Solid Mechanics Test 2

Number of Questions: 30 **Time: 75 min.**

Directions for questions 1 to 30: Select the correct alternative from the given choices.

1. If Poisson's ratio of a material is 0.5, modulus of elasticity of the material is

(A) $\frac{1}{3}$ times the shear modulus
(B) 3 times the shear modulus
(C) 4 times the shear modulus
(D) equal to the shear modulus

2. A circular shaft subjected to torsion undergoes a twist of 1° in a length of 1.6 m. If the maximum shear stress induced is 10,000 N/cm^2 and if modulus of rigidity is 8×10^6 N/cm^2 then radius of the shaft is

(A) $\frac{27}{\pi}$ cm (B) $\frac{36}{\pi}$ cm
(C) $\frac{\pi}{36}$ cm (D) $\frac{\pi}{27}$ cm

3. A 2 m long mild steel bar of 2000 mm^2 cross sectional area is subjected to an axial load of 40 kN. If Young's modulus for the shaft is 2×10^5 N/mm^2, extension of the shaft in mm is

(A) 0.5 mm (B) 1 mm
(C) 0.2 mm (D) 2 mm

4. A steel bar of 1 m length is heated from 30°C to 60°C. Coefficient of linear expansion is 12×10^{-6}/°C and Young's modulus is 2×10^5 MN/m^2. Stress developed in the bar is

(A) 18 N/mm^2 (B) zero
(C) 36 N/mm^2 (D) 72 N/mm^2

5. Slope of a beam under load is

(A) rate of change of deflection
(B) rate of change of bending moment
(C) rate of change of bending moment x flexural rigidity
(D) rate of change deflection x flexural rigidity

6. Relationship between modulus of elasticity E, modulus of rigidity G and bulk modulus K is

(A) $E = \frac{6KG}{3K+G}$ (B) $E = \frac{9KG}{3K+G}$
(C) $E = \frac{3K+G}{6KG}$ (D) $E = \frac{3K+G}{9KG}$

7. A column with 80mm diameter is fixed at both the ends. If the crippling load calculated by Rankine formula is 750 kN then what will be the crushing load of the column. The length of column is 8m. E = 180 GPa

(A) 949.4 kN (B) 994.4 kN
(C) 317.77 kN (D) 984.6 kN

8. A bar of 3m in length 30 mm breadth and 20 mm thickness is subjected to a compressive stress of 50 kN/m^2. What will be the final volume of the bar if the poisson's ratio is 0.30 and modulus of rigidity is 90 GN/m^2.

(A) will increase by 0.4615 mm^3
(B) will decrease by 0.5625 mm^3
(C) will decrease by 0.4615 mm^3
(D) will increase by 0.5625 mm^3

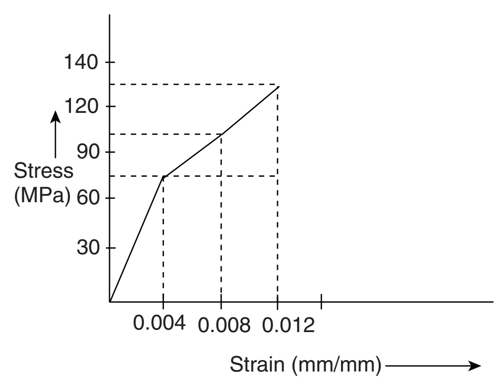
9. When 'C' is the modulus of rigidity and 'q' is the intensity of shear stress, the strain energy due to shear is given by

(A) $\frac{q}{2C} \times$ volume of block
(B) $\frac{q^2}{2C} \times$ volume of block
(C) $\sqrt{\frac{q^2}{2C}} \times$ volume of block
(D) $\frac{q}{2} \times$ Volume of block

10. According to maximum shear stress failure criterion, yielding in material occurs when maximum shear stress is

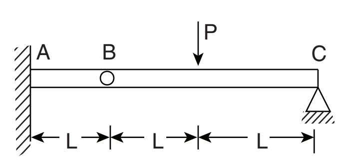
(A) $\frac{1}{2}$ yield stress (B) $\sqrt{2}$ yield stress
(C) $\frac{\sqrt{2}}{3}$ yield stress (D) 2 yield stress

11. Stress-strain behaviour of a material is shown in the figure. Proof Resilience in Nm/m^3 is



(A) 10×10^4 (B) 15×10^4
(C) 76×10^4 (D) 130×10^4

12.

