Theory of Machines and Vibrations – Short Notes

Instantaneous Centre of Velocity (I-centre)

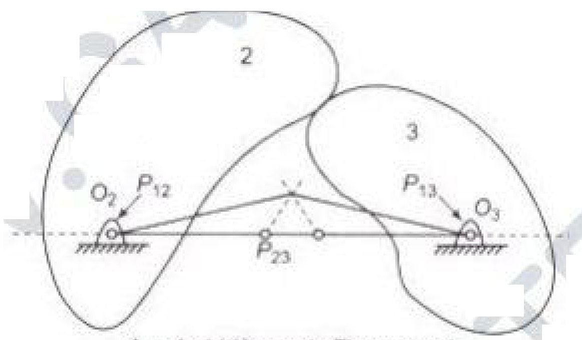
- The instantaneous centre of velocity can be defined as a point which has no velocity with respect to the fixed link.

Centro

- Instantaneous centre is also called centro
- Primary Centro One which can be easily located by a mere observation of the mechanism.
- Secondary Centro Centros that cannot be easily located

Aronhold-Kennedy Theorem of Three Centre

- It state that if three bodies are in relative motion with respect to one another, the three relative instantaneous centers of velocity ar collinear.



Aronhold-Kennedy Theorem of three centre 1, 2 and 3

Number of Centros in a Mechanism

- For a mechanism of n links, the number of centros (Instantaneous centre) N is

$$N = \frac{1}{2}n(n-1)$$

Linkages are the basic building blocks of all mechanisms

- Links: rigid member having nodes.
- Node: attachment points.
- Binary link: 2 nodes
- Ternary link: 3 nodes
- Quaternary link: 4 nodes
- Joint: connection between two or more links (at their nodes) which allows motion; (Joints also called kinematic pairs)

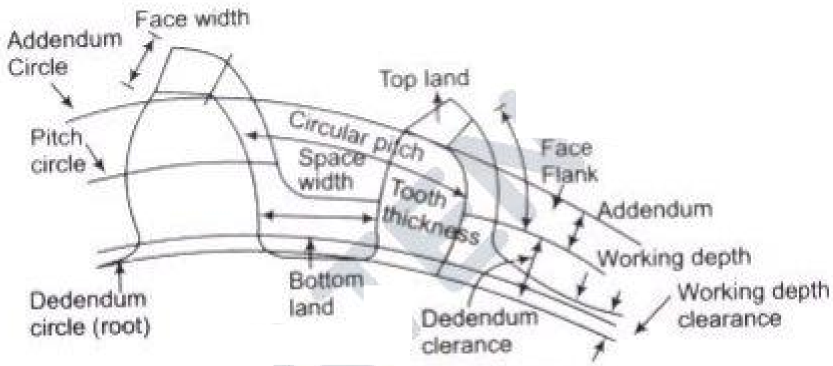
D'Alembert's Principle and Inertia Forces

- D'Alembert's principle states that the reverse-effective forces and torques and the external forces and torques on a body together give statical equilibrium

$$F + (-ma_G) = 0$$

$$T_{eG} + (-I_G\alpha) = 0$$

Gear Terminology



A schematic gear diagram

Circular Pitch (p):

- It is a distance measured along the circumference of the pitch circle from a point on one tooth to the corresponding point on the adjacent tooth.

$$p = \frac{\pi d}{T}$$

Diametrical Pitch (P)

- It is the number of teeth per unit length of the pitch circle diameter in inches.

$$P = \frac{T}{d}$$

Module (m)

- It is the ratio of pitch diameter in mm to the number of teeth. The term is used SI units in place of diametrical pitch.

$$m = \frac{d}{T}$$

$$\Rightarrow p = \pi m$$

Gear Ratio (G)

- It is the ratio of the number of teeth on the gear to that on the pinion.

$$G = \frac{T}{t}$$

where, T = number of teeth on the gear

t = number of teeth on the pinion

Velocity Ratio

- The velocity ratio is defined as the ratio of the angular velocity of the follower to the angular velocity of the driver gear

$$VR = \frac{\omega_2}{\omega_1} = \frac{N_2}{N_1} = \frac{d_1}{d_2} = \frac{T_1}{T_2}$$

Gear Train

- A gear train is a combination of gears used to transmit motion from one shaft to another. Gear trains are used to speed up or stepped down the speed of driven shaft. The following are main types of gear trains.

