\theta = \dfrac{\frac{-1}{2}}{\frac{-\sqrt{3}}{2}} = \frac{1}{\sqrt{3}}$

Question-9

Find the values of the other five trigonometric functions in the following problem: sin $\theta - \frac{3}{5}, \theta$ is quadrant II

Solution:

sinθ and cosecθ positive; tanθ , cosθ , cotθ and secθ all are negative.

$\sin^2\theta + \cos^2\theta = 1$

$\cos^2\theta = 1 - \sin^2\theta = 1 - \left(\frac{3}{5}\right)^2 = \frac{25-9}{25} = \frac{16}{25}$

$\cos\theta = -\frac{4}{5}$

$\therefore \tan\theta = \dfrac{\frac{3}{5}}{\frac{4}{5}} = -\frac{3}{4}$

$\cot\theta = \dfrac{\frac{4}{5}}{\frac{3}{5}} = -\frac{4}{3}$

$\text{cosec}\,\theta = \frac{1}{\sin\theta} = \dfrac{1}{\frac{3}{5}} = \frac{5}{3}$

$\sec\theta = \frac{1}{\cos\theta} = \dfrac{1}{-4/5} = -5/4$

Question-10

Find the values of the other five trigonometric functions in the following problem: tanθ = $\frac{3}{4}$, θ is quadrant III

Solution:

sinθ , cosθ , cosecθ and secθ are negative in III rd quadrant tanθ and cotθ are positive in IIIrd quadrant.

$1+\tan^2\theta = \sec^2\theta$ $1+\frac{\sin^2\theta}{\cos^2\theta} = \frac{1}{\cos^2\theta}$

$1+\left(\frac{3}{4}\right)^2 = \sec^2\theta$

$\frac{16+9}{16} = \frac{25}{16} = \sec^2\theta$

$\sec\theta = \frac{-5}{4}$

$\cos\theta = \frac{-4}{5}$

$\sin^2\theta = 1-\left(\frac{4}{5}\right)^2 = \frac{9}{25}$

$\therefore \sin\theta = \frac{-3}{5}$

$\text{cosec}\,\theta = \frac{-5}{3}$

$\cot\theta = \frac{4}{3}$

Question-11

Find the values of the other five trigonometric functions in the following problem: sec θ = 13/5, θ lies in fourth quadrant.

Solution:

sin θ , tan θ , cosec θ and cot θ are negative in IV th quadrant sec θ and cos θ are positive in IVth quadrant.

sec θ = 13/5

cos θ = 1/sec θ = 5/13

$\sin^2\theta = 1-\left(\frac{5}{13}\right)^2 - \frac{144}{169}$

$\therefore \sin\theta = \frac{-12}{13}$

$\text{cosec}\,\theta = \frac{-13}{12}$

$\therefore \tan\theta = \frac{\frac{-12}{13}}{\frac{5}{13}} = -\frac{12}{5}$

$\cot\theta = \frac{\frac{5}{13}}{-\frac{12}{13}} = -\frac{5}{12}$

Question-12

Find the value of the following trigonometric function: sin 765°

Solution:

$\sin 765^\circ = \sin \frac{17\pi}{4} = \sin\left(4\pi + \frac{\pi}{4}\right) = \sin \frac{\pi}{4} = \frac{1}{\sqrt{2}}$

Question-13

Find the value of the following trigonometric function: cosec(-1410°)

Solution:

$\text{cosec}(-1410^\circ) = -\text{cosec}\, 1410^\circ = -\text{cosec}\,(8\pi - 30^\circ) = \text{cosec}\, 30^\circ = 2$

Question-14

Find the value of the following trigonometric function: tan $\frac{13\pi}{3}$

Solution:

$\tan \frac{13\pi}{3} = \tan\left(4\pi + \frac{\pi}{3}\right) = \tan \frac{\pi}{3} = \sqrt{3}$

Question-15

Find the value of the following trigonometric function: cot$\left(-\frac{15\pi}{4}\right)$

