অষ্টম অধ্যায়	ত্রিকোণমিতিৰ পৰিচয় (TRIGONOMETRY)

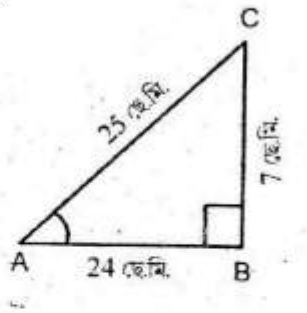
অনুশীলনী – 8.1

প্রশ্ন 1. ΔABC ত্রিভুজৰ B কোণ সমকোণ আৰু $AB = 24cm, BC = 7cm$ হ'লে তলত দিয়াবিলাক উলিওৱা ।

$(i) sinA, cosA$ $(ii) sinC, cosC$

সমাধান ঃ

দিয়া আছে ঃ $AB = 24cm,\ BC = 7cm, \angle B = 90^0$



$\therefore AC^2 = AB^2 + BC^2$ (পিথাগোৰাচৰ সূত্রমতে)

$\Rightarrow AC^2 = (24)^2 + (7)^2$

$\Rightarrow AC^2 = 576 + 49 = 625$

$\Rightarrow AC = \sqrt{625} = 25cm.$

$(i) \sin A = \frac{BC}{AC} = \frac{7cm}{25cm} = \frac{7}{25}$

$\cos A = \frac{AB}{AC} = \frac{24cm}{25cm} = \frac{24}{25}$ (উত্তৰ)

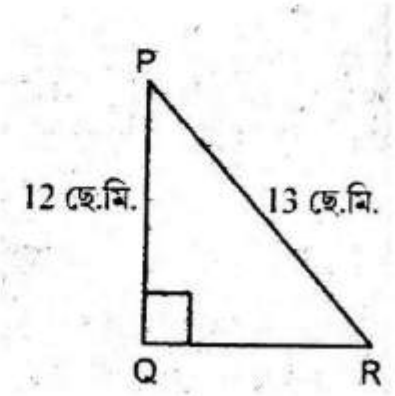
$(ii) \sin C = \frac{AB}{AC} = \frac{24cm}{25cm} = \frac{24}{25}$

$\cos C = \frac{BC}{AC} = \frac{7cm}{25cm} = \frac{7}{25}$ (উত্তৰ)

প্রশ্ন 2. চিত্র 8.13 ৰ পৰা $tanP - cotR$ নির্ণয় কৰা ।

সমাধান ঃ

$PR = 13cm, PQ = 12cm$



$\therefore PR^2 = PQ^2 + QR^2$

$\Rightarrow (23)^2 = (12)^2 + (QR)^2$

$\Rightarrow QR^2 = 169 - 144 = 25$

$\Rightarrow QR = \sqrt{25} = 5cm$

$\therefore \tan P = \frac{RQ}{QP} = \frac{5}{12}$

$\therefore \cot R = \frac{RQ}{PQ} = \frac{5}{12}$

$\therefore \tan P - \cot R = \frac{5}{12} - \frac{5}{12} = 0$

প্রশ্ন 3. যদি $sinA = \frac{3}{4}$, তেন্তে $cosA$ আৰু $tanA$ উলিওৱা ।

সমাধান ঃ

ABC এটা সমকোণী ত্রিভুজ । $\angle B = 90^0$

$\therefore sin\ A = \frac{3}{4} = \frac{BC}{AC} = k$ ধৰা হ'ল । [সমানতাৰ ধ্রুৱক]

$\therefore BC = 3k, AC = 4k$

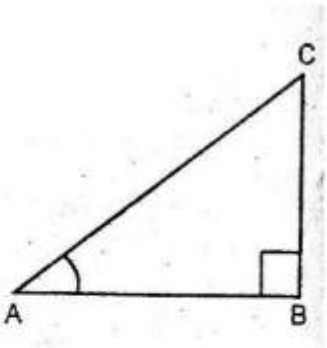
$\therefore AC^2 = AB^2 + BC^2$ (পিথাগোৰাচৰ সূত্রমতে)

$\Rightarrow (4k)^2 = AB^2 + (3k)^2$

$\Rightarrow 16k^2 - 9k^2 = AB^2$

$\Rightarrow AB^2 = 7k^2$

$\Rightarrow AB = \sqrt{7k}$



$\therefore cos\ A = \frac{AB}{AC} = \frac{\sqrt{7k}}{4k} = \frac{\sqrt{7}}{4}$

$tan\ A = \frac{BC}{AB} = \frac{3k}{\sqrt{7k}} = \frac{3}{\sqrt{7}}$ (উত্তৰ)

প্রশ্ন 4. দিয়া আছে যে, $15\ cotA = 8$, তেন্তে $sinA$ আৰু $secA$ উলিওৱা ।

সমাধান ঃ

ABC এটা সমকোণী ত্রিভুজ । $\angle B = 90^0$

$\therefore 15\ cot\ A = 8$

$\Rightarrow cost\ A = \frac{8}{15} = \frac{AB}{BC} = k$ ধৰা হ'ল । $k \rightarrow$ [সমানতাৰ ধ্রুৱক]

