Exponents and Powers

Numbers with Negative Exponents

The multiplicative inverse of a number *a* is the number, which when multiplied with *a*, gives 1 as the product. The multiplicative inverse of a number is also called its **reciprocal.**

How can we find the multiplicative inverse of numbers with negative exponents?

Two important results are:

$$\therefore a^{-x} = \frac{1}{a^x}$$

In general, we can say that a^{-m} is the multiplicative inverse of a^m and vice-versa.

Let us discuss some examples based on this concept.

Example 1:What are the values of

(i) 2^{-5}

(ii) 6^{-3}

(iii) 1^{-17}

Solution:

(i) $2^{-5} = \frac{1}{2^5} = \frac{1}{32}$

(ii) $6^{-3} = \frac{1}{6^3} = \frac{1}{216}$

(iii) $1^{-17} = \frac{1}{1^{17}} = \frac{1}{1} = 1$

Example 2:Write the multiplicative inverse of

(i) 10^{-16}

(ii) 6^{-39}

Solution:

We know that the multiplicative inverse of a^{-m} is a^{m}.

(i) The multiplicative inverse of 10^{-16} is 10^{16}.

(ii) The multiplicative inverse of 6^{-39} is 6^{39}.

Expanded Form of Decimal Numbers

Consider the number 3487. How can you write it in the expanded form?

It can be written in the expanded form as

$3487 = 3000 + 400 + 80 + 7$

You can also write this number in the expanded form using exponents as

$3487 = 3 \times 10^3 + 4 \times 10^2 + 8 \times 10^1 + 7 \times 10^0$

What if the number has a decimal part? For example, consider a number such as 3487.65. How do you write such a number in the expanded form?

r this question.

Similarly, we can write the number 605.1294 in the expanded form as

$605.1294 = 6 \times 10^2 + 5 \times 10^0 + 1 \times 10^{-1} + 2 \times 10^{-2} + 9 \times 10^{-3} + 4 \times 10^{-4}$

Let us now discuss some examples.

Expand the following numbers using exponents.

(i) 12.9482

(ii) 10573.02

Solution:

(i) The number 12.9482 can be expanded as

$1 \times 10^1 + 2 \times 10^0 + 9 \times 10^{-1} + 4 \times 10^{-2} + 8 \times 10^{-3} + 2 \times 10^{-4}$

(ii) The number 10573.02 can be expanded as

$1 \times 10^4 + 0 \times 10^3 + 5 \times 10^2 + 7 \times 10^1 + 3 \times 10^0 + 0 \times 10^{-1} + 2 \times 10^{-2}$

$= 1 \times 10^4 + 5 \times 10^2 + 7 \times 10^1 + 3 \times 10^0 + 2 \times 10^{-2}$

Laws of Exponents of Integers

We know that for a non-zero integer a, $a^m \times a^n = a^{m+n}$, where m and n are natural numbers. Does this law also hold for negative exponents? Let us verify this by taking $a = 10$, $m = -3$, and $n = -2$.

$$
\begin{aligned}
10^{-2} \times 10^{-3} &= \frac{1}{10^2} \times \frac{1}{10^3} && \left(a^{-m} = \frac{1}{a^m}\right) \\
&= \frac{1}{10^2 \times 10^3} \\
&= \frac{1}{10^{2+3}} && \left(a^m \times a^n = a^{m+n}\right) \\
&= \frac{1}{10^5} \\
&= 10^{-5} && \left(a^{-m} = \frac{1}{a^m}\right) \\
&= 10^{(-2)+(-3)} && \{(-2)+(-3) = -5\}
\end{aligned}
$$

As seen in the above example, for a non-zero integer a, the relation $a^m \times a^n = a^{m+n}$ holds true for negative exponents as well.

