4 Introduction to Set

- **Let's learn new :**
- **Identification of set :**

Dear students, have you ever seen herd of cattle ?

have you seen flock of sheep ?

have you ever seen caravan of camel ?

have you even seen such type of group anywhere ?

- In all these groups, the number of members is more than one or two. But number of members are definite. For example, 10, 15, 20, 28,... etc. Therefore, group of definite elements (members) i.e. **A set is a collection of well specified object. Set is an undefined term.**
- **The brackets { } are used to express a set**
  (1) The group of birds like sparrow, parrot, crow are written in set form as {Sparrow, Parrot, Crow}.
  (2) The group of flowers like rose, marigold, hibiscus, bougainvillea are denoted in set forms {rose, marigold, hibiscus, bougainvillea}.
  (3) The group of numbers line 1, 2, 3, 4, 5 are denoted in set form as {1, 2, 3, 4, 5}.
- Each of the well specified objects forming a set is called a member or an element of the set.
- Each member of a set is separated by commas (,). In the last comma and fullstop is not denoted. No element or member of a set is repeated.
- The symbol $\in$ (Belongs to) is used to denote that an object is a member of set and the symbol $\notin$ (Does not Belong to) is used to denote that an object is not a member of set.

Methods of describing a set : (1) The listing method (2) property method

Sets are denoted by capital letters A, B, C, ..., X, Y, Z, etc.

For example, A = {1, 2, 3, 4, 5}, X = {Mango, Asopalav, Pipal, Gulmohor}

Here, the natural numbers from 1 to 5 are written in the set form. This method of denoting the set is called listing method.

If this list is to show briefly then first five natural number can be written as : A = $\{x / x$ is first five natural numbers$\}$. This method of denoting (writing) a set is called the property method. If all the members of a set have some common properties then that property is denoted by P(x) and the set is denoted by $\{x/p(x)\}$.

- **Some special sets :**

Complete the following table :

No.	The listing method	The property method
1		B = $\{x / x$ is an even prime number$\}$
2.	X = {2, 4, 6, 8, 10}	
3.	Y = {–2, –1, 0, 1, 2}	
4.		C = $\{x / x < 6, x \in N\}$
5.		Z = $\{x / x$ is tools of geometry box$\}$
6.	M = {a, b, c, d, e, f}	

(1) Which is the set of having one and only one member ?

(2) Which is the set of having members more than 5 ?

(3) Which two sets have equal numbers of members ?

- **Some special sets :**

Empty Set :

The set without any member is called the empty set. It is denoted by symbol ϕ (phi) or "{ }".

For example : A = $\{x / x$ is a prime number less than 2, $x \in N\}$, then A = ϕ or A = { }

B = $\{x / x$ is a prime of female chief minister of Gujarat$\}$, then B = ϕ or B = { }

Singleton Set :

A set having only one element (member) is called the singleton set.

For example : P = $\{x / x$ is an odd prime number less than 5$\}$

P = {3}

Finite Set :

If the number of members in a set is a definite non-negative integer, then the set is called a finite set. For example, A = {1, 2, 3, ... 10}

Here, 1 to 10 natural numbers are included in set A, which can be counted. Therefore, the number of members of set A is 10 which is definite. Therefore, the set A is a finite set.

The number of members of a set A is denoted by $n(A)$.

Here, the number of members of set A is 10. Therefore, $n(A) = 10$

- The empty set is also a finite set.

Infinite Set :

A set which is not finite is an infinite set.

For example : A = {x/x is a natural number} $\therefore$ A = {1, 2, 3, 4, ...}

Here, there is no end of the list of members in set A. Such set is called an infinite set. To an infinite set after writing some members, generally three points are denoted.

- The set of natural numbers is denoted by a special symbol N.

  N = {1, 2, 3, ...}
- The set of whole numbers is denoted by a special symbol W.

  W = {0, 1, 2, 3, ...}
- The set of integers is denoted by a special symbol Z.

  Z = {..., −3, −2, −1, 0, 1, 2, 3, ...}
- The set of rational numbers (quotients) is denoted by a special symbol Q.