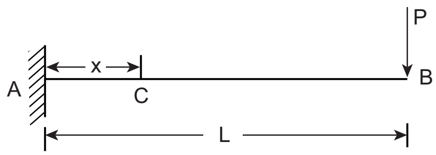


A beam is made of 2 bars AB and BC hinged at B, fixed at A and simply supported at C. If it is loaded at mid point of BC as shown in figure, bending moment at A is

(A) PL (B) $\frac{PL}{2}$

(C) $2PL$ (D) $\frac{2}{3}PL$

13. A cantilever beam is loaded as shown in the figure



If L is the length of the beam and EI, the flexural rigidity, slope at point C at a distance x from fixed end is

(A) $\frac{Px}{\text{EI}}(2L-x)$ (B) $-\frac{Px}{2\text{EI}}(2L-x)$

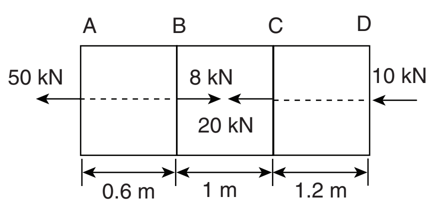
(C) $\frac{Px}{2\text{EI}}(L-x)$ (D) $-\frac{Px}{\text{EI}}(L-x)$

14. The extension of a circular bar tapering uniformly from diameter d_1 to d_2 is same as that of a uniform circular bar of same length, under same load. Diameter of the uniform bar is

(A) $\sqrt{d_1 d_2}$ (B) $\sqrt{d_1^2 - d_2^2}$

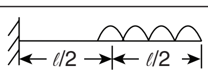
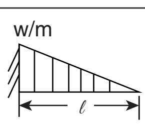
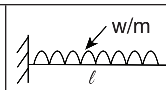
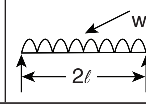
(C) $\frac{d_1 + d_2}{2}$ (D) $\frac{d_1 - d_2}{2}$

15. A brass bar having a cross sectional area of 1000 mm^2 is subjected to axial forces as shown in the figure. The total change in length of the bar is.
Take $E = 1.05 \times 10^5$ N/mm^2.



(A) – 0.15 mm (B) + 0.15 mm
(C) – 0.1143 mm (D) + 0.1143 mm

16. Match the following List – I (Loaded beam) and List – II (Maximum bending moment).

	List – I		List – II
a.		1.	$\frac{w\ell^2}{2}$
b.		2.	$\frac{w\ell^2}{6}$
c.		3.	$\frac{3}{8}w\,\ell^2$
d.		4.	$\frac{w\ell^2}{4}$

Codes:

	a	b	c	d		a	b	c	d
(A)	3	2	4	1	(B)	3	4	2	1
(C)	1	2	3	4	(D)	4	3	2	1

Directions for questions 17 and 18: A circular bar made of C.I is to resist on occasional torque of 2.2 kNm acting in transverse plane. The allowable stresses in compression, tension and shear are 100, 50, 35 MN/m^2 respectively.
Take $G = 40$ GN/m^2.

17. The diameter of the bar will be

(A) 64.8 mm (B) 68.4 mm
(C) 66.8 mm (D) 67.4 mm

18. The angle of twist under the applied torque per meter length of bar will be

(A) 1.86° (B) 1.26°
(C) 1.46° (D) 1.16°

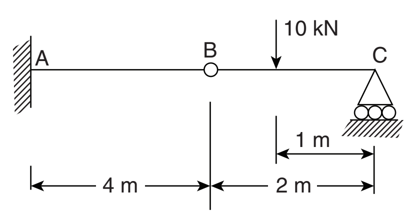
19. If diameter of long column is reduced by 20% then percentage of reduction in Euler buckling is