In fact, all the relations for positive exponents hold true for negative exponents as well. For non-zero integers a and b, and integers m and n. The laws of exponents (Note that these laws are valid for both positive and negative exponents) can be summarized as:

(i)	$a^m \times a^n = a^{m+n}$
(ii)	$a^m \div a^n = a^{m-n}$
(iii)	$a^{-m} = \frac{1}{a^m}$
(iv)	$(a^m)^n = a^{mn}$
(v)	$a^m \times b^m = (ab)^m$
(vi)	$\frac{a^m}{b^m} = \left(\frac{a}{b}\right)^m$
(vii)	$a^0 = 1$

Let us now discuss some examples where we need to use these laws of exponents.

Example 1:

Find the value of $\left(\frac{1}{3}\right)^{-2}$ **.**

Solution:

$$\left(\frac{1}{3}\right)^{-2} = \frac{1^{-2}}{3^{-2}} \qquad \left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$$

$$= \frac{\left(\frac{1}{1^2}\right)}{\left(\frac{1}{3^2}\right)} \qquad \left(a^{-m} = \frac{1}{a^m}\right)$$

$$= 1 \times \frac{3^2}{1}$$

$$= 9$$

Example 2:Simplify the expression $(7)^2 \div (7)^{11}$ using positive exponents.

Solution:

$(7)^2 \div (7)^{11} = 7^{2-11}$ $(a^m \div a^n = a^{m-n})$

$= 7^{-9}$

$= \frac{1}{7^9}$ $\left(a^{-m} = \frac{1}{a^m}\right)$

$= \frac{1^9}{7^9}$ $\left(1^9 = 1\right)$

$= \left(\frac{1}{7}\right)^9$ $\left(\frac{a^m}{b^m} = \left(\frac{a}{b}\right)^m\right)$

Thus, the simplified form of the expression $(7)^2 \div (7)^{11}$ is $\left(\frac{1}{7}\right)^9$.

Example 3:Find the value of the expression $\left(\frac{1}{6}\right)^{-2} + \left(\frac{1}{7}\right)^{-1} + \left(\frac{1}{11}\right)^{-1}$.

Solution:

$\left(\frac{1}{6}\right)^{-2} + \left(\frac{1}{7}\right)^{-1} + \left(\frac{1}{11}\right)^{-1}$

$= \frac{1^{-2}}{6^{-2}} + \frac{1^{-1}}{7^{-1}} + \frac{1^{-1}}{11^{-1}}$ $\left(\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}\right)$

$= \frac{6^2}{1^2} + \frac{7^1}{1^1} + \frac{11^1}{1^1}$ $\left(a^{-m} = \frac{1}{a^m}\right)$

$= 36 + 7 + 11$

$= 54$

Hence, the value of the expression $\left(\frac{1}{6}\right)^{-2} + \left(\frac{1}{7}\right)^{-1} + \left(\frac{1}{11}\right)^{-1}$ is 54.

Example 4:Find the value of x for the equation $(-3)^x \div (-3)^{-2} = (-3)^5$.

Solution:

$(-3)^x \div (-3)^{-2} = (-3)^5$

$(-3)^{x-(-2)} = (-3)^5$ $(a^m \div a^n = a^{m-n})$

$(-3)^{x+2} = (-3)^5$

Equating the powers of −3 on both sides:

$x + 2 = 5$

$x = 3$

Thus, the value of x for the equation $(-3)^x \div (-3)^{-2} = (-3)^5$ is 3.

Example 5:Evaluate the following expressions.

(i) $\left[\dfrac{a^0 \div 1}{2}\right] \times \dfrac{4^2 \times 5}{2^3}$

(ii) $\dfrac{\left(x^2\right)^{-3} \times 125 \times 2^8}{2^{11} \times 5^3 \times x^{-7}}$

Solution:

(i)

$$\left[\frac{a^0 \div 1}{2}\right] \times \frac{4^2 \times 5}{2^3}$$

$$= \left(\frac{1 \div 1}{2}\right) \times \frac{\left(2^2\right)^2 \times 5}{2^3} \qquad \left[a^0 = 1 \text{ and } 4 = 2^2\right]$$

$$= \frac{1}{2} \times \frac{2^4 \times 5}{2^3} \qquad \left[\left(a^m\right)^n = a^{mn}\right]$$

$$= \frac{2^4 \times 5}{2^1 \times 2^3}$$

$$= \frac{2^4 \times 5}{2^{1+3}}$$

$$= \frac{2^4 \times 5}{2^4}$$

$$= 5$$

Thus, the value of the expression $\left[\frac{a^0 \div 1}{2}\right] \times \frac{4^2 \times 5}{2^3}$ is 5.

