Design of Question Paper
Mathematics - Class XII

Time : 3 hours **Max. Marks : 100**

Weightage of marks over different dimensions of the question paper shall be as follows :

A. Weightage to different topics/content units

S.No.	Topics	Marks
1.	Relations and functions	10
2.	Algebra	13
3.	Calculus	44
4.	Vectors & three-dimensional Geometry	17
5.	Linear programming	06
6.	Probability	10
	Total	**100**

B. Weightage to different forms of questions

S.No.	Forms of Questions	Marks for each question	No. of Questions	Total Marks
1.	Very Short Answer questions (VSA)	01	10	10
2.	Short answer questions (SA)	04	12	48
3.	Long answer questions (LA)	06	07	42
	Total		**29**	**100**

C. Scheme of Options

There will be no overall choice. However, an internal choice in any four questions of four marks each and any two questions of six marks each has been provided.

D. Difficulty level of questions

S.No.	Estimated difficulty level	Percentage of marks
1.	Easy	15
2.	Average	70
3.	Difficult	15

Based on the above design, separate sample papers along with their blue prints and Marking schemes have been included in this document. About 20% weightage has been assigned to questions testing higher order thinking skills of learners.

Class XII
MATHEMATICS
Blue-Print I

S.No.	TOPIC	VSA (1)Mark	SA (4) Marks	LA (6) Marks	Total
1. (a)	Relations & Functions	1 (1)	4 (1)	-	5 (2) } 10 (4)
(b)	Inverse Trigonometric functions	1 (1)	4 (1)	-	5 (2)
2. (a)	Matrices	2 (2)	-	6 (1)	8 (3) } 13 (5)
(b)	Determinants	1 (1)	4 (1)	-	5 (2)
3. (a)	Continuity & Differentiability	-	8 (2)	-	8 (2) } 19 (5)
(b)	Applications of Derivatives	1 (1)	4 (1)	6 (1)	11 (3)
(c)	Integrals	1 (1)	12 (3)	-	13 (4) } 19 (5)
(d)	Applications of Integrals	-	-	6 (1)	6 (1)
(e)	Differential Equations	-	-	6 (1)	6 (1) 6 (1)
4. (a)	Vectors	2 (2)	4 (1)	-	6 (3) } 17 (6)
(b)	Three Dimensional Geometry	1 (1)	4 (1)	6 (1)	11 (3)
5.	Linear Programming	-	-	6 (1)	6 (1) } 6 (1)
6.	Probability	-	4 (1)	6 (1)	10 (2) } 10 (2)
	Total	10 (10)	48 (12)	42 (7)	**100 (29)**

Sample Question Paper - I
MATHEMATICS
Class XII

Time : 3 Hours **Max. Marks : 100**

General Instructions

1. All questions are compulsory.

2. The question paper consist of 29 questions divided into three sections A, B and C. Section A comprises of 10 questions of one mark each, section B comprises of 12 questions of four marks each and section C comprises of 07 questions of six marks each.

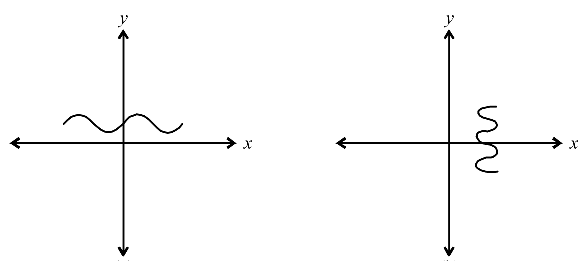
3. All questions in Section A are to be answered in one word, one sentence or as per the exact requirement of the question.

4. There is no overall choice. However, Internal choice has been provided in 04 questions of four marks each and 02 questions of six marks each. You have to attempt only one of the alternatives in all such questions.

5. Use of calculators is not permitted. You may ask for logarithmic tables, if required.

SECTION - A

1. Which one of the following graphs represent the function of x ? Why ?



(a) (b)

2. What is the principal value of

$$\cos^{-1}\left(\cos\frac{2\pi}{3}\right)+\sin^{-1}\left(\sin\frac{2\pi}{3}\right) \ ?$$

3. A matrix A of order 3×3 has determinant 5. What is the value of $|3A|$?

4. For what value of x, the following matrix is singular ?

$$\begin{bmatrix} 5-x & x+1 \\ 2 & 4 \end{bmatrix}$$

5. Find the point on the curve $y = x^2 - 2x + 3$, where the tangent is parallel to x-axis.

6. What is the angle between vectors $\vec{a}$ & $\vec{b}$ with magnitude $\sqrt{3}$ and 2 respectively ? Given $\vec{a} . \vec{b} = 3$.

7. Cartesian equations of a line AB are.

$$\frac{2x-1}{2} = \frac{4-y}{7} = \frac{z+1}{2}$$

Write the direction ratios of a line parallel to AB.

8. Write a value of $\int e^{3\log x}\left(x^4\right)dx$

9. Write the position vector of a point dividing the line segment joining points A and B with position vectors $\vec{a}$ & $\vec{b}$ externally in the ratio

1 : 4, where $\vec{a} = 2\hat{i} + 3\hat{j} + 4\hat{k}$ and $\vec{b} = -\hat{i} + \hat{j} + \hat{k}$

10. If $A = \begin{bmatrix} 2 & 1 & 4 \\ 4 & 1 & 5 \end{bmatrix}$ and $B = \begin{bmatrix} 3 & -1 \\ 2 & 2 \\ 1 & 3 \end{bmatrix}$

Write the order of AB and BA.

SECTION - B

11. Show that the function f : **R** $\to$ **R** defined by $f(x) = \frac{2x-1}{3}$, $x \in$ **R** is one-one and onto function. Also find the inverse of the function f.

OR

Examine which of the following is a binary operation

(i) $a * b = \frac{a+b}{2}$, $a, b \in N$

(ii) $a * b = \frac{a+b}{2}$, $a, b \in Q$

for binary operation check the commutative and associative property.

