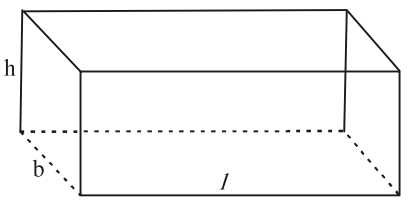
MENSURATION-II

An object which occupies space has usually three dimensions : length, breadth, and depth. Such an object is usually called a *solid*.

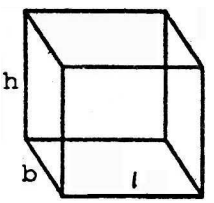
Given below are some commonly known solids:

CUBOID



1. Volume = $(l \times b \times h)$
2. Total surface area = $2(lb + bh + lh)$
3. Diagonal = $\sqrt{l^2 + b^2 + h^2}$
4. Area of 4 walls of a room = $2 \times h\ (l + b)$

CUBE



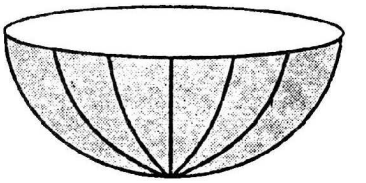
In cube $l = h = b$

1. Volume = $(l)^3$
2. $l = \sqrt[3]{\text{volume}}$
3. Total surface area = $6\ (l)^2$
4. Diagonal = $l\sqrt{3}$

SPHERE

(*i*) Let radius of sphere = r

1. Volume = $\frac{4}{3}\ \pi\ r^3$
2. Total surface area = $4\pi r^2$

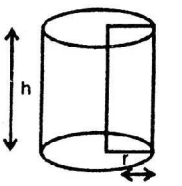


(*ii*) Hemi-sphere (half-sphere)

1. Volume = $\frac{2}{3}\pi r^3$
2. Curved surface area = $2\pi r^2$
   Total surface area = $3\pi r^2$

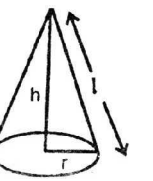
CYLINDER

1. Volume = $\pi\ r^2 h$
2. Curved surface area = $2\pi rh$
3. Total surface area
   $= 2\ \pi\ r\ (r + h)$



CONE

1. Slant height = $l = \sqrt{r^2 + h^2}$
2. Volume $= \frac{1}{3}\ \pi\ r^2 h$
3. Curved surface area = πrl
4. Total surface area = $\pi r\ (r + l)$



Example 1 : A trench is 10 m long, 5 m broad, and $3\frac{1}{3}$ m deep. The earth dugout from this trench is evenly spread on a road which is 200 m long and 5 m broad. Find the height of the earth spread on the road.

(*a*) $16\frac{2}{3}$ cm (*b*) $6\frac{2}{3}$ cm

(*c*) $16\frac{3}{5}$ cm (*d*) $15\frac{3}{5}$ cm

(*e*) None of these

Solution. (*a*) : Volume of the trench

$$= 10 \times 5 \times \frac{10}{3} = \frac{500}{3} \text{ cu m}$$

The area of the road = 200 × 5 = 1000 sq m.

$\therefore$ Height of the earth spread = $\frac{500}{3} \times \frac{1}{1000}$

$$= \frac{1}{6} \text{ m} = 16\frac{2}{3} \text{ cm}$$

Example 2 : How many bricks will be required to build a wall 30 m long, 30 cm thick and 5 m high with a provision of 2 doors, each 2.5 m × 1.2 m; each brick being 20 cm × 16 cm × 8 cm when 1/9 of the wall is filled with lime?

(*a*) 15000 (*b*) 25000

(*c*) 35000 (*d*) 500

(*e*) None of these

Solution. (*a*) : Area of wall = 30 × 5 = 150 sq m

Area of 2 doors = 2 × 2.5 × 1.2 = 6 sq m

$\therefore$ Remaining area of wall = 150 – 6 = 144 sqm.

$$\text{Volume of wall} = 144 \times \frac{30}{100} = \frac{216}{5} \text{ cu m}$$

$$\text{Lime used in the wall} = \frac{216}{5} \times \frac{1}{9} = \frac{24}{5} \text{ cu m}$$

$\therefore$ Volume of the bricks used in the wall

$$= \frac{216}{5} - \frac{24}{5} = \frac{192}{5} \text{ cu m}$$

$$\text{Volume of 1 brick} = \frac{20}{100} \times \frac{16}{100} \times \frac{8}{100} \text{ cu cm}$$

$$= \frac{8}{3125} \text{ cu m.}$$

$$\therefore \text{ Number of bricks} = \frac{\frac{192}{5}}{\frac{8}{3125}} = \frac{192 \times 3125}{40}$$

$$= 15000.$$

Example 3 : The inner diameter of a hollow metallic sphere is 18" and its thickness is 2". Find the weight of the sphere, if the weight of 1 cu ft metal is 486 lbs.

