*"Statistically, the probability of any one of us being here is so small that the mere fact of our existence should keep us all in a state of contented dazzlement."*

**– Lewis Thomas**

# 1

# Probability

**Contents :**

## 1.1 Introduction

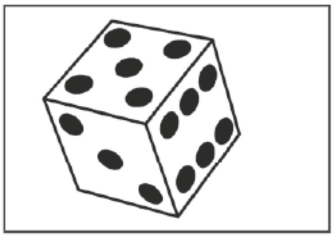
Many events occur in our day-to-day life. We can definitely say for many events that these events will certainly happen. For example, each person taking birth will die, a fruit freely falling from a tree will fall on ground, if the profit per item of a trader is ₹ 10 then he will earn a profit of ₹ 500 by selling 50 items, if a person invests ₹ 1,00,000 in a nationalised bank at an annual interest rate of 7.5 percent then the interest received will be ₹ 7,500, etc. These events are certain but some events are such that we can not be definitely say in advance whether they will happen. For example, getting head on the upper side after tossing a balanced coin, getting number 3 on the upper side of a die when a six faced unbiased die is thrown, the new baby to be born will be a boy, an item produced in a factory is non-defective, what will be the total rainfall in a certain region in the current year, what will be the wheat production in a state in the current year, what will be the result of a cricket match played between teams of two countries, etc. We cannot say with certainty that these events will definitely occur. It is not possible to give precise prediction about the occurrance of such events. We can intuitively get some idea about possibility of happening (or not happening) for these events but there is uncertainty regarding happening (or not happening) of these events. We accept that the occurrence (or non-occurrence) of these events depends upon an unknown element which is called chance. Such events which depend on chance are called random events. Probability is used to numerically express the possibility of these uncertain events. We shall study the theory of probability, the classical definition of probability, its statistical definition and the illustrations showing utility of probability. Now, let us see the explanation of certain terms which are useful to study probability.

## 1.2 Random Experiment and Sample Space

### 1.2.1 Random Experiment

Let us consider the following experiments :

**Experiment 1 :** Toss a balanced coin. Any one outcome is obtained out of two posible outcomes (i) Head-*H* (ii) Tail-*T* for this experiment. (We assume that the coin does not stand on its edge.) Thus, *'H'* and *'T'* are the only possible outcomes for the experiment of tossing a coin. But which of the outcomes will be obtained among these two outcomes cannot be said with certainty before conducting the experiment.



**Experiment 2 :** Throw a balanced die with six faces marked with numbers 1, 2, 3, 4, 5, 6 on it. Note the number appearing on its upper face. Any one outcome among the six possible outcomes 1, 2, 3, 4, 5, 6 will be obtained. There are only six possible outcomes 1, 2, 3, 4, 5, 6 for this experiment of throwing a die but which of the six outcomes will be obtained cannot be said with certainty before conducting the experiment.


**Experiment 3 :** Suppose there is a wheel marked with 10 numbers 0, 1, 2, .... , 9 and a pointer is kept against it. If this wheel is rotated with hand, it will spin and become stable after some time. When the wheel stops, any one of the numbers 0, 1, 2, .... , 9 will appear against the pointer. This number is the winning number. There are total ten possible outcomes 0, 1, 2, .... , 9 for this experiments. But which of the ten numbers will be obtained as a winning number cannot be said with certainty before conducting the experiment.

The experiments 1, 2, 3 shown above are called random experiments. A random experiment is defined as follows. The experiment which can be independently repeated under indentical conditions and all its possible outcomes are known but which of the outcomes will appear cannot be predicted with certainty before conducting the experiment is called a random experiment. The following characteristics of the random experiment can be deduced from its definition :

(1) A random experiment can be independently repeated under almost identical conditions.

(2) All possible outcomes of the random experiment are known but which of the outcomes will appear cannot be predicted before conducting the experiment.

(3) The random experiment results into a certain outcome.

### 1.2.2 Sample Space

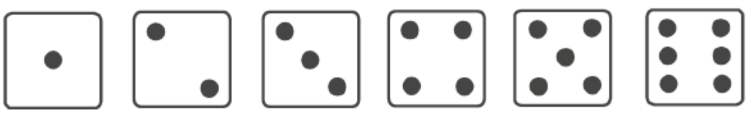
The set of all possible outcomes of a random experiment is called a sample space of that random experiment. The sample space is generally denoted by $U$ or $S$. The elements of sample space are called sample points.

The sample space of the random experiment in the earlier discussion can be obtained as follows :

**Experiment 1 :** Toss a balanced coin. There are total two possible outcomes for this random experiment : $H$ and $T$. Thus, the Sample Space can be written here as $U = \{H, T\}$ or $U = \{T, H\}$.

Head - H    Tail - T

**Experiment 2 :** Throw a balanced die with six faces with numbers 1, 2, 3, 4, 5, 6 on it. There are total six possible outcomes for this random experiment : 1, 2, 3, 4, 5, 6. Thus, the sample space is $U = \{1, 2, 3, 4, 5, 6\}$.



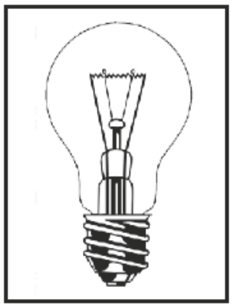
1 2 3 4 5 6

**Experiment 3 :** To decide the winning number by rotating a wheel marked with numbers 0, 1, 2, .... , 9. There are total ten possible outcomes for this random experiment. Thus, the sample space $U = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$.

0 1 2 3 4 5 6 7 8 9

**Finite Sample Space :** If the total number of possible outcomes in the sample space is finite then it is called a finite sample space. For example, the sample spaces of all the three random experiments given above are finite sample spaces.

**Infinite Sample Space :** If the total number of possible outcomes in the sample space of a random experiment is infinite then it is called an infinite sample space. For example, if the life of electric bulbs ($L$) from a production is recorded in hours then it is a real number. The value of $L$ will be 0 or more. Thus, there will be infinite possible outcomes for an experiment of measuring life of bulbs. The sample space will be $U = \{L \mid L \geq 0,\ L \in R\}$. If the maximum life of electric bulbs is assumed to be 700 hours, the sample space will be $U = \{L \mid 0 \leq L \leq 700;\ L \in R\}$ which is an Infinite sample space.



Now we shall see some more illustrations of sample space of a random experiment.

**Illustration 1 : Two balanced coins are tossed simultaneously. Write the sample space of this random experiment.**

We shall consider any one of the two coins here as the first coin and the other as the second coin. The outcome of this experiment will be as shown in the following diagram.

If we denote the head as $H$ and the tail as $T$, the sample space will be as follows :

$U = \{HH,\ HT,\ TH,\ TT\}$

Any one of the outcomes out of $H$ and $T$ can be obtained on the first coin. Thus, this action can be done in two ways and the other coin can also show one of the outcomes $H$ and $T$ which can also be done in two ways. According to the fundamental principle of counting for multiplication, the total number of outcomes will be $2 \times 2 = 2^2 = 4$. It should be noted here that the sample space for the experiment of tossing one balanced coin two times will also be the same as above.

**Illustration 2 : Two balanced dice are thrown where each die has numbers 1 to 6 on the six sides. Write the sample space of this experiment.**

We shall consider any one die as the first die and the other will be called the second die. The number on the first die will be shown as $i$ and the number on the second die will be shown as $j$. The following sample space will be obtained by denoting the pair of numbers on the two dice as $(i,\ j)$ where $i,\ j = 1, 2, 3, 4, 5, 6$.

$$U = \{(1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6),$$
$$(2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6),$$
$$(3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (3, 6),$$
$$(4, 1), (4, 2), (4, 3), (4, 4), (4, 5), (4, 6),$$
$$(5, 1), (5, 2), (5, 3), (5, 4), (5, 5), (5, 6),$$
$$(6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6)\}$$

**OR**

$$U = \{(i,\ j);\ i, j = 1, 2, 3, 4, 5, 6\}$$

Any one of the integers 1 to 6 can be shown on the upper side of the first die which can occur in 6 ways and the second die can also show one of the integers among 1 to 6 which will also occur in 6 ways. The total number of outcomes will be $6 \times 6 = 6^2 = 36$ according to fundamental principle of counting for multiplication. Similarly, the sample space for the random experiment of throwing three balanced dice simultaneously will have $6^3 = 216$ total outcomes.

**Illustration 3 : Write the sample space of the random experiment of finding the number of defective items while testing the quality of 1000 items produced in a factory.**

If the defective items are found among 1000 items produced in the factory then the number of defective items in the production can be 0, 1, 2, .... , 1000. Thus, the sample sapce will be as follows :

$$U = \{0, 1, 2, ...., 1000\}$$

**Illustration 4 : Write the sample space of random experiment of randomly selecting three numbers from the first four natural numbers.**

If three numbers are selected simultaneously from the first four natural numbers 1, 2, 3, 4 then those three numbers can be $(1, 2, 3)$, $(1, 2, 4)$, $(1, 3, 4)$ or $(2, 3, 4)$. Thus, the sample space of the random experiment will be as follows :

$$U = \{(1, 2, 3), (1, 2, 4), (1, 3, 4), (2, 3, 4)\}$$

3 numbers are to be selected here from the 4 numbers which has ${}^4C_3 = 4$ combinations. Thus, the total number of outcomes for this random experiment is 4.

**Illustration 5 : Write the sample space of a random experiment of randomly selecting any one number from the natural numbers.**

The natural numbers are 1, 2, 3, ..... If one number is randomly selected from these numbers then the sample space will be as follows :

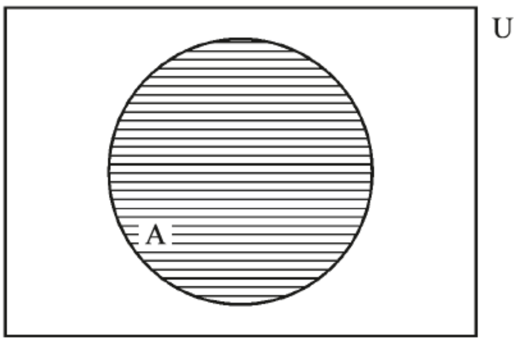
$$U = \{1, 2, 3, 4, ....\}$$

It should be noted here that this is an infinite sample space.

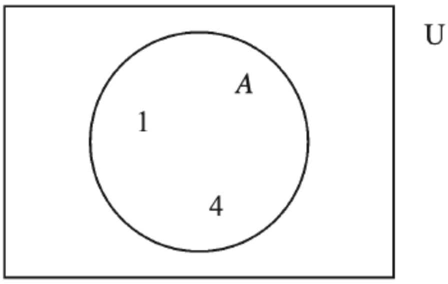
## 1.3 Events : Certain Event, Impossible Event, Special Events

We will study the different types of events by first understanding the meaning of an event.

**(1) Event :** A subset of the sample space of a random experiment is called an event. The events are generally denoted by letters *A, B, C,* ... or as $A_1$, $A_2$, $A_3$, .... The set formed by the sample points showing favourable outcomes of an event *A* will be a subset of the sample space *U*. Thus, any event *A* associated with the random experiment is the subset of sample space *U*. This is denoted as $A \subset U$.



For example, the sample space of a random experiment of throwing a balanced die is $U = \{1, 2, 3, 4, 5, 6\}$. If the event of obtaining a complete square as a number on the upper side of the die is denoted by $A$ then event $A = \{1, 4\}$.




Now, we shall show that an event is a subset of the sample space by taking a few examples of events in the random experiment of throwing two balanced dice.

- $A_1$ = the sum of numbers on the dice is 6.