(ii)

$$\frac{\left(x^2\right)^{-3} \times 125 \times 2^8}{2^{11} \times 5^3 \times x^{-7}}$$

$$= \frac{(x)^{-6} \times (5)^3 \times (2)^8}{(2)^{11} \times (5)^3 \times (x)^{-7}} \qquad \left[\left(a^m\right)^n = a^{mn}\right]$$

$$= \frac{x^{-6}}{x^{-7}} \times \frac{(5)^3}{(5)^3} \times \frac{(2)^8}{2^{11}}$$

$$= x^{(-6)-(-7)} \times (5)^{3-3} \times 2^{8-11} \qquad \left[\frac{a^m}{a^n} = a^{m-n}\right]$$

$$= x^{-6+7} \times 5^0 \times 2^{-3}$$

$$= x^1 \times 1 \times 2^{-3} \qquad \left[5^0 = 1\right]$$

$$= x \times \frac{1}{2^3} \qquad \left[a^{-m} = \frac{1}{a^m}\right]$$

$$= \frac{x}{8}$$

Thus, the value of the expression $\frac{\left(x^2\right)^{-3} \times 125 \times 2^8}{2^{11} \times 5^3 \times x^{-7}}$ is $\frac{x}{8}$.

Expressing Very Small Numbers in Standard Form and Vice-versa

Let us consider the following examples:

1. The mass of the earth is 5970000000000000000000000 kg.

2. The thickness of a wire is 0.000003 m.

3. The charge of an electron is 0.00000000000000000016 Coulomb.

What can you say about these numbers?

Here, the mass of the earth is represented by a very large number, whereas the thickness of a wire, the average radius of a red blood cell, and the charge of an electron are represented by very small numbers.

In example (3), there are 18 zeroes between the decimal point and the digit 1.

In example (1), there are 22 zeroes after 597.

When we write such numbers, we might easily make a mistake in writing the exact number of zeroes. If we are using these values in some calculation, then making a mistake in writing the number will lead to an incorrect answer.

For this reason, it is easier to write a number in its standard form. Writing a number in its standard form does not cause its value to change and this method is also less time consuming.

We express a number in standard form as $\boldsymbol{x \times 10^{y}}$. Here, x is a number that lies between 1 and 10. When $x < 1$, the value of the exponent (y) is a negative integer and when $x > 1$, the value of the exponent (y) is a non-negative integer.

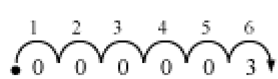
Let us now learn to write small numbers in standard form with the help of an example. Let us take the number 0.000003.

$$0.000003 = \frac{3}{1000000} = \frac{3}{10^6} = 3 \times 10^{-6}, \text{ which is the required standard form.}$$

However, this method is a little time consuming. The easier method is:

In the number 0.000003, there are 5 zeroes and the digit 3 after the decimal point.

Here, we have to shift the decimal point towards the right and put it after the digit 3. To do this, we have to move the decimal point 6 times towards the right, as shown in the following figure.



Since we moved 6 times towards the right, the exponent will carry a negative sign and the value of the exponent will be −6.

Thus, we can write the number 0.000003 in standard form as 3×10^{-6}.

We discussed in example (3) that the charge of an electron is 0.00000000000000000016 Coulomb.

In this number, there are 18 zeroes and the digits 1 and 6 after the decimal point. In this case, we have to shift the decimal point after the digit 1. Thus, we have to move 19 steps towards the right.

Hence, we can write the charge of an electron in standard form as 1.6×10^{-19} Coulomb.

Let us also see how to convert a number from its standard form to its standard notation.