(A) 4 (B) 36
(C) 49 (D) 59

20. A shaft subjected to a maximum bending stress of 80 N/mm^2 and maximum shearing stress equal to 30 N/mm^2 at a particular section. If the yield point in the tension of the material is 280 N/mm^2, and maximum shear stress theory of failure is used, then the factor of safety obtained will be

(A) 2.5 (B) 2.8
(C) 3.0 (D) 3.5

21. A Cantilever beam AB is connected to another beam BC with a pin joint at B as shown in the figure. For the loading as shown in the figure, the magnitude of bending moment at A (in kN-m) is



(A) 5 (B) 10
(C) 20 (D) 50

22. A solid metal tube with modulus of elasticity E and Poisson's ratio μ is constrained on all faces. It is heated so that temperature rises uniformly. If coefficient of thermal expansion is α, the compressive stress developed in the cube due to the heating is

(A) $\dfrac{E\alpha\Delta T}{2(1-2\mu)}$ (B) $\dfrac{E\alpha\Delta T}{(1-2\mu)}$

(C) $\dfrac{2E\alpha\Delta T}{(1-2\mu)}$ (D) $\dfrac{E\alpha\Delta T}{3(1-2\mu)}$

23. A bar of length L, breadth b and thickness t is subjected to an axial pull of P. If e_x is the strain in the direction of pull, volumetric strain produced is (μ = Poisson's ratio)
(A) $e_x(1 + 2\mu)$
(B) $e_x(1 - 2\mu)$
(C) $e_x(1 + \mu)$
(D) $e_x(1 - \mu)$

24. A simply supported beam of length L has a cross section of depth d and width $\dfrac{d}{2}$. If it is loaded with a uniformly distributed load of w/unit length, maximum deflection is (Young's modulus = E)

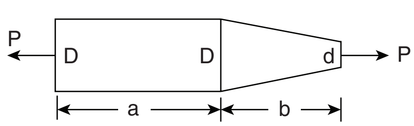
(A) $\dfrac{5}{8}\dfrac{wL^4}{Ed^4}$ (B) $\dfrac{5}{16}\dfrac{wL^4}{Ed^4}$

(C) $\dfrac{5}{8}\dfrac{wL^3}{Ed^4}$ (D) $\dfrac{5}{16}\dfrac{wL^3}{ed^4}$

25. When a material is subjected to uniaxial tension, to avoid failure due to shear in 45° planes, the shear strength of the material should be atleast
(A) half the tensile strength
(B) $\dfrac{1}{\sqrt{2}}$ times tensile strength
(C) tensile strength
(D) $\dfrac{3}{4}$ times tensile strength

26. At a point in a strained material, direct stresses 120 N/mm^2 (tensile) and 100 N/mm^2 (compressive) are acting. If major principal stress is 150 N/mm^2, maximum shearing stress at the point is
(A) 87 N/mm^2
(B) 140 N/mm^2
(C) 130 N/mm^2
(D) 280 N/mm^2

27.



A bar is having uniform diameter D for a length a and tapering diameter from D to d for a length b as shown in figure. If the bar is subjected to an axial pull P, the extension produced is

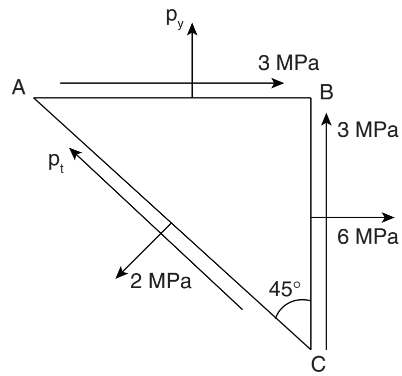
(A) $\dfrac{4P}{\pi DE}\left(\dfrac{a}{D}+\dfrac{b}{d}\right)$

(B) $\dfrac{4P}{\pi dE}\left(\dfrac{a}{D}+\dfrac{b}{d}\right)$

(C) $\dfrac{2P}{\pi DE}\left(\dfrac{a}{D}+\dfrac{b}{d}\right)$

(D) $\dfrac{2P}{\pi dE}\left(\dfrac{a}{D}+\dfrac{b}{d}\right)$

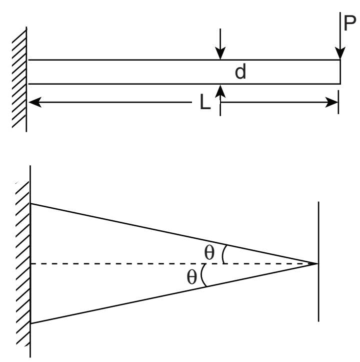
28.