  $\therefore\ A_1 = \{(1, 5), (2, 4), (3, 3), (4, 2), (5, 1)\}$

- $A_2$ = the numbers on the dice are same.

  $\therefore\ A_2 = \{(1, 1), (2, 2), (3, 3), (4, 4), (5, 5), (6, 6)\}$

- $A_3$ = the sum of numbers on the dice is more than 9.

  $\therefore\ A_3 = \{(4, 6), (5, 5), (5, 6), (6, 4), (6, 5), (6, 6)\}$

All these subsets are called events.

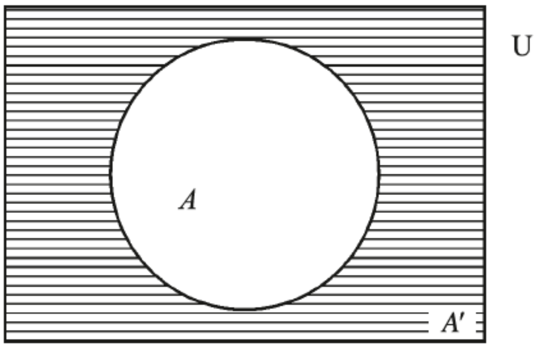
**(2) Impossible Event :** The special subset $\phi$ or $\{\ \}$ of the sample space of a random experiment is called an impossible event. Impossible event is an event which never occurs. It is denoted by $\phi$ or $\{\ \}$.

For example, the event of getting both head ($H$) and tail ($T$) on a balanced coin is an impossible event.

**(3) Certain Event :** The special subset $U$ of the sample space of random experiment is called a certain event. The certain event is an event which always occurs. It is denoted by $U$.

For example, the day next to Saturday is Sunday, the number on the upper side die when a balanced die is thrown is less than 7, etc. are certain events.

**(4) Complementary Event :** Suppose $U$ is a finite sample space and $A$ is one of its events. The set of all the outcomes or elements of $U$ which are not in the event $A$ is called as complementary event of $A$. The complementary event of event $A$ is denoted by $A'$, $\overline{A}$, $A^c$. We will use the notation $A'$ for complementary event of $A$.




$A'$ = Complementary event of event $A$.

= Non-occurrence of event $A$.

= $U - A$.

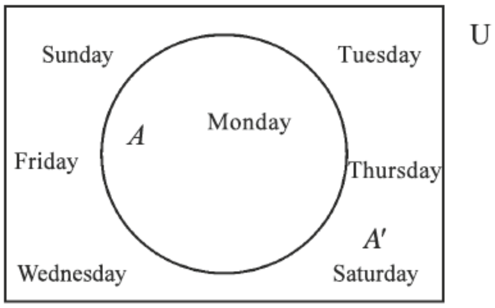
For example, the sample space of the random experiment of finding the day when a cargo ship will reach port $Y$ after leaving from port $X$ will be as follows.

$U$ = {Sunday, Monday, Tuesday, Wednesday, Thursday, Friday, Saturday}

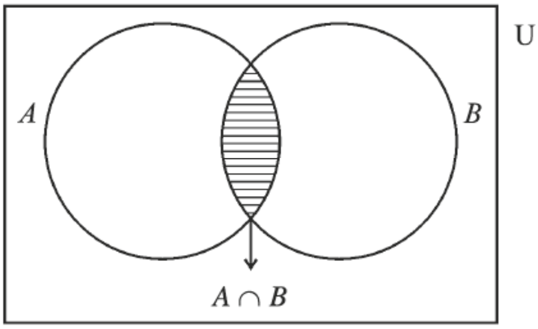
Suppose $A$ denotes that this ship reaches port $Y$ on Monday. Then the set of days except Monday will be the set of outcomes of event $A'$.

$A$ = {Monday}

$A' = U - A$ = {Sunday, Tuesday, Wednesday, Thursday, Friday, Saturday}



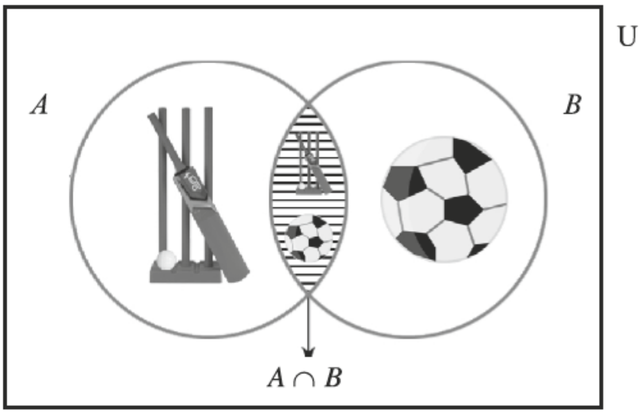

**(5) Intersection of Events :** Suppose $A$ and $B$ are two events of a finite sample space $U$. The event where events $A$ and $B$ occur simultaneously is called the intersection of two events $A$ and $B$. It is denoted by $A \cap B$.



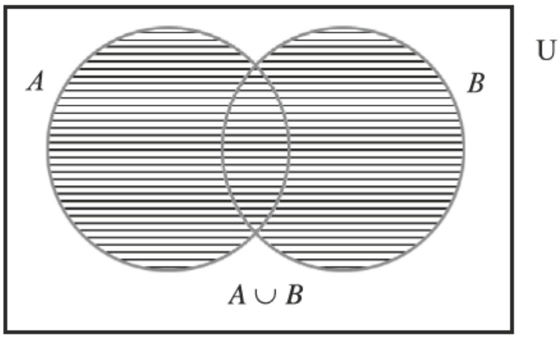

$A \cap B$ = Intersection of two events $A$ and $B$

= Simultaneous occurrence of events $A$ and $B$

For example, some of the students studying in a class of a school are the members of school cricket team and some students are members of school football team. Let us denote the event that a student is a member of cricket team by event $A$ and the event that a student is a member of football team by $B$. If one student is randomly selected from this class then the event that the student is a member of school cricket and football team is called $A \cap B$, the intersection of events $A$ and $B$.




**(6) Union of Events :** Suppose $A$ and $B$ are any two events of a finite sample space $U$. The event where the event $A$ occurs or the event $B$ occurs or both the events $A$ and $B$ occur is called the union of events $A$ and $B$. It is denoted by $A \cup B$.

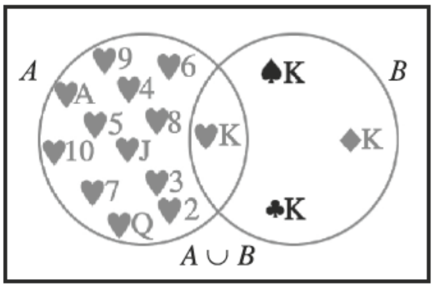



$A \cup B$ = Union of events $A$ and $B$

= Event $A$ occurs or event $B$ occurs or both events $A$ and $B$ occur together

= At least one of the event $A$ and $B$ occurs.

For example, the event $A \cup B$ of getting heart card (say event $A$) or a king (say event $B$) when a card is drawn randomly from a pack of 52 cards will be as follows :




$$A = \{H_A, H_2, H_3, H_4, H_5, H_6, H_7, H_8, H_9, H_{10}, H_J, H_Q, H_K\}$$

$$B = \{S_K, D_K, C_K, H_K\}$$

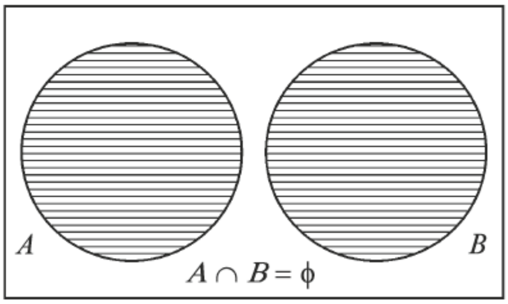
$$A \cup B = \{H_A, H_2, H_3, H_4, H_5, H_6, H_7, H_8, H_9, H_{10}, H_J, H_Q, H_K, S_K, D_K, C_K\}$$

Thus, the occurrence of $A \cup B$ is selecting any one of these 16 cards.

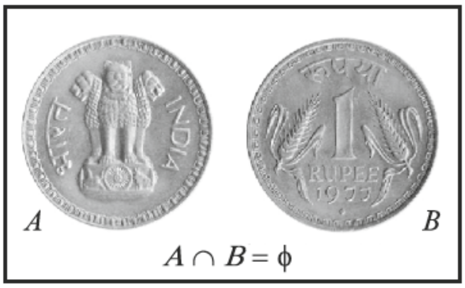
**The suits and types of cards are shown as follows :**

| | | | |
|---|---|---|---|
| Spade - $S$ | Diamond - $D$ | Club - $C$ | Heart - $H$ |
| Ace - $A$ | King - $K$ | Queen - $Q$ | Jack - $J$ |

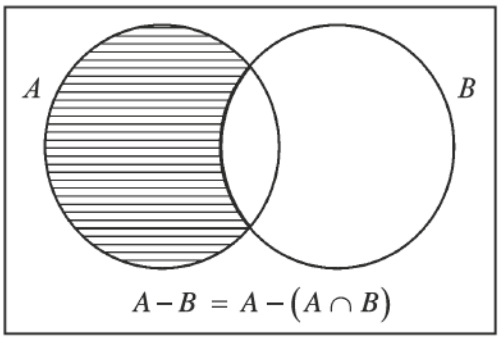
**(7) Mutually Exclusive Events :** Suppose $A$ and $B$ are any two events of a finite sample space $U$. Events $A$ and $B$ do not occur together which means $A \cap B = \phi$ or in other words, event $B$ does not occur when event $A$ occurs and event $A$ does not occur when event $B$ occurs then the events $A$ and $B$ are called mutually exclusive events.




For example, toss a balanced coin. Denote the outcome $H$ on the coin as event $A$ and the outcome $T$ on the coin as $B$. We get $A = \{H\}$ and $B = \{T\}$. It is clear that $A \cap B = \phi$ because when we get $H$ in a trial, it is not possible to get the outcome $T$ in the same trial and vice versa, when we get $T$ in a trial, it is not possible to get the outcome $H$ in the same trial in the random experiment of tossing a balanced coin. Thus, these two events cannot occur simultaneously.




**(8) Difference Event :** Suppose $A$ and $B$ are any two events of a finite sample space $U$. The set of elements or outcomes where event $A$ happens but event $B$ does not happen is called the difference of events $A$ and $B$. It is denoted by $A - B$. It is clear from the venn diagram given here that



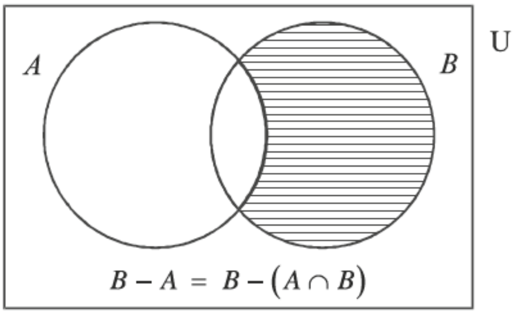

$$A - B = A \cap B' = A - (A \cap B) = (A \cup B) - B$$

$A - B$ = Difference of events $A$ and $B$

= Event $A$ happens but event $B$ does not happen

= Only $A$ happens out of events $A$ and $B$.