$\therefore AB = 8k, BC = 15k$

$\therefore AC^2 = AB^2 + BC^2$

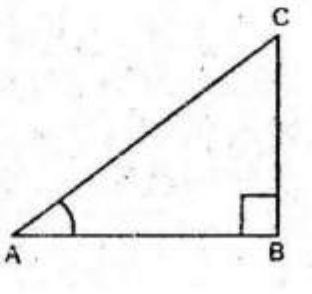
$\Rightarrow AC^2 = (8k)^2 + (15k)^2$

$\Rightarrow AC^2 = 64k^2 + 225k^2$

$\Rightarrow AC^2 = 289k^2$

$\Rightarrow AC = \sqrt{289k^2}$

$\Rightarrow AC = 17k$



$\therefore cos\ A = \frac{BC}{AC} = \frac{15k}{17k} = \frac{15}{17}$

$sec\ A = \frac{AC}{AB} = \frac{17k}{8k} = \frac{17}{8}$ (উত্তৰ)

প্রশ্ন 5. দিয়া আছে যে, $sec\theta = \frac{13}{12}$, আন ত্রিকোণমিতিক অনুপাতবোৰ গণনা কৰা ।

সমাধান ঃ

ABC এটা সমকোণী ত্রিভুজ $\angle B = 90^0, \angle ABC = \theta$ (সূক্ষ্মকোণ)

$\Rightarrow sec\,\theta = \frac{13}{12} = \frac{AC}{AB} = k$ ধৰা হ'ল । $k \rightarrow$ এটা [ধ্রুবক]

$\therefore AC = 13k, AB = 12k$

$\therefore AC^2 = AB^2 + BC^2$

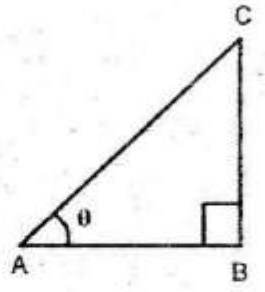
$\Rightarrow (13k)^2 = (12k)^2 + BC^2$

$\Rightarrow 169k^2 = 144k^2 + BC^2$

$\Rightarrow BC^2 = 169k^2 - 144k^2$

$\Rightarrow BC^2 = 25k^2$

$\Rightarrow BC = \sqrt{25k^2} = 5k$



$\therefore sin\,\theta = \frac{BC}{AC} = \frac{5k}{13k} = \frac{5}{13}$

$cos\,\theta = \frac{AB}{AC} = \frac{12k}{13k} = \frac{12}{13}$

$sin\,\theta = \frac{BC}{AB} = \frac{5k}{12k} = \frac{5}{12}$

$cosec\,\theta = \frac{AC}{BC} = \frac{13k}{5k} = \frac{13}{5}$

$cot\,\theta = \frac{AB}{BC} = \frac{12k}{5k} = \frac{12}{5}$

প্রশ্ন 7. যদি তেন্হে মান উলিওৱা ।

$(i)\frac{(1+sin\,\theta)(1-sin\,\theta)}{(1+cos\,\theta)(1-cos\,\theta)}$ $(ii)cot^2\theta$

সমাধান ঃ

$(i)\angle ABC = \theta$ [সূক্ষ্মকোণ] । ABC এটা সমকোণী ত্রিভুজৰ $\angle C = 90^0$

$\therefore cot\,\theta = \frac{7}{8}$ [প্রদত্ত]

$\Rightarrow$BC $COT\,\theta = \frac{7}{8} = \frac{BC}{AC}$

$\therefore \frac{BC}{AC} = \frac{7}{8} = k$ ধৰা হ'ল । $k \rightarrow$ এটা [ধ্রুবক]

$\therefore BC = 7k, AC = 8k$

$\therefore AB^2 = AC^2 + BC^2$

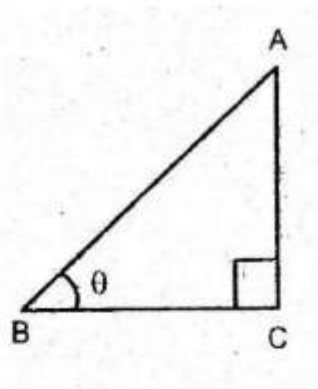
$$\Rightarrow AB^2 = (8k)^2 + (7k)^2 = 64k^2 + 49k^2$$

$$\Rightarrow AB^2 = 113k^2$$

$$\Rightarrow AB = \sqrt{113k^2} = \sqrt{113}k$$

$$\therefore \sin\theta = \frac{AC}{AB} = \frac{8k}{\sqrt{113}k} = \frac{8}{\sqrt{113}}$$

$$\cos\theta = \frac{BC}{AB} = \frac{7k}{\sqrt{113}k} = \frac{7}{\sqrt{113}}$$



$$\therefore = \frac{(1+\sin\theta)(1-\sin\theta)}{(1+\cos\theta)(1-\cos\theta)}$$

$$= \frac{1-\sin^2\theta}{1-\cos^2\theta}$$

$$= \frac{1-\left(\frac{8}{\sqrt{113}}\right)^2}{1-\left(\frac{7}{\sqrt{113}}\right)^2} = \frac{1-\frac{64}{113}}{1-\frac{49}{113}}$$

$$= \frac{\frac{113-64}{113}}{\frac{113-49}{113}} = \frac{\frac{49}{113}}{\frac{64}{113}} = \frac{49}{113} \times \frac{113}{64} = \frac{49}{64} \qquad (Ans.)$$