At a point in a stressed body stresses acting are as shown in the figure. Value of P_y is

(A) –8 MPa (B) 8 MPa
(C) –4 MPa (D) 4 MPa

29.

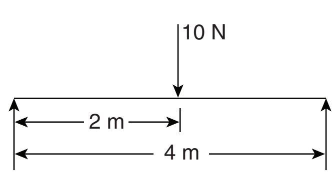


A cantilever beam of varying width and constant depth is loaded as shown in the figure. Maximum bending stress at the fixed end of the beam is

(A) $\dfrac{PL}{d^2\tan\theta}$ (B) $\dfrac{2P}{d^2\tan\theta}$

(C) $\dfrac{3P}{d^2\tan\theta}$ (D) $\dfrac{3PL}{d^2\tan\theta}$

30.



A beam with cross section 10 cm width and 20 cm depth is loaded as shown in the figure. Maximum shear stress at a section 1 m away from end is

(A) 0 (B) 0.375 MPa
(C) 3.75 MPa (D) 37.5 MPa

Answer Keys

1. B	**2.** B	**3.** C	**4.** B	**5.** A	**6.** B	**7.** C	**8.** C	**9.** B	**10.** A
11. B	**12.** B	**13.** B	**14.** A	**15.** C	**16.** A	**17.** B	**18.** C	**19.** D	**20.** B
21. C	**22.** B	**23.** B	**24.** B	**25.** A	**26.** B	**27.** A	**28.** A	**29.** C	**30.** B

Hints and Explanations

1. $E = 2G(1 + \mu)$
$= 2G(1 + 0.5) \Rightarrow E = 3G$ Choice (B)

2. $L = 1.6$ m

$\theta = 1° = \dfrac{\pi}{180}$ radian

$G = 8 \times 10^6$ N/cm^2 = 8×10^{10} N/m^2

$\tau = 10{,}000$ N/cm^2 = 1×10^8 N/m^2

$$\frac{\tau}{R} = \frac{G\theta}{L}$$

$$\Rightarrow \quad R = \frac{\tau L}{G\theta} = \frac{10^8 \times 1.6 \times 180}{8 \times 10^{10} \times \pi}\text{m} = \frac{0.36}{\pi}\text{m} = \frac{36}{\pi}\text{cm.}$$

Choice (B)

3. $L = 2$ m = 200 mm; $A = 2000$ mm^2;
$P = 40$ kN = 40×10^3 N
$E = 2 \times 10^5$ N/mm^2

$$\text{Extension } \delta L = \frac{PL}{AE} = \frac{40 \times 10^3 \times 2000}{2000 \times 2 \times 10^5} = 0.2 \text{ mm.}$$

Choice (C)

4. As the expansion of the bar is not blocked, no stress is developed. Choice (B)

7. If both the ends fixed

$$L_e = \frac{L}{2} = \frac{8}{2} = 4\text{m}$$

$E = 180 \times 10^9$N/m^2

$$\text{Euler's load } P_E = \frac{\pi^2 EI}{L_e^2} = \frac{\pi^2 \times 180 \times 10^9 \times I}{4^2}$$

$d = 80$mm = 80×10^{-3}m

$$I = \frac{\pi}{64}d^4 = \frac{\pi}{64}\left(80 \times 10^{-3}\right)^4 = 2.01 \times 10^{-6}\text{m}^4$$

$$P_E = \frac{\pi^2 \times 180 \times 10^9 \times 2.01 \times 10^{-6}}{16 \times 1000} = 223.2 \text{ kN}$$

$$\frac{1}{P_R} = \frac{1}{P_E} + \frac{1}{P_C}$$

$$\frac{1}{750} = \frac{1}{223.2} + \frac{1}{P_C}$$

$$\frac{1}{P_C} = \frac{1}{750} - \frac{1}{223.2} = \frac{223.2 - 750}{750 \times 223.2}$$