  $Q = \left\{\frac{p}{q} \mid p \in Z, q \in N\right\}$
- N, W, Z and Q, all these are infinite sets.
- Why is the set of leaves of a tree is called an infinite set ? Why ? Think !

1. Fill in the blanks with appropriate symbol $\in$ or $\notin$:

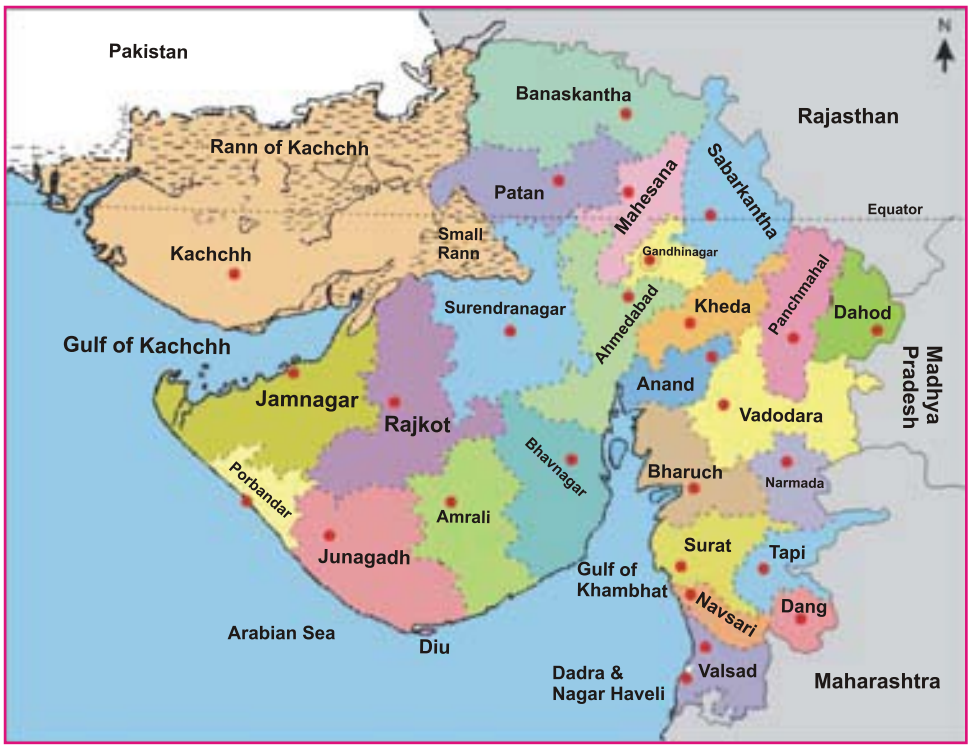
(1) 3 ..... {1, 2, 3, 4}

(2) 100 ..... {1, 2, 3, ..., 99}

(3) 5 ..... $\{x / x$ is a multiple of 10$\}$ (5) 0 ..... $\{x / x$ is a natural number$\}$

(4) 2 ..... $\{x / x$ is a prime factor of 15$\}$

2. On the basis of given map write the given groups by the listing method :



No.	Groups	The listing method
1.	The district of Gujarat starting with 'A'	
2.	Such a district of Gujarat whose boundary touches Uttar Pradesh	
3.	The bay lying in Gujarat	
4.	The smallest district	
5.	The largest district	

3. Answer the following questions on the basis of groups given in the above Table :

(1) Which are the singleton sets ?

(2) Which are the finite sets ?

(3) Which one is the empty set ?

Subset :

If every member of the set B is a member of the set A, then B is called a subset of A.

For example, A = $\{x / x$ is a positive integer less than 10$\}$

= $\{1, 2, 3, ..., 9\}$

B = $\{1, 2, 3, 6\}$

Here, each and every member of set B lies in the set A. Therefore, set B is called subset of set A. Symbolically it is denoted as B $\subset$ A.

Think : In every set, each and element of that set is member of itself, then can we say every set is a subset of itself.