(ii) $\therefore \cot\theta = \frac{7}{8}$ [প্রদত্ত]

$$\Rightarrow \cot^2\theta = \left(\frac{7}{8}\right)^2$$

$$\Rightarrow \cot^2\theta = \frac{49}{64} \quad (Ans.)$$

প্রশ্ন 9. ΔABC ৰ B কোণ সমকোণ । যদি $\tan A = \frac{1}{\sqrt{3}}$ তেন্তে তলৰ মান বিলাক উলিওৱা –

$(i) \sin A \cos C + \cos A \sin C$ $\qquad$ $(ii) \cos A \cos C - \sin A \sin C$

সমাধান ঃ

ABC এটা সমকোণী ত্রিভুজৰ $\angle B = 90^0$

$\angle A = \theta$ [সূক্ষ্মকোণ] ।

$\therefore \tan A = \frac{1}{\sqrt{3}}$ [দিয়া আছে]

$\therefore \tan A = \frac{BC}{AB} = \frac{1}{\sqrt{3}} = k$ ধৰা হ'ল । $k \rightarrow$ এটা [ধ্রুবক]

$\therefore BC = k, AB = \sqrt{3}k$

$\therefore AC^2 = AB^2 + BC^2$

$\Rightarrow AC^2 = (\sqrt{3}k)^2 + (k)^2 = 3k^2 + k^2 = 4k^2$

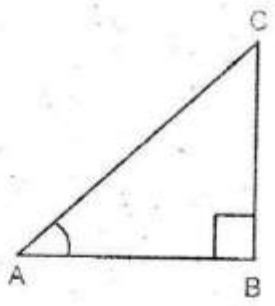
$\Rightarrow AC = \sqrt{4k^2} = 2k$

$$\therefore \sin A = \frac{BC}{AC} = \frac{k}{2k} = \frac{1}{2}$$

$$\cos A = \frac{AB}{AC} = \frac{\sqrt{3}k}{2k} = \frac{\sqrt{3}}{2}$$

$$\sin C = \frac{AC}{AC} = \frac{\sqrt{3}k}{2k} = \frac{\sqrt{3}}{2}$$

$$\cos C = \frac{BC}{AC} = \frac{k}{2k} = \frac{1}{2}$$



$(i) \sin A \cos C + \cos A \sin C$

$$= \frac{1}{2} \times \frac{1}{2} + \frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2}$$

$$= \frac{1}{4} + \frac{3}{4} = \frac{1+3}{4} = \frac{4}{4} = 1$$

$(ii) \cos A \cos C + \sin A \sin C$

$$= \frac{\sqrt{3}}{2} \times \frac{1}{2} - \frac{1}{2} \times \frac{\sqrt{3}}{2}$$

$$= \frac{\sqrt{3}}{4} - \frac{\sqrt{3}}{4} = 0 \quad \text{(উত্তৰ)}$$

প্ৰশ্ন 10. ΔPQR কোণ Q সমকোণ আৰু $PR + QR = 25cm$ আৰু $PQ = 5cm$; $\cos P$ আৰু $\tan P$ ৰ মান উলিওৱা ।

সমাধান ঃ

PQR এটা সমকোণী ত্ৰিভুজৰ । $\angle \theta = 90^0$ আৰু $\angle P \rightarrow$ সূক্ষ্মকোণ।

$$PR + QR = 25cm \text{ আৰু } PQ = 5cm.$$

$\therefore QR$ এটা সমকোণী ত্ৰিভুজৰ পৰা পাওঁ –

$$PR^2 = QR^2 + PQ^2$$

$$\Rightarrow PR^2 = (25 - PR)^2 + (5)^2$$

$$\Rightarrow PR^2 = 625 - 50PR + PR^2 + 25$$

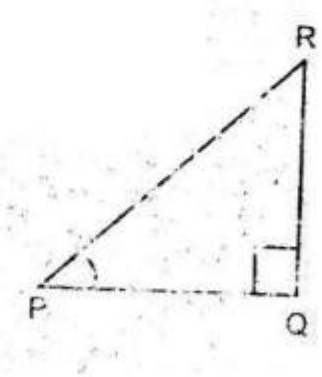
$$\Rightarrow PR^2 - PR^2 = 650 - 50PR$$

$$\Rightarrow 0 = 650 - 50PR$$

$$\Rightarrow 50PR = 650$$

$$\Rightarrow PR = \frac{650}{50} = 13cm.$$

$$\therefore QR = 25 - PR = 25 - 13 = 12cm$$



$$\left.\begin{aligned} \therefore sinP &= \frac{QR}{PR} = \frac{12cm}{13cm} = \frac{12}{13} \\ \therefore cosP &= \frac{PQ}{PR} = \frac{5cm}{13cm} = \frac{5}{13} \\ \therefore tanP &= \frac{QR}{Pq} = \frac{12cm}{5cm} = \frac{12}{5} \end{aligned}\right\} \text{(উত্তৰ)}$$