Similarly, for two events $A$ and $B$ of a finite sample space $U$, the set of elements or outcomes where $B$ happens but $A$ does not happen is called as the difference of event $B$ and event $A$. It is denoted by $B - A$. It is clear from the venn diagram given here that,



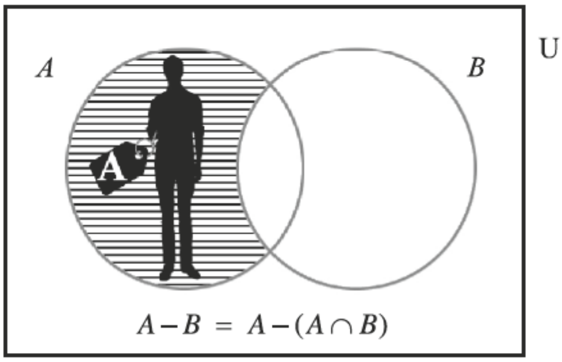

$$B - A \;=\; A' \cap B \;=\; B - (A \cap B) \;=\; (A \cup B) - A$$

$B - A$ = Difference of events $B$ and $A$

= Event $B$ happens but event $A$ does not happen

= Only event $B$ happens out of events $A$ and $B$

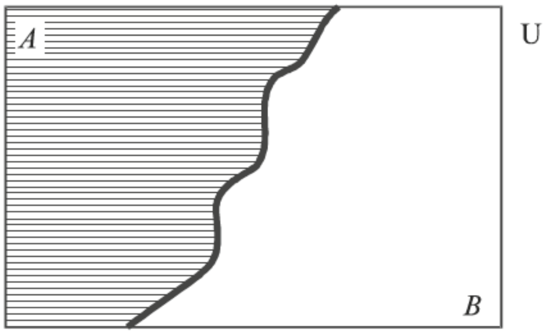
For example, two employees $A$ and $B$ among the employees working in an office are friends. Denote the presence of employee $A$ in the office as event $A$ and the presence of employee $B$ in the office as event $B$. On a ctertain day, if it is said that only the employee $A$ is present in the office out of employees $A$ and $B$ then it is clear that among two employees $A$ and $B$, employee $A$ is present but employee $B$ is not present. Thus, it is called the difference of two events $A - B$ for events $A$ and event B. Here,




$A - B$ = Only employee $A$ is present in the office among the employees $A$ and $B$

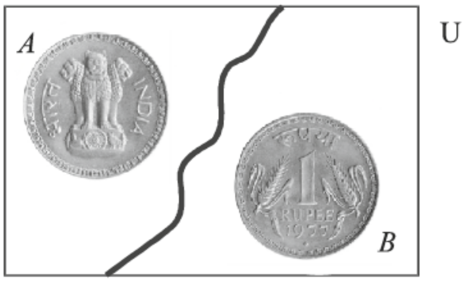
$B - A$ = Only employee $B$ is present in the office among the employees $A$ and B

**(9) Exhaustive Events :** If the group of favourable outcomes of events of random experiment is the sample space then the events are called exhaustive events. Suppose $A$ and $B$ are any two events of a sample space $U$. The events $A$ and $B$ are called the exhaustive events if the union $A \cup B$ of the two events $A$ and $B$ is the sample space $U$, that is $A \cup B = U$.




For example, denote the outcome $H$ as event $A$ and the outcome $T$ as event $B$ when a balanced coin is tossed. It is clear in this case that $A = \{H\}$, $B = \{T\}$ and $A \cup B = \{H, T\} = U$.

$\therefore$ $A$ and $B$ are exhaustive events.



**(10) Mutually Exclusive and Exhaustive Events :** Suppose $A$ and $B$ are two events of a finite sample space $U$. These two events $A$ and $B$ are called the mutually exclusive and exhaustive events if $A \cap B = \phi$ and $A \cup B = U$. It should be noted here that all the mutually exclusive events need not be exhaustive events and similarly, all the exhaustive events need not be the mutually exclusive events.

For example, consider the sample space $U = \{1, 2, 3, 4, 5, 6\}$ of the experiment of throwing a balanced die. Let the event $A$ = getting odd number on the die $= \{1, 3, 5\}$ and event $B$ = getting even number on the die $= \{2, 4, 6\}$. It is clear that $A \cap B = \phi$ and $A \cup B = U$. Thus, the events $A$ and $B$ are mutually exclusive and exhaustive.

**(11) Elementary Events :** The events formed by all the subsets of single elements of the sample space $U$ of a random experiment are called the elementary events. The elementary events are mutually exclusive and exhaustive.

For example, consider the sample space $U = \{H, T\}$ for the random experiment of tossing a balanced coin. The events $A = \{H\}$ and $B = \{T\}$ having single elements are the elementary events. Since $A \cap B = \phi$ and $A \cup B = U$ in this case, it can be said that the elementary events are mutually exclusive and exhaustive.

**Illustration 6 : There are 3 yellow and 2 pink flowers in a basket. One flower is randomly selected from this basket. Denote the selection of yellow flower as an event $A$ and the selection of pink flower as the event $B$. Find the sets representing the following events and answer the given questions.**

**(1)** $U$ **(2)** $A$ **(3)** $B$ **(4)** $A'$ **(5)** $B'$ **(6)** $A \cap B$ **(7)** $A \cup B$ **(8)** $A \cap B'$ **(9)** $A' \cap B$

**(10) State the elementary events of the sample space for this random experiment.**

**(11) Can it be said that the events $A$ and $B$ are mutually exclusive events ? Give reason.**

**(12) Can it be said that the events $A$ and $B$ are exhaustive events ? Give reason.**

We will denote the 3 yellow flowers in the basket as $Y_1, Y_2, Y_3$ and the 2 pink flowers as $P_1, P_2$. The sets representing the required events will be as follows :

(1) $U = \{Y_1, Y_2, Y_3, P_1, P_2\}$

(2) $A = \{Y_1, Y_2, Y_3\}$

(3) $B = \{P_1, P_2\}$

(4) $A' = U - A = \{Y_1, Y_2, Y_3, P_1, P_2\} - \{Y_1, Y_2, Y_3\}$

$= \{P_1, P_2\}$

(5) $B' = U - B = \{Y_1, Y_2, Y_3, P_1, P_2\} - \{P_1, P_2\}$

$= \{Y_1, Y_2, Y_3\}$

(6) $A \cap B = \{Y_1, Y_2, Y_3\} \cap \{P_1, P_2\}$

$= \phi$

(7) $A \cup B = \{Y_1, Y_2, Y_3\} \cup \{P_1, P_2\}$

$= \{Y_1, Y_2, Y_3, P_1, P_2\}$

(8) $A \cap B' = \{Y_1, Y_2, Y_3\} \cap \{Y_1, Y_2, Y_3\}$

$= \{Y_1, Y_2, Y_3\}$

**OR**

$A \cap B' = A - (A \cap B)$

$= \{Y_1, Y_2, Y_3\} - \phi$

$= \{Y_1, Y_2, Y_3\}$

(9) $A' \cap B = \{P_1, P_2\} \cap \{P_1, P_2\}$

$= \{P_1, P_2\}$

**OR**

$A' \cap B = B - (A \cap B)$

$= \{P_1, P_2\} - \phi$

$= \{P_1, P_2\}$

(10) The elementary events are the subsets with one element. If we denote the different elementary events as $E_1, E_2, E_3, \ldots$ then

$E_1 = \{Y_1\}, \ E_2 = \{Y_2\}, \ E_3 = \{Y_3\}, \ E_4 = \{P_1\}, \ E_5 = \{P_2\}$

(11) The events $A$ and $B$ can be called mutually exclusive events because according to the definition of mutually exclusive events, the events $A$ and $B$ are called the mutually exclusive events if $A \cap B = \phi$. It can be seen from the answer to the question 6 that $A \cap B = \phi$.

(12) The events $A$ and $B$ can be called exhaustive events because according to the definition of exhaustive events, the $A$ and $B$ are called the exhaustive events if $A \cup B = U$. It can be seen from the answer to the question 7 that $A \cup B = U$.

**Illustration 7 : The events $A$ and $B$ of a random experiment are as follows :**

$$A = \{1, 2, 3, 4\}, \quad B = \{-1, 0, 1\}$$

**If the sample space $U = A \cup B$ then find the sets showing the following events.**

**(1) $B'$ (2) $A' \cap B$ (3) $A - B$**

Here, $A = \{1, 2, 3, 4\}$

$B = \{-1, 0, 1\}$

$U = A \cup B = \{1, 2, 3, 4\} \cup \{-1, 0, 1\}$

$= \{-1, 0, 1, 2, 3, 4\}$

(1) $B' = U - B$

$= \{-1, 0, 1, 2, 3, 4\} - \{-1, 0, 1\}$

$= \{2, 3, 4\}$

(2) $A' \cap B = B - (A \cap B)$

First we find $A \cap B$,

$A \cap B = \{1, 2, 3, 4\} \cap \{-1, 0, 1\}$
$= \{1\}$

**Alternate Method :**

$A' = U - A = \{-1, 0, 1, 2, 3, 4\} - \{1, 2, 3, 4\} = \{-1, 0\}$

$\therefore \; A' \cap B = \{-1, 0\} \cap \{-1, 0, 1\} = \{-1, 0\}$

Now, $A' \cap B = B - (A \cap B)$

$= \{-1, 0, 1\} - \{1\}$

$= \{-1, 0\}$

(3) $A - B = \{1, 2, 3, 4\} - \{-1, 0, 1\}$
$= \{2, 3, 4\}$

**Illustration 8 : One number is randomly selected from the first 50 natural numbers. Find the sets showing the following events.**

**(1) The number selected is a multiple of 5 or 7.**

**(2) The number selected is a multiple of both 5 and 7.**

**(3) The number selected is a multiple of 5 but not a multiple of 7.**

**(4) The number selected is only a multiple of 7 out of 5 and 7.**

If one number is selected from the first 50 natural numbers then the group of all possible outcomes of this experiment, which is the sample space $U$, is as follows :

$U = \{1, 2, 3, ....., 50\}$

Event $A$ = Selected number is a multiple of 5

$= \{5, 10, 15, 20, 25, 30, 35, 40, 45, 50\}$

Event $B$ = Selected number is a multiple of 7

$= \{7, 14, 21, 28, 35, 42, 49\}$

Now, the required events are as follows :

(1) The event of selecting a number which is a multiple of 5 or 7 = $A \cup B$

$\therefore \; A \cup B = \{5, 7, 10, 14, 15, 20, 21, 25, 28, 30, 35, 40, 42, 45, 49, 50\}$

(2) The event of selecting a number which is a multiple of both 5 and 7 = $A \cap B$

$\therefore \; A \cap B = \{35\}$

(3) The event of selecting a number which is a multiple of 5 but not of 7 $= A \cap B'$

$$\therefore \; A \cap B' = A - (A \cap B)$$
$$= \{5, 10, 15, 20, 25, 30, 35, 40, 45, 50\} - \{35\}$$
$$= \{5, 10, 15, 20, 25, 30, 40, 45, 50\}$$

(4) The event of selecting a number which is only a multiple of 7 out of 5 and 7 $= A' \cap B$

$$\therefore \; A' \cap B = B - (A \cap B)$$
$$= \{7, 14, 21, 28, 35, 42, 49\} - \{35\}$$
$$= \{7, 14, 21, 28, 42, 49\}$$

**Illustration 9 : The events $A_1$ and $A_2$ of a random experiment are defined as follows. Find the sets showing union event $A_1 \cup A_2$ and intersection event $A_1 \cap A_2$.**

$A_1 = \{x \mid x = -1, 0, 1\}, \quad A_2 = \{x \mid x = 1, 2, 3\}$

It is given that $A_1 = \{-1, 1, 0\}$ and $A_2 = \{1, 2, 3\}$.