A = $\{a, b, c\}$ and B = $\{x, b, c\}$, then can we say that set A is subset of set B ? Why ?

- **Keep in mind :** Every set is a subset of itself, i.e. A $\subset$ A.
  - The empty set is a subset of each and every set, i.e. $\phi \subset$ A.
  - Every set has at least two subsets, except the empty set.
  - The empty set has only one subset, i.e., itself only.
  - If A is not a subset of set B, then it is written as A $\not\subset$ B.

Number of subsets of a given set : Look and understand

No.	Set	Number of member in set	Subsets	Number subsets	Number of subsets in exponent form
1.	A = { }	0	{ }	1	2^0
2.	B = $\{x\}$	1	{ }, $\{x\}$	2	2^1
3.	C = $\{p, q\}$	2	{ }, $\{p\}$ $\{q\}$, $\{p, q\}$	4	2^2
4.	D = $\{a, b, c\}$	3	{ }, $\{a\}$, $\{b\}$ $\{c\}$, $\{a, b\}$ $\{b, c\}$, $\{c, a\}$ $\{a, b, c\}$	8	2^3

On the basis of above Table it can be said that number subsets of set having number of members n in itself = 2^n.

Think :

(1) How many subsets are there in a set having four members in it ? .....

(2) How many subsets are there in a set having five members in it ? .....

- **Equal Set :**

If set A and B contain the same elements, then set A and set B are said to be equal sets. Symbolically it is written as A = B.

For example : A = $\{x / x$ is a natural number less than 5$\}$, A = $\{1, 2, 3, 4\}$

B = $\{x / x$ is a factor of 12 less than 5$\}$, B = $\{1, 2, 3, 4\}$

Here, set A and set B have the same elements, therefore set A and set B are equal sets.

$\therefore$ A = B

Additionally here A $\subset$ B and B $\subset$ A.

$\therefore$ A $\subset$ B and B $\subset$ A, then A = B

- **One to One Correspondence :**

Suppose there are 10 students in a class. A unique (one and only one) roll number is given to each student.

(1) Utsav or 1 $\leftrightarrow$ Utsav

(2) Vijay or 2 $\leftrightarrow$ Vijay

(3) Chahana or 3 $\leftrightarrow$ Chahana

. . .
. . .
. . .
. . .

(10) Rehana or 10 $\leftrightarrow$ Rehana

Therefore, there is only one number from 1 to 10 corresponding to each student, which is his roll number and no student have two roll numbers because there will not be two students with same roll number. Such correspondence is called one to one correspondence.

Now, A = $\{1, 2, 3\}$ and B = $\{a, b, c\}$, then there can be 6 correspondence between them.

(i)	(ii)	(iii)	(iv)	(v)	(vi)
$1 \leftrightarrow a$	$1 \leftrightarrow a$	$1 \leftrightarrow b$	$1 \leftrightarrow b$	$1 \leftrightarrow c$	$1 \leftrightarrow c$
$2 \leftrightarrow b$	$2 \leftrightarrow c$	$2 \leftrightarrow a$	$2 \leftrightarrow c$	$2 \leftrightarrow a$	$2 \leftrightarrow b$
$3 \leftrightarrow c$	$3 \leftrightarrow b$	$3 \leftrightarrow c$	$3 \leftrightarrow a$	$3 \leftrightarrow b$	$3 \leftrightarrow a$

- **Equivalent Set :**

If the number of the elements of two finite sets is the same, then they are called equivalent sets. Its symbol is '$\sim$'.

$A = \{1, 4, 6\}$ $\quad$ $B = \{x, y, z\}$

$n(A) = 3$ $\quad$ $n(B) = 3$

Here, $n(A) = n(B)$. Therefore set A and set B are equivalent sets.

Symbolically they are written as $A \sim B$.

- **Universal Set :**

Generally, in the discussion about sets, all sets are assumed to be the subsets of a definite set. This definite set with respect to its subsets is called the universal set. The symbol U is used for universal set.