(*a*) 701.5 lbs (*b*) 700.0 lbs

(*c*) 709.5 lbs (*d*) 710.5 lbs

(*e*) None of these

Solution. (*c*) : Inner radius of the sphere = $\frac{18}{2}$ = 9 inches

Inner volume of the sphere = $\frac{4}{3}\pi r^3$

$$= \frac{4}{3}\pi \times 9^3 \text{ cu inches}$$

Outer radius of the sphere = 9 + 2 = 11 inches

$\therefore$ Outer volume of the sphere

$$= \frac{4}{3}\pi \times 11^3 \text{ cu inches}$$

$\therefore$ Volume of the metal used in the sphere

$$= \frac{4}{3}\pi(11^3 - 9^3) \text{ cu inches}$$

$$= \frac{4}{3} \times \frac{22}{7}(1331 - 729)$$

$$= \frac{88 \times 602}{21} = 2522.67 \text{ cu inches}$$

$\therefore$ Weight of the sphere

$$= \frac{2522.67}{12 \times 12 \times 12} \times 486 = 709.5 \text{ lbs.}$$

Example 4 : The volume of a cone is equal to that of a cylinder whose height is 9 cm and diameter 60 cm. What is the radius of the base of cone if its height is 108 cm?

(*a*) 10 cm (*b*) 15 cm

(*c*) 20 cm (*d*) 25 cm

(*e*) None of these

Solution. (*b*) : Volume of cylinder = $\pi r^2 h$

$$= \frac{22}{7} \times 30 \times 30 \times 9 \text{ cu cm}$$

$$\text{Volume of a cone} = \frac{1}{3}\pi r^2 \times \text{height}$$

$$= \frac{1}{3} \times \frac{22}{7} \times r^2 \times 108 \text{ cu cm}$$

$\because \frac{22}{7} \times 30 \times 30 \times 9 = \frac{1}{3} \times \frac{22}{7} \times r^2 \times 108$

$$\therefore r^2 = \frac{30 \times 30 \times 9 \times 3}{108} = 225 \text{ or } r = 15 \text{ cm}$$

Example 5 : There is a cubical room whose length is 10 m. How many students can it accommodate if each student requires 5 cu m of space?

(*a*) 100 (*b*) 150
(*c*) 200 (*d*) 250
(*e*) None of these

Solution. (*c*) : Volume of room = $(10)^3$ cu m
= 1000 cu m

Space required for a student = 5 cu m

$\therefore$ Required number of students $= \frac{1000}{5} = 200.$

Example 6 : Three cubes whose edges are 3 cm, 4 cm, and 5 cm respectively are melted to form a single cube. The surface of the new cube will be :

(*a*) 100 sq m (*b*) 216 sq m
(*c*) 200 sq m (*d*) 150 sq m
(*e*) None of these

Soltuion. (*b*) : Volume of 1st, 2nd and 3rd cube is 27, 64 and 125 cu cm.

Total volume of all three cubes = 27 + 64 + 125
= 216 cu cm

$\therefore$ Edge of new cube = $\sqrt[3]{216} = 6$ cm

$\therefore$ Surface of the new cube = $6 \times (6)^2 = 216$ sq cm.

Example 7 : Find the length of the longest rod that can be placed in a room 12 m long 9 m broad and 8 m high.

(*a*) 16 m (*b*) 17 m
(*c*) 15 m (*d*) 12 m
(*e*) None of these

Solution. (*b*) : The longest rod that can be placed in the room is equal to the length of its diagonal.

$\therefore$ Length of the longest rod

$= \sqrt{12^2 + 9^2 + 8^2} = \sqrt{144 + 81 + 64}$

$= \sqrt{289} = 17$ m

Example 8 : Find the volume of a right circular cone whose height is 24 cm and diameter of the base is 20 cm.