$P_C = 317.77$ kN Choice (C)

8. $\ell = 3$m = 3000 mm
$b = 30$ mm = 30×10^{-3} m
$t = 20$ mm = 20×10^{-3}m
$\sigma = 50$ kN/m^2
$\mu = 0.3$
$G = 90 \times 10^9$ N/m^2
$V = \ell b\, t = 3000 \times 30 \times 20$
$= 18 \times 10^5$ mm^3
$E = 2G(1 + \mu)$
$E = 2 \times 90 \times 10^9 (1 + 0.3) = 234 \times 10^9$ N/m^2
$E = 3K(1 - 2\mu)$
$234 \times 10^9 = 3K(1 - 0.6)$

$$K = \frac{234 \times 10^9}{3 \times 0.4} = 195 \times 10^9$$

$$K = \frac{\text{Compressive stress}}{\text{volumetric strain}}$$

$$195 \times 10^9 = \frac{50 \times 10^3}{\left(\dfrac{dv}{v}\right)}$$

$$\frac{dv}{v} = \frac{50 \times 10^3}{195 \times 10^9} = 0.256 \times 10^{-6}$$

$dv = 0.4615$ mm^3
Will decrease by 0.4615 mm^3. Choice (C)

11. Proof resilience is the strain energy at elastic limit

$$= \frac{1}{2}(\text{stress} \times \text{strain} \times \text{volume})$$

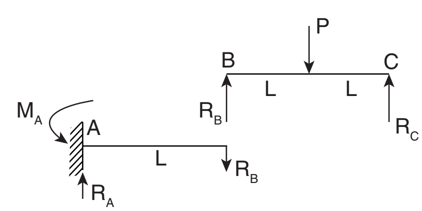
Proof resilience/ unit volume

$$= \frac{1}{2} \times \text{stress} \times \text{strain at elastic limit}$$

$$= \frac{1}{2} \times 75 \times 10^6 \times 0.004 = 15 \times 10^4 \text{ Nm/m}^3$$

Choice (B)

12.



Free body diagram is as shown above

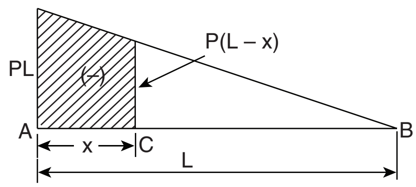
$$R_B = R_C = \frac{P}{2}$$

Bending moment at A

$$M_A = R_B \times L = \frac{PL}{2}.$$

Choice (B)

13.



BM diagram

BM diagram

Slope at C

$Q_C = Q_C - Q_A$ as $QA = 0$

$$= \frac{\text{Area of bending moment diagram between } A \text{ and } C}{EI}$$

$$= \frac{-[PL + P(L-x)]x}{2EI} = \frac{-(2PL - Px)}{2EI}x$$

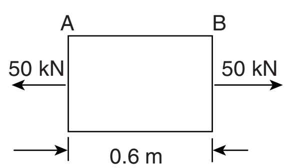
$$= \frac{-Px}{2EI}(2L - x)$$ Choice (B)

14. Extension $= \dfrac{PL}{AE}$

$$\therefore \frac{PL}{\frac{\pi d^2}{4}E} = \frac{PL}{\frac{\pi d_1 d_2}{4}E}$$

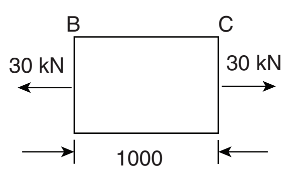
$\Rightarrow \; d = \sqrt{d_1 d_2}$ Choice (A)

15.