For example, with reference to the set of all students of the school, the set of the player of Kho-kho team, the set of players of Kabaddi team, the set of members of the prayer committee, the set of students of VIII Standard etc are the subsets of the set of the students of school. Therefore, in this reference the set of the students of the school is the universal set.

1. Fill in the blanks using symbol $\subset$ or $\not\subset$:

(1) N ..... Z

(2) $\{3, 1, -1\}$ ..... N

(3) Z ..... Q

(4) $\left\{\frac{1}{2}, \frac{2}{3}, \frac{3}{4}\right\}$ ..... Q

2. Write the following with symbols :

(1) 3 lies in the set A.

(2) $\frac{1}{2}$ does not lie in the set A.

(3) The set A is an empty set.

(4) Set A and set B are equal sets.

(5) Set C is subset of set D.

(6) Set B and set C are equivalent sets.

(7) Set A is not subset of set B.

(8) $\{0\}$ is a subset of set B.

3. A = $\{x / x$ is an even natural number less than 10$\}$ and B = $\{-2, -3, -4, -5\}$, then whether they are equal sets or equivalent set ? Write with symbol.

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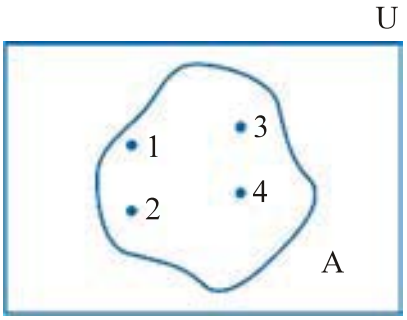
Venn Diagram :

According to the method of British logician John Venn, a universal set is expressed by the interior part of a rectangle and other sets (subsets of the universal set) are expressed by freehand interior part of the rectangle. This pictorial representation of the sets are known as Venn Diagram.

Venn diagram of a single set :

A = $\{1, 2, 3, 4\}$



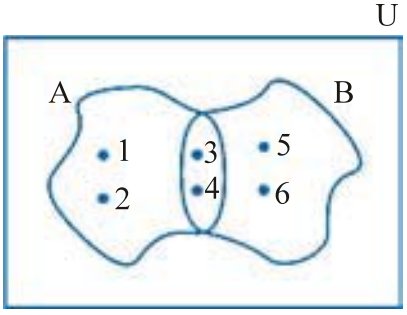
Venn diagrams of two sets :

There are four possibilities for Venn diagrams of two sets.

(1) When one or more than one members between the two sets are common.

A = $\{1, 2, 3, 4\}$

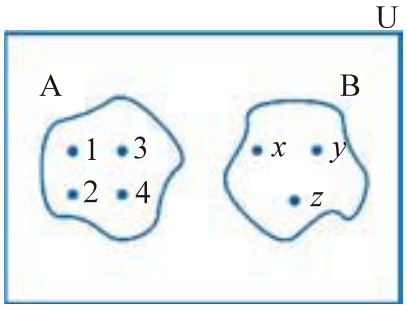
B = $\{3, 4, 5, 6\}$



(2) When both sets are disjoint sets

A = $\{1, 2, 3, 4\}$

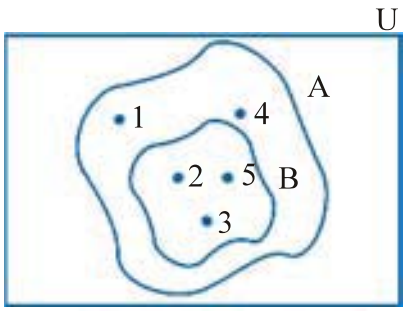
B = $\{x, y, z\}$



(3) When out of two sets one set is subset of the other.

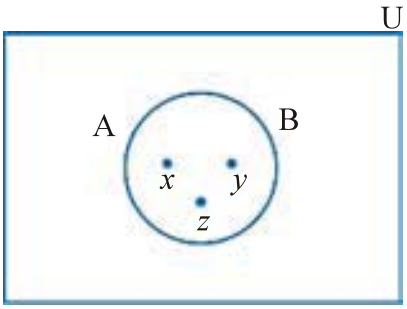
A = $\{1, 2, 3, 4, 5\}$

B = $\{2, 3, 5\}$



(4) When both are equal sets.