12. Prove that

$$\tan^{-1}\left(\frac{63}{16}\right) = \sin^{-1}\left(\frac{5}{13}\right) + \cos^{-1}\left(\frac{3}{5}\right)$$

13. Using elementary transformations, find the inverse of

$$\begin{bmatrix} 2 & -6 \\ 1 & -2 \end{bmatrix}$$

OR

Using properties of determinants, prove that

$$\begin{vmatrix} -bc & b^2+bc & c^2+bc \\ a^2+ac & -ac & c^2+ac \\ a^2+ab & b^2+ab & -ab \end{vmatrix} = (ab+bc+ca)^3$$

14. Find all the points of discontinuity of the function **f** defined by

$$f(x) = \begin{matrix} x+2, & x \le 1 \\ x-2, & 1 < x < 2 \\ 0, & x \ge 2 \end{matrix}$$

15. If $x^p y^q = (x+y)^{p+q}$, prove that $\frac{dy}{dx} = \frac{y}{x}$

OR

Find $\frac{dy}{dx}$, if $y = \tan^{-1}\left[\frac{\sqrt{1+x^2}+\sqrt{1-x^2}}{\sqrt{1+x^2}-\sqrt{1-x^2}}\right], 0 < |x| < 1$

16. Evaluate $\int \frac{(x^2+1)(x^2+4)}{(x^2+3)(x^2-5)} dx$

17. A water tank has the shape of an inverted right circular cone with its axis vertical and vertex lower most. Its semi-vertical angle is $\tan^{-1}\left(\frac{1}{2}\right)$. Water is poured into it at a constant rate of 5 cubic meter per minute. Find the rate at which the level of the water is rising at the instant when the depth of water in the tank is 10m.

18. Evaluate the following integral as limit of sum $\int_1^2 (3x^2-1)dx$

19. Evaluate $\int_0^{\pi/2} \log \sin x \, dx$

20. Find the vector equation of the line parallel to the line $\frac{x-1}{5} = \frac{3-y}{2} = \frac{z+1}{4}$ and passing through (3, 0, –4). Also

find the distance between these two lines.

21. In a regular hexagon ABC DEF, if $\vec{AB} = \vec{a}$ and $\vec{BC} = \vec{b}$, then express $\vec{CD}, \vec{DE}, \vec{EF}, \vec{FA}, \vec{AC}, \vec{AD}, \vec{AE}$ and $\vec{CE}$ in terms of $\vec{a}$ and $\vec{b}$.

22. A football match may be either won, drawn or lost by the host country's team. So there are three ways of forecasting the result of any one match, one correct and two incorrect. Find the probability of forecasting at least three correct results for four matches.

OR

A candidate has to reach the examination centre in time. Probability of him going by bus or scooter or by other means of transport are $\frac{3}{10}, \frac{1}{10}, \frac{3}{5}$ respectively. The probability that he will be late is $\frac{1}{4}$ and $\frac{1}{3}$ respectively, if he travels by bus or scooter. But he reaches in time if he uses any other mode of transport. He reached late at the centre. Find the probability that he travelled by bus.

SECTION - C

23. Find the matrix P satisfying the matrix equation

$$\begin{bmatrix} 2 & 1 \\ 3 & 2 \end{bmatrix} P \begin{bmatrix} -3 & 2 \\ 5 & -3 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 2 & -1 \end{bmatrix}$$

24. Find all the local maximum values and local minimum values of the function

$$f(x) = \sin 2x - x, \qquad -\frac{\pi}{2} < x < \frac{\pi}{2}$$

OR

A given quantity of metal is to be cast into a solid half circular cylinder (i.e., with rectangular base and semicircular ends). Show that in order that the total surface area may be minimum, the ratio of the length of the cylinder to the diameter of its circular ends is $\pi : (\pi + 2)$.

25. Sketch the graph of

$$f(x) = \begin{cases} |x-2|+2, & x \le 2 \\ x^2 - 2, & x > 2 \end{cases}$$

Evaluate $\int_0^4 f(x)\,dx$. What does the value of this integral represent on the graph ?

26. Solve the following differential equation $(1 - x^2)\frac{dy}{dx} - xy = x^2$, given $y = 2$ when $x = 0$

27. Find the foot of the perpendicular from P(1, 2, 3) on the line

$$\frac{x-6}{3}=\frac{y-7}{2}=\frac{z-7}{-2}$$

Also obtain the equation of the plane containing the line and the point (1, 2, 3)

28. Let X denote the number of colleges where you will apply after your results and P(X = x) denotes your probability of getting admission in x number of colleges. It is given that

$$P(X=x)=\begin{cases} kx & \text{if } x=0 \text{ or } 1 \\ 2kx & \text{if } x=2 \\ k(5-x) & \text{if } x=3 \text{ or } 4 \end{cases}, \quad \text{k is +ve constant}$$

(a) Find the value of k.

(b) What is the probability that you will get admission in exactly two colleges?

(c) Find the mean and variance of the probability distribution.

OR

Two bags A and B contain 4 white 3 black balls and 2 white and 2 black balls respectively. From bag A two balls are transferred to bag B. Find the probability of drawing

(a) 2 white balls from bag B ?

(b) 2 black balls from bag B ?

(c) 1 white & 1 black ball from bag B ?

29. A catering agency has two kitchens to prepare food at two places A and B. From these places 'Mid-day Meal' is to be supplied to three different schools situated at P, Q, R. The monthly requirements of the schools are respectively 40, 40 and 50 food packets. A packet contains lunch for 1000 students. Preparing capacity of kitchens A and B are 60 and 70 packets per month respectively. The transportation cost per packet from the kitchens to schools is given below :

Transportation cost per packet (in rupees)		
To	**From**	
	A	**B**
P	5	4
Q	4	2
R	3	5

How many packets from each kitchen should be transported to school so that the cost of transportation is minimum ? Also find the minimum cost.

MARKING SCHEME
SAMPLE PAPER - I
Mathematics - XII

Q. No.	Value Points	Marks
	SECTION A	
1.	(a)	$\frac{1}{2}$
$\therefore$	for every value of x there is unique y	$\frac{1}{2}$
2.	π	1
3.	135	1
4.	3	1
5.	(1, 2)	1
6.	$\pi/_6$	1
7.	(1, - 7, 2) or their any multiple	1
8.	$\frac{\mathbf{x}^8}{8}+c$	1
9.	$3\hat{i}+\frac{11}{3}\hat{j}+5\hat{k}$	1
10.	order of AB is 2 x 2	$\frac{1}{2}$
	order of BA is 3 x 3	$\frac{1}{2}$