Union of events $A_1 \cup A_2 = \{-1, 0, 1, 2, 3\}$

Intersection of events $A_1 \cap A_2 = \{1\}$

**Illustration 10 : A factory produces screws of different lengths. The length (in cm) of screw is denoted by $x$. The events $A_1$ and $A_2$ are defined as follows in the experiment of finding the length of selected screws. Find the events showing union event $A_1 \cup A_2$ and intersection event $A_1 \cap A_2$.**

$A_1 = \{x \mid 0 < x < 1\}, \quad A_2 = \{x \mid \frac{1}{2} \le x < 2\}$

If is given that $A_1 = \{x \mid 0 < x < 1\}$ and $A_2 = \{x \mid \frac{1}{2} \le x < 2\}$.

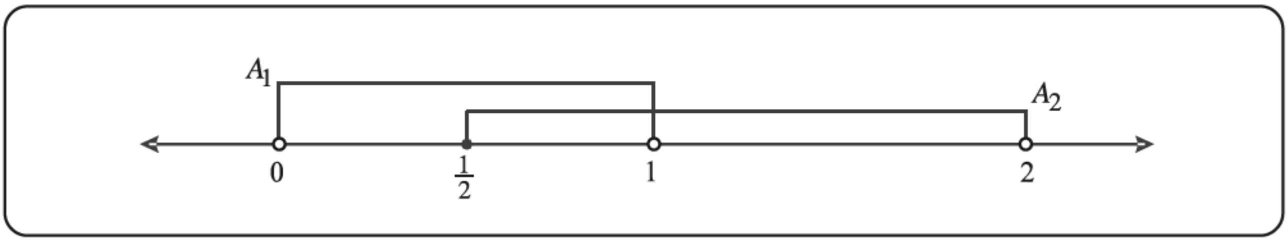
Union of events $A_1 \cup A_2 = \{x \mid 0 < x < 2\}$

$= (0, 2)$ (interval form)

Intersection of events $A_1 \cap A_2 = \{x \mid \frac{1}{2} \le x < 1\}$

$= \left[\frac{1}{2}, 1\right)$ (interval form)

See the following diagram carefully for better explanation of $A_1 \cup A_2$ and $A_1 \cap A_2$.



## Exercise 1.1

**1.** State the sample space for the following random experiments :

(1) A balanced coin is thrown three times.

(2) A balanced die with six sides and a balanced coin are tossed together.

(3) Two persons are to be selected from five persons *a, b, c, d, e*.

**2.** Write the sample space for the marks (in integers) scored by a student appearing for an examination of 100 marks and state the number of sample points in it.

**3.** Write the sample space for randomly selecting one minister and one deputy minister from four persons.

**4.** A balanced coin in thrown in a random experiment till the first head is obtained. The experiment is terminated with a trial of first head. Write the sample space of this experiment and state whether it is finite or infinite.

**5.** Write the sample space for the experiment of randomly selecting three numbers from the first five natural numbers.

**6.** The sample space of a random experiment of selecting a number is $U = \{1, 2, 3, ....., 20\}$. Write the sets showing the following events :

(1) The selected number is odd number

(2) The selected number is divisible by 3

(3) The selected number is divisible by 2 or 3.

**7.** One family is selected from the families having two children. The sex (male or female) of the children from this family is noted. State the sample space of this experiment and write the sets showing the following events :

(1) Event $A_1$ = One child is a female

(2) Event $A_2$ = At least one child is a female.

**8.** Two six faced balanced dice are thrown simultaneously. State the sample space of this random experiment and hence write the sets showing the following events :

(1) Event $A_1$ = The sum of numbers on the dice is 7

(2) Event $A_2$ = The sum of numbers on the dice is less than 4

(3) Event $A_3$ = The sum of numbers on the dice is divisible by 3

(4) Event $A_4$ = The sum of numbers on the dice is more than 12.

**9.** Two numbers are selected at random from the first five natural numbers. The sum of two selected numbers is at least 6 is denoted by event $A$ and the sum of two selected numbers is even is denoted by event $B$. Write the sets showing the following events and answer the given questions :

(1) $U$ (2) $A$ (3) $B$ (4) $A \cup B$ (5) $A \cap B$ (6) $A'$ (7) $A - B$ (8) $A' \cap B$

(9) Can it be said that the events $A$ and $B$ are mutually exclusive ? Give reason.

(10) State the number of sample points in the sample space of this random experiment.

**10.** Three female employees and two male employees are working in an office. One employee is selected from the employees of this office for training. The event that the employee selected for the training is a female is denoted by $A$ and the event that this employee is a male is denoted by $B$. Find the sets showing the following events and answer the given questions :

(1) $U$ (2) $A$ (3) $B$ (4) $A \cup B$ (5) $A \cap B$ (6) $A' \cap B$

(7) Can it be said that the events $A$ and $B$ are mutually exclusive ? Give reason.

(8) Can it be said that the events $A$ and $B$ are exhaustive ? Give reason.

**11.** One card is randomly drawn from a pack of 52 cards. If drawing a spade card is denoted by event $A$ and drawing a card from ace to ten (non-face card) is denoted by $B$ then write the sets showing the following events :

(1) $U$ (2) $A$ (3) $B$ (4) $A \cup B$ (5) $A \cap B$ (6) $B'$

**12.** The events $A_1$ and $A_2$ of a random experiment are as follows. Find the sets showing the union event $A_1 \cup A_2$ and intersection event $A_1 \cap A_2$.

$$A_1 = \{x \mid 0 < x < 5\}, \qquad A_2 = \{x \mid -1 < x < 3, x \text{ is an integer}\}$$

**13.** The events $A_1$ and $A_2$ of a random experiment are as follows. Find the sets showing the union event $A_1 \cup A_2$ and intersection event $A_1 \cap A_2$.

$$A_1 = \{x \mid 2 \le x < 6,\ x \in N\}, \qquad A_2 = \{x \mid 3 < x < 9,\ x \in N\}$$

**14.** The sample space $U$ of a random experiment and its event $A$ are defined as follows. Find the complementary event $A'$ of $A$.

$$U = \{x \mid x = 0, 1, 2, \ldots\ldots, 10\}, \qquad A = \{x \mid x = 2, 4, 6\}$$

**15.** The sample space $U$ of a random experiment and its event $A$ are defined as follows. Find the complementary event $A'$ of $A$.

$$U = \{x \mid 0 < x < 1\}, \qquad A = \{x \mid \tfrac{1}{2} \le x < 1\}$$

*

After getting aquainted with the random experiment, sample space and different events, we shall now study the probability. We shall begin with the definition of probability.

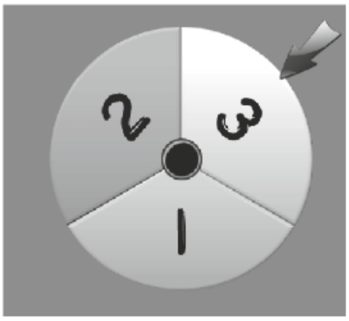
## 1.4 Mathematical Definition of Probability

To understand the mathematical definition of probability, we shall first understand the two important terms namely equiprobable events and favourable outcomes.

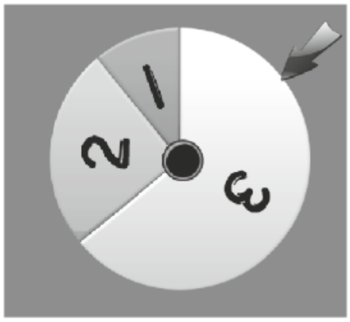
**Equiprobable Events :** If there is no apparent reason to believe that out of one or more events of a random experiment, any one event is more or less likely to occur than the other events then the events are called as equiprobable events.

For example, a manufacturer of a certain item has two machines $M_1$ and $M_2$ in his factory for the production of items. Both the machines produce the same number of items during a day. The lots are made of the produced goods by properly mixing the items produced on both the machines during the day. An item randomly selected from such a lot is made on machine $M_1$ or machine $M_2$ are the elementary events which are equiprobable.

Similarly, the wheels $A$ and $B$ marked with numbers 1, 2, 3 as shown in the following pictures are rotated by hand. The number against the pointer is noted down when the wheels stop rotating after some time. It is clear from the picture that all the three numbers on wheel $A$ will come against the pointer are equiprobable events. But the numbers 1, 2 and 3 coming against the pointer for wheel $B$ are not equiprobable events.




**Wheel *A***




**Wheel *B***

**Favourable Outcomes :** If some outcomes out of all the elementary outcomes in the sample space of random experiment indicate the occurrence of a certain event $A$ then these outcomes are called the favourable outcomes of the event $A$. For example, a card is drawn from a pack of 52 cards. If event $A$ denotes that the card drawn is a face card then the set of favourable outcomes is as follows :

$$A = \left\{S_K, D_K, C_K, H_K, S_Q, D_Q, C_Q, H_Q, S_J, D_J, C_J, H_J\right\}$$

Thus, 12 outcomes are favourable for event $A$.

**Mathematical Definition of probability :** Suppose there are total $n$ outcomes in the finite sample space of a random experiment which are mutually exclusive, exhaustive and equiprobable. If $m$ outcomes among them are favourable for an event $A$ then the probability of the event $A$ is $\frac{m}{n}$. The probability of event $A$ is denoted by $P(A)$.

$$P(A) = \text{Probability of event } A$$

$$= \frac{\text{Favourable outcomes of event } A}{\text{Total number of mutually exclusive, exhaustive and equi-probable outcomes of sample space}}$$

$$= \frac{m}{n}$$

Both the numbers $m(\geq 0)$ and $n(> 0)$ are integers and $m \leq n$. It should be noted here that $n$ can not be zero and infinity. The mathematical definition of probability is also called the classical definition.

The assumptions of the mathematical definition are as follows :

(1) The number of outcomes in the sample space of the random experiment is finite.

(2) The number of outcomes in the sample space of the random experiment is known.

(3) The outcomes in the sample space of the random experiment are equi-probable.

We will accept some of the following important results about probability without proof :

(1) The range for the value of probability $P(A)$ for any event $A$ in the sample space $U$ is 0 to 1. Thus, $0 \leq P(A) \leq 1$.

(2) The probability of an impossible event is zero. Earlier we have denoted an impossible event by $\phi$. Hence, $P(\phi) = 0$.

(3) The probability of certain event is always 1. Earlier we have denoted a certain event by $U$. Hence, $P(U) = 1$.

(4) The probability of complementary event $A'$ of event $A$ in the sample space $U$ is $P(A') = 1 - P(A)$.

(5) If $A \subset B$ for two events $A$ and $B$ in the sample space of a random experiment then

- $P(A) \leq P(B)$
- $P(B - A) = P(B) - P(A)$

(6) For two events $A$ and $B$ in the sample space of a random experiment,

- $P(A \cap B) \leq P(A)$ $\quad [\because A \cap B \subset A]$
- $P(A \cap B) \leq P(B)$ $\quad [\because A \cap B \subset B]$
- $P(A) \leq P(A \cup B)$ $\quad [\because A \subset A \cup B]$
- $P(B) \leq P(A \cup B)$ $\quad [\because B \subset A \cup B]$
- $P(A' \cap B') = P(A \cup B)' = 1 - P(A \cup B)$
- $P(A' \cup B') = P(A \cap B)' = 1 - P(A \cap B)$
- $P(A - B) = P(A \cap B') = P(A) - P(A \cap B)$
- $P(B - A) = P(A' \cap B) = P(B) - P(A \cap B)$
- $0 \leq P(A \cap B) \leq P(A) \leq P(A \cup B) \leq P(A) + P(B)$

We shall now consider illustrations of finding probability of different events using the mathematical definition.