$A = \{x, y, z\}$ and $B = \{x, y, z\}$



Think : How many possibilities are there for Venn diagram of three sets ?

- **Complement of a set :**

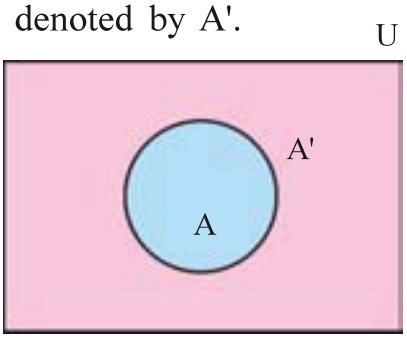
The set of all those elements which are present in the universal set U but not present in the set A, is called complement of the set A. It is denoted by A'.

Therefore $A' = \{x / x \in U \text{ and } x \notin A\}$

In short $A' = U - A$

$U = \{1, 2, 3, 4, 5\}$ and $A = \{1, 2\}$

$A' = \{3, 4, 5\}$



Think : (1) U' = ... (2) (A')' = ...

Set operations : There are two types of set opeations : (1) Union (2) Intersection.

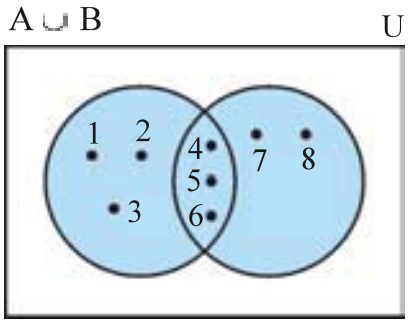
- **Union :**

The set consisting of all elements of the set A and the set B is called the union of the sets A and B and it is denoted by $A \cup B$.

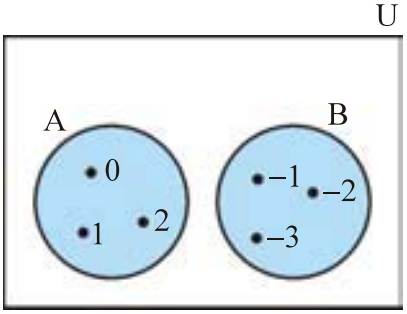
$A \cup B = \{x / x \in A \text{ or } x \in B\}$

For example, $A = \{1, 2, 3, 4, 5, 6\}$ and set $B = \{4, 5, 6, 7, 8\}$

$A \cup B = \{1, 2, 3, 4, 5, 6, 7, 8\}$



In the diagram, $A \cup B$ is denoted by shaded portion. The elements lying in both set A and set B are not written two times but they are shown in the intersection part of two circles.



For example, : $A = \{1, 0, 2\}$

$B = \{-2, -3, -1\}$, then

$A \cup B = \{1, 0, 2, -1, -2, -3\}$

Intersection :

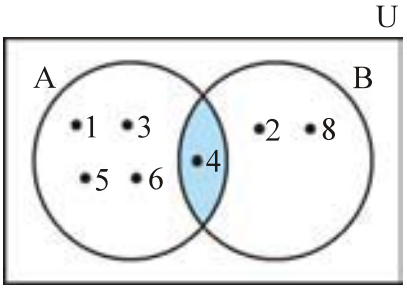
The set consisting of elements which are in both A and B is called the intersection of A and B. It is denoted by $A \cap B$.

$A \cap B = \{x \mid x \in A \text{ and } x \in B\}$

For example, : $A = \{1, 3, 4, 5, 6\}$ and

$B = \{2, 4, 8\}$, then

$A \cap B = \{4\}$

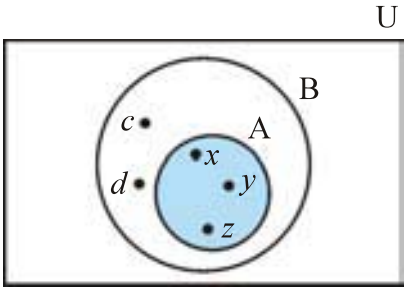


In this figure the shaded region shows $A \cap B$.