Q. No.	Value Points	Marks

SECTION B

11. $f(x) = \frac{2x-1}{3}, x \in \mathbb{R}$

To show *f* is one-one

Let $x_1, x_2 \in \mathbb{R}$ s.t. $x_1 \neq x_2$

$\Rightarrow$ $2x_1 \neq 2x_2$

$\Rightarrow$ $2x_1 - 1 \neq 2x_2 - 1$

$\Rightarrow$ $\frac{2x_1 - 1}{3} \neq \frac{2x_2 - 1}{3}$ ½

$\Rightarrow$ $f(x_1) \neq f(x_2)$ ½

$\Rightarrow$ *f* is one-one

To show *f* is onto

Let $y = \frac{2x-1}{3}, y \in \mathbb{R}$ (codomain of f)

or $3y = 2x - 1$

or $x = \frac{3y+1}{2} \in \mathbb{R}$ 1

$\therefore$ for all $y \in$ R (codomain of *f*), there exist

$x = \frac{3y+1}{2} \in \mathbb{R}$ (codomain of *f*), such that

$$f(x) = f\left(\frac{3y+1}{2}\right) = \frac{2\left(\frac{3y+1}{2}\right) - 1}{3} = y$$

$\Rightarrow$ every element in codomain of *f* has its pre-image in the domain of *f*. ½

$\Rightarrow$ *f* is onto.

To find f^{-1}

Let $f(x) = y, \quad x = \frac{3y+1}{2}$

$\Rightarrow$ $f^{-1}(y) = x$

Q. No. **Value Points** **Marks**

$\Rightarrow$ $f^{-1}(y) = \dfrac{3y+1}{2}$

$\therefore$ f^{-1} : R $\to$ R given by

$f^{-1}(y) = \dfrac{3y+1}{2}$ 1½

OR

(i) $a * b = \dfrac{a+b}{2}, \; a, b \in N$

$\forall$ $a, b \in N$ $\dfrac{a+b}{2}$ may or may not belong to N. ½

$\therefore$ a * b is not always natural no.

$\therefore$ '*' is not a binary operation on N ½

(ii) $a * b = \dfrac{a+b}{2}, \; a, b \in Q$

$\forall$ $a, b \in Q$

$\dfrac{a+b}{2} \in Q$ ½

$\Rightarrow$ $a * b \in Q$.

$\Rightarrow$ '*' is a binary operation on Q ½

(iii) For $a * b = \dfrac{a+b}{2}, \quad a, b \in Q$

$a * b = \dfrac{a+b}{2} = \dfrac{b+a}{2} = b * a$ ½

$\Rightarrow$ * is commutative ½

(iv) $(a * b) * c \quad = \left(\dfrac{a+b}{2}\right) * c$

$\forall$ a, b, c, $\in$ Q.

$= \dfrac{\dfrac{a+b}{2} + c}{2}$

$= \dfrac{a+b+2c}{4}$

Q. No. **Value Points** **Marks**

$$a * (b * c) = a * \left(\frac{b+c}{2}\right)$$

$$= \frac{a + \left(\frac{b+c}{2}\right)}{2}$$

$$= \frac{2a+b+c}{4}$$

$(a * b) * c \neq a * (b * c) \ \forall\ a, b, c, \in Q$ ½

$\therefore$ '$*$' is not associative, ½

12. Let $\sin^{-1}\left(\frac{5}{13}\right) = x$ & $\cos^{-1}\left(\frac{3}{5}\right) = y$

$\Rightarrow$ $\sin x = \frac{5}{13}$ & $\cos y = \frac{3}{5}$

& $\cos x = \frac{12}{13}$ & $\sin y = \frac{4}{5}$

$\Rightarrow$ $\tan x = \frac{5}{13}$ & $\tan y = \frac{4}{3}$ (1 + 1)

$$\tan(x+y) = \frac{\tan x + \tan y}{1 - \tan x \tan y}$$ (½)

$$\tan(x+y) = \frac{63}{16}$$ (1)

$\Rightarrow$ $x + y = \tan^{-1}\left(\frac{63}{16}\right)$ (½)

$\Rightarrow$ $\sin^{-1}\left(\frac{12}{13}\right) + \cos^{-1}\left(\frac{3}{5}\right) = \tan^{-1}\left(\frac{63}{16}\right)$

Q. No.	Value Points	Marks

Sol.13. Let $A = \begin{bmatrix} 2 & -6 \\ 1 & -2 \end{bmatrix}$

$A = I A$ (½)

$$\begin{bmatrix} 2 & -6 \\ 1 & -2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A$$

$R_1 \leftrightarrow R_2$

$$\Rightarrow \begin{bmatrix} 1 & -2 \\ 2 & -6 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} A \qquad (½)$$

$R_2 \to R_2 - 2R_1$ (1)

$$\Rightarrow \begin{bmatrix} 1 & -2 \\ 0 & -2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 1 & -2 \end{bmatrix} A$$

$R_1 \to R_1 - R_2 (1)$

$$\Rightarrow \begin{bmatrix} 1 & 0 \\ 0 & -2 \end{bmatrix} = \begin{bmatrix} -1 & 3 \\ 1 & -2 \end{bmatrix} A$$

$R_2 \to -\frac{1}{2} R_2$ (½)

$$\Rightarrow \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} -1 & 3 \\ -\frac{1}{2} & 1 \end{bmatrix} A$$

$$\therefore \quad A^{-1} = \begin{bmatrix} -1 & 3 \\ -\frac{1}{2} & 1 \end{bmatrix} \qquad (½)$$

OR

Operate $R_1 \to aR_1, R_2 \to bR_2, R_3 \to cR_3$

$$\begin{vmatrix} -bc & b^2+bc & c^2+bc \\ a^2+ac & -ac & c^2+ac \\ a^2+ab & b^2+ab & -ab \end{vmatrix}$$

$$= \frac{1}{abc} \begin{vmatrix} -abc & ab^2+abc & ac^2+abc \\ a^2b+abc & -abc & bc^2+bac \\ a^2c+abc & b^2c+abc & -abc \end{vmatrix} \qquad 1$$

Take a, b, c common from C_1, C_2, C_3 respectively 1

$$= \begin{vmatrix} -bc & ab+ac & ac+ab \\ ab+bc & -ac & bc+ba \\ ac+bc & bc+ac & -ab \end{vmatrix}$$

$R_1 \to R_1 + R_2 + R_3$

$$= \begin{vmatrix} ab+bc+ac & ab+bc+ac & ab+bc+ac \\ ab+bc & -ac & bc+ba \\ ac+bc & bc+ac & -ab \end{vmatrix}$$

$$= (ab+bc+ca) \begin{vmatrix} 1 & 1 & 1 \\ ab+bc & -ac & bc+ba \\ ac+bc & bc+ac & -ab \end{vmatrix}$$

$C_1 \to C_1 - C_3, \qquad C_2 \to C_2 - C_3$

$$= (ab+bc+ca) \begin{vmatrix} 0 & 0 & 1 \\ 0 & -(ab+bc+ca) & bc+ba \\ ac+bc+ab & bc+ac+ab & -ab \end{vmatrix}$$ 1½

On expanding by R_1 we get ½

$$= (ab+bc+ca)^3$$

Sol.14. Being a polynomial function $f(x)$ is continuous at all point for $x < 1$, $1 < x < 2$ and $x \geq 2$. 1

Thus the possible points of discontinuity are $x = 1$ and $x = 2$.