অনুশীলনী – 8.2

প্রশ্ন 1. তলত দিয়া বিলাকৰ মান উলিওৱা –

$$(i) 2tan^2 45^0 + cos^2 30^0 - sin^2 60^0$$

$$(ii) \frac{cos\,45^0}{sec\,30^0 + cosec30^0} \qquad (iii) \frac{sin30^0 + tan\,45^0 - cosec\,60^0}{sec\,30^0 + cos\,^{60}0 + cot\,45^0}$$

$$(iv) \frac{5\,cos^2\,60^0 + 4\,sec^2\,30^0 - tan^2\,45^0}{sec\,30^0 + cos\,^{60}0 + cot\,45^0}$$

সমাধান ঃ $(i) 2tan^2 45^0 + cos^2 30^0 - sin^2 60^0$

$$= 2(1)^2 + \left(\frac{\sqrt{3}}{2}\right)^2 - \left(\frac{\sqrt{3}}{2}\right)^2$$

$$= 2 \times 1 + \frac{3}{4} - \frac{3}{4} = 2 \qquad \text{(উত্তৰ)}$$

সমাধান ঃ $(ii) \frac{cos\,45^0}{sec\,30^0 + cosec30^0}$

$$= \frac{\frac{1}{\sqrt{2}}}{\frac{2}{\sqrt{3}} + 2} = \frac{\frac{1}{\sqrt{2}}}{\frac{2+2\sqrt{3}}{\sqrt{3}}}$$

$$= \frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2+2\sqrt{3}} = \frac{\sqrt{3}}{\sqrt{2}(2+2\sqrt{3})}$$

$$\frac{\sqrt{3}}{2\sqrt{2}(1+\sqrt{3})} = \frac{\sqrt{3}(\sqrt{3}-1)}{2\sqrt{2}(\sqrt{3}+1)(\sqrt{3}-1)}$$

$$= \frac{\sqrt{2}\times\sqrt{3}(\sqrt{3}-1)}{2\sqrt{2}\times\sqrt{2}(3-1)}$$

$$= \frac{\sqrt{6}(\sqrt{3}-1)}{8} = \frac{3\sqrt{2}-\sqrt{6}}{8}$$

সমাধান ঃ $(iii) \frac{sin30^0 + tan\,45^0 - cosec\,60^0}{sec\,30^0 + cos\,^{60}0 + cot\,45^0}$

$$= \frac{\frac{1}{2}+1-\frac{2}{\sqrt{3}}}{\frac{2}{\sqrt{3}}+\frac{1}{2}+1} = \frac{\frac{3}{2}-\frac{2}{\sqrt{3}}}{\frac{2}{\sqrt{3}}+\frac{3}{2}}$$

$$= \frac{3\sqrt{3}-4}{4+3\sqrt{3}} = \frac{(3\sqrt{3}-4)(3\sqrt{3}-4)}{(3\sqrt{3}+4)(3\sqrt{3}-4)}$$

$$= \frac{27+16-24\sqrt{3}}{27-16} = \frac{43-24\sqrt{3}}{11} \quad \text{(উত্তৰ)}$$

সমাধান ঃ $(iv)\ \frac{5\cos^2 60^0+4\sec^2 30^0-\tan^2 45^0}{\sec 30^0+\cos {}^{60}0+\cot 45^0}$

$$= \frac{5\times\left(\frac{1}{2}\right)^2+4\left(\frac{1}{\sqrt{3}}\right)^2-(1)^2}{\left(\frac{1}{2}\right)^2+\left(\frac{\sqrt{3}}{2}\right)^2} = \frac{\frac{5}{4}+4\times\frac{4}{3}-1}{\frac{1}{4}+\frac{3}{4}}$$

$$= \frac{\frac{5}{4}+\frac{16}{3}-1}{\frac{4}{4}} = \frac{\frac{15+64-12}{12}}{1} = \frac{67}{12} \quad \text{(উত্তৰ)}$$

প্ৰশ্ন 3. যদি $\tan(A+B)=\sqrt{3}$ আৰু $\tan(A-B)=\frac{1}{\sqrt{3}};\ 0^0 < A+B \leq 90^0; A > B,$ তেন্তে A আৰু B উলিওৱা ।

সমাধান ঃ $\tan(A+B)=\sqrt{3}=\tan 60^0$

$$\Rightarrow A+B=60^0 \ldots\ldots\ldots\ldots\ldots\ldots\ldots (i)$$

$$\tan(A-B)=\frac{1}{\sqrt{3}}=\tan 30^0$$

$$\Rightarrow A-B=30^0 \ldots\ldots\ldots\ldots\ldots (ii)$$

$$\therefore A+B=60^0$$

$$A-B=30^0$$

(যোগকৰি) $2A=90^0$

$$\Rightarrow A=45^0$$

এতিয়া, $A=45^0$, (i) নং সমীকৰণত বহুৱাই পাওঁ ঃ

$$A+B=60^0$$

$$\Rightarrow 15^0+B=60^0$$

$$\Rightarrow B=60^0-45^0=15^0$$

$$\left.\begin{aligned}\therefore A &= 45^0\\ B &= 15^0\end{aligned}\right\} \quad \text{(উত্তৰ)}$$