Solution:

$\cot\left(-\frac{15\pi}{4}\right) = -\cot\left(\frac{15\pi}{4}\right) = -\cot\left(4\pi - \frac{\pi}{4}\right) = \cot \frac{\pi}{4} = 1$

Question-16

Prove that: $\sin^2\frac{\pi}{6} + \cos^2\frac{\pi}{3} - \tan^2\frac{\pi}{4} = -\frac{1}{2}$

Solution:

L.H.S $= \sin^2\frac{\pi}{6} + \cos^2\frac{\pi}{3} - \tan^2\frac{\pi}{4}$

$= \frac{1}{4} + \frac{1}{4} - 1$

$= \frac{1}{2} - 1$

$= -\frac{1}{2}$ = R.H.S

Question-17

Prove that: $2\sin^2\frac{\pi}{6} + \text{cosec}\frac{7\pi}{6}\cos^2\frac{\pi}{3} = 0$

Solution:

L.H.S $= 2\sin^2\frac{\pi}{6} + \text{cosec}\frac{7\pi}{6}\cos^2\frac{\pi}{3}$

$= 2\times\frac{1}{4} + \text{cosec}\left(\pi+\frac{\pi}{6}\right)\times\frac{1}{4}$

$= \frac{1}{2} - \text{cosec}\frac{\pi}{6}\times\frac{1}{4}$

$= \frac{1}{2} - 2\times\frac{1}{4}$

$= 0 =$ R.H.S

Question-18

Prove that: $3\cos^2\frac{\pi}{4} + \sec\frac{2\pi}{3} + 5\tan^2\frac{\pi}{3} = \frac{29}{2}$

Solution:

L.H.S $= 3\cos^2\frac{\pi}{4} + \sec\left(\pi-\frac{\pi}{3}\right) + 5\tan^2\frac{\pi}{3}$

$= 3\times\frac{1}{2} - \sec\frac{\pi}{3} + 5\tan^2\frac{\pi}{3}$

$= \frac{3}{2} - 2 + 5\times(\sqrt{3})^2$

$= \frac{3}{2} - 2 + 15$

$= 13 + \frac{3}{2}$

$= \frac{29}{2}$

$=$ R.H.S

Question-19

Prove that: $\cot^2\frac{\pi}{6} + \text{cosec}\frac{5\pi}{6} + 3\tan^2\frac{\pi}{6} = 6$

Solution:

L.H.S $= \cot^2\frac{\pi}{6} + \text{cosec}\frac{5\pi}{6} + 3\tan^2\frac{\pi}{6}$

$= \cot^2\frac{\pi}{6} + \text{cosec}\left(\pi-\frac{\pi}{6}\right) + 3\tan^2\frac{\pi}{6}$

$= \cot^2\frac{\pi}{6} + \text{cosec}\frac{\pi}{6} + 3\tan^2\frac{\pi}{6}$

$= (\sqrt{3})^2 + 2 + 3\left(\frac{1}{\sqrt{3}}\right)^2$

$= 3 + 2 + 1$

$= 6$

$=$ R.H.S

Question-20

Prove that: $2\sin^2\frac{3\pi}{4} + 2\cos^2\frac{\pi}{4} + 2\sec^2\frac{\pi}{3} = 10$

Solution:

L.H.S $= 2\sin^2\frac{3\pi}{4} + 2\cos^2\frac{\pi}{4} + 2\sec^2\frac{\pi}{3}$

$= 2\sin^2\left(\pi - \frac{\pi}{4}\right) + 2\cos^2\frac{\pi}{4} + 2\sec^2\frac{\pi}{3}$

$= 2\sin^2\frac{\pi}{4} + 2\cos^2\frac{\pi}{4} + 2\sec^2\frac{\pi}{3}$

$= 2\times \frac{1}{2} + 2\times \frac{1}{2} + 2\times 4$

$= 1 + 1 + 8$

$= 10$

= R.H.S

Question-21

Show that:

$\cos 70^o \cos 10^o + \sin 70^o \sin 10^o = ½$

Solution:

We know that

$\cos(\theta - \varphi) = \cos\theta \cos\varphi + \sin\theta \sin\varphi$ ∴ L.H.S $= \cos 70^o \cos 10^o + \sin 70^o \sin 10^o$

$= \cos(70^o - 10^o)$

$= \cos 60^o$

$= 1/2$

Question-22

Show that:

$\cos 130^o \cos 40^o + \sin 130^o \sin 40^o = 0$

Solution:

We know that

$\cos(\theta - \varphi) = \cos\theta \cos\varphi + \sin\theta \sin\varphi$ ∴ L.H.S $= \cos 130^o \cos 40^o + \sin 130^o \sin 40^o$

$= \cos(130^o - 40^o)$

$= \cos 90^o$

$= 0$

Question-23

Show that:

$$\sin(40^\circ + \theta)\cos(10^\circ + \theta) - \cos(40^\circ + \theta)\sin(10^\circ + \theta) = ½$$

Solution:

We know that

$\sin(\theta - \varphi) = \sin\theta\cos\varphi - \cos\theta\sin\varphi$L.H.S $= \sin(40^\circ + \theta)\cos(10^\circ + \theta) - \cos(40^\circ + \theta)\sin(10^\circ + \theta)$

$= \sin 30^\circ$

$= 1/2$

= R.H.S

Question-24

Prove that:

$$\cos\left(\frac{\pi}{4}-\theta\right)\cos\left(\frac{\pi}{4}-\phi\right) - \sin\left(\frac{\pi}{4}-\theta\right)\sin\left(\frac{\pi}{4}-\phi\right) = \sin(\theta + \varphi)$$

Solution:

We know that

$\cos(\theta + \varphi) = \cos\theta\cos\varphi - \sin\theta\sin\varphi$L.H.S $= \cos\left(\frac{\pi}{4}-\theta\right)\cos\left(\frac{\pi}{4}-\phi\right) - \sin\left(\frac{\pi}{4}-\theta\right)\sin\left(\frac{\pi}{4}-\phi\right)$

$= \cos\left(\frac{\pi}{4}-\theta+\frac{\pi}{4}-\phi\right)$

$= \cos\left(\frac{\pi}{2}-(\theta+\phi)\right)$

$= \sin(\theta + \varphi)$

= R.H.S

Question-25

Prove that:

$$\frac{\tan\left(\frac{\pi}{4}+x\right)}{\tan\left(\frac{\pi}{4}-x\right)} = \left(\frac{1+\tan x}{1-\tan x}\right)^2$$

Solution:

$$\text{L.H.S} = \frac{\tan\left(\frac{\pi}{4}+x\right)}{\tan\left(\frac{\pi}{4}-x\right)}$$

$$= \frac{\dfrac{\tan\frac{\pi}{4}+\tan x}{1-\tan\frac{\pi}{4}\tan x}}{\dfrac{\tan\frac{\pi}{4}-\tan x}{1+\tan\frac{\pi}{4}\tan x}}$$

$$= \frac{\dfrac{1+\tan x}{1-\tan x}}{\dfrac{1-\tan x}{1+\tan x}}$$

$$= \left(\frac{1+\tan x}{1-\tan x}\right)^2$$

= R.H.S

Question-26

Prove that :

$$\frac{\cos(\pi+\theta)\cos(-\theta)}{\sin(\pi-\theta)\cos\left(\frac{\pi}{2}+\theta\right)} = \cot^2\theta$$

Solution:

L.H.S = $\frac{\cos(\pi+\theta)\cos(-\theta)}{\sin(\pi-\theta)\cos\left(\frac{\pi}{2}+\theta\right)}$

$= \frac{(-\cos\theta)\cos\theta}{\sin\theta(-\sin\theta)}$

$= \cot^2\theta$ = R.H.S

Question-27

$\cos\theta + \sin(270^o + \theta) - \sin(270^o - \theta) + \cos(180^o + \theta) = 0$.