<u>To check continuity at $x = 1$</u>

$$\left.\begin{array}{l} \underset{x \to 1^-}{\text{lt}} f(x) = \underset{x \to 1}{\text{lt}}\ x + 2 = \\ \underset{x \to 1^+}{\text{lt}} f(x) = \underset{x \to 1}{\text{lt}}\ x - 2 = - \\ f(1) = 3. \\ \text{since, } \underset{x \to 1^-}{\text{lt}} f(x) = f(1) \neq \underset{x \to 1^+}{\text{lt}} f(x \\ \therefore \quad f(x) \text{ is not continuous at } x = \end{array}\right\}$$ 1

Q. No.	Value Points	Marks

To check continuity at $x = 2$

$$\lim_{x\to 2^-} f(x) = \lim_{x\to 2^-} x - 2 = 0$$

$$\lim_{x\to 2^+} f(x) = \lim_{x\to 2} 0 = 0$$

$$f(2) = 0 \qquad (1)$$

since $\lim_{x\to 2^-} f(x) = \lim_{x\to 2^+} f(x) = f(2) = 0$

$\therefore$ $f(x)$ is continuous at $x = 2$.

$\therefore$ The only point of discontinuity is $x = 1$. (1)

Sol.15. $x^p y^q = (x+y)^{p+q}$

Take log on both sides

$$p \log x + q \log y = (p+q)\log(x+y)$$

$$\frac{p}{x} + \frac{q}{y}\cdot\frac{dy}{dx} = \frac{p+q}{x+y}\left(1 + \frac{dy}{dx}\right) \qquad (2)$$

or $$\frac{p}{x} - \frac{p+q}{x+y} = \frac{dy}{dx}\left(\frac{p+q}{x+y} - \frac{q}{y}\right) \qquad (1)$$

or $$\frac{px + py - px - qx}{x(x+y)} = \frac{dy}{dx}\left(\frac{py + qy - qx - qy}{y(x+by)}\right)$$

or $$\frac{py - qx}{x} = \frac{dy}{dx}\left(\frac{py - qx}{y}\right)$$

or $$\frac{y}{x} = \frac{dy}{dx} \qquad (1)$$

OR

$$y = \tan^{-1}\left[\frac{\sqrt{1+x^2} + \sqrt{1-x^2}}{\sqrt{1+x^2} + \sqrt{1-x^2}}\right]$$

Q. No.	Value Points	Marks

Put $x^2 = \cos\theta$

$$y = \tan^{-1}\left[\frac{\sqrt{1+\cos\theta}+\sqrt{1-\cos\theta}}{\sqrt{1+\cos\theta}-\sqrt{1-\cos\theta}}\right]$$ (½)

$$= \tan^{-1}\left[\frac{\cos\frac{\theta}{2}+\sin\frac{\theta}{2}}{\cos\frac{\theta}{2}-\sin\frac{\theta}{2}}\right]$$ (1)

$$= \tan^{-1}\left[\frac{1+\tan\frac{\theta}{2}}{1-\tan\frac{\theta}{2}}\right]$$

$$= \tan^{-1}\left[\tan\left(\frac{\pi}{4}+\frac{\theta}{2}\right)\right]$$

$$y = \frac{\pi}{4}+\frac{\theta}{2}$$ (1)

or $$y = \frac{\pi}{4}+\frac{1}{2}\cos^{-1}\left(x^2\right)$$ (½)

$$\frac{dy}{dx} = -\frac{1}{2}\left(\frac{2x}{\sqrt{1-x^4}}\right)$$

$$= \frac{-x}{\sqrt{1-x^4}}$$ (1)

Sol. 16. $$\int \frac{(x^2+1)(x^2+4)}{(x^2+3)(x^2-5)}\,dx$$

Consider

$$\frac{(x^2+1)(x^2+4)}{(x^2+3)(x^2-5)} = \frac{(t+1)(t+4)}{(t+3)(t-5)} \quad \text{where} \quad t = x^2$$

$$= 1+\frac{7t+19}{(t+3)(t-5)}$$ (1)

Q. No. **Value Points** **Marks**

Consider

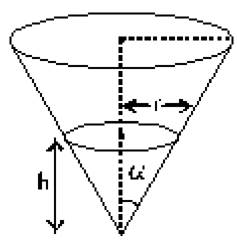
$$\frac{7t+19}{(t+3)(t-5)} = \frac{A}{t+3} + \frac{B}{t-5}$$

$$A = \frac{1}{4}, \quad B = \frac{27}{4}$$ (1)

$\therefore$ $$\int \frac{(x^2+1)(x^2+4)}{(x^2+3)(x^2-5)}\, dx = \int dx + \frac{1}{4}\int \frac{dx}{x^2+3} + \frac{27}{4}\int \frac{dx}{x^2-5}$$

$$= x + \frac{1}{4\sqrt{3}}\tan^{-1}\left(\frac{x}{\sqrt{3}}\right) + \frac{27}{8\sqrt{5}}\log\left|\frac{x-\sqrt{5}}{x+\sqrt{5}}\right| + c$$ (2)

Sol.17.