**Illustration 11 : If two balanced coins are tossed, then find the probability of (1) getting one head and one tail and (2) getting at least one head.**

The sample space for the random experiment of tossing two balanced coins is as follows :

$U = \{HH, HT, TH, TT\}$

$\therefore$ No. of mutually exclusive, exhaustive and equi-probable outcomes $n = 4$.

(1) If $A$ denotes the event of getting one head $H$ and one tail $T$ then $HT$ and $TH$ are two favourable outcomes of event $A$. Thus, $m = 2$.

From the mathematical definition of probability,

$$P(A) = \frac{m}{n}$$

$$= \frac{2}{4}$$

$$= \frac{1}{2}$$

Required probability $= \frac{1}{2}$

(2) If $B$ denotes the event of getting at least one head then $HT, TH, HH$ are the favourable outcomes of event $B$. Hence, the number of favourable outcomes $m = 3$ for even $B$.

From the mathematical definition of probability,

$$P(B) = \frac{m}{n}$$

$$= \frac{3}{4}$$

Required probability $= \frac{3}{4}$

**Illustration 12 : Two balanced dice marked with numbers 1 to 6 are thrown simultaneously. Find the probability that (1) sum of numbers on both the dice is 7 (2) sum of numbers on both the dice is more than 10 (3) sum of number on both the dice is at the most 4 (4) both the dice show same numbers (5) sum of numbers on both the dice is 1 (6) sum of numbers on both the dice is 12 or less.**

The sample space for throwing two balanced dice simultaneously is as follows :

$U = \{(i,\ j)\ ;\ i,\ \ i, j = 1, 2, 3, 4, 5, 6\}$

$\therefore$ Total number of outcomes $n = 36$.

(1) If $A_1$ denotes that sum of the numbers on the dice is 7 then there are total 6 outcomes $(1, 6)$, $(2, 5)$, $(3, 4)$, $(4, 3)$, $(5, 2)$, $(6, 1)$ favourable for this event $A_1$. Thus, the number of favourable outcomes $m = 6$ for event $A_1$. Probability of event $A_1$

$P(A_1) = \frac{m}{n}$

$= \frac{6}{36}$

$= \frac{1}{6}$

$\therefore$ Required probability $= \frac{1}{6}$

(2) If $A_2$ denotes the event that the sum of numbers on two dice is more than 10 then $(5, 6)$, $(6, 5)$, $(6, 6)$ are the favourable outcomes of event $A_2$. Thus, the number of favourable outcomes $m = 3$ for even $A_2$. Probability of $A_2$

$P(A_2) = \frac{m}{n}$

$= \frac{3}{36}$

$= \frac{1}{12}$

Required probability $= \frac{1}{12}$

(3) If $A_3$ denotes the event that the sum of numbers on two dice is at the most 4 then total 6 outcomes $(1, 1)$, $(1, 2)$, $(1, 3)$, $(2, 1)$, $(2, 2)$, $(3, 1)$ are favourable outcomes of event $B$. Thus, the number of favourable outcomes $m = 6$ for event $A_3$. Probability of event $A_3$

$P(A_3) = \frac{m}{n}$

$= \frac{6}{36}$

$= \frac{1}{6}$

Required probability $= \frac{1}{6}$

(4) Event $A_4$ = both the dice show the same numbers.

$\therefore$ Total 6 outcomes $(1, 1)$, $(2, 2)$, $(3, 3)$, $(4, 4)$, $(5, 5)$, $(6, 6)$ are favourable for the event $A_4$. Thus, the number of favourable outcomes $m = 6$ for event $A_4$. Probability of event $A_4$

$P(A_4) = \frac{m}{n}$

$= \frac{6}{36}$

$= \frac{1}{6}$

Required probability $= \frac{1}{6}$

(5) Let $A_5$ be the event that the sum of numbers on two dice is 1. It is obvious that not a single outcome in the sample space is favourable for $A_5$. Hence, the number of favourable outcomes $m = 0$ for event $A_5$. Probability of event $A_5$

$$P(A_5) = \frac{m}{n}$$

$$= \frac{0}{36}$$

$$= 0$$

Required probability $= 0$

(The probability of impossible event is always 0.)

(6) Let $A_6$ be the event that the sum of numbers on two dice is 12 or less. It is obvious that all the outcomes in the sample space are favourable for event $A_6$. Hence, the number of favourable outcomes $m = 36$ for the event $A_6$. Probability of event $A_6$

$$P(A_6) = \frac{m}{n}$$

$$= \frac{36}{36}$$

$$= 1$$

Required probability $= 1$

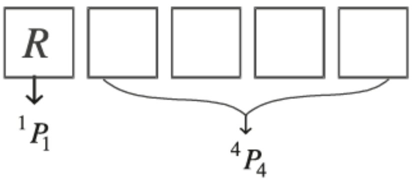
(The probability of certain event is always 1.)

**Illustration 13 : Find the probability of getting *R* in the first place in all possible arrangements of each and every letter of the word *RUTVA*.**

There are 5 letters *R, U, T, V, A* in the word *RUTVA*. These five letters can be arranged in ${}^5P_5 = 5! = 120$ different ways. Thus, total number of outcomes $n = 120$.

Event of getting *R* in the first place of the arrangement $= A$.

The favourable outcomes of event *A* are obtained as follows :




*R* can be arranged in the first place in ${}^1P_1$ ways and the remaining four letters *U, T, V, A* in the rest of the four places can be arranged in ${}^4P_4$ ways. According to the fundamental principle of multiplication, there will be ${}^1P_1 \times {}^4P_4$ arrangements of getting *R* in the first place. Hence, the number of favourable outcomes for event *A* will be

$$m = {}^1P_1 \times {}^4P_4 = 1! \times 4! = 1 \times 24 = 24.$$

Probability of event $A$ $\quad P(A) = \frac{m}{n}$

$$= \frac{24}{120}$$

$$= \frac{1}{5}$$

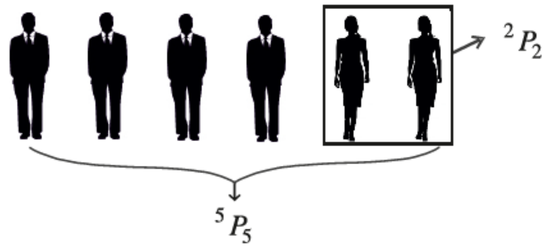
Required probability $= \frac{1}{5}$

**Illustration 14 : Four male employees and two female employees working in a government department are sent one by one in turns to the training centre for training. Find the probability that the two female employees go successively for the training.**

Total 6 persons, 4 males and 2 females can be sent for training at the training centre one by one in ${}^6P_6 = 6! = 720$ ways. Thus, total number of outcomes will be $n = 720$.

Event of two female employees go successively for training = $A$.

The favourable outcomes of event $A$ can be obtained as follows :




Considering the two female employees going successively for the training as one person, total 5 persons can be arranged in ${}^5P_5$ ways and two female employees can be arranged among themselves in ${}^2P_2$ ways in each of these arrangements.

Thus, the number favourable outcomes of event $A$ is $m = {}^5P_5 \times {}^2P_2$

$= 5! \times 2!$

$= 120 \times 2$

$= 240$

Probability of event $A$ $P(A) = \frac{m}{n}$

$= \frac{240}{720}$

$= \frac{1}{3}$

Required probability $= \frac{1}{3}$

**Illustration 15 : Find the probability of having 53 Thursdays in a leap year.**

There are 366 days in a leap year where we have 52 complete weeks (52×7=364 days) and 2 additional days. Each day appears once in each week and thus each day will appear 52 times in 52 weeks. Now, the additional 2 days can be as follows which gives the sample space for this experiment.

$U$ = {Sunday-Monday, Monday-Tuesday, Tuesday-Wednesday, Wednesday-Thursday, Thursday-Friday, Friday-Saturday, Saturday-Sunday}

Thus, total number of outcomes will be $n = 7$.

Event $A$ = leap year has 53 Thursdays.

Wednesday-Thursday and Thursday-Friday are the 2 favourable outcomes of event $A$ from the above 7 outcomes. Thus, $m = 2$.

Probability of event $A$ $\quad P(A) = \frac{m}{n}$

$= \frac{2}{7}$

Required probability $= \frac{2}{7}$

**Illustration 16 : There are 2 officers, 3 clerks and 2 peons among the 7 employees working in the cash department of a bank. A committee is formed by randomly selecting two employees from the employees of this department. Find the probability that there are**

**(1) two peons**

**(2) two clerks**

**(3) One officer and one clerk among the two employees selected in the committee.**

There are 7 employees working in the cash department of the bank. If the employees are randomly selected from them then the total number of mutually exclusive, exhaustive and equi-probable outcomes will be $n = {}^7C_2 = \frac{7 \times 6}{2 \times 1} = 21$.

(1) Event of selecting two peons $= A$

Selecting 2 peons from the 2 peons and not selecting any employee from the remaining 5 employees will be the favourable outcomes of event $A$.

The number of such outcomes will be $m = {}^2C_2 \times {}^5C_0 = 1 \times 1 = 1$.

Probability of event $A$ $\quad P(A) = \frac{m}{n}$

$= \frac{1}{21}$

Required probability $= \frac{1}{21}$

(2) Event of selecting two clerks $= B$

Selecting 2 clerks from the 3 clerks and not selecting any employee from the remaining four employees will be the favourable outcomes of event $B$.

The number of such outcomes will be $m = {}^3C_2 \times {}^4C_0 = 3 \times 1 = 3$.

Probability of event $B$ $\quad P(B) = \frac{m}{n}$

$= \frac{3}{21}$

$= \frac{1}{7}$

Required probability $= \frac{1}{7}$

(3) Event of selecting one officer and one clerk = $C$

Selecting 1 officer from 2 officers, one clerk from three clerks and not selecting any peon from two peons will be the favourable outcomes of event $C$.

The number of such outcomes will be $m = {}^2C_1 \times {}^3C_1 \times {}^2C_0 = 2 \times 3 \times 1 = 6$.

Probability of event $C$ $P(C) = \frac{m}{n}$

$= \frac{6}{21}$

$= \frac{2}{7}$

Required probability $= \frac{2}{7}$

**Illustration 17 : A box contains 20 items and 10% of them are defective. Three items are randomly selected from this box. Find the probability that,**

**(1) two items are defective**

**(2) two items are non-defective**

**(3) all three items are non-defective among the three selected items.**

There are 20 items wherein 10% that is 20 × 10 % = 2 items are defective and the rest 18 are non-defective. 3 items are selected from this box of 20 items at random. Hence, the total number of outcomes in the sample space will be $n = {}^{20}C_3 = \frac{20 \times 19 \times 18}{3 \times 2} = 1140$.

(1) Event of getting two defective items among three selected items = $A$

Selecting 2 items from 2 defective items and selecting 1 item from the 18 non-defective items will be the favourable outcomes for the event $A$.

The number of such outcomes $m = {}^2C_2 \times {}^{18}C_1 = 1 \times 18 = 18$.

Probability of event $A$ $P(A) = \frac{m}{n}$

$= \frac{18}{1140}$

$= \frac{3}{190}$

Required probability $= \frac{3}{190}$

(2) Event of getting two non-defective items among three selected items = $B$

Selecting 2 items from 18 non-defective items and selecting one item from 2 defective items will be the favourable outcomes of the event $B$.

The number of such outcomes $m = {}^{18}C_2 \times {}^2C_1 = 153 \times 2 = 306$.