Let us convert 5.93×10^{-5} into its standard notation. The exponent is negative. This means that we are talking about a very small number. Since the value of the exponent is -5, we need to move the decimal five places towards the left. We can do this as

$$0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 5.93 \times 10^{-5}$$

Thus, we can write the number 5.93×10^{-5} in its standard notation as 0.0000593.

We can also add and subtract two numbers written in standard form. For this, their exponents should be the same.

For example, let us find the value of $4.28 \times 10^{-3} + 1.27 \times 10^{-4}$. Here, the two exponents are different.

First of all, we have to make the exponents same.

We can write 4.28×10^{-3} as $4.28 \times 10 \times 10^{-4} = 42.8 \times 10^{-4}$

$\therefore 4.28 \times 10^{-3} + 1.27 \times 10^{-4} = 42.8 \times 10^{-4} + 1.27 \times 10^{-4}$

$= (42.8 + 1.27) \times 10^{-4}$

$= 44.07 \times 10^{-4}$

$= 4.407 \times 10^{-3}$

Let us now look at some examples to understand this concept better.

Example 1:

Express the numbers appearing in the following statements in standard form.

(a) The average radius of a red blood cell is 0.0000035 mm.

(b) 1 nanometer is equal to $\frac{1}{1000000000}$ **m.**

Solution:

(a) The given number is 0.0000035.

To shift the decimal point after the digit 3, we will move 6 steps towards the right.

$\therefore$ 0.0000035 = 3.5×10^{-6}

Hence, we can also express the average radius of a red blood cell as 3.5×10^{-6} mm.

(b) The given number is $\frac{1}{1000000000}$.

$\frac{1}{1000000000}$ can also be written as 1×10^{-9}.

Hence, the given value can also be written as: 1 nanometer = 1×10^{-9} m.

So far, we have learnt the conversion of a small number into the standard form. The manner in which we can convert a standard form of numbers into the usual form can be explained by taking a concrete example.

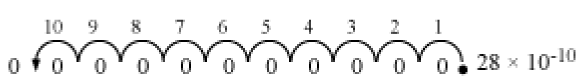
Let us consider an example based on the conversion of a number into its usual form.

Example 2:

Write 0.28×10^{-10} in standard notation.

Solution:

The given number is 0.28×10^{-10}. Since the value of the exponent is −10, the decimal point needs to be shifted 10 places towards its left. This can be done as



Thus, 0.28×10^{-10} can be expressed in standard notation as 0.000000000028.

Example 3:There are two types of sheets: A and B. The thickness of sheets A and B are 0.1×10^{-8} cm and 0.2×10^{-10} cm respectively. Find the difference between the thicknesses of the sheets of each category. If 4 sheets of type A and 3 sheets of type B are taken, then what is the total thickness of the sheets?

Solution:

Thickness of type A sheet = 0.1×10^{-8} cm

Thickness of type B sheet = 0.2×10^{-10} cm

Difference between the thickness of the two sheets

$= 0.1 \times 10^{-8} \text{ cm} - 0.2 \times 10^{-10} \text{ cm}$

$= 0.1 \times 10^{2} \times 10^{-10} \text{ cm} - 0.2 \times 10^{-10} \text{ cm}$

$= (10 - 0.2) \times 10^{-10} \text{ cm}$

$= 9.8 \times 10^{-10} \text{ cm}$

If 4 type A sheets and 3 type B sheets are taken, then the total thickness of the sheets is

$= 4 \times 0.1 \times 10^{-8} \text{ cm} + 3 \times 0.2 \times 10^{-10} \text{ cm}$

$= 0.4 \times 10^{-8} \text{ cm} + 0.6 \times 10^{-10} \text{ cm}$

$= 0.4 \times 10^{2} \times 10^{-10} \text{ cm} + 0.6 \times 10^{-10} \text{ cm}$

$= 40 \times 10^{-10} \text{ cm} + 0.6 \times 10^{-10} \text{ cm}$

$= (40 + 0.6) \times 10^{-10} \text{ cm}$

$= 40.6 \times 10^{-10} \text{ cm}$

$= 4.06 \times 10 \times 10^{-10} \text{ cm}$

$= 4.06 \times 10^{-9} \text{ cm}$

Hence, the difference between the thickness of the two types of sheets is 9.8×10^{-10} cm, and the total thickness of the 7 sheets is 4.06×10^{-9} cm.