Let r = radius of cone formed by water at any time
h = height of cone formed by water at any time

Given $\alpha = \tan^{-1}\left(\frac{1}{2}\right)$

$\therefore$ $\tan\alpha = \frac{1}{2}$

Also $\tan\alpha = \frac{r}{h}$

$\Rightarrow$ $h = 2r$ (1)

Volume of this cone

$$V = \frac{1}{3}\pi r^2 h$$

$$= \frac{\pi}{3}\left(\frac{h}{2}\right)^2 h$$

Q. No.	Value Points	Marks

$$v = \frac{\pi}{12} h^3$$ (1)

$$\frac{dv}{dt} = \frac{\pi}{12}\left(3h^2\right)\frac{dh}{dt}$$ (1)

$$= \frac{\pi}{4} h^2 \frac{dh}{dt}$$

But $\frac{dv}{dt} = 5\ m^3/\text{minute}$

$\therefore$ $5 = \frac{\pi}{4} h^2 \frac{dh}{dt}$

or $\frac{dh}{dt} = \frac{20}{\pi(10)^2}$ when $h = 10m$

or $\frac{dh}{dt} = \frac{1}{5\pi}$ m/minute (1)

18. For $\int_1^2 \left(3x^2 - 1\right) dx$

$a = 1, \quad b = 2, \quad h = \frac{1}{n}$ as $n \to \infty, \quad h \to 0$ (½)

$f(x) = 3x^2 - 1$

$$\int_a^b f(x)dx = \lim_{h\to 0} h\left[f(a) + f(a+h) + \ldots + f(a+(n-1)h)\right]$$ (½)

$$\int_1^2 \left(3x^2 - 1\right)dx = \lim_{h\to 0} h\left[3n + 3h^2\left(1^2 + 2^2 + \ldots + (n-1)^2\right) + 6h\left(1 + 2 + \ldots + (n-1)\right) - n\right]$$ (1)

$$= \lim_{h\to 0} h\left[2n + 3h^2\frac{(n)(n-1)(2n-1)}{6} + 3h(n)(n-1)\right]$$ (1)

$$= \lim_{h\to 0} h\left[\frac{2}{h} + \frac{3h^2(1-h)(2-h)}{6h^3} + 3h\left(\frac{1}{h}\right)\left(\frac{1-h}{h}\right)\right]$$

Q. No.	Value Points	Marks

$= 2+\frac{1}{2}(2)+3$ (1)

$= 6$

19. Given $I = \int_0^{\pi/2} \log \sin x \, dx$

$I = \int_0^{\pi/2} \log \cos x \, dx$ $\left(\because \int_0^a f(a-x)dx = \int_0^a f(x)dx \right)$ (1)

$\therefore$ $2I = \int_0^{\pi/2} (\log \sin 2x \, dx - \log 2) dx$

$I = \frac{1}{2}\int_0^{\pi/2} \log \sin 2x \, dx - \frac{\pi}{4}\log 2$ .... (1) (1)

Consider

$I_1 = \int_0^{\pi/2} \log \sin 2x \, dx$

$= \frac{1}{2}\int_0^{\pi} \log \sin t \, dt$

Put $2x = t$

$dx = \frac{dt}{2}$

$x = 0 \Rightarrow t = 0$

$x = \frac{\pi}{2} \Rightarrow t = \pi$

$= \frac{1}{2}.2\int_0^{\pi/2} \log \sin t \, dt$ $\left(\because \int_0^{2a} f(x)dx = 2\int_0^a f(x)dx \text{ if } f(2a-x) = f(x) \right)$

$= \int_0^{\pi/2} \log \sin t \, dt$

$I_1 = \int_0^{\pi/2} \log \sin x \, dx$ .... (2) (1)

Q. No.	Value Points	Marks

From (1) and (2)

$$\left.\begin{aligned} &I = \frac{1}{2}\int_0^{\pi/2} \log\sin dx - \frac{\pi}{4}\log 2 \\ &\mathbf{I - \frac{1}{2}I = -\frac{\pi}{4}\log 2} \\ &\mathbf{I = -\frac{\pi}{2}\log 2.} \end{aligned}\right\}$$ (1)

20. Given line

$$\frac{x-1}{5} = \frac{3-y}{2} = \frac{z+1}{4}$$ (½).

or, $$\frac{x-1}{5} = \frac{y-3}{-2} = \frac{z-(-1)}{4}$$ .... (i)

is passing through (1, 3, –1) and has D.R. 5, –2, 4 .

Equations of line passing through (3, 0, –4) and parallel to given line is

$$\frac{x-3}{5} = \frac{y-0}{-2} = \frac{z+4}{4}$$ .... (ii)

Vector equations of line (i) & (ii)

$$\vec{r} = \hat{i} + 3\hat{j} - \hat{k} + \lambda\left(5\hat{i} - 2\hat{j} + 4\hat{k}\right)$$

$$\vec{r} = 3\hat{i} - 4\hat{k} + \mu\left(5\hat{i} - 2\hat{j} + 4\hat{k}\right)$$ (1)

$\therefore$ $$\vec{a}_2 - \vec{a}_1 = 2\hat{i} - 3\hat{j} - 3\hat{k}$$ (½)

$$\left|\vec{b}\right| = \sqrt{(5)^2 + (-2)^2 + (4)^2} = \sqrt{45} = 3\sqrt{5}.$$

Also $$\vec{b} \times \left(\vec{a}_2 - \vec{a}_1\right) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 5 & -2 & 4 \\ 2 & -3 & -3 \end{vmatrix}$$

$$= 18\hat{i} + 23\hat{j} - 11\hat{k}$$

Q. No. **Value Points** **Marks**

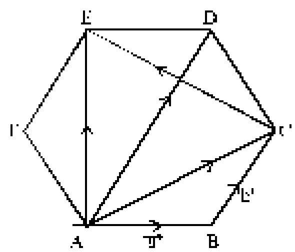
$\therefore \quad \left|\vec{b} \times \left(\vec{a}_2 - \vec{a}_1\right)\right| = \sqrt{(18)^2 + (23)^2 + (11)^2} = \sqrt{974}$ (1)

$\therefore$ Distance between two parallel lines.

$$= \frac{\left|\vec{b} \times \left(\vec{a}_2 - \vec{a}_1\right)\right|}{\left|\vec{b}\right|}$$ (½)

$$= \frac{\sqrt{974}}{\sqrt{45}} = \sqrt{\frac{974}{45}} \text{ units}$$ (½)

21.