Probability of event $B$ $\quad P(B) = \frac{m}{n}$

$$= \frac{306}{1140}$$

$$= \frac{51}{190}$$

Required probability $= \frac{51}{190}$

(3) Event of getting all three non-defective items $= C$

Selecting 3 items from 18 non-defective items and not selecting any item from the defective items will be the favourable outcomes of event $C$.

The number of such outcomes $m = {}^{18}C_3 \times {}^{2}C_0 = 816 \times 1 = 816$.

Probability of event $C$ $\quad P(C) = \frac{m}{n}$

$$= \frac{816}{1140}$$

$$= \frac{68}{95}$$

Required probability $= \frac{68}{95}$

**Illustration 18 : A box contains 10 chits of which 3 chits are eligible for a prize. A boy named Kathan randomly selects two chits from this box. Find the probability that Kathan gets the prize.**

There are 10 chits of which 3 chits are eligible for a prize and 7 chits are not eligible for prize. If two chits are randomly selected from these 10 chits then the number of mutually exclusive, exhaustive and equiprobable outcomes in the sample space will be $n = {}^{10}C_2 = \frac{10 \times 9}{2} = 45$.

Event of Kathan getting prize $= A$

$\therefore$ Event that Kathan does not get prize $= A'$

The outcomes in which Kathan will draw 2 chits at random from the 7 chits which are not eligible for prize will be the favourable outcomes of the event $A'$.

The number of such outcomes $m = {}^{7}C_2 = 21$.

Probability of $A'$ $\quad P(A') = \frac{m}{n}$

$$= \frac{21}{45}$$

$$= \frac{7}{15}$$

Now $\quad P(A) = 1 - P(A')$

$$= 1 - \frac{7}{15}$$

$$= \frac{8}{15}$$

Thus, probability that Kathan gets prize $= \frac{8}{15}$

**Limitations :** The limitations of the mathematical definition of probability are as follows :

(1) The probability of an event cannot be found by this definition if there are infinite outcomes in the sample space of a random experiment.

(2) The probability of an event cannot be found by this definition if the total number of outcomes in the sample space of a random experiment are not known.

(3) The probability of an event cannot be found by this definition if the elementary outcomes in the sample space of a random experiment are not equi-probable.

(4) The word 'equi-probable' is mentioned in the mathematical definition of probability. Equi-probable events are the events with same probability. Thus, the word probability is used in the definition of probability.

## Exercise 1.2

**1.** A balanced coin is tossed three times. Find the probability of the following events :

(1) Getting all three heads
(2) Not getting a single head
(3) Getting at least one head
(4) Getting more than one head
(5) Getting at the most one head
(6) Getting less than two heads
(7) Getting head and tail alternately
(8) Getting more number of heads than tails

**2.** Two balanced dice are thrown simultaneously. Find the probability of the following events :

(1) The sum of numbers on the dice in 6
(2) The sum of numbers on the dice is not more than 10
(3) The sum of numbers on the dice is a multiple of 3
(4) The product of numbers on the dice is 12

**3.** One family is randomly selected from the families having two children. Find the probability that

(1) One child is a girl and one child is a boy.
(2) At least one child is a girl among the two children of the selected family.

(**Note :** Assume that the chance of the child being a boy or girl is same.)

**4.** One number is selected at random from the first 100 natural numbers. Find the probability that this number is divisible by 7.

**5.** The sample space for a random experiment of selecting numbers is $U = \{1, 2, 3, ....., 120\}$ and all the outcomes in the sample space are equiprobable. Find the probability that the number selected is

(1) a multiple of 3
(2) not a multiple of 3
(3) a multiple of 4
(4) not a multiple of 4
(5) a multiple of both 3 and 4.

**6.** Find the probability of getting $R$ in the first place and $M$ in the last place when all the letters of the word *RANDOM* are arranged in all possible ways.

**7.** Find the probability of getting vowels in the first, third and sixth place when all the letters of the word *ORANGE* are arranged in all possible ways.

**8.** Five members of a family, husband, wife and three children, are randomly arranged in a row for a family photograph. Find the probability that the husband and wife are seated next to each other.

**9.** Seven speakers $A$, $B$, $C$, $D$, $E$, $F$, $G$ are invited in a programme to deliver speech in random order. Find the probability that speaker $B$ delivers speech immediately after speeker $A$.

**10.** Find the probability of having 5 Mondays in the month of February of a leap year.

**11.** Find the probability of having 53 Fridays in a year which is not a leap year.

**12.** Find the probability of having 5 Tuesdays in the month of August of any year.

**13.** 4 couples (husband-wife) attend a party. Two persons are randomly selected from these 8 persons. Find the probability that the selected persons are,

(1) husband and wife (2) one man and one woman

(3) one man and one woman who are not husband and wife.

**14.** 8 workers are employed in a factory and 3 of them are excellent in efficiency where as the rest of them are moderate in efficiency. 2 workers are randomly selected from these 8 workers. Find the probability that,

(1) both the workers have excellent efficiency

(2) both the workers have moderate efficiency

(3) one worker is excellent and one worker is moderate in efficiency.

**15.** Two cards are drawn from a well shuffled pack of 52 cards. Find the probability that,

(1) both the cards are of different colour (2) both the cards are face cards

(3) one of the two cards is a king.

**16.** 3 bulbs are defective in a box of 10 bulbs. 2 bulbs are randomly selected from this box. These bulbs are fixed in two bulb-holders installed in a room. Find the probability that the room will be lighted after starting the electric supply.

**17.** For two events $A$ and $B$ in the sample space of a random experiment, $P(A) = 0.6$, $P(B) = 0.5$ and $P(A \cap B) = 0.15$. Find

(1) $P(A')$ (2) $P(B - A)$ (3) $P(A \cap B')$ (4) $P(A' \cup B')$

**18.** For two events $A$ and $B$ in the sample space of a random experiment, $P(A') = 2P(B') = 3P(A \cap B) = 0.6$. Find the probability of difference events $A - B$ and $B - A$.

*

## 1.5 Law of Addition of Probability

The rule of obtaining the probability of the occurrence of at least one of the event $A$ and $B$ in the sample space of a random experiment is called the law of addition of probability. We have seen earlier that the occurrence of at least one of the events $A$ and $B$ is denoted by $A \cup B$, the union of events $A$ and $B$. Hence we can say that the law of addition of probability is the rule of obtaining the probability of $A \cup B$, the union of events $A$ and $B$. This rule is stated as follows and we will accept it without proof :

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

The law of addition of probability can also be used for obtaining the probability of union of more than two events. The law of addition of probability for $A \cup B \cup C$, the union of three events $A$, $B$ and $C$ is as follows :

$$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(A \cap C) - P(B \cap C) + P(A \cap B \cap C)$$

Some of the important results obtained from this rule are as follows :

(1) If the events $A$ and $B$ in the sample space of a random experiment are mutually exclusive then $A \cap B = \phi$ and $P(A \cap B) = 0$. Hence,

$$P(A \cup B) = P(A) + P(B)$$

(2) If three events $A$, $B$ and $C$ in the sample space of a random experiment are mutually exclusive then,

$A \cap B = \phi$, $A \cap C = \phi$, $B \cap C = \phi$, $A \cap B \cap C = \phi$ and

$P(A \cap B) = P(A \cap C) = P(B \cap C) = P(A \cap B \cap C) = 0$. Hence,

$$P(A \cup B \cup C) = P(A) + P(B) + P(C)$$

(3) If two events $A$ and $B$ in the sample space of a random experiment are mutually exclusive and exhaustive then $A \cap B = \phi$ and $A \cup B = U$. As $P(\phi) = 0$ and $P(U) = 1$, $P(A \cap B) = 0$ and $P(A \cup B) = 1$.

$$P(A \cup B) = P(A) + P(B) = 1$$

(4) If three events $A$, $B$ and $C$ in the sample space of a random experiment are mutually exclusive and exhaustive then,

$$P(A \cup B \cup C) = P(A) + P(B) + P(C) = 1$$

**Illustration 19 : A number is randomly selected from the first 50 natural numbers. Find the probability that it is a multiple of 2 or 3.**

If one number is randomly selected from the first 50 natural numbers then the number of mutually exclusive, exhaustive and equi-probable outcomes in the sample space of this random experiment will be $n = {}^{50}C_1 = 50$.

If event $A$ denotes that the number selected is a multiple of 2 and event $B$ denotes that the number selected is a multiple of 3 then the event that the selected number is a multiple of 2 or 3 will be denoted by $A \cup B$. (This event can also be denoted as $B \cup A$. According to set theory, $A \cup B = B \cup A$). To find the probability of $A \cup B$, the union of events $A$ and $B$ by the law of addition of probability, we will first find $P(A), P(B)$ and $P(A \cap B)$.

$A$ = Event that the selected number is a multiple of 2

= {2, 4, 6,..., 50}

Hence, the number of favourable outcomes of event $A$ will be $m$ = 25.

Probability of event $A$ $P(A) = \frac{m}{n}$

$= \frac{25}{50}$

$B$ = Event that the selected number is a multiple of 3

= {3, 6, 9,..., 48}

Hence, the number of favourable outcomes of event $B$ will be $m$ = 16.

Probability of event $B$ $P(B) = \frac{m}{n}$

$= \frac{16}{50}$

$A \cap B =$ Event that the selected number is a multiple of 2 and 3 that is multiple the LCM of 2 and 3 which is 6.

= {6, 12, 18,..., 48}

Hence, the number of favourable outcomes of event $A \cap B$ will be $m$ = 8.

Probability of event $A \cap B$ $P(A \cap B) = \frac{m}{n}$

$= \frac{8}{50}$

From the law of addition of probability,

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$= \frac{25}{50} + \frac{16}{50} - \frac{8}{50}$$

$$= \frac{25 + 16 - 8}{50}$$

$$= \frac{33}{50}$$

Required probability $= \frac{33}{50}$

**Illustration 20 : One card is randomly selected from a pack of 52 cards. Find the probability that the selected card is**

**(1) club or queen card**

**(2) neither a club nor a queen card.**

If one card is randomly selected from a pack of 52 cards then the number of mutually exclusive, exhaustive and equi-probable outcomes in the sample space of this random experiment $n = {}^{52}C_1 = 52$.

Event that the selected card is a club card = $A$

Event that the selected card is a queen = $B$

(1) Event that the selected card is club or queen card = $A \cup B$

To find the probability of event $A \cup B$ by the law of addition of probability, we will first find $P(A), P(B)$ and $P(A \cap B)$.

$A$ = Event that the selected card is club card.

There are 13 club cards in a pack of 52 cards. Thus, the number of favourable outcomes of event $A$ is $m = 13$.

Probability of event $A$ $\quad P(A) = \frac{m}{n}$

$$= \frac{13}{52}$$

$B =$ Event that the selected card is a queen card.

There are 4 queen cards in a pack of 52 cards. Thus, the number of favourable outcomes of event $B$ is $m = 4$.

Probability of event $B$ $\quad P(B) = \frac{m}{n}$

$$= \frac{4}{52}$$

$A \cap B =$ Event that the selected card is club and queen card that is a club queen.

There is only 1 card in the pack of 52 cards which is club queen. Hence, the number of favourable outcomes of $A \cap B$ is $m = 1$.