For example, $A = \{x, y, z\}$

$B = \{x, y, z, c, d\}$, then

$A \cap B = \{x, y, z\} = A$



If $A \subset B$, then $A \cap B = A$

Example 1 : $A = \{1, 2, 3, 4\}$

$B = \{x / x \text{ is an integer from } -3 \text{ to } 3\}$, then represent

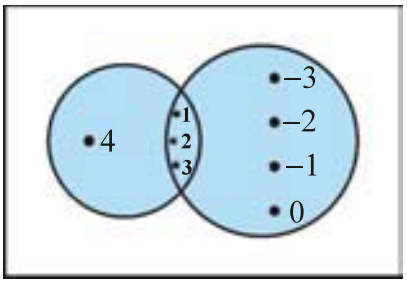
$A \cup B$ and $A \cap B$ by Venn diagram.

Solution : $A \cup B = \{1, 2, 3, 4\} \cup \{-3, -2, -1, 0, 1, 2, 3\}$

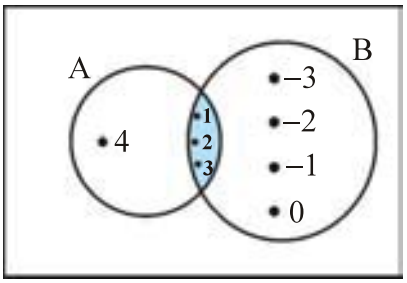
$= \{-3, -2, -1, 0, 1, 2, 3, 4\}$

$A \cap B = \{1, 2, 3, 4\} \cap \{-3, -2, -1, 0, 1, 2, 3\} = \{1, 2, 3\}$

$A \cup B$ U

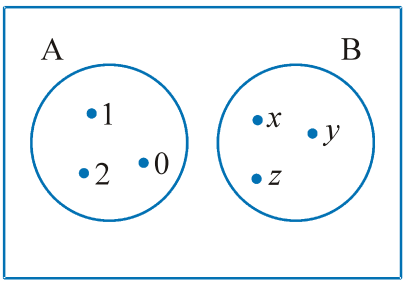


$A \cap B$ U



Example 2 : If $A = \{1, 0, 2\}$ and $B = \{x, y, z\}$, then show by Venn diagram such that $A \cap B = \{\} = \phi$.

Solution :



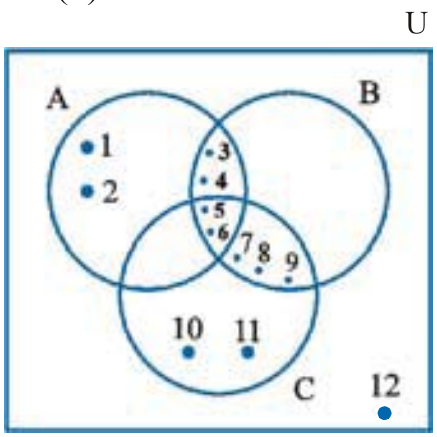
Keep in mind : For two non-empty sets A and if $A \cap B = \phi$, then A and B are called **Disjoint Sets**.

Example 3 : $U = \{x / x \in N, 1 \leq x \leq 12\}$, $A = \{1, 2, 3, 4, 5, 6\}$
$B = \{3, 4, 5, 6, 7, 8, 9\}$, $C = \{5, 6, 7, 8, 9, 10, 11\}$

Draw a Venn diagram representing these sets. From that, find the following sets :

(1) $B \cup C$ (2) $A \cap C$ (3) $A \cap B$

(4) $(A \cup B) \cap C$ (5) $(A \cap C) \cup (B \cap C)$ (6) $A \cap B \cap C$



(1) $B \cup C = \{3, 4, 5, 6, 7, 8, 9, 10, 11\}$

(2) $A \cap C = \{5, 6\}$

(3) $A \cap B = \{3, 4, 5, 6\}$

(4) $A \cup B = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$
$C = \{5, 6, 7, 8, 9, 10, 11\}$
$(A \cup B) \cap C = \{5, 6, 7, 8, 9\}$