From fig. $\vec{DE} = -\vec{a}$ (½)

$\vec{EF} = -\vec{b}$ (½)

$\vec{AC} = \vec{AB} + \vec{BC}$

$= \vec{a} + \vec{b}$ (½)

$\vec{AD} = 2\vec{BC} = 2\vec{b}$ (½)

$\vec{AD} = \vec{AC} + \vec{CD}$

$\Rightarrow \quad \vec{CD} = \vec{AD} - \vec{AC}$

$= \vec{b} - \vec{a}$ (½)

$\vec{FA} = -\vec{CD} = \vec{a} - \vec{b}$ (½)

$\vec{CE} = \vec{CD} + \vec{DE} = \vec{b} - 2\vec{a}$ (½)

Q. No.	Value Points	Marks

$$\vec{AE} = \vec{AD} + \vec{DE} = 2\vec{b} - \vec{a}$$ (½)

22. P (Correct forecast) $= \frac{1}{3}$ (1)

P (Incorrect forecast) $= \frac{2}{3}$

P (At least three correct forecasts for four matches)

= P(3 correct) + P(4 correct) (½)

$$= {}^4c_3 \left(\frac{1}{3}\right)^3 \left(\frac{2}{3}\right)^1 + {}^4c_4 \left(\frac{1}{3}\right)^4$$ (1+1)

$$= \frac{8}{81} + \frac{1}{81}$$

$$= \frac{9}{81} = \frac{1}{9}$$ Ans. (½)

OR

Let E : Candidate Reaches late

A_1 : Candidate travels by bus

A_2 : Candidate travels by scooter

A_3 : Candidate travels by other modes of transport

$$P(A_1) = \frac{3}{10}, \quad P(A_2) = \frac{1}{10}, \quad P(A_3) = \frac{3}{5}$$ (½)

$$P(E/A_1) = \frac{1}{4}, \quad P(E/A_2) = \frac{1}{3}, \quad P(E/A_3) = 0$$ (1½)

∴ By Baye's Theorem

$$P(A_1/E) = \frac{P(A_1)P(E/A_1)}{P(A_1)P(E/A_1) + P(A_2)P(E/A_2) + P(A_3)P(E/A_3)}$$ (1)

$$= \frac{\frac{3}{10} \times \frac{1}{4}}{\frac{3}{40} + \frac{1}{30} + 0}$$

$$= \frac{9}{13}$$ (1)

Q. No.	Value Points	Marks

SECTION C

23. Given $\begin{bmatrix} 2 & 1 \\ 3 & 2 \end{bmatrix} P \begin{bmatrix} -3 & 2 \\ 5 & -3 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 2 & -1 \end{bmatrix}$

Let $R = \begin{bmatrix} 2 & 1 \\ 3 & 2 \end{bmatrix}$ then $|R| = 1$ (½)

$S = \begin{bmatrix} -3 & 2 \\ 5 & -3 \end{bmatrix}$ then $|S| = -1$ (½)

$Q = \begin{bmatrix} 1 & 2 \\ 2 & -1 \end{bmatrix}$

Since R and S are non-singular matrices

$\therefore$ R^{-1} and S^{-1} exist.

$$R^{-1} = \frac{\text{Adj } R}{|R|} = \begin{bmatrix} 2 & -1 \\ -3 & 2 \end{bmatrix}$$ (1)

$$S^{-1} = \frac{\text{Adj } S}{|S|} = \begin{bmatrix} 3 & 2 \\ 5 & 3 \end{bmatrix}$$ (1)

Now given

$$\left.\begin{aligned} RPS &= Q \\ R^{-1}(RPS) &= R^{-1}Q \\ (R^{-1}R)PS &= R^{-1}Q \\ PS &= R^{-1}Q \\ PSS^{-1} &= R^{-1}QS^{-1} \\ P &= R^{-1}QS^{-1} \end{aligned}\right\} \quad (\because R^{-1}R = I \quad I.P = P)$$ (1)

$$\therefore \quad \left.\begin{aligned} P &= \begin{bmatrix} 2 & -1 \\ -3 & 2 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 2 & -1 \end{bmatrix}\begin{bmatrix} 3 & 2 \\ 5 & 3 \end{bmatrix} \\ &= \begin{bmatrix} 25 & 15 \\ -37 & -22 \end{bmatrix} \end{aligned}\right\}$$ (2)

Q. No.	Value Points	Marks

24. $f(x) = \sin 2x - x \qquad -\frac{\pi}{2} < x < \frac{\pi}{2}$

$f'(x) = 2\cos 2x - 1$ (½)

$f'(x) = 0 \Rightarrow \cos 2x = \frac{1}{2}$

or $2x = -\frac{\pi}{3}, \frac{\pi}{3}$ (1)

or $x = -\frac{\pi}{6}, \frac{\pi}{6}$

$f''(x) = -4\sin 2x$ (½)

$f''(x) = 2\sqrt{3} > 0$ at $x = -\frac{\pi}{6}$ (1)

$\Rightarrow$ $x = -\frac{\pi}{6}$ is point of local minima

$f''(x) = -2\sqrt{3} < 0$ at $x = \frac{\pi}{6}$ (1)

$\Rightarrow$ $x = \frac{\pi}{6}$ is point of local maxima

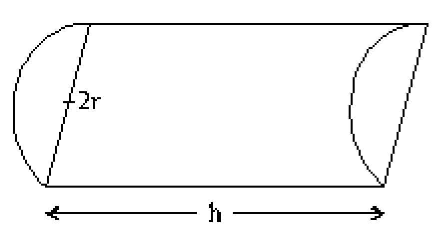
$\therefore$ Local minimum value is

$f\left(-\frac{\pi}{6}\right) = \frac{-\sqrt{3}}{2} + \frac{\pi}{6}$ (1)

Local maximum value is

$f\left(\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2} - \frac{\pi}{6}$ (1)

OR



Q. No.	Value Points	Marks

Let h = length of cylinder

r = radius of semi-circular ends of cylinder (½)

$$v = \frac{1}{2}\pi r^2 h$$

S = Total surface area of half circular cylinder

= 2 (Area of semi circular ends) + Curved surface area of half circular cylinder + Area of rectangular base. (1)

$$= 2\left(\frac{1}{2}\pi r^2\right) + \frac{1}{2}(2\pi r h) + 2rh \quad (½)$$

$$= \pi r^2 + (\pi + 2) rh$$

$$= \pi r^2 + (\pi + 2) r \cdot \frac{2v}{\pi r^2} \quad (1)$$

$$\frac{ds}{dr} = 2\pi r - \frac{2v(\pi+2)}{\pi}\left(\frac{1}{r^2}\right)$$

$$\frac{ds}{dr} = 0 \Rightarrow r^3 = \frac{(\pi+2)v}{\pi^2} \quad (1)$$

$$\frac{d^2 s}{dr^2} = 2\pi + \frac{2v(\pi+2)}{\pi} \cdot \frac{2}{r^3} > 0 \quad (1)$$

$\therefore$ S is minimum when $r^3 = \frac{(\pi+2)v}{\pi^2}$

$$= \frac{(\pi+2)}{\pi^2}\left(\frac{1}{2}\pi r^2 h\right)$$

$$\Rightarrow \quad r = \frac{\pi+2}{2\pi} . h$$

$$\therefore \quad \frac{h}{2r} = \frac{\pi}{\pi+2} \quad (1)$$

Which is required result.