Probability of $A \cap B$ $\quad P(A \cap B) = \frac{m}{n}$

$$= \frac{1}{52}$$

From the law of addition of probability,

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$= \frac{13}{52} + \frac{4}{52} - \frac{1}{52}$$

$$= \frac{13 + 4 - 1}{52}$$

$$= \frac{16}{52}$$

$$= \frac{4}{13}$$

Required probability $= \frac{4}{13}$

Event $A \cup B$ can be easily explained by the following diagram :

| **Suit** | **Type of Card** | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | **A** | **2** | **3** | **4** | **5** | **6** | **7** | **8** | **9** | **10** | **J** | **Q** | **K** |
| ♠ | ♠A | ♠2 | ♠3 | ♠4 | ♠5 | ♠6 | ♠7 | ♠8 | ♠9 | ♠10 | ♠J | ♠Q | ♠K |
| ♦ | ♦A | ♦2 | ♦3 | ♦4 | ♦5 | ♦6 | ♦7 | ♦8 | ♦9 | ♦10 | ♦J | ♦Q | ♦K |
| ♣ | ♣A | ♣2 | ♣3 | ♣4 | ♣5 | ♣6 | ♣7 | ♣8 | ♣9 | ♣10 | ♣J | ♣Q | ♣K |
| ♥ | ♥A | ♥2 | ♥3 | ♥4 | ♥5 | ♥6 | ♥7 | ♥8 | ♥9 | ♥10 | ♥J | ♥Q | ♥K |

(2) Event that the selected card is not of club = $A'$

Event that the selected card is not queen = $B'$

Hence, the event that the selected card is neither club nor queen is $A' \cap B'$

Thus, the probability of $A' \cap B'$

$$P(A' \cap B') = P(A \cup B)'$$

$$= 1 - P(A \cup B)$$

$$= 1 - \frac{4}{13}$$

$$= \frac{9}{13}$$

Required probability $= \frac{9}{13}$

**Illustration 21 : 3 persons from medical profession and 5 persons from engineering profession offer services at a social organization. 2 persons are randomly selected from these persons with the purpose of forming a committee. Find the probability that both the persons selected belong to the same profession.**

There are in all $3 + 5 = 8$ persons. Hence, 2 persons can be selected in $^8C_2 = \frac{8 \times 7}{2 \times 1} = 28$ ways.

Thus, the total number of mutually exclusive, exhaustive and equi-probable outcomes in the sample space is $n = 28$.

Event that both the persons selected belong to medical profession = $A$

Even that both the persons selected belong to the engineering profession = $B$

Event that both the persons selected belong to the same profession = $A \cup B$

The two events $A$ and $B$ can not occur together that is $A \cap B = \phi$

Thus, the events $A$ and $B$ are mutually exclusive. Hence, from the law of addition of probability,

$$P(A \cup B) = P(A) + P(B)$$

For which we first find $P(A)$ and $P(B)$.

$A$ = Event that both the persons selected belong to medical profession.

The number of favourable outcomes of $A$ is $m = {}^3C_2 = 3$.

Probability of event $A$ $P(A) = \frac{m}{n}$

$$= \frac{3}{28}$$

$B =$ Event that both the persons selected belong to engineering profession.

The number of favourable outcomes of $B$ is $m = {}^5C_2 = 10$.

Probability of event $B$ $P(B) = \frac{m}{n}$

$$= \frac{10}{28}$$

Now,

$$P(A \cup B) = P(A) + P(B)$$

$$= \frac{3}{28} + \frac{10}{28}$$

$$= \frac{3+10}{28}$$

$$= \frac{13}{28}$$

Required probability $= \frac{13}{28}$

**Illustration 22 : The probability that a person from a group reads newspaper $X$ is 0.55, the probability that he read newspaper $Y$ is 0.69 and the probability that he reads both the newspaper $X$ and $Y$ is 0.27. Find the probability that a person selected at random from this group.**

**(1) reads at least one of the newspapers $X$ and $Y$.**

**(2) does not read any of the newspapers $X$ and $Y$.**

**(3) reads only one of the newspapers $X$ and $Y$.**

If the event that a person from the group reads newspaper $X$ is denoted by event $A$ and reads newspaper $Y$ by event $B$ then the given information can be shown as follows :

$P(A) = 0.55,\ P(B) = 0.69,\ P(A \cap B) = 0.27$

(1) Event that the selected person reads at least one of the newspapers $= A \cup B$

From the law of addition of probability,

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$= 0.55 + 0.69 - 0.27$$

$$= 0.97$$

Required probability = 0.97

(2) Event that the selected person does not read newspaper $A = A'$

Event that the selected person does not read newspaper $B = B'$

Hence, event that the selected person does not read any of the newspaper $X$ and $Y = A' \cap B'$.

Probability of $A' \cap B'$

$$P(A' \cap B') = P(A \cup B)'$$

$$= 1 - P(A \cup B)$$

$$= 1 - 0.97$$

$$= 0.03$$

Required probability = 0.03

(3) If the event that the selected person reads only one of the newspapers $X$ and $Y$ is denoted by $C$ then the event $C$ can occur as follows :

The person reads newspaper $X$ (event $A$) and does not read newspaper $Y$ (event $B'$)

**OR**

The person does not read newspaper $X$ (event $A'$) and reads newspaper $Y$ (event $B$)

Thus $C = (A \cap B') \cup (A' \cap B)$

Since the events $A \cap B'$ and $A' \cap B$ are mutually exclusive,

$$
\begin{aligned}
P(C) &= P(A \cap B') + P(A' \cap B) \\
&= \left[P(A) - P(A \cap B)\right] + \left[P(B) - P(A \cap B)\right] \\
&= [0.55 - 0.27] + [0.69 - 0.27] \\
&= 0.28 + 0.42 \\
&= 0.7
\end{aligned}
$$

Required probability = 0.7

**Illustration 23 : For two events $A$ and $B$ in the sample space of a random experiment $P(A) = 2P(B) = 4P(A \cap B) = 0.6$. Find the probability of the following events :**

**(1) $A' \cap B'$ (2) $A' \cup B'$ (3) $A - B$ (4) $B - A$**

It is given that $P(A) = 2P(B) = 4P(A \cap B) = 0.6$. Hence,

| | | |
|---|---|---|
| $P(A) = 0.6$ | $2P(B) = 0.6$<br>$\therefore P(B) = 0.3$ | $4P(A \cap B) = 0.6$<br>$\therefore P(A \cap B) = 0.15$ |

(1) Probability of event $A' \cap B'$

$$
\begin{aligned}
&= P(A' \cap B') \\
&= P(A \cup B)' \\
&= 1 - P(A \cup B) \\
&= 1 - \left[P(A) + P(B) - P(A \cap B)\right] \\
&= 1 - [0.6 + 0.3 - 0.15] \\
&= 1 - 0.75 \\
&= 0.25
\end{aligned}
$$

Required probability = 0.25

(2) Probability of event $A' \cup B'$

$$
\begin{aligned}
&= P(A' \cup B') \\
&= P(A \cap B)' \\
&= 1 - P(A \cap B) \\
&= 1 - 0.15 \\
&= 0.85
\end{aligned}
$$

Required probability = 0.85

(3) Probability of event $A-B$ $= P(A-B)$

$= P(A)-P(A\cap B)$

$= 0.6-0.15$

$= 0.45$

Required probability $= 0.45$

(4) Probability of event $B-A$ $= P(B-A)$

$= P(B)-P(A\cap B)$

$= 0.3-0.15$

$= 0.15$

Required probability $= 0.15$

**Illustration 24 : For two events $A$ and $B$ in the sample space of a random experiment $P(A')=0.3$, $P(B)=0.6$ and $P(A\cup B)=0.83$. Find $P(A\cap B')$ and $P(A'\cap B)$.**

Here, $P(A')=0.3 \therefore P(A)=1-P(A')=1-0.3=0.7$

$P(B)=0.6$ and $P(A\cup B)=0.83$

First we will find $P(A\cap B)$.

$P(A\cup B)=P(A)+P(B)-P(A\cap B)$

$\therefore\ 0.83=0.7+0.6-P(A\cap B)$

$\therefore\ P(A\cap B)=0.7+0.6-0.83$

$\therefore\ P(A\cap B)=0.47$

Now,

$P(A\cap B')=P(A)-P(A\cap B)$

$= 0.7 - 0.47$

$= 0.23$

Required probability $= 0.23$

$P(A'\cap B)=P(B)-P(A\cap B)$

$= 0.6 - 0.47$

$= 0.13$

Required probability $= 0.13$

**Illustration 25 : Two events $A$ and $B$ in the sample space of a random experiment are mutually exclusive. If $3P(A)=4P(B)=1$ then find $P(A\cup B)$.**

Since $3P(A)=4P(B)=1$

| | |
|---|---|
| $3P(A)=1$ | $4P(B)=1$ |
| $\therefore\ P(A)=\frac{1}{3}$ | $\therefore\ P(B)=\frac{1}{4}$ |

As the events $A$ and $B$ are mutually exclusive $(A\cap B=\phi)$,

$$P(A\cup B)=P(A)+P(B)$$

$$=\frac{1}{3}+\frac{1}{4}$$

$$=\frac{7}{12}$$

Required probability $=\frac{7}{12}$

**Illustration 26 : For three mutually exclusive and exhaustive events $A$, $B$ and $C$ in the sample space of a random experiment $2P(A)=3P(B)=4P(C)$. Find $P(A\cup B)$ and $P(B\cup C)$.**

Taking $2P(A)=3P(B)=4P(C)=x$,

| | | |
|---|---|---|
| $2P(A)=x$ | $3P(B)=x$ | $4P(C)=x$ |
| $\therefore\ P(A)=\frac{x}{2}$ | $\therefore\ P(B)=\frac{x}{3}$ | $\therefore\ P(C)=\frac{x}{4}$ |

Since $A$, $B$ and $C$ are mutually exclusive and exhaustive events,

$$P(A\cup B\cup C)=P(A)+P(B)+P(C)=1$$

$$\therefore\ \frac{x}{2}+\frac{x}{3}+\frac{x}{4}=1$$

$$\therefore\ \frac{6x+4x+3x}{12}=1$$

$$\therefore\ 13x=12$$

$$\therefore\ x=\frac{12}{13}$$

Thus,

$$P(A)=\frac{x}{2}=\frac{\frac{12}{13}}{2}=\frac{6}{13}$$

$$P(B)=\frac{x}{3}=\frac{\frac{12}{13}}{3}=\frac{4}{13}$$

$$P(C)=\frac{x}{4}=\frac{\frac{12}{13}}{4}=\frac{3}{13}$$

Now, the probability of required events,

$$P(A \cup B) = P(A) + P(B)$$

$$= \frac{6}{13} + \frac{4}{13}$$

$$= \frac{10}{13}$$

Required probability $= \frac{10}{13}$

$$P(B \cup C) = P(B) + P(C)$$

$$= \frac{4}{13} + \frac{3}{13}$$

$$= \frac{7}{13}$$

Required probability $= \frac{7}{13}$

## Exercise 1.3

**1.** 2 cards are drawn from a pack of 52 cards. Find the probability that both the cards drawn are

(1) of the same suit

(2) of the same colour.

**2.** 3 books of Statistics and 4 of Mathematics are arranged on a shelf. Two books are randomly selected from these books. Find the probability that both the books selected are of the same subject.

**3.** One card is randomly drawn from a pack of 52 cards. Find the probability that it is

(1) Spade card or ace (2) Neighter spade nor ace.

**4.** A number is selected from the natural number 1 to 100. Find the probability of the event that the selected number is a multiple of 3 or 5.

**5.** Two balanced dice are thrown simultaneously. Find the probability that the sum of numbers on two dice is a multiple of 2 or 3.

**6.** The probability that the price of potato rises in the vegetable market during festive days in 0.8. The probability that the price of onion rises is 0.7. The probability of rise in price of both potato and onion is 0.6. Find the probability of rise in price of at least one of the two, potato and onion.