Simple Gear Train

- Series of gears, capable of receiving and transmitting motion from one gear to another is called a simple gear train.

Train value

$$= \frac{N_3}{N_1} = \frac{T_1}{T_3}$$

$$= \frac{\text{Number of teeth on driving gears}}{\text{Number of teeth on driver gear}}$$

Speed ratio

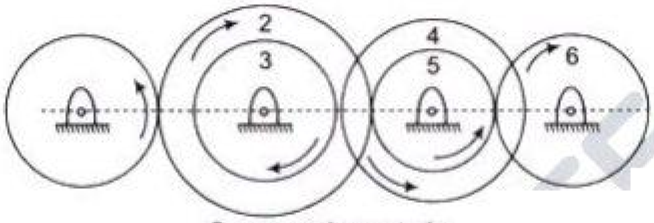
$$= \frac{1}{\text{Train value}}$$

Gears-and-gear-trains

- The intermediate gears have no effect on the speed ratio and therefore they are known as idlers.

Compound Gear Train

- When a series of gears are connected in such a way that two or more gears rotate about an axis with the same angular velocity.



Compound gear train

Train value

$$= \frac{\text{Product of number of teeth on driving gears}}{\text{Product of number of teeth on driven gears}}$$

Planetary or Epicyclic Gear Train

- A gear train having a relative motion of axes is called a planetary or an epicyclic gear train. In an epicyclic train, the axis of at least one of the gears also moves relative to the frame.
- If the arm a is fixed the wheels S and P constitute a simple train. However if the wheel S is fixed so that arm a can rotate about the axis of S. The P would be moved around S therefore it is an epicyclic train

Flywheel

- A flywheel is used to control the variations in speed during each cycle of an operation. A flywheel acts as a reservoir of energy which stores energy during the period when the supply of energy is more than the requirement and releases the energy during the period when the supply energy is less than the requirement.

Maximum fluctuation of energy (e),

$$e = (\triangle KE) = \frac{1}{2} I(\omega_{\max}^2 - \omega_{\min}^2)$$

$$e = I\omega(\omega_1 - \omega_2) \left(\omega = \frac{\omega_1 + \omega_2}{2}\right)$$

$$e = I\omega^2 k_s \qquad \left[k_s = \frac{\omega_1 - \omega_2}{2}\right]$$

$$k_s = \frac{e}{\frac{1}{2} I\omega^2 \times 2}, \; k_s = \frac{e}{2E}$$

where,

ωmax and ωmin are the maximum and minimum angular speed respectively.

E = kinematic energy of the flywheel at mean speed.

Flywheel in Punching Press

- Generally, flywheel is used to reduce fluctuation of speed where the load on the crank shaft constant while the applied torque varies.
- However, the flywheel can also be used to reduce fluctuation of speed when the torque is constant but load varies during the cycle *e.g.*, in punching press in riveting machine.
- Let *E* be energy required for one punch energy supplied to crank shaft from the motor during punching

$$= E\left(\frac{\theta_2 - \theta_1}{2\pi}\right)$$

Governors

The function of a governor is to maintain or regulate the speed of an engine within specified limits whenever there is variation of load.

Types of Governors

The broadly classification of the governors are given below.