Q. No.	**Value Points**	**Marks**

25. $f(x)\begin{cases} -(x-2)+2 & x \le 2 \\ x^2-2, & x > 2 \end{cases}$

or $f(x)=\begin{cases} 4-x & x \le 2 \\ x^2-2, & x > 2 \end{cases}$ (1)

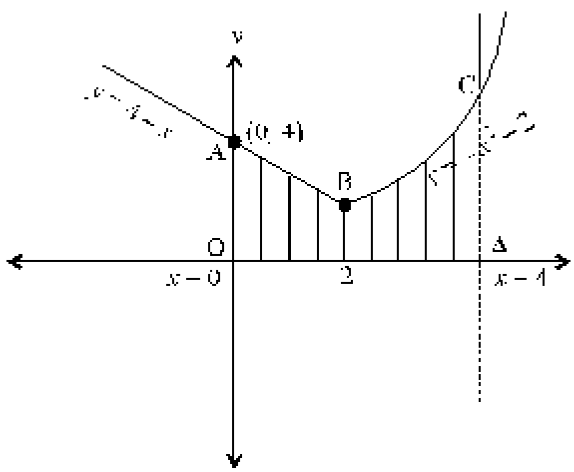
To sketch the graph of above function following tables are required.

For $f(x)=4-x,\ \ x\le 2$ & for $f(x)=x^2-2,\ x\ge 2$

x	-1	0	1	2
y	5	4	3	2

Also $f(x) = x^2 - 2$ represent parabolic curve.

x	2	3	4	5	6
y	2	7	14	23	34



(2)

Area $= \int_0^4 f(x)\,dx = \int_0^2 (4-x)\,dx + \int_2^4 (x^2-2)\,dx$

$$= 4x - \frac{x^2}{2}\Big|_0^2 + \frac{x^3}{3} - 2x\Big|_2^4$$ (2)

$$= 6 + \frac{44}{3} = \frac{62}{3} \text{ sq. units}$$

Q. No.	Value Points	Marks

On the graph $\int_0^4 f(x)\,dx$ represents the area bounded by x-axis the lines $x = 0$; $x = 4$ and the curve $y = f(x)$.

i.e. area of shaded region shown in fig. (1)

26. $$(1-x^2)\frac{dy}{dx} - xy = x^2$$

or $$\frac{dy}{dx} - \frac{x}{1-x^2}\cdot y = \frac{x^2}{1-x^2}$$

$$P = -\frac{x}{1-x^2}\quad , \quad Q = \frac{x^2}{1-x^2}$$ (½)

I.F. $$= e^{\int P\,dx} = e^{\int -\frac{-x}{1-x^2}dx}$$

$$= e^{\frac{1}{2}\log(1-x^2)}$$

$$= \sqrt{1-x^2}$$ (1)

$\therefore$ Solution of diff. equation is

$$y\sqrt{1-x^2} = \int \frac{x^2}{1-x^2}\cdot\sqrt{1-x^2}\,dx$$ (1)

$$= \int\left(\frac{1}{\sqrt{1-x^2}} - \sqrt{1-x^2}\right)dx$$

$$= \sin^{-1}x - \left(x\sqrt{1-x^2} + \frac{1}{2}\sin^{-1}x\right) + c$$ (1)

$$y\sqrt{1-x^2} = \frac{1}{2}\sin^{-1}x - x\sqrt{1-x^2} + c$$ (1)

When $x = 0, \quad y = 2$ (1)

$2 = c$

$\therefore$ Solution is

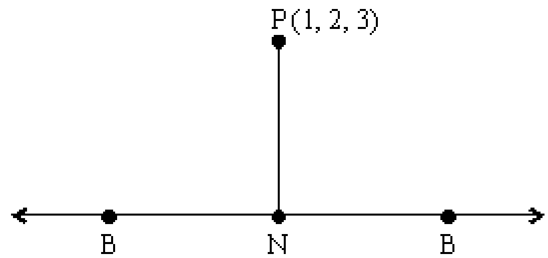
$$y\sqrt{1-x^2} = \frac{1}{2}\sin^{-1}x - x\sqrt{1-x^2} + 2$$ (½)

Q. No.	Value Points	Marks

27. The given line is

$$\frac{x-6}{3}=\frac{y-7}{2}=\frac{z-7}{-2}=\lambda(\text{say}) \qquad \text{..................} \text{ (i)}$$

Let N be the foot of the perpendicular from P(1, 2, 3) to the given line



Coordinates of N = $(3\lambda + 6, 2\lambda + 7 -2\lambda + 7)$ (½)

D.R. of NP $\quad 3\lambda + 5 , 2\lambda + 5 , -2\lambda + 4$ (1)

D.R. of AB $\quad 3 , 2 , -2$

Since $\quad$ NP $\perp$ AB

$\therefore \quad 3(3\lambda + 5) + 2(2\lambda + 5) - 2(-2\lambda + 4) = 0$

or $\quad \lambda = -1$ (1)

$\therefore$ Coordinates of foot of perpendicular N are (3, 5, 9) (½)

Equation of plane containing line (i) and point (1, 2, 3) is (½)

Equation of plane containing point (6, 7, 7) & (1, 2, 3) and parallel to line with D.R. 3 , 2 , –2 is

$$\begin{vmatrix} x-6 & y-7 & z-7 \\ -5 & -5 & -4 \\ 3 & 2 & -2 \end{vmatrix}=0 \qquad (1\tfrac{1}{2})$$

or, $\quad 18x - 22y + 5z + 11 = 0.$ (1)

28. Given

x	$P(x)$
0	0
1	k
2	4k
3	2k
4	k
	$\Sigma p_i = 8k$

(1)

But $\quad \Sigma\, p_i = 1$ (1)

$\Rightarrow \quad \boxed{k = \frac{1}{8}}$

$\therefore$ Probability distribution is

x_i	p_i	$p_i x_i$	$p_i x_i^2$
0	0	0	0
1	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$
2	$\frac{1}{2}$	1	2
3	$\frac{1}{4}$	$\frac{3}{4}$	$\frac{9}{4}$
4	$\frac{1}{8}$	$\frac{1}{2}$	2

(1)