প্রশ্ন 4. তলত দিয়াবিলাক সত্য নে অসত্য কোৱা । তোমাৰ উত্তৰৰ যুক্তি দাঙি ধৰা –

$(i) sin(A+B) = sin\,A + sin\,B.$

$(ii) sin\theta$ ৰ মান বাঢ়ি যায় যদি θ ৰ মান বাঢ়ে ।

$(iii) cos\theta$ ৰ মান বাঢ়ি যায় যদি θ ৰ মান বাঢ়ে ।

$(iv)\theta$ ৰ সকলো মানৰ বাবে $sin\theta = cos\theta$

$(v) A = 0^0$ ৰ বাবে $cotA$ সংজ্ঞাবদ্ধ নহয় ।

সমাধান ঃ

(i) $sin(A+B) = sin\,A + sin\,B$ অশুদ্ধ ।

কাৰণ, যেতিয়া, $A = 60^0, B = 30^0$ ধৰা হয় তেতিয়া –

$\therefore$ বাঁওপক্ষ $= sin\,(A+B) = sin(60^0 + 30^0) = sin90^0 = 1$

সোঁপক্ষ $= sin\,A + sin\,B = sin\,60^0 + sin30^0 = \frac{\sqrt{3}}{2} + \frac{1}{2} \neq 1$

অর্থাৎ, $L.H.S. \neq R.H.S.$

সমাধান ঃ

(ii) সত্য ।

$\therefore sin\,0^0 = \frac{1}{2}$

$sin\,45^0 = \frac{1}{\sqrt{2}} = 0.7$ (প্ৰায়)

$sin\,60^0 = \frac{\sqrt{3}}{2} = 0.87$ (প্ৰায়)

আৰু, $sin\,90^0 = 1$

অর্থাৎ, $sin\theta$ বৃদ্ধি পালে, θ -ৰ মান 0^0-ৰ পৰা 90^0 লৈকে বৃদ্ধি পায় ।

সমাধান ঃ

(iii) অসত্য ।

লক্ষ্য কৰিলে দেখা যায় যে, –

$cos\,0^0 = 1$

$cos\,30^0 = \frac{\sqrt{3}}{2} = 0.87$ (প্ৰায়)

$cos\,45^0 = \frac{1}{\sqrt{2}} = 0.7$ (প্ৰায়)

$cos\,60^0 = \frac{1}{2}$ $\quad cos\,90^0 = 0$

$\therefore$ দেখা যায় যে θ-ৰ মান হ্ৰাস পালে, θ-ৰ মান 0^0-ৰ পৰা 90^0 লৈকে বৃদ্ধি পায় ।

সমাধান ঃ

(iv) অসত্য ।

$\therefore \sin 30^0 = \frac{1}{2}, \cos 30^0 = \frac{\sqrt{3}}{2}$

$\therefore \sin 30^0 \neq \cos 30^0$

অৰ্থাৎ, $\sin 45^0 = \cos 45^0$ হয় ।

সমাধান ঃ

(v) সত্য ।

কাৰণ, $\cot 0^0 = \frac{1}{\tan 0^0} = \frac{1}{0}$ সংজ্ঞাবদ্ধ নহয় অৰ্থাৎ অৰ্থহীন ।

অনুশীলনী - 8.3

প্রশ্ন 1. মান নির্ণয় কৰা ।

$(i) \frac{\sin 18^0}{\cos 72^0}$ $(ii) \frac{\tan 26^0}{\cot 64^0}$

$(iii) \cos 48^0 - \sin 42^0$ $(iv) \operatorname{cosec} 31^0 - \sec 59^0$

সমাধান ঃ

$(i) \frac{\sin 18^0}{\cos 72^0} = \frac{\sin 18^0}{\cos(90^0 - 18^0)} = \frac{\sin 18^0}{\sin 18^0} = 1$ (উত্তৰ)

$[\therefore \cos(90^0 - \theta) = \sin \theta]$

সমাধান ঃ

$(ii) \frac{\tan 26^0}{\cot 64^0} = \frac{\tan 26^0}{\cos(90^0 - 26^0)} = \frac{\tan 26^0}{\tan 26^0} = 1$ (উত্তৰ)

$[\therefore \cot(90^0 - \theta) = \tan \theta]$

সমাধান ঃ

$(iii) \cos 48^0 - \sin 42^0$

$= \cos(90^0 - 42^0) - \sin 42^0$ $[\therefore \cot(90^0 - \theta) = \tan \theta]$

$= \sin 42^0 - \sin 42^0$

$= 0$ (উত্তৰ)

সমাধান ঃ

$(iv) cosec 31^0 - sec 59^0$

$= codec\ 31^0 - sec(90 - 31^0)$ $\quad [\because sec(90^0 - \theta) = cosec\ \theta]$

$= cosec 31^0 - cosec\ 31^0$

$= 0$ (উত্তৰ)