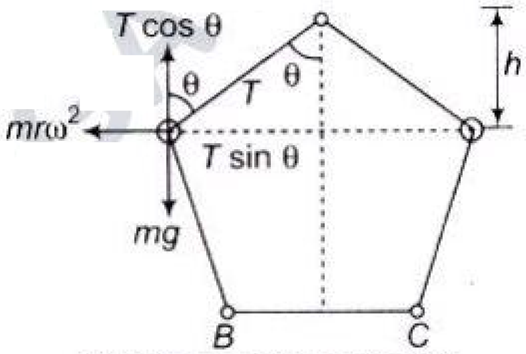
Centrifugal Governor

- In this type of governor, the action of governor depends upon the centrifugal effects produced by the masses of two balls.

Inertia Governor

- In this type of governor, positions of the balls are effected by the forces set up by an angular acceleration or deceleration of the given spindle in addition to centrifugal forces on the balls.

Pendulum Type Watt Governor



Simple conical governor

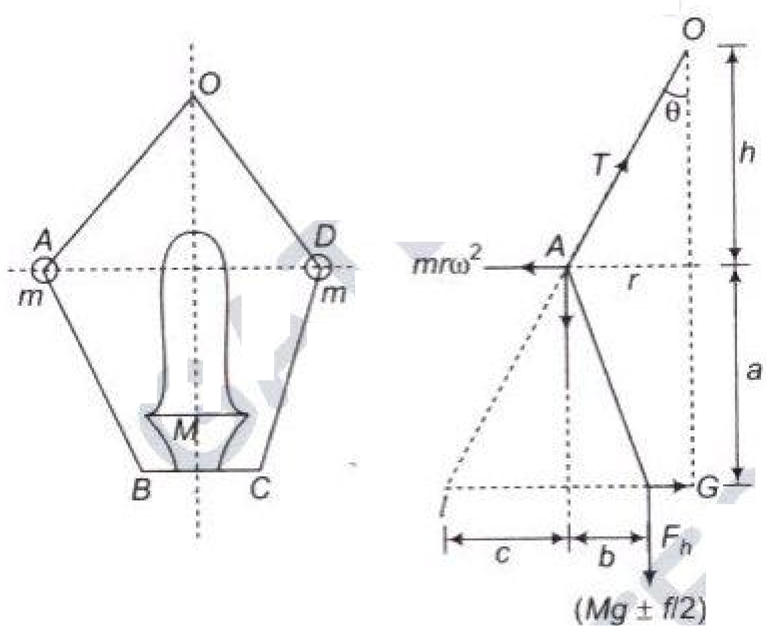
height of each bal

$$h = \frac{g}{w^2}$$

Porter Type Governor

Porter governor can be shown as

$$h = \frac{895}{N^2}\left(\frac{2mg + (mg \pm f)(1+k)}{2mg}\right)$$



Wilson Hartness Governor

- **Main spring force**

$$F'_{S_2} - F'_{S_1} = 4s(r_2 - r_1)$$

- **Net auxiliary spring force**

$$F'_{S_2} - F'_{S_1} = h_2\ s_a = \left(h_1 \frac{y}{x}\right)\frac{s}{a}$$

$$= (r_2 - r_1)\frac{b}{a}\frac{y}{x}\ s_a$$

Pickering Governor

$$f = \frac{m(e + f)\omega^2}{192\ EI}$$

Where, E = modulus of elasticity of the spring material

I = moment of inertia of the cross-section of the spring about neutral axis

Sensitiveness of a Governor

- The governor is said to be sensitive when it readily responds to a small change of speed.
  - Sensitiveness of a governor is defined as the ratio of difference between the maximum and minimum speeds to the mean equilibrium speed.

$$\text{Sensitiveness} = \frac{\text{range of speed}}{\text{mean speed}}$$

$$= 2\frac{(N_2 - N_1)}{N_1 + N_2}$$

where, N = mean speed

$N1$ = minimum speed corresponding to full load conditions

$N2$ = maximum speed corresponding to no load conditions.

Huning

- Sensitiveness of a governor is a desirable quality. However, if a governor is too sensitive, it may fluctuate continuously. This phenomenon of fluctuation is known as hunting.

