Short Answer Type Questions

Q. 1. How many terms are free from radical signs in the expansion of $\left(x^{1/5} + y^{1/10}\right)^{55}$. [DDE – 2017]

Sol. The general term in the expansion of $\left(x^{1/5} + y^{1/10}\right)^{55}$ is given by

$T_{r+1} = {}^{55}C_r \left(x^{1/5}\right)^{55-r} \left(y^{1/10}\right)^{r}$

$\Rightarrow T_{r+1} = {}^{55}C_r\, x^{11-\frac{r}{5}}\, y^{r/10}$

Clearly, T_{r+1} will be free from radical signs, if $\frac{r}{5}$ and $\frac{r}{10}$ are integers for $0 \leq r \leq 55$.

$\therefore$ r = 0, 10, 20, 30, 40, 50.

Hence, there are 6 terms in the expansion of $\left(x^{1/5} + y^{1/10}\right)^{55}$ which are independent of radical sign.

Q. 2. Find the constant term in expansion $\left(x - \frac{1}{x}\right)^{10}$ [DDE – 2017]

Sol. General term of the expansion $\left(x - \frac{1}{x}\right)^{10}$ is given by

$T_{r+1} = {}^{10}C_r (x)^{10-r} \left(\frac{-1}{x}\right)^{r}$

$= {}^{10}C_r (x)^{10-2r} (-1)^r$

$= (-1)^{r}\, {}^{10}C_r\, x^{10-2r}$

For the constant term, put 10 – 2r = 0

We get, r = 5

Hence, $T^{5+1} = T_6$ i.eth term of the expansion is constant.

$T_6 = T_{5+1} = {}^{10}C_5 (-1)^5 x^{10-10}$

$= -\frac{10!}{5!5!}$

$= -\frac{10 \times 9 \times 8 \times 7 \times 6 \times 5!}{5 \times 4 \times 3 \times 2 \times 1 \times 5!}$

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Q. 3. Find the 4th term from the end in the expansion of $\left(\frac{x^3}{2} - \frac{2}{x^3}\right)^{9}$. [DDE – 2017]

Sol. Since r^{th} term from the end in the expansion of $(a + b)^n$ is $(n – r + 2)^{th}$ term from the beginning. Therefore 4^{th} term from the end is 9 – 4 + 2, i. e., 7^{th} term from the beginning, which was given by

$$T_7 = {}^9C_6 \left(\frac{x^3}{2}\right)^3 \left(\frac{-2}{x^2}\right)^6 = {}^9C_3 \frac{x^9}{8}\frac{64}{x^{12}}$$

$$= \frac{9\times8\times7}{3\times2\times1} \times \frac{8}{x^3} = \frac{672}{x^3}$$

OR

4^{th} term from the end of $\left(\frac{x^2}{2} - \frac{2}{x^3}\right)^9$

= 4^{th} term from the beginning $\left(\frac{-2}{x^3} + \frac{x^2}{2}\right)^9$

$$\therefore T_4 = {}^9C_4 \left(\frac{-2}{x^3}\right)^{9-3} \left(\frac{+x^2}{2}\right)^3$$

$$= {}^9C_3 \frac{2^6}{x^9}, \frac{x^6}{8}$$

$$= \frac{9.8.7}{3.2.1}\, \frac{64}{x^3\times8}$$

$$= \frac{672}{x^3}$$

Q. 4. Find the coefficient of x^5 in the expansion of $(1 + x)^{21}$ $\{1 + (1 + x) \ldots + (1 - x)9\}$
[DDE – 2017]

Sol. Given,

$$(1 + x)^{21} \{1 + (1 + x) + \ldots (1 - x)^9\}$$

$$= (1 + x)^{21} \left\{\frac{(1+x)^{10} - 1}{(1+x) - 1}\right\} \quad \text{(by sum of G. P)}$$

$$= \frac{1}{x}[(1 + x)^{31} - (1 + x)^{21}]$$

$\therefore$ Coefficient of x^5 in this expression = coefficient of x^6 in the expansion $[(1 + x)^{31} - (1 + x)^{21}]$

= ${}^{31}C_6 – {}^{21}C_6$

Q. 5. Write the last two digits of the number 3^{400} [DDE – 2017]

Sol. Given, 3^{400}

= $(1 + 2)^{400}$

$= {}^{400}C_0 (1)^{400-0} (2)^0 + {}^{400}C_1 (1)^{400-1} (2)^1 + {}^{400}C_2 (1)^{400-2} (2)^2 + \ldots + {}^{400}C_{400} (1)^{400-400} (2)^{400}$

We observe that all term expect the first one are divisible by 100, so the remainder when $3^{400} + 100$ is same as the $[{}^{400}C_0 (1)^{400} (2)^0] \div 100 = 1 \div 100$