Solution:

L.H.S = $\cos\theta + \sin(270^o + \theta) - \sin(270^o - \theta) + \cos(180^o + \theta)$

$= \cos\theta - \cos\theta + \cos\theta - \cos\theta = 0$ = R.H.S

Question-28

$\cos\left(\frac{3\pi}{2}+\theta\right)\cos(2\pi + \theta)\left[\cot\left(\frac{3\pi}{2}-\theta\right) + \cot(2\pi+\theta)\right] = 1$.

Solution:

L.H.S = $\cos\left(\frac{3\pi}{2}+\theta\right)\cos(2\pi + \theta)\left[\cot\left(\frac{3\pi}{2}-\theta\right) + \cot(2\pi+\theta)\right]$

$= (-\sin\theta)(-\cos\theta)(\tan\theta + \cot\theta)$

$= \sin\theta\cos\theta(\tan\theta + \cot\theta)$

$= \sin^2\theta + \cos^2\theta = 1$ = R.H.S

Question-29

$\sin(n + 1)x \sin(n + 2)x + \cos(n + 1)x\cos(n + 2)x = \cos x$.

Solution:

L.H.S = $\sin(n + 1)x \sin(n + 2)x + \cos(n + 1)x\cos(n + 2)x$

$= \cos[(n + 1)x - (n + 2)x]$

$= \cos(-x)$

$= \cos x$

= RH.S

Question-30

Find the value of:

(i) cos 210^o

(ii) sin 225^o

(iii) tan 330^o

(iv) cot (-315^o)

Solution:

(i)cos 210^o = cos (180^o + 30^o) = - cos 30^o = - $\frac{\sqrt{3}}{2}$

(ii) sin 225^o = sin (270^o + 45^o) = cos 45^o = -$\frac{1}{\sqrt{2}}$

(iii) tan 330^o = tan (360^o - 30^o) = - tan 30^o = -$\frac{1}{\sqrt{3}}$

(iv) cot (-315^o) = cot (315^o) = cot (360^o – 45^o) = cot 45^o = 1

Question-31

Find the value of tan(α + β), given that

cot α = ½, α ∈ $\left(\pi, \frac{3\pi}{2}\right)$ and sec β = -5/3, β ∈ $\left(\frac{\pi}{2}, \pi\right)$

Solution:

cot α = ½, α ∈ $\left(\pi, \frac{3\pi}{2}\right)$

and sec β = -5/3, β ∈ $\left(\frac{\pi}{2}, \pi\right)$

tanβ = -4/3

tan(α + β) = $\frac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta}$

$= \frac{2 - \frac{4}{3}}{1 - 2\times\frac{-4}{3}}$

$= \frac{\frac{2}{3}}{\frac{11}{3}}$

= 2/11

Question-32

Prove the following identity:

$\sin(150^\circ + x) + \sin(150^\circ - x) = \cos x$

Solution:

We know that

$2\sin\theta \cos\varphi = \sin(\theta + \varphi) + \sin(\theta - \varphi)$

L.H.S = $\sin(150^\circ + x) + \sin(150^\circ - x)$

$= 2\sin 150^\circ \cos x$

$= 2\sin(180^\circ - 30^\circ) \cos x$

$= 2\sin 30^\circ \cos x$

$= 2(1/2)\cos x$

$= \cos x$

= R.H.S

Question-33

Prove the following identity:

$\cos\left(\frac{3\pi}{4} + x\right) - \cos\left(\frac{3\pi}{4} - x\right) = -\sqrt{2}\sin x$

Solution:

We know that

$-2\sin\theta \sin\varphi = \cos(\theta + \varphi) - \cos(\theta - \varphi)$

L.H.S = $-2\sin\frac{3\pi}{4}\sin x = -2\sin\left(\pi - \frac{\pi}{4}\right)\cos x = -2\sin\frac{\pi}{4}\sin x = -2\times\frac{1}{\sqrt{2}}\sin x = -\sqrt{2}\sin x$ =