Isochronism

- If a governor is at equilibrium only for a particular speed, it is called isochronous governor, for which we can say that an isochronous governor is infinitely sensitive.

$$\frac{dF}{dr} = m\omega^2$$

Stability

- A governor is said to be stable if it brings the speed of the engine to the required value and there is not much hunting. Obviously, the stability and sensitivity are two opposite characteristics.

Cam:

- A cam is a mechanical member used to impart desired motion (displacement) to a follower by direct contact (either point or line contact).
- Cam mechanisms belong to higher pair mechanism.
- A driver member known as cam.
- A driven member called the follower.
- A frame is one which supports the cam and guides the follower.

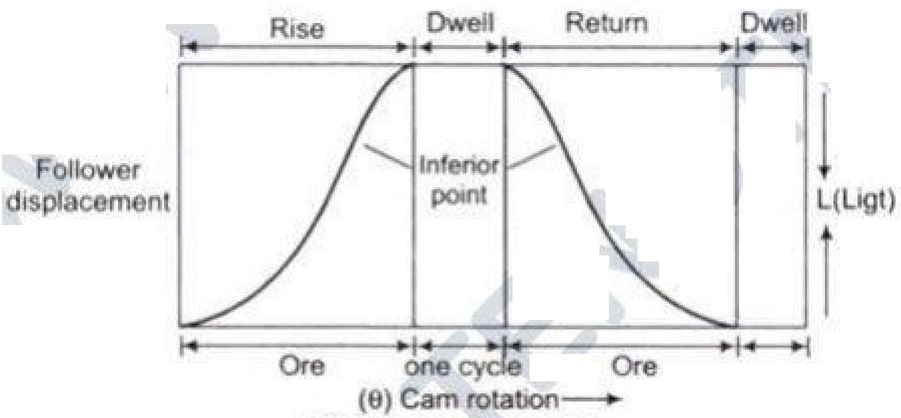
Key Points for Cams

- For a roller follower, the trace point is at the roller centre.
- For a flat-face follower, it is at the point of contact between the follower and cam surface when the contact is along the base circle of the cam.
- During a complete rotation, the pressure angle varies from its maximum to its minimum value.
- The greater the pressure angle, the higher will be side thrust and consequently the changes of the translating follower jamming in its guide will increase.
- It is not desirable to increase the pressure angle

Follower Displacement Diagram

The following terms are used with reference to the angular motion of the cam

- **Angle of Ascent (φ*a*):** It is the angle through which the cam turns during the time the follower rises.
- **Angle of Dwell (*f*)** Angle of dwell is the angle through which the cam turns while the follower remains stationery at the highest or the lowest position.



Displacement diagram

Balancing

- Balancing is defined as the process of designing a machine in which unbalance force is minimum.

- If the centre of mass of rotating machines does not lie on the axis of rotation, the inertia force is given by $F_1 = m\omega^2 e$

m = mass of the machine

ω = angular speed of the machine

e = eccentricity i.e., the distance from the centre of mass to the axis of rotation

Internal and external Balancing

- The shaft can be completely balancing by adding a massm_1 at a distance e_1 from the axis of rotation diametrically opposite to m so that,

$$m\omega 2e = m_1\omega^2 e_1$$

Static Balancing

- If a shaft carries a number of unbalanced masses such that the centre of mass of the system is said to be statically balanced

Dynamic Balancing

- A system of rotating masses in dynamic balance when there does not exist any resultant centrifugal force as well as resultant couple.

Vibrations

Time Period for Simple Pendulum

Vibrations: Vibration refers to mechanical oscillations about an equilibrium point. In its simplest form, vibration can be considered to be the oscillation or repetitive motion of an object around an equilibrium position.