(5) $A \cap C = \{5, 6\}$, $B \cap C = \{5, 6, 7, 8, 9\}$
$(A \cap C) \cup (B \cap C) = \{5, 6, 7, 8, 9\}$

(6) For $A \cap B \cap C$, taking intersection $A \cap B$ as in (3) with C, we will obtain the result.
$A \cap B \cap C = \{5, 6\}$

- Please do activity by game of BINGO under the guidance of the teacher.

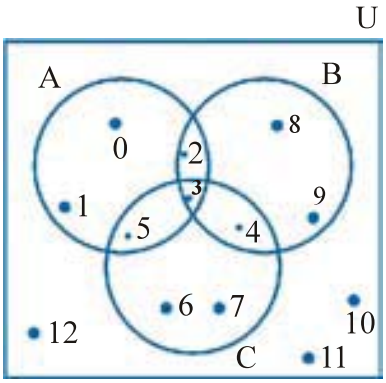
1. Calculate as asked :

(1) $U = \{x / x$ is name of months of English calendar$\}$,
$A = \{$March, May, July, June$\}$, then find A'.

(2) $U = \{x / x$ is main colour of Rainbow$\}$, $R = \{$Violet, Red, Yellow$\}$, then find R'.

(3) $U = \{x \in N / x \leq 9\}$, $A = \{2, 3, 5\}$
$B = \{4, 5, 7\}$, then find A', (A')', B' and (B')'.

2. Find the following results from the Venn diagram :



(1) $A \cup B =$ .....

(2) $A \cap B =$ .....

(3) $(A \cap C) \cup B =$ .....

(4) $(A \cup C) \cup B =$ .....

(5) $U =$ .....

3. Find union and intersection of the given sets and also show them by Venn diagram.

$A = \{x / x$ is a natural number less than $5\}$, $B = \{x / 3 < x < 7; x \in N\}$

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1. Fill in the blanks using proper symbol $\in$, $\notin$, $\subset$, $\not\subset$, $\sim$ or $=$:

(1) 6 ..... {1, 2, 4, 6}

(2) {20} ..... {20, 30, 40}

(3) 7 ..... $\{x / x$ is a prime natural number$\}$

(4) 9 ..... $\{x / x$ is a multiple of 18$\}$

(5) {1, 2, 3} ..... N.

(6) {−1, 1, 0} ..... N.

(7) If A = {*a, b, c*} and B = {1, 2, 3}, then A ..... B.

2. Show the following groups by the listing method and by the property method :

No.	Groups	The listing method	The property method
1.	Multiplies of 5		
2.	Prime numbers between 21 and 30		
3.	Positive integers less than 6		
4.	Factors of 21		

3. State which of the following sets are empty sets and which are singleton sets :

(1) $\{x / x$ is a prime number less than $3\}$

(2) {5}

(3) $\{x / x + 1 = 1, x \in N\}$

(4) $\{x / x$ is the additive identity$\}$ or $\{x / x$ is a neutral demant for addition1$\}$

4. Which of the following are finite sets and infinite sets, write them :

(1) The set of citizens of India.

(2) The set of numbers of three digits more than 100.