R.H.S

Question-34

Prove the following identity:

$\cos\left(\frac{\pi}{4} + x\right) + \cos\left(\frac{\pi}{4} - x\right) = \sqrt{2}\cos x$

Solution:

We know that

$2\cos\theta \cos\varphi = \cos(\theta + \varphi) + \cos(\theta - \varphi)$

L.H.S = $2\cos\frac{\pi}{4}\cos x = 2\times\frac{1}{\sqrt{2}}\cos x = \sqrt{2}\cos x$ = R.H.S

Question-35

$\sin 2x + 2\sin 4x + \sin 6x = 4\cos^2 x \sin 4x$

Solution:

L.H.S = $\sin 2x + 2\sin 4x + \sin 6x$

$= 2\sin\frac{6x+2x}{2}\cos\frac{6x-2x}{2} + 2\sin 4x$

$= 2\sin 4x\cos 2x + 2\sin 4x$

$= 2\sin 4x(\cos 2x + 1)$

$= 4\sin 4x\cos^2 x$

= R.H.S

Question-36

$\sin^2 6x - \sin^2 4x = \sin 2x \sin 10x$

Solution:

L.H.S = $\sin^2 6x - \sin^2 4x$

[Using the formulae $2\sin^2 x = 1 - \cos 2x$]

$= \frac{1-\cos 12x}{2} - \frac{1-\cos 8x}{2}$

$= -\frac{1}{2}(\cos 12x - \cos 8x)$

$= -\frac{1}{2}[\cos(10x + 2x) - \cos(10x - 2x)]$

[Using the formulae $-2\sin\theta \sin\varphi = \cos(\theta + \varphi) - \cos(\theta - \varphi)$]

$= \frac{1}{2} \times 2 \sin 10x \sin 2x$

$= \sin 10x \sin 2x$

Question-37

$\cos^2 2x - \cos^2 6x = \sin 4x \sin 8x$

Solution:

L.H.S = $\cos^2 2x - \cos^2 6x$

[Using the formulae $2\cos^2 x = 1 + \cos 2x$]

$= \frac{1+\cos 4x}{2} - \frac{1+\cos 12x}{2}$

$= -\frac{1}{2}(\cos 12x - \cos 4x)$

$= -\frac{1}{2}[\cos(8x + 4x) - \cos(8x - 4x)]$

[Using the formulae $-2\sin\theta \sin\varphi = \cos(\theta + \varphi) - \cos(\theta - \varphi)$]

$= \frac{1}{2} \times 2 \sin 8x \sin 4x$

$= \sin 4x \sin 8x$

Question-38

cos7x + cos5x + cos3x + cosx = 4 cosx cos2x cos4x

Solution:

L.H.S = cos7x + cosx + cos5x + cos3x

= cos(4x + 3x) + cos(4x − 3x) + cos(4x + x) + cos(4x − x)

[Using the formula 2cosθ cosφ = cos(θ + φ) + cos(θ - φ)]

= 2cos4xcos3x + 2cos4xcosx

= 2cos4x(cos3x + cosx)

= 2cos4x(cos3x + cosx)

[Using the formula cosx + cosy = 2 cos $\frac{x+y}{2}$ cos $\frac{x-y}{2}$]

= 4cos4xcos $\frac{3x+x}{2}$ cos $\frac{3x-x}{2}$]

= 4cos4xcos2xcosx

= R.H.S

Question-39

cot4x(sin5x + sin3x) = cotx(sin5x − sin3x)

Solution:

L.H.S = cot4x(sin5x + sin3x)

[Using the formula sin5x + sin3x = 2 sin $\frac{x+y}{2}$ cos $\frac{x-y}{2}$]

= 2cot4x sin $\frac{5x+3x}{2}$ cos $\frac{5x-3x}{2}$

= 2cot4x sin $\frac{8x}{2}$ cos $\frac{2x}{2}$

= 2cot4x sin4x cosx

= 2cos4x cosx

R.H.S = cotx(sin5x − sin3x)