প্রশ্ন 3. যদি $tan 2A = cot(A - 18^0)$, য'ত $2A$ সূক্ষ্মকোণ, তেন্তে A ৰ মান উলিওৰা ।

সমাধান ঃ $tan 2A = cot(A - 18^0)$

$= cot(90^0 - 2A) = cot(A - 18^0)$ $\quad [\because cot\ (90^0 - \theta) = tan\ \theta]$

$\Rightarrow 90^0 - 2A = A - 18$

$\Rightarrow -2A - A = -18^0 - 90^0$

$\Rightarrow -3A = -108^0$

$\Rightarrow A = \frac{-108^0}{-3} = 36^0$

$\therefore A = 36^0$ (উত্তৰ)

প্রশ্ন 4. যদি $tan\ A = cotB$, প্রমাণ কৰা যে $A + B = 90^0$

সমাধান ঃ

$\therefore tan\ A = cotB$

$\Rightarrow tan\ A = tan(90^0 - B)$ $\quad [\because tan\ (90^0 - \theta) = tan\ \theta]$

$\Rightarrow A = 90^0 - B$

$\Rightarrow A + B = 90^0$ (উত্তৰ)

প্রশ্ন 5. যদি $sec\ 4A = cosec(A - 20^0)$, য'ত $4A$ সূক্ষ্মকোণ, তেন্তে A ৰ মান উলিওৰা ।

সমাধান ঃ

$\therefore sec\ 4A = cosec(A - 20^0)$

$\Rightarrow cosec(90^0 - 4A) = cosec(A - 20^0)$ $\quad [\because cosec\ (90^0 - \theta) = sec\ \theta]$

$\Rightarrow 5A = 110^0$

$\Rightarrow A = \frac{110^0}{5} \Rightarrow A = 22^0$ (উত্তৰ)

প্রশ্ন 6. যদি A, B আৰু C কোণকেইটা ABC ত্ৰিভুজৰ অন্তঃকোণ হয়, তেন্তে দেখুওৱা যে –

$$sin\left(\frac{B+C}{2}\right) = cos\frac{A}{2}$$

সমাধান ঃ

আমি জানো যে, ΔABC ত্ৰিভুজৰ $\angle A + \angle B + \angle C = 180^0$

$\Rightarrow B + C = 180^0 - a$

$\Rightarrow \frac{B+C}{2} = \frac{180^0 - a}{2}$

$\Rightarrow \frac{B+C}{2} = 90^0 - \frac{A}{2}$

$\Rightarrow sin\left(\frac{B+C}{2}\right) = sin\left(90^0 - \frac{A}{2}\right)$ [উভয় পক্ষত লৈ পাওঁ ।]

$\Rightarrow sin\left(\frac{B+C}{2}\right) = cos\frac{A}{2}$ [$\therefore sin\left(90^0 - \theta\right) = cos\theta$]

[দেখুওৱা হ'ল]

অনুশীলনী −8.4

প্রশ্ন 1. $sinA$, $secA$ আৰু $tanA$ এই ত্ৰিকোণমিতিক অনুপাত কেইটাক $cotA$ ৰ দ্বাৰা প্ৰকাশ কৰা ।

সমাধান ঃ

$\therefore cosec^2A - cot^{2A} = 1$

$\Rightarrow cosec^2A = 1 + cot^2A$

$\Rightarrow \left(\frac{1}{sinA}\right)^2 = 1 + cot^2A$

$\Rightarrow sin^2A = \frac{1}{1+cot^2A}$

$\Rightarrow sinA = \pm\frac{1}{\sqrt{1+cot^2A}}$

ইয়াত, ঋণাত্মক মান আমি বৰ্জন কৰিম ।

$\therefore sinA = \frac{1}{\sqrt{1+cot^2A}}$

আকৌ, $sec^2A - tan^2A = 1$

$\Rightarrow sec^2A = 1 + tan^2A$

$$\Rightarrow sec^2A = 1 + \frac{1}{cot^2A} = \frac{cot^2A+1}{cot^2A}$$

$$\Rightarrow sec\,A = \pm\sqrt{\frac{1+cot^2A}{cot^2A}} = \pm\frac{\sqrt{1+cot^2A}}{cot\,A}$$

ইয়াত ঋণাত্মক মান গ্রহণযোগ্য নহয় ।

$$\therefore sec\,A = \frac{\sqrt{1+cot^2A}}{cot\,A}$$

$$\therefore tan\,A = \frac{1}{cot\,A}$$

প্রশ্ন 2. $secA$ ৰ সহায়ত $\angle A$ কোণৰ আন সকলোবিলাক ত্রিকোণমিতিক অনুপাত লিখা ।