Comparison Of Very Small And Very Large Numbers Expressed In Standard Form

Consider the following examples.

1. Size of a bacterium is 0.0000005 m.

2. Size of a plant cell is 0.00001275m.

Which one of the two is smaller in size?

We can compare very small numbers after converting them into standard form very easily.

The size of the bacterium in standard form is 5×10^{-7} m and the size of the plant cell in standard form is 1.275×10^{-5} m.

Now, let us make the exponents of these two values same.

We can write 1.275×10^{-5} m as 127.5×10^{-7} m.

Since the exponents are same in both the numbers, we can easily compare their decimals.

Now, 5 is less than 127.5.

Thus, we can say that the size of bacterium is smaller than the size of plant cell.

Can we compare the size of plant cell and size of bacterium?

$$\frac{\text{Size of plant cell}}{\text{Size of bacterium}}$$
$$= \frac{1.275 \times 10^{-5}}{5 \times 10^{-7}}$$
$$= \frac{1.275}{5} \times \frac{10^{-5}}{10^{-7}}$$
$$= 0.255 \times 10^{2}$$
$$= 25.5$$

Thus, the size of the plant cell is approximately 25 times the size of bacterium.

In the same way, we can compare very large numbers. Firstly, we write the numbers in standard form and then we make the exponents same. Now, we can compare the numbers by comparing their decimals.

Let us compare 2.4×10^{11} and 4×10^{8}.

The numbers are already given in standard form. Now, let us make the exponents same.

2.4×10^{11} can be written as $2.4 \times 10^{3} \times 10^{8}$ i.e., 2400×10^{8}.

Now, $2400 > 4$

Therefore, 2.4×10^{11} is greater than 4×10^{8}.

Also,

$$\frac{2.4\times10^{11}}{4\times10^{8}}=\frac{2.4\times10^{8}\times10^{3}}{4\times10^{8}}$$
$$=\frac{2400}{4}$$
$$=600$$

Thus, 2.4×10^{11} is 600 times of 4×10^{8}.

Let us discuss some examples based on the comparison of very small and very large numbers.

Example 1:The thickness of two sheets are 0.1×10^{-8} cm and 0.2×10^{-10} cm respectively. Which sheet is thinner? Compare the thickness of the sheets.

Solution:

We can compare two numbers in exponential form, if the exponents are same.

We can write 0.2×10^{-10} as $0.2 \times 10^{-2} \times 10^{-8} = 0.002 \times 10^{-8}$

Now, the thickness of the sheets are 0.1×10^{-8} and 0.002×10^{-8} respectively.

Clearly, 0.002×10^{-8} is smaller. Therefore, the sheet with thickness 0.2×10^{-10} is thinner.

On comparing the thickness of the sheets, we obtain

$$\frac{\text{Thickness of first sheet}}{\text{Thickness of second sheet}}=\frac{0.1\times10^{-8}\text{ cm}}{0.2\times10^{-10}\text{ cm}}=\left(\frac{0.1}{0.2}\right)\times\left(\frac{10^{-8}}{10^{-10}}\right)=\frac{1}{2}\times100=50$$

Therefore, the thickness 0.1×10^{-8} cm of the first sheet is 50 times the thickness

0.2×10^{-10} cm of the second sheet.

Example 2:The speed of light in vacuum is 3×10^{8} m/s and in glass is 2.26×10^{8} m/s. Find the difference of speed of light in these two mediums.

Solution:

Speed of light in vacuum = 3×10^{8} m/s

Speed of light in glass = 2.26×10^{8} m/s

Difference = $(3 \times 10^8 \text{ m/s}) - (2.26 \times 10^8 \text{ m/s})$

$= (3 - 2.26) \times 10^8$ m/s

$= 0.74 \times 10^8$ m/s

$= 7.4 \times 10^7$ m/s

Thus, the difference between the speed of light in vacuum and glass is 7.4×10^7 m/s.