Vibrations or mechanical oscillations are of many types as given below

- **Free Vibration** (Natural vibration) Vibration over an interval of time during which the system is free from excitation is known as free vibration.
- **Damped and Undamped Vibration:** Energy of a vibrating system is gradually dissipated by friction and other resistance.
- **Forced Vibration** When a repeated force continuously acts on a system, the vibrations are said to be forced.
- **Linear Vibration**: If all the basic components of a vibratory system-the spring, the mass, and the damper, behave linearly, the resulting vibration is known as linear vibration. The differential equations that govern the behaviour of vibratory linear systems are linear. Therefore, the principle of superposition holds.
- Nonlinear vibration: vibration: If however, however, any of the basic components behave nonlinearly, the

- **Harmonic Vibration:** Vibration in which the motion is a sinusoidal function of time.
- **Fundamental Vibration:** Harmonic component of a vibration with the lowest frequency.
- **Steady State Vibration:** When the particles of the body move in steady state condition or continuing period vibration is called steady state vibration.
- **Transient Vibration:** Vibratory motion of a system other than steady state.
- **Longitudinal Vibration:** Vibration parallel to the longitudinal axis of a member
- **Transverse Vibration:** Vibration in a direction perpendicular to the longitudinal axis or central plane of a member.
- **Torsional Vibration:** Vibration that involves torsion of a member.

Mode of Vibration: Configuration of points of a SHM is called the mode of vibration.

Natural Frequency: Frequency of free simple harmonic vibration of an undamped linear system.

Time Period: Time taken for one oscillation is called time period.

Simple Pendulum: If time period of the pendulum is 1s, then pendulum is called simple pendulum.

$$\left|\frac{\theta}{\alpha}\right| = \frac{l}{g}\left(where,\ \alpha = \frac{d^2\theta}{dt^2}\right)$$

Time period is given by

$$T = 2\pi\sqrt{\left|\frac{\theta}{\alpha}\right|}$$

$$T = 2\pi\sqrt{\frac{l}{g}}$$

(for small amplitude sin θ ≈ θ)

$$T = 2\pi\sqrt{\frac{l}{g}\left(1 + \frac{\theta_0^3}{16}\right)}$$

[large amplitude (θ_0)]

Free Vibration of Damped One Degree-of-Freedom Systems

- Damping factor

$$\zeta = \frac{c}{2m\omega_n} = \frac{c}{2\sqrt{km}}$$

- **The system response when under-damped: ξ < 1**

$$x(t) = e^{-\xi\omega_n t}(x_0 \cos\omega_d t + \frac{\dot{x}_0 + \xi\omega_n x_0}{\omega_d}\sin\omega_d t)$$

- **The system response when critically damped: ξ = 1**

$$x(t) = e^{-\omega_n t}(x_0 + (\dot{x}_0 + \omega_n x_0)t)$$

- **The system response when over-damped: ξ > 1**

$$x(t) = C_1 e^{(-\xi+\sqrt{\xi^2-1})\omega_n t} + C_2 e^{(-\xi-\sqrt{\xi^2-1})\omega_n t}$$

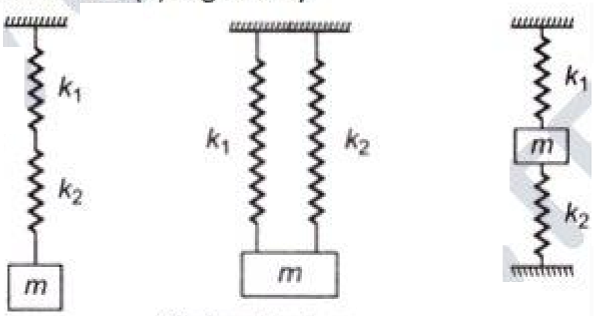
C_1 and C_2 are the constants that are lengthy in closed-form. They can be found numerically by the initial conditions.