সমাধান ঃ

আমি জানো যে, $sin^2A + cos^2A = 1$

$$\Rightarrow sin^2A = 1 - cos^2A$$

$$\Rightarrow sin^2A = 1 - \frac{1}{sec^2A} = \frac{sec^2A-1}{sec^2A}$$

$$\Rightarrow sinA = \pm\frac{\sqrt{sec^2A-1}}{secA}$$

ইয়াত ঋণাত্মক মান গ্রহণযোগ্য নহয় ।

$$\therefore sin\,A = \frac{\sqrt{sec^2A-1}}{sec\,A}$$

$$cos\,A = \frac{1}{sec\,A}$$

$$1 + tan^2A = sec^2A$$

$$\Rightarrow tan^2A = sec^2A - 1$$

$$\Rightarrow tanA = \pm\sqrt{sec^2A-1}$$

ইয়াত ঋণাত্মক মান গ্রহণযোগ্য নহয় ।

$$\therefore tan\,A = \sqrt{sec^2A-1}$$

$$cosec\,A = \frac{1}{sin\,A} = \frac{1}{\frac{\sqrt{sec^2A-1}}{sec\,A}} = \frac{secA}{\sqrt{sec^2A-1}}$$

$$cot\,A = \frac{1}{tanA} = \frac{1}{\sqrt{sec^2A-1}}$$

প্রশ্ন 3. মান নির্ণয় কৰা –

$(i)\frac{sin^2 63^0+sin^2 27^0}{cos^2 17^0+cos^2 73^0}$ $(ii) sin25^0\, cos\, 65^0 + cos\, 25^0 65^0$

সমাধান ঃ

$(i)\frac{sin^2 63^0+sin^2 27^0}{cos^2 17^0+cos^2 73^0}$

$=\frac{sin^2(90^0-27^0)+sin^2 27^0}{cos^2 17^0+cos^2(90^0-17^0)}$ $\left[\begin{matrix} sin(90^0-\theta)=cos\theta \text{ আৰু} \\ \cos(90^0-\theta)=sin\theta \end{matrix}\right]$

$=\frac{cos^2 27^0+sin^2 27^0}{cos^2 17^0+sin^2 17^0}=\frac{1}{1}=1$ (উত্তৰ)

সমাধান ঃ

$(ii) sin25^0\, cos\, 65^0 + cos\, 25^0 65^0$

$= sin\, 25^0 \times cos(90^0 - 25^0) + cos\, 25^0 \times sin(90^0 - 25^0$

$\therefore \left[cos\,(90^0-\theta) = sin\,\theta \text{ and } sin(90^0-\theta) = cos\,\theta\right]$

$= sin\, 25^0 \times sin25^0 + cos\, 25 \times cos\, 25$

$= sin^2 25^0 + cos^2 25^0$

$= 1$

প্রশ্ন 5. তলৰ অভেদ কেইটা প্ৰমাণ কৰা যদিহে ইয়াত কোণ বিলাক সূক্ষ্ম কোণ আৰু যাৰ বাবে অভেদ কেইটা সংজ্ঞাবদ্ধ হয় –

$(i)(cosec\,\theta - cot\,\theta)^2 = \frac{1-cos\,\theta}{1+cos\,\theta}$

$(ii)\frac{cos\,A}{1+sin\,A}+\frac{1+sin\,A}{cos\,A}=2sec\,A$

$(iii)\frac{tan\,\theta}{1-cot\,\theta\theta}+\frac{cot\,\theta}{1-tan\,\theta}=1+sec\,\theta\, cosec\,\theta$

[ইংগিত ঃ ইয়াত থকা পদবোৰ $sin\theta$ আৰু $cos\theta$ ত প্ৰকাশ কৰা ।

$(iv)\sqrt{\frac{1+sin\,A}{1-sin\,A}}=secA+tanA$

$(v)(sin\,A+cosecA)^2+(cosA+secA)^2=7+tan^2 A+cot^2 A$

সমাধান ঃ $(i)(cosec\,\theta - cot\,\theta)^2 = \frac{1-cos\,\theta}{1+cos\,\theta}$

বাঁওপক্ষ $= (cosec\,\theta - cot\theta)^2$

$=\left(\frac{1}{sin\,\theta}-\frac{cos\,\theta}{sin\,\theta}\right)^2$

$$= \left(\frac{1-\cos\theta}{\sin\theta}\right)^2$$

$$= \frac{(1-\cos\theta)^2}{\sin^2\theta} = \frac{(1-\cos\theta)^2}{1-\cos^2\theta}$$

$$= \frac{\cancel{(1-\cos\theta)}(1-\cos\theta)}{(1+\cos\theta)\cancel{(1-\cos\theta)}}$$

$$= \frac{1-\cos\theta}{1+\cos\theta}$$ সৌঁপক্ষ (প্রমাণিত)

সমাধান ঃ $(ii)\ \frac{\cos A}{1+\sin A} + \frac{1+\sin A}{\cos A} = 2\sec A$

বাঁওপক্ষ $= \frac{\cos A}{1+\sin A} + \frac{1+\sin A}{\cos A}$

$$= \frac{\cos^2 A + (1+\sin A)^2}{\cos A(1+\sin A)}$$

$$= \frac{\cos^2 A + 1 + \sin^2 A + 2\sin A}{\cos A(1+\sin A)}$$

$$= \frac{1+1+2\sin A}{\cos A(1+\sin A)}$$

$$= \frac{2+2\sin A}{\cos A(1+\sin A)}$$

$$= \frac{2\cancel{(1+\sin A)}}{\cos A\cancel{(1+\sin A)}}$$

$$= 2 \times \frac{1}{\cos A} = 2\sec A =$$ সৌঁপক্ষ (প্রমাণিত)