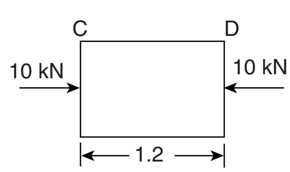


$$\delta l_1 = \frac{p\ell}{AE} = \frac{50 \times 10^3 \times 600}{1000 \times 1.05 \times 10^5}$$

$\delta l_1 = 0.2857$ mm



$$\delta l_2 = \frac{p\ell}{AE} = \frac{30 \times 10^3 \times 1000}{1000 \times 1.05 \times 10^5} = -\,0.2857\text{mm } (-)$$



$$\delta l_3 = \frac{p_l}{A_E} = \frac{10 \times 10^3 \times 1200}{1000 \times 1.05 \times 10^5} = -\,0.1143 \text{ mm}$$

$\delta l = \delta l_1 + \delta l_2 + \delta l_3 = -0.1143$ mm Choice (C)

17. $T = 2.2$ kNm

$\sigma_C = 100$ MN/m^2

$\sigma_t = 50$ MN/m^2

$\tau = 35$ MN/m^2

$G = 40$ GN/m^2

$$\frac{T}{J} = \frac{\tau_{\max}}{R} = \frac{2.2 \times 10^3}{\frac{\pi}{32}d^4} = \frac{35 \times 10^6}{\frac{d}{2}}$$

$d = 68.4$ mm Choice (B)

18. $$\frac{\tau_{\max}}{R} = \frac{G\theta}{L} = \frac{35 \times 10^6}{\left(\frac{68.4}{2000}\right)} = \frac{40 \times 10^9 \times \theta}{1}$$

$\theta = 0.026$ radians $\times\ 180/\pi$

$\theta = 1.46°$ Choice (C)

19. $F_{\text{euler}} = \dfrac{\pi^2 EI}{L^2}$

$F \propto I$

$I = \dfrac{\pi}{64}d^4$

$F \propto d^4$

$$\frac{F_1 - F_2}{F_1} = \frac{d_1^4 - d_2^4}{d_1^4}$$

$$D_2 = 0.8d_1 = \frac{d_1^4 - (0.8d_1)^4}{d_1^4} = \left(\frac{1 - 0.8^4}{1}\right) \times 100 = 59\%$$

Choice (D)

20. $\sigma_b = 80$ N/mm^2

$\tau = 30$ N/mm^2

$\sigma_y = 280$ N/mm^2

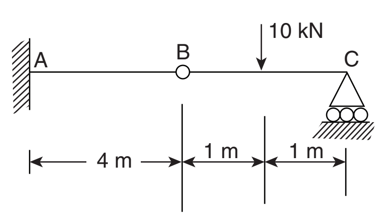
$$\tau_{\max} = \frac{1}{2}\sqrt{\sigma^2 + 4\tau^2} = \frac{280}{\text{FS} \times 2}$$

$$= \frac{1}{2}\sqrt{80^2 + 4(30)^2} = \frac{280}{\text{FS} \times 2} = 50 = \frac{280}{\text{FS}}$$

F . S = factor of safety $= \dfrac{280}{50 \times 2} = 2.8$ Choice (B)

21. Bending moment at hinge B

$M_B = 0$



Considering the portion BC, reaction at C,

$R_c = 5$ kN

Let M_A be the bending moment at A

Taking moments about A

$10 \times 5 - 5 \times 6 = 20$ k Nm. Choice (C)

22. Let side length be L

Elongation due to heating $= \alpha \Delta T L$

As elongation is blocked compressive strain in the x-direction

$$e_x = \alpha\,\Delta T = \frac{\sigma_x}{E} - \frac{\mu\sigma_y}{E} - \mu\frac{\sigma_z}{E}$$

As $\sigma_x = \sigma_y = \sigma_z = \sigma$

$$e_x = \frac{\sigma}{E}[1-2\mu]$$

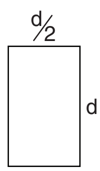
i.e., $\frac{\sigma}{E}[1-2\mu] = \alpha\Delta T \Rightarrow \sigma = \frac{E\alpha\Delta T}{(1-2\mu)}$ (compressive)

Choice (B)

23. Volumetric strain

$e_v = e_x + e_y + e_z = e_x + 2(-e_x\,.\,\mu) = e_x(1-2\mu)$ Choice (B)

24.