(3) A = {x / x is a number whose unit digit is 7}

(4) {x / x is a prime number}

5. Classify the following sets are equal sets and equivalent set and write them in symbolic form :

(1) P = {a, b, c}, Q = {x, y, z}

(2) F = { }, G = {x / x is a four digit number less than 1000}

(3) A = {1, 4, 9, 16}, B = {x / x is a perfect square number less than 25}

(4) D = {p, q, r}, E = {r, q, p}

(5) A = {1, 2, 3}, B = {7, 8, 9}

6. If U = N and A = {1, 2, 3,..., 10}, then find A' and (A')'.

7. Write all one-to-one correspondence between the given two sets :

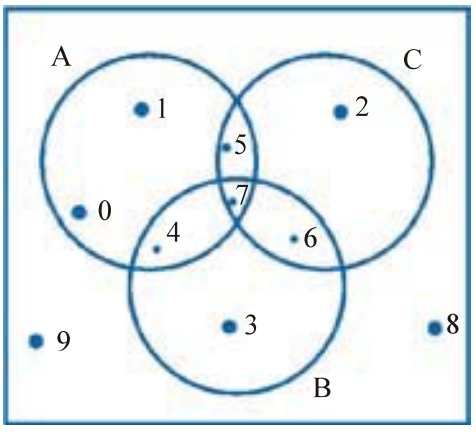
A = {x, y} and B = {a, b}

8. Draw Venn diagram for given sets and write union and intersection for each set :

(1) A = {x, y, z, w}, B = {a, b, c, x, y}

(2) S = {5, 10, 15}, R = {10, 15, 25, 20}

9. From the given Venn diagram find the following results :



(1) A ∩ B

(2) (A ∪ B) ∪ (B ∪ C)

(3) A ∩ (B ∪ C)

(4) (A ∪ C) ∩ B

(5) U

Answers

Practice 1

1. (1) ∈ (2) ∉ (3) ∉ (4) ∉ (5) ∉

Practice 2

1. (1) $\subset$ (2) $\not\subset$ (3) $\subset$ (4) $\subset$

2. (1) $3 \in A$ (2) $\frac{1}{2} \notin A$ (3) $A = \phi$ (4) $C = B$
(5) $C \subset D$ (6) $B \sim C$ (7) $A \not\subset B$ (8) $\{0\} \subset B$

3. (1) Equivalent sets, $A \sim B$

Practice 3

1. (1) A' = {January, February, March, April, May, June, July, August, September, October, November, December}
(2) R' = {Blue, Indigo, Green, Orange}
(3) A' = {1, 4, 6, 7, 8, 9}, (A')' = A, B' = {1, 2, 3, 6, 8, 9}, (B')' = B

2. (1) {0, 1, 2, 3, 4, 5, 8, 9} (2) {2, 3} (3) {2, 3, 4, 5, 8, 9}
(4) {0, 1, 2, ..., 9} (5) {0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12}

3. $A \cap B = \{4\}$, $A \cup B = \{1, 2, 3, 4, 5, 6\}$

Exercise

1. (1) $\in$ (2) $\subset$ (3) $\notin$ (4) $\notin$ (5) $\subset$ (6) $\not\subset$ (7) $\sim$

2. (1) **Listing method :** {5, 10, 15, ...}, **Property method :** {x / x is a multiple of 5}
(2) **Listing method :** {23, 29},
Property method : {x / x is a prime number between 21 and 30}
(3) **Listing method :** {1, 2, 3, 4, 5},
Property method : {x / x is positive integer less than 6}
(4) **Listing method :** {1, 3, 7, 21}, **Property method :** {x / x is a factor of 21}

3. (1) Singleton set (2) Singleton set (3) Empty set (4) Singleton set

4. (1) Finite set (2) Finite set (3) Infinite set (4) Infinite set

5. (1) $P \sim Q$ (2) $F = G$ (3) $A = B$ (4) $D = E$ (5) $A \sim B$

6. A' = {11, 12, 13,...}, (A')' = A

7. $x \leftrightarrow a$, $x \leftrightarrow b$, $y \leftrightarrow a$, $y \leftrightarrow b$

8. (1) $A \cup B = \{a, b, c, x, y, z, w\}$, $A \cap B = \{x, y\}$
(2) $S \cup R = \{5, 10, 15, 20, 25\}$, $S \cap R = \{10, 15\}$

9. (1) {4, 7} (2) {0, 1, 2, 3, 4, 5, 6, 7} (3) {4, 5, 7} (4) {4, 6, 7}
(5) {0, 1, 2, 3, 4, 5, 6, 7, 8, 9}