- **Spring Block System**

$$ma = -kx$$

$$\left|\frac{x}{a}\right| = \frac{m}{k} \Rightarrow T = 2\pi\sqrt{\frac{m}{k}}$$

- **Equivalent force constant** (*k*) is given by



Spring block system

$$\frac{1}{k} = \frac{1}{k_1} + \frac{1}{k_2}$$

$$k = \frac{k_1 k_2}{k_1 + k_2}$$

$$k = k_1 + k_2$$

$$k = k_1 + k_2$$

- If spring has a mass m_s and a mass *m* is suspended from it,

$$T = 2\pi\sqrt{\frac{m + \frac{m_s}{3}}{k}}$$

Mass spring system

$$m\ddot{x} + kx = 0$$

$\omega_n = \sqrt{k/m}$: Natural frequency

$T = \dfrac{2\pi}{\omega_n}$: Period of motion

Damping

Any influence which tends to dissipate the energy of a system.

Damping Factor or Damping Ratio: It is the ratio of actual to critical damping coefficient.

$$\xi = \sqrt{\frac{(c/2m)^2}{k/m}} = \frac{c}{2\sqrt{mk}}$$

- ξ **=** 1, the damping is known as critical under critical damping condition Critical damping coefficient

$$C_c = 2\xi\sqrt{mk}$$
$$\Rightarrow C_c = 2\,m\omega_n$$

- ξ < 1 *i.e.*, system is underdamped.

$$\alpha_{1,2} = \left(-\xi \pm i\sqrt{1-\xi^2}\right)\omega_n$$

$$x = Ae^{\left(-\xi + i\sqrt{1-\xi^2}\right)\omega_n t} + Be^{\left(-\xi - i\sqrt{1-\xi^2}\right)\omega_n t}$$

- Damped frequency

$$\omega_d = \sqrt{1-\xi^2}\;\omega_n$$

Logarithmic Decrement

- In an underdamped system, arithmetic ratio of two successive oscillations is called logarithmic decrement (constant).

Since,

$$\frac{X_n}{X_{n+1}} = e^{\xi\omega_n T_d} = \frac{X_0}{X_1} = \frac{X_1}{X_2} = \frac{X_2}{X_3}$$

Logarithmic decrement,

$$\delta = In\left(\frac{X_n}{X_{n+1}}\right) = In\, e^{\xi\omega_n T_d}\ \delta = \xi\omega_n T_d$$

$$\delta = \frac{2\pi\xi}{\sqrt{1-\xi}}$$

$$\delta = \frac{1}{n}\ \text{ln}\left(\frac{X_0}{X_n}\right)$$

Forced Vibration

- Amplitude of the steady state response is given by

case of steady state response first term zero ($e^{-\infty} = 0$).

$$A = \frac{F_0}{\sqrt{(k - m\omega^2)^2 + (c\omega)^2}}$$

$$A = \frac{F_0 / k}{\sqrt{\left[1-\left(\frac{\omega}{\omega_n}\right)^2\right]^2 + \left(2\,\xi\,\frac{\omega}{\omega_n}\right)^2}}\ \tan\phi = \left(\frac{c\omega}{k - m\omega^2}\right)$$

Magnification Factor

- Ratio of the amplitude of the steady state response to the static deflection under the action force F0 is known as magnification factor

$$MF = \frac{\frac{F_0}{\sqrt{(k - m\omega^2)^2 + (c\omega)^2}}}{F_0 / k} = \frac{k}{\sqrt{(k - m\omega^2)^2 + (c\omega)^2}}$$

$$= \frac{1}{\sqrt{\left[1-\left(\frac{\omega}{\omega_n}\right)^2\right]^2 + \left(2\,\xi\,\frac{\omega}{\omega_n}\right)^2}}$$

Resonance

- When the frequency of external force(driving frequency) is equal to the natural frequency of a vibrating body, the amplitude of vibration becomes excessively large. This phenomenon is known as resonance

Critical Speed

- Critical or whirling or whipping speed is the speed at which the shaft tends to vibrate violently in transverse direction

Critical speed essentially depends on

- The eccentricity of the C.G of the rotating masses from the axis of rotation of the shaft.
- Diameter of the disc
- Span (length) of the shaft, and
- Type of supports connections at its ends