(*a*) $2514\frac{2}{7}$ cu cm (*b*) $2004\frac{1}{5}$ cu cm
(*c*) $2510\frac{3}{5}$ cu cm (*d*) $2156\frac{3}{7}$ cu cm
(*e*) None of these

Solution. (*a*) : Volume of cone = $\frac{1}{3}\pi r^2 h$

$= \frac{1}{3} \times \frac{22}{7} \times 10 \times 10 \times 24$

$= \frac{17600}{7}$ cu cm. $= 2514\frac{2}{7}$ cu cm.

Example 9 : Find the height of a right circular cone which is formed by melting a solid cylinder 3.5 m high and 2 m in radius. The radius of the base of the cone being equal to the radius of the cylinder.

(*a*) 11.5 m (*b*) 12.5 m
(*c*) 10.6 m (*d*) 10.5 m
(*e*) None of these

Solution. (*d*) : Volume of the cylinder

$= \frac{22}{7} \times 2 \times 2 \times 3.5 = 44$ cu m

Radius of the base of the cone = 2 m

$\because \quad \frac{1}{3}\pi r^2 h = 44$

$\because \quad h = \frac{3 \times 44 \times 7}{22 \times 2 \times 2} = \frac{21}{2} = 10.5$ m

Example 10 : A spherical iron shell with 21 cm external diameter weighs $22775\frac{5}{21}$ grams. Find the thickness of the shell if the metal weighs 10 gram per cu m.

(*a*) 0.5 cm (*b*) 1.0 cm
(*c*) 2.0 cm (*d*) 2.5 cm
(*e*) None of these

Solution. (*c*) : Let internal radius be *r* cm.

Internal volume = $\frac{4}{3}\pi r^3$ cu cm.

External radius = $\frac{21}{2}$ cm.

$\therefore$ External volume

$= \frac{4}{3} \times \frac{22}{7} \times \frac{21}{2} \times \frac{21}{2} \times \frac{21}{2} = 4851$ cm

$\therefore$ Volume of the metal of the shell

$= \frac{478280}{21} \times \frac{1}{10} = \frac{47828}{21}$ cu cm.

Internal volume of the metal of the shell

$= \left(4851 - \frac{47828}{21}\right) = \left(\frac{101871 - 47828}{21}\right)$

$= \frac{54043}{21}$ cu cm.

$\frac{4}{3}\pi r^3 = \frac{54043}{21}$

$$\frac{4}{3} \times \frac{22}{7} \times r^3 = \frac{54043}{21}$$

$$\therefore \quad r^3 = \frac{54043 \times 3 \times 7}{21 \times 4 \times 22}$$

or $r^3 = 614.125$

or $r = 8.5$ cm

$\therefore$ Thickness $= (10.5 - 8.5)$

$= 2$ cm.

EXERCISE

1. A wall 8 m long 6 m high and 22.5 cm thick is made up of bricks each measuring (25 cm × 11.25 cm × 6 cm). The number of bricks required is :
(*a*) 6000 (*b*) 5600
(*c*) 6400 (*d*) 7200
(*e*) None of these

2. The maximum length of rod that can be kept in a rectangular box of dimensions 8 cm × 6 cm × 2 cm, is :
(*a*) $2\sqrt{13}$ cm (*b*) $2\sqrt{14}$ m
(*c*) $2\sqrt{26}$ cm (*d*) $10\sqrt{2}$ m
(*e*) None of these

3. A rectangular block 6 cm × 12 cm × 15 cm is cut up into exact number of equal cubes. The least possible number of cubes will be :
(*a*) 6 (*b*) 11
(*c*) 33 (*d*) 40
(*e*) None of these

4. Three cubes of iron whose edges are 6 cm, 8 cm and 10 cm respectively are melted and formed into a single cube. The edge of the new cube formed is :
(*a*) 12 cm (*b*) 14 cm
(*c*) 16 cm (*d*) 18 cm
(*e*) None of these

5. The surface area of a cube is 600 cm. The length of its diagonal is :
(*a*) $10/\sqrt{3}$ cm (*b*) $10/\sqrt{2}$ cm
(*c*) $10\sqrt{3}$ cm (*d*) $10\sqrt{2}$ cm
(*e*) None of these