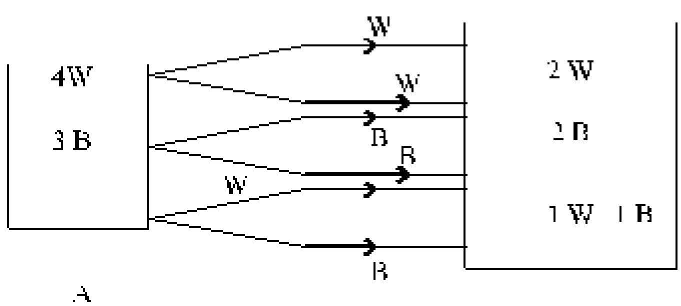
Probability of getting admission in two colleges $= \frac{1}{2}$ (1)

$$\text{Mean} = \mu = \Sigma p_i x_i = \frac{19}{8}$$ (1)

$$\text{Variance} = \sigma^2 = \Sigma p_i x_i^2 - \mu^2 = \frac{51}{8} - \left(\frac{19}{8}\right)^2$$

$$= \frac{47}{64}$$ (1)

OR



Three cases arise, when 2 balls from bag A are shifted to bag B.

Q. No. **Value Points** **Marks**

Case 1 : If 2 white balls are transferred from bag A.

$$P(W_A W_A) = \frac{4}{7} \cdot \frac{2}{6} = \frac{2}{7} \qquad (1)$$

Case 2 : If 2 black balls are transferred from bag A

$$P(B_A B_A) = \frac{3}{7} \cdot \frac{2}{6} = \frac{1}{7} \qquad (1)$$

Case 3 : If 1 white and 1 black ball is transferred from bag A

$$P(W_A B_A) = 2\left(\frac{4}{7} \cdot \frac{3}{6}\right) = \frac{4}{7} \qquad (1)$$

(a) Probability of drawing 2 white balls from bag B

$= P(W_A W_A). P(W_B W_B) + P(B_A B_A). P(W_B W_B) + P(W_A B_A).P(W_B.W_B)$

$$= \frac{2}{7}\left(\frac{4}{6} \cdot \frac{3}{5}\right) + \frac{1}{7}\left(\frac{2}{6} \cdot \frac{1}{5}\right) + \frac{4}{7}\left(\frac{3}{6} \cdot \frac{2}{5}\right) = \frac{5}{21} \qquad (1)$$

(b) Probability of drawing 2 black balls from bag B

$= P(W_A W_A). P(B_B B_B) + P(B_A.B_A). P(B_B B_B) + P(W_A B_A).P(B_B.B_B)$

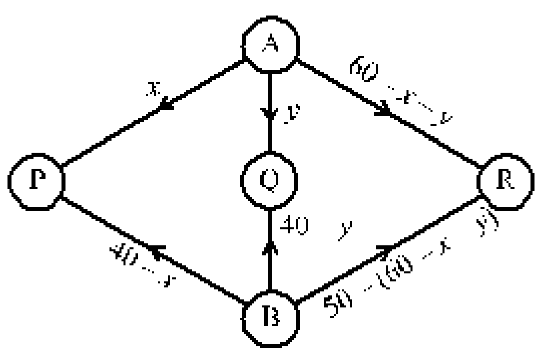
$$= \frac{2}{7}\left(\frac{2}{6} \cdot \frac{1}{5}\right) + \frac{1}{7}\left(\frac{4}{6} \cdot \frac{3}{4}\right) + \frac{4}{7}\left(\frac{3}{6} \cdot \frac{2}{5}\right) \qquad (1)$$

$$= \frac{4}{21}$$

(c) Probability of drawing 1 white and 1 black ball from bag B

$$= \frac{2}{7}\left(\frac{4}{6} \cdot \frac{2.2}{5}\right) + \frac{1}{7}\left(\frac{2.2}{6} \cdot \frac{4}{5}\right) + \frac{4}{7}\left(\frac{2.3}{6} \cdot \frac{3}{5}\right) = \frac{4}{7} \qquad (1)$$

29.



Q. No.	**Value Points**	**Marks**

Let x no. of packets from kitchen A are transported to P and y of packets from kitchen A to Q. Then only $60 - x - y$ packets can be transported to R from A.

Similarily from B $40 - x$ packets can be transported to P and $40 - y$ to Q. Remaining requirement of R i.e. $50 - (60 - x - y)$ can be transported from B to Q.

$\therefore$ Constraints are (1)

$$\left.\begin{array}{l} 40 - x \geq 0 \\ 40 - y \geq 0 \\ 60 - x - y \geq 0 \\ -10 + x + y \geq 0 \\ x \geq 0, y \geq 0 \end{array}\right\}$$ (1)

Objective function is:

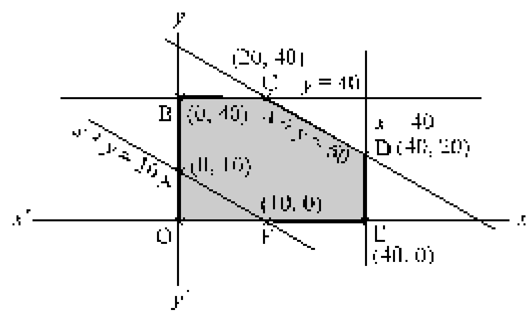
Minimise. $z = 5x + 4y + 3(60 - x - y) + 4(40 - x) + 2(40 - y) + 5(x + y - 10)$

$\therefore$ L.P.P. is

To Minimise. $z = 3x + 4y + 370$ (1)

subject to constraints

$$\left.\begin{array}{l} x \leq 40 \\ y \leq 40 \\ x + y \leq 60 \\ x + y \geq 10 \\ x \geq 0,\ y \geq 0 \end{array}\right\}$$



(1)

Feasible Region is A B C D E F A with corner points

A (0, 10) $z = 3(0) + 4(10) + 370 = 410$

B (0, 40) $z = 3(10) + 4(40) + 370 = 530$

C (20, 40) $z = 3(20) + 4(40) + 370 = 590$

D (40, 20) $z = 3(40) + 4(20) + 370 = 570$

E (40, 0) $z = 3(40) + 4(0) + 370 = 490$

F (10, 0) $z = 3(10) + 4(0) + 370 = 400$ (1)

$\therefore$ $x = 10, y = 0$ gives minimum cost of transportion.

Thus No. of packets can be transported as follows (1)

	A	B
P	10	30
Q	0	40
R	50	0

Minimum cost of transportation is Rs. 400.