Moment of inertia $= \frac{bd^3}{12} = \frac{\left(\frac{d}{2}\right)\cdot d^3}{12} = \frac{d^4}{24}$

Maximum deflection $= \frac{5wL^4}{384EI} = \frac{5}{384}\frac{wL^4}{E}\times\frac{24}{d^4}$

$$= \frac{5}{16}\frac{wL^4}{Ed^4}$$

Choice (B)

25. Maximum shear stress $= \sqrt{\left(\frac{p_x + p_y}{2}\right)^2 + q^2}$

$$= \sqrt{\left(\frac{p_x}{2}\right)^2 + 0} = \frac{p_x}{2}$$

Maximum shear stress $= \frac{1}{2}$ the uni axial tensile stress

Choice (A)

26. $p_x = 120$ N/mm², $p_y = -100$ N/mm²

$p_1 = 150$ N/mm²

$p_1 + p_2 = p_x + p_y = 120 - 100 = 20$ N/m²

$\Rightarrow\ 150 + p_2 = 20 \Rightarrow p_2 = -130$

Maximum shearing stress $= \frac{p_1 - p_2}{2}$

$$= \frac{(150+130)}{2} = 140 \text{ N/mm}^2$$

Choice (B)

27. Mean area for tapering portion $= \frac{\pi Dd}{4}$

Total extension produced $= \frac{Pa}{\frac{\pi D^2{}_E}{4}} + \frac{Pb}{\frac{\pi Dd_E}{4}}$

$$= \frac{4P}{\pi DE}\left(\frac{a}{D} + \frac{b}{d}\right)$$

Choice (A)

28. Normal stress on plane AC

$$p_n = \frac{p_x + p_y}{2} + \frac{p_x - p_y}{2}\cos 2\theta + q\sin 2\theta$$

$p_n = 2$MPa; $p_x = 6$ MPa

$q = 3$ MPa; $\theta = 45°$

$$\therefore\ 2 = \frac{6 + p_y}{2} + 0 + 3\times 1 = 3 + \frac{p_y}{2} + 3$$

$$\Rightarrow\ \frac{p_y}{2} = -4 \Rightarrow p_y = -8 \text{ MPa}$$

$= 8$ MPa (compressive) Choice (A)

29. Maximum bending stress $f = \frac{M}{Z}$

where M = bending moment

and Z = section modulus

At fixed end,

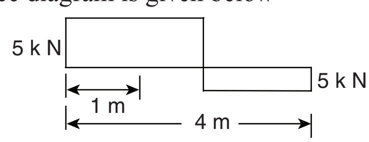
$M = P \times L$

$$Z = \frac{bd^2}{6} = \frac{2L\tan\theta d^2}{6}$$

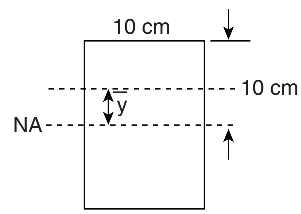
$$\therefore\ f = \frac{P\times L\times 6}{2Ld^2\tan\theta} = \frac{3P}{d^2\tan\theta}$$

Choice (C)

30. Shear force diagram is given below



Shear force at a distance 1m from end $F = 5$kN



$\bar{y} = 50$ mm

Area above neutral axis

$a = (100)^2$ mm²

breadth $b = 100$ mm

shear stress is maximum at neutral axis and is given by

$$q = \frac{F}{bI}a\bar{y}$$

$$q_{\max} = \frac{5\times 10^3 \times (100)^2 \times 50}{100\times\frac{100\times(200)^3}{12}} = 0.375 \text{ MPa}$$

Alternately,

Average shear stress $q_{av} = \frac{F}{bd}$

Maximum shear stress

$$q_{\max} = q_{av}\times 1.5 = \frac{F}{bd}\times 1.5 = \frac{5\times 10^3\times 1.5}{100\times 200} = 0.375 \text{ MPa}$$

Choice (B)