সমাধান ঃ $(iii)\ \frac{\tan\theta}{1-\cot\theta\theta} + \frac{\cot\theta}{1-\tan\theta} = 1 + \sec\theta\ \mathrm{cosec}\ \theta$

বাঁওপক্ষ $= \frac{\tan\theta}{1-\cot\theta} + \frac{\cot\theta}{1-\tan\theta}$

$$= \frac{\frac{1}{\cot\theta}}{1-\cot\theta} + \frac{\cot\theta}{1-\frac{1}{\cot\theta}}$$

$$= \frac{1}{\cot\theta(1-\cot\theta)} + \frac{\cot\theta}{\frac{\cot\theta-1}{\cot\theta}}$$

$$= \frac{1}{\cot\theta\,(1-\cot\theta} + \frac{\cot^2\theta}{\cot\theta-1}$$

$$= \frac{1}{\cot\theta(1-\cot\theta)} - \frac{\cot^2\theta}{1-\cot\theta}$$

$$= \frac{1-\cot^3\theta}{\cot\theta(1-\cot\theta)}$$

$$= \frac{(1-\cot\theta)(1+\cot\theta+\cot^2\theta}{\cot\theta(1-\cot\theta)}$$

$$= \frac{1+\cot\theta+\cot^2\theta}{\cot\theta}$$

$$= \frac{1}{\cot\theta} + 1 + \frac{\cot^2\theta}{\cot\theta}$$

$$= \frac{1}{\cot\theta} + 1 + \cot\theta$$

$$= \frac{\sin\theta}{\cos\theta} + 1 + \frac{\cos\theta}{\sin\theta}$$

$$= \frac{\sin^2\theta+\sin\theta\cos\theta+\cos^2\theta}{\sin\theta.\cos\theta}$$

$$= \frac{(\sin^2\theta+\cos^2\theta)+\sin\theta\cos\theta}{\sin\theta.\cos\theta}$$

$$= \frac{1+\sin\theta\cos\theta}{\sin\theta\cos\theta}$$

$$= \frac{1}{\sin\theta\cos\theta} + \frac{\sin\theta.\cos\theta}{\sin\theta.\cos\theta}$$

$$= \frac{1}{\sin\theta} \times \frac{1}{\cos\theta} + 1$$

$$= \operatorname{cosec}\theta.\sec\theta + 1$$

$$= 1 + \sec\theta.\operatorname{cosec}\theta = \text{সোঁপক্ষ (প্রমাণিত)।}$$

সমাধান ঃ $(iv)\sqrt{\frac{1+\sin A}{1-\sin A}} = \sec A + \tan A$

বাঁওপক্ষ $= \sqrt{\frac{1+\sin A}{1-\sin A}}$

$$= \sqrt{\frac{(1+\sin A)(1+\sin A)}{(1-\sin A)(1+\sin A)}}$$

$$= \sqrt{\frac{(1+\sin A)^2}{1-\sin^2 A}}$$

$$= \sqrt{\frac{(1+\sin A)^2}{\cos^2 A}}$$

$$= \frac{1+\sin A}{\cos A}$$

$$= \frac{1}{\cos A} + \frac{\sin A}{\cos A}$$

$$= \sec A + \tan A = \text{সোঁপক্ষ (প্রমাণিত)।}$$

সমাধান ঃ $(v)(\sin A + \operatorname{cosec} A)^2 + (\cos A + \sec A)^2 = 7 + \tan^2 A + \cot^2 A$

বাওঁপক্ষ $= (\sin A + \operatorname{cosec} A)^2 + (\cos A + \sec A)^2$

$$= \sin^2 + 2.\sin A.\operatorname{cosec} A + \operatorname{cosec}^2 A$$

$$+\cos^2 + 2\cos A.\sec A + \sec^2 A$$

$$= \sin^2 A + \cos^2 A + 2.\sin A \times \frac{1}{\sin A}$$

$$+\operatorname{cosec}^2 A + 2\cos A \times \frac{1}{\sec A} + \sec^2 A$$

$$= 1 + 2 + \operatorname{cosec}^2 A + 2 + \sec^2 A$$

$$= 5 + 1 + \cot^2 A + 1 + \tan^2 A$$

$= 7 + \tan^2 A + \cot^2 A$ = সোঁপক্ষ (প্রমাণিত)।

প্রশ্ন 6. যদি $A = 30^0, B = 60^0$ তেন্হে দেখুওরা যে –

$(i) \sin(A + B) = \sin A \cos B + \cos A \sin B$

সমাধান ঃ $LHS = \sin(A + B)$

$= \sin(30 + 60)$

$= \sin 90 = 1$

$RHS = \sin A \cos B + \cos A \sin B$

$= \sin 30 \cos 60 + \cos 30 \sin 60$

$= \frac{1}{2} \times \frac{1}{2} + \frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2}$

$= \frac{1}{4} + \frac{3}{4} = \frac{1+3}{4} = \frac{4}{4} = 1$

$\therefore LHS = RHS.$