[Using the formula sin5x - sin3x = 2 cos $\frac{x+y}{2}$ sin $\frac{x-y}{2}$]

= 2cotx cos $\frac{5x+3x}{2}$ sin $\frac{5x-3x}{2}$

= 2cotx cos $\frac{8x}{2}$ sin $\frac{2x}{2}$

= 2cos4x cosx

∴ L.H.S = R.H.S

Question-40

tan3xtan2xtanx = tan3x – tan2x – tanx.

Solution:

tan3x = tan(2x + x)

= (tan2x + tanx)/(1 - tan2xtanx)

tan3x(1 - tan2xtanx) = tan2x + tanx

tan3x - tan3xtan2xtanx = tan2x + tanx

∴tan3xtan2xtanx = tan3x – tan2x – tanx.

Question-41

Prove that $\frac{\cos 9x - \cos 5x}{\sin 17x - \sin 3x} = \frac{-\sin 2x}{\cos 10x}$

Solution:

L.H.S = $\frac{\cos 9x - \cos 5x}{\sin 17x - \sin 3x}$

$= \frac{-2\sin\frac{9x+5x}{2}\sin\frac{9x-5x}{2}}{2\cos\frac{17x+3x}{2}\sin\frac{17x-3x}{2}}$

$= \frac{-\sin\frac{14x}{2}\sin\frac{4x}{2}}{\cos\frac{20x}{2}\sin\frac{14x}{2}}$

$= \frac{-\sin 7x \sin 2x}{\cos 10x \sin 7x}$

$= \frac{-\sin 2x}{\cos 10x}$

= R.H.S

Question-42

Prove that $\frac{\sin 5x + \sin 3x}{\cos 5x + \cos 3x}$ **= tan4x**

Solution:

L.H.S = $\frac{\sin 5x + \sin 3x}{\cos 5x + \cos 3x}$

$= \frac{2\sin\frac{5x+3x}{2}\cos\frac{5x-3x}{2}}{2\cos\frac{5x+3x}{2}\cos\frac{5x-3x}{2}}$

$= \frac{\sin\frac{8x}{2}}{\cos\frac{8x}{2}}$

$= \frac{\sin 4x}{\cos 4x}$

= tan4x

= R.H.S

Question-43

Prove that $\frac{\sin x - \sin y}{\cos x + \cos y} = \tan\frac{x-y}{2}$

Solution:

L.H.S = $\frac{\sin x - \sin y}{\cos x + \cos y}$

$= \frac{2\cos\frac{x+y}{2}\sin\frac{x-y}{2}}{2\cos\frac{x+y}{2}\cos\frac{x-y}{2}}$

$= \frac{\sin\frac{x-y}{2}}{\cos\frac{x-y}{2}}$

$= \tan\frac{x-y}{2}$

= R.H.S

Question-44

Prove that (sinA + sin3A)/(cosA + cos3A) = tan2A

Solution:

L.H.S = (sinA + sin3A)/(cosA + cos3A) = 2sin2AcosA/2cos2AcosA = tan2A
= R.H.S

Question-45

Prove that $\frac{\sin x + \sin y}{\cos x + \cos y} = \tan\frac{x+y}{2}$

Solution:

L.H.S = $\frac{\sin x + \sin y}{\cos x + \cos y}$

$= \frac{2\sin\frac{x+y}{2}\cos\frac{x-y}{2}}{2\cos\frac{x+y}{2}\cos\frac{x-y}{2}}$

$= \frac{\sin\frac{x+y}{2}}{\cos\frac{x+y}{2}}$

$= \tan\frac{x+y}{2}$

= R.H.S

Question-46

Prove that (tan5θ+ tan3θ)/(tan5θ - tan3θ)= 4cos2θcos4θ

Solution:

L.H.S $= \dfrac{\dfrac{\sin 5\theta\cos 3\theta+\sin 3\theta\cos 5\theta}{\cos 5\theta\cos 3\theta}}{\dfrac{\sin 5\theta\cos 3\theta-\sin 3\theta\cos 5\theta}{\cos 5\theta\cos 3\theta}}$

$= \dfrac{\dfrac{\sin 8\theta}{\cos 5\theta\cos 3\theta}}{\dfrac{\sin 2\theta}{\cos 5\theta\cos 3\theta}}$

[sin(A+B) = sinAcosB + cosAsinB and sin (A-B) = sinAcosB - cosAsinB]

$= \dfrac{\sin 8\theta}{\sin 2\theta}$

$= \dfrac{2\sin 4\theta\cos 4\theta}{\sin 2\theta}$

$= \dfrac{2\times 2\sin 2\theta\cos 2\theta\cos 4\theta}{\sin 2\theta}$ (sin2θ = 2sinθ cosθ)

= 4 cos2θ cos4θ

= R.H.S

Hence proved

Question-47

Prove that $\dfrac{\sin x-\sin 3x}{\sin^2 x-\cos^2 x}$ =2sinx

Solution:

L.H.S $= \dfrac{\sin x-\sin 3x}{\sin^2 x-\cos^2 x}$

$= \dfrac{\sin 3x-\sin x}{\cos^2 x-\sin^2 x}$

$= \dfrac{2\cos\frac{3x+x}{2}\sin\frac{3x-x}{2}}{\cos 2x}$

[Using the identity cos2x = $\cos^2 x - \sin^2 x$ and sinx – siny = 2cos $\frac{x+y}{2}$ sin $\frac{x-y}{2}$]

$= \dfrac{2\cos 2x\sin x}{\cos 2x}$

= 2sinx

= R.H.S

Question-48

Prove that $(3\sin A - \sin 3A)^{2/3} + (3\cos A + \cos 3A)^{2/3} = 4^{2/3}$

Solution:

L.H.S $=(3\sin A - \sin 3A)^{2/3} + (3\cos A + \cos 3A)^{2/3}$

$=[3\sin A - (3\sin A - 4\sin^3 A)]^{2/3} + [3\cos A + (4\cos^3 A - 3\cos A)]^{2/3}$

$=(4\sin^3 A)]^{2/3} + [4\cos^3 A]^{2/3}$

$= 4^{2/3}(\sin^2 A + \cos^2 A)^{2/3}$

$= 4^{2/3}\ 1^{2/3}$

$= 4^{2/3}$

= R.H.S

Question-49

Prove that $\frac{(\sin 7x+\sin 5x)+(\sin 9x+\sin 3x)}{(\cos 7x+\cos 5x)+(\cos 9x+\cos 3x)} = \tan 6x$

Solution:

L.H.S = $\frac{(\sin 7x+\sin 5x)+(\sin 9x+\sin 3x)}{(\cos 7x+\cos 5x)+(\cos 9x+\cos 3x)}$

$= \frac{2\sin\frac{7x+5x}{2}\cos\frac{7x-5x}{2} + 2\sin\frac{9x+3x}{2}\cos\frac{9x-3x}{2}}{2\cos\frac{7x+5x}{2}\cos\frac{7x-5x}{2} + 2\cos\frac{9x+3x}{2}\cos\frac{9x-3x}{2}}$

$= \frac{2\sin 6x\cos x + 2\sin 6x\cos 3x}{2\cos 6x\cos x + 2\cos 6x\cos 3x}$

$= \frac{2\sin 6x(\cos x + \cos 3x)}{2\cos 6x(\cos x + \cos 3x)}$

$= \tan 6x$

= R.H.S

Question-50

Prove that $\cos 4x = 1 - 8\sin^2 x\cos^2 x$.

Solution:

L.H.S = $\cos 4x = 1-2\sin^2 2x = 1-2(2\sin x\cos x)^2 = 1-8\sin^2 x\cos^2 x$ = R.H.S

Hence proved.