6. A beam 9 m long, 40 cm wide and 20 cm high is made up of iron which weighs 50 kg per cubic metre. The weight of the beam is :
(*a*) 56 kg (*b*) 48 kg
(*c*) 36 kg (*d*) 27 kg
(*e*) None of these

7. The sum of the length, breadth and depth of a cuboid is 19 cm and its diagonal is $5\sqrt{5}$ cm. Its surface area is :
(*a*) 361 cm^2 (*b*) 125 cm^2
(*c*) 236 cm^2 (*d*) 486 cm^2
(*e*) None of these

8. Given that 1 cu cm of marble weighs 25 gms, the weight of a marble block 28 cm in width and 5 cm thick is 112 kg. The length of the block is :
(*a*) 36 cm (*b*) 37.5 cm
(*c*) 32 cm (*d*) 26.5 cm
(*e*) None of these

9. The volume of a wall, 5 times as high as it is broad and 8 times as long as it is high, is 12.8 cu metres. The breadth of the wall is :
(*a*) 30 cm (*b*) 40 cm
(*c*) 22.5 cm (*d*) 25 cm
(*e*) None of these

10. In a shower 5 cm of rain falls. The volume of water that falls on 1.5 hectares of ground is :
(*a*) 75 cu m (*b*) 750 cu m
(*c*) 7500 cu m (*d*) 75000 cu m
(*e*) None of these

EXPLANATORY ANSWERS

1. (*c*) : Number of bricks $= \dfrac{800 \times 600 \times 22.5}{25 \times 11.25 \times 6}$

$= 6400.$

2. (*c*) : Required length $= \sqrt{(8^2 + 6^2 + 2^2)}$ cm

$= \sqrt{104} = 2\sqrt{26}$ cm

3. (*d*) : Volume of rectangular block = $(6 \times 12 \times 15)$ = 1080 cm^3
The side of largest cube = HCF of 6 cm, 12 cm, 15 cm = 3 cm
Volume of cube=$(3 \times 3 \times 3)$ cm^3 = 27 cm^3
Number of cubes = (1080/27) = 40.

4. (*a*) : Volume of the new cube = $[6^3 + 8^3 + (10)^3]$
= 1728 cu cm
Let the edge of new cube be a cm
Then, $a^3 = 1728 = (4 \times 4 \times 4 \times 3 \times 3 \times 3)$
$\Rightarrow \quad a = 12$ cm

5. (*c*) : $6a^2 = 600 \Rightarrow a^2 = 100$ or $a = 10$ cm
So, diagonal = $\sqrt{3}a = 10\sqrt{3}$ cm.

6. (*c*) : Volume $= (9 \times \frac{40}{100} \times \frac{20}{100})$ cu m $= \frac{18}{25}$ cu m
So, weight of the beam $= \left(\frac{18}{25} \times 50\right)$ kg
= 36 kg.

7. (*c*) : $(l + b + h) = 19$ and $\sqrt{l^2 + b^2 + h^2} = 5\sqrt{5}$
and so $(l^2 + b^2 + h^2) = 125$
Given $(l + b + h)^2 = 19^2$
Now $(l^2 + b^2 + h^2) + 2(lb + bh + lh) = 361$
$\Rightarrow 2(lb + bh + lh) = (361 - 125) = 236$
So, surface area = 236 cm^2.

8. (*c*) : Let length = x cm
Then, $x \times 28 \times 5 \times \frac{25}{1000} = 112$
So, $x = \frac{112 \times 1000}{28 \times 5 \times 25} = 32$ cm
So, length of block = 32 cm

9. (*b*) : Let, breadth = x metres . Then,
height = $5x$ metres
and length = $40x$ metres
So, $x \times 5x \times 40x = 12.8$
or $x^3 = \frac{12.8}{200} = \frac{128}{2000} = \frac{64}{1000}$
So, $x = \frac{4}{10}$ m $= \left(\frac{4}{10} \times 100\right)$ cm = 40 cm

10. (*b*) : Area = (1.5×10000) sq . metres
= 15000 sq metres.
Depth = 5/100 m = 1/20 m
So, Volume = (Area × Depth)
= $(15000 \times 1/20) = 750$ cu m.