**7.** Two aircrafts drop bomb to destroy a bridge. The probability that a bomb dropped from the first aircraft hits the target is 0.9 and the probability that a bomb from the second aircraft hits the target is 0.7. The probability of one bomb dropped from both the aircrafts hitting the target is 0.63. The bridge is destroyed even if one bomb drops on it. Find the probability that the bridge is destroyed.

8. The probability that a teenager coming to a restaurant for dinner orders pizza is 0.63. The probability of ordering cold-drink is 0.54. The probability that the teenager orders at least one out of pizza and cold-drink is 0.88. Find the probability that the teenager coming for dinner on a certain day orders only one of the two items from pizza and cold-drink.

9. If $A$ and $B$ are mutually exclusive and exhaustive events in a sample space $U$ and $P(A)=2P(B)$ then find $P(A)$.

10. Three events $A$, $B$ and $C$ in a sample space are mutually exclusive and exhaustive. If $4P(A)=5P(B)=3P(C)$ then find $P(A\cup C)$ and $P(B\cup C)$.

11. Find $P(A\cup B\cup C)$ using the following information about three events $A$, $B$ and $C$ in a sample space.

$P(A)=0.65$, $P(B)=0.45$, $P(C)=0.25$, $P(A\cap B)=0.25$, $P(A\cap C)=0.15$, $P(B\cap C)=0.2$,

$P(A\cap B\cap C)=0.05$

12. Three events $A$, $B$ and $C$ in a sample space are mutually exclusive and exhaustive. If $P(C')=0.8$ and $3P(B)=2P(A')$ then find $P(A)$ and $P(B)$.

*

## 1.6 Conditional Probability and Law of Multiplication of Probability

### 1.6.1 Conditional Probability

Suppose $U$ is a finite sample space and $A$ and $B$ are any two events in it. The probability of occurrence of event $B$ under the condition that $A$ occurs is called the conditional probability. If the occurrence of event $B$ under the condition that event $A$ occurs is denoted by $B/A$ then the probability $P(B/A)$ of the conditional event $B/A$ is called the conditional probability. This probability is obtained using the following formula :

$$P(B/A) = \frac{P(A\cap B)}{P(A)}; \quad P(A)\neq 0$$

Similarly, if the occurrence of event $A$ under the condition that event $B$ occurs is denoted by $A/B$ then the probability $P(A/B)$ of the conditional event $A/B$ is obtained using the following formula :

$$P(A/B) = \frac{P(A\cap B)}{P(B)}; \quad P(B)\neq 0$$

Suppose a company produces a certain type of item in its two different factories $A_1$ and $A_2$. One item is randomly selected from a store selling the items produced by this company. Let us denote the event that the selected item is defective as $D$.

- If the selected item is produced at factory $A_1$ then the event that it is defective is denoted by $D/A_1$.
- If the selected item is produced at factory $A_2$ then the event that it is defective is denoted by $D/A_2$.

Thus,

$P(D/A_1)$ = Probability of occurrence of $D$ under the condition that event $A_1$ has occurred and
$P(D/A_2)$ = Probability of occurrence of $D$ under the condition that event $A_2$ has occurred

### 1.6.2 Independent Events

Suppose $A$ and $B$ are any two events in a finite sample space $U$. If the probability of occurrence of event $A$ does not change due to occurrence (or non-occurrence) of event $B$ then the events $A$ and $B$ are called the independent events.

Thus, if $P(A) = P(A/B) = P(A/B')$ and $P(B) = P(B/A) = P(B/A')$ the events $A$ and $B$ are called independent events.

For example, Event $A$ = First throw of a balanced die shows number 1.

Event $B$ = Second throw of a balanced die shows an even number.

It can be said here that the probability of getting an even number in the second throw of the die does not change because the first throw had shown the number 1. This fact can be easily understood by the following calculation :

The total number of outcomes by throwing the dice two times is $n = 6 \times 6 = 36$.

$A$ = Event that the first throw of a balanced die shows the number 1.

The number of favourable outcomes of $A$ is $m = 6$.

Probability of event $A$ $P(A) = \frac{m}{n}$

$= \frac{6}{36}$

$B$ = Event that the second throw of balanced die shows an even number.

The number of favourable outcomes of $B$ is $m = 18$

Probability of event $B$ $P(B) = \frac{m}{n}$

$= \frac{18}{36}$

$= \frac{1}{2}$

$A \cap B$ = Event that the first throw of a balanced die shows the number 1 and the second throw shows even number.

The number of favourable outcomes of $A \cap B$ is $m = 3$.

Probability of event $A \cap B$ $P(A \cap B) = \frac{m}{n}$

$= \frac{3}{36}$

Now, if the first throw of the die shows number 1 then the probability $P(B/A)$ for the event $B/A$ of getting an even number in the second throw can be obtained as follows :

$$P(B/A) = \frac{P(A \cap B)}{P(A)}$$

$$= \frac{\frac{3}{36}}{\frac{6}{36}}$$

$$= \frac{1}{2}$$

Since we get $P(B) = P(B/A)$, we say that the events $A$ and $B$ are independent.

### 1.6.3 Law of Multiplication of Probability

If $A$ and $B$ are the two events in a sample space $U$ then the rule of obtaining the probability of simultaneous occurrence of events $A$ and $B$ is called the law of multiplication of probability.

For example, Event $A$ = Getting head when a coin is tossed for the first time.

Event $B$ = Getting head when a coin is tossed for the second time.

If the coin is tossed two times then the probability of getting head both the times that is event $A \cap B$ can be obtained by the law of multiplication of probability. The law of multiplication of probability is as follows :

$$P(A \cap B) = P(A) \times P(B/A); \; P(A) \neq 0$$

$$P(A \cap B) = P(B) \times P(A/B); \; P(B) \neq 0$$

Some of the important results deduced from this rule are as follows which will be accepted without proof.

(1) If $A$ and $B$ are independent events then $P(A \cap B) = P(A) \times P(B)$

(2) If $A$ and $B$ are independent events then

(i) The events $A'$ and $B'$ are also independent. Hence, $P(A' \cap B') = P(A') \times P(B')$

(ii) The events $A$ and $B'$ are also independent. Hence, $P(A \cap B') = P(A) \times P(B')$

(iii) The events $A'$ and $B$ are also independent. Hence, $P(A' \cap B) = P(A') \times P(B)$

### 1.6.4 Selection with Replacement and without Replacement

When the units are to be randomly selected one by one from the population, the selection can be done in two ways :

(1) **Selection with replacement :** If the selection of a unit from the population in any trial is done by replacing the unit selected in the previous trial back to the population then that selection is called the selection with replacement.

(2) **Selection without Replacement :** If the selection of a unit from the population in any trial is done by not replacing the unit selected in the previous trial back to the population then that selection is called the selection with replacement.

**Illustration 27 : A balanced coin is tossed twice. If the first toss of the coin shows head then find the probability of getting head in both the tosses.**

The sample space of the random experiment of tossing a balanced coin twice is $U = \{HH, HT, TH, TT\}$, where the first symbol shows the outcome of the first toss of the coin and the second symbol shows the outcome of the second toss of the coin. The total number of outcomes in this sample space is $n = 4$.

If $A$ denote the event of getting head in the first toss of the coin and $B$ denotes the event that both the tosses result in head then we have to find $P(B/A)$, probability of $B/A$.

Event $A$ = First toss shows head

= $\{HH, HT\}$

Hence, the number of favourable outcomes of $A$ is $m = 2$.

Probability of event $A$ $P(A) = \frac{m}{n}$

$= \frac{2}{4}$

Event $B$ = Head is shown in both the tosses

= $\{HH\}$

Hence, the number of favourable outcomes of $B$ is $m = 1$.

Probability of event $B$ $P(B) = \frac{m}{n}$

$= \frac{1}{4}$

Event $A \cap B$ = Getting head in the first toss and getting head in both the tosses of the coin (we have $B \subset A$.)

= $\{HH\}$

Hence, the number of favourable outcomes of $A \cap B$ is $m = 1$.

Probability of $A \cap B$ $P(A \cap B) = \frac{m}{n}$

$= \frac{1}{4}$

Now,

$$P(B/A) = \frac{P(A \cap B)}{P(A)}$$

$$= \frac{\frac{1}{4}}{\frac{2}{4}}$$

$$= \frac{1}{2}$$

Required probability $= \frac{1}{2}$

**Illustration 28 : A factory has received an order to prepare 50,000 units of an item in a certain time period. The probability of completing this work in the given time is 0.75 and the probability that the workers will not declare strike during that time period is 0.8. The probability that this work will be completed during the given period and the workers will not declare strike is 0.7. Find the probability that**

**(1) The work will be completed as per schedule under the condition that the workers have not declared strike.**

**(2) Find the probability that the workers do not declare strike in the given period knowing that the work is completed as per schedule.**

If we denote event $A$ that the work will be completed as per schedule and event $B$ that the workers will not declare strike then the given information can be written as follows :

$P(A) = 0.75, \ P(B) = 0.8, \ P(A \cap B) = 0.7$

(1) Event that the work will be completed in the given period under the condition that the workers do not declare strike = $A/B$

Probability of $A/B$ from the definition of condition probability,

$$P(A/B) = \frac{P(A \cap B)}{P(B)}$$

$$= \frac{0.7}{0.8}$$

$$= \frac{7}{8}$$

Required probability = $\frac{7}{8}$

(2) If it is given that the work is completed as per schedule then the event that the workers do not declare strike = $B/A$

Probability of $B/A$ from the definition of conditional probability,

$$P(B/A) = \frac{P(A \cap B)}{P(A)}$$

$$= \frac{0.7}{0.75}$$

$$= \frac{14}{15}$$

Required probability = $\frac{14}{15}$

**Illustration 29 : If $P(A') = \frac{7}{25}$, $P(B/A) = \frac{5}{12}$ and $P(A/B) = \frac{1}{2}$ for two events $A$ and $B$ in the sample space of a random experiment then find $P(A \cap B)$ and $P(B)$.**

It is given that $P(A') = \frac{7}{25}$, $P(B/A) = \frac{5}{12}$ and $P(A/B) = \frac{1}{2}$.

$$P(A) = 1 - P(A')$$

$$= 1 - \frac{7}{25}$$

$$= \frac{18}{25}$$

We will find $P(A \cap B)$ from the formula of $P(B/A)$.

$$P(B/A) = \frac{P(A \cap B)}{P(A)}$$

$\therefore \quad \frac{5}{12} = \frac{P(A \cap B)}{\frac{18}{25}}$

$\therefore \quad \frac{5}{12} \times \frac{18}{25} = P(A \cap B)$

$\therefore \quad P(A \cap B) = \frac{3}{10}$

Required probability $= \frac{3}{10}$

Now, we will find $P(B)$ by substituting $P(A/B)$ and $P(A \cap B)$ in the formula of $P(A/B)$.

$$P(A/B) = \frac{P(A \cap B)}{P(B)}$$

$$\therefore \quad \frac{1}{2} = \frac{\frac{3}{10}}{P(B)}$$

$$\therefore \quad P(B) = \frac{\frac{3}{10}}{\frac{1}{2}}$$

$$= \frac{3 \times 2}{10 \times 1}$$

$$= \frac{3}{5}$$

Required probability $= \frac{3}{5}$

**Illustration 30 : A medicine is tested on a group of rabbits and mice to know its effect. It was observed that 7 rabbits show the effect of medicine in a group of 10 rabbits who were given the medicine and 5 mice show the effect of medicine in a group of 9 mice who were given the medicine. One animal is selected at random from each group. Find the probability that (1) both the selected animals show the effect of medicine and (2) one of the two selected animals shows the effect of medicine and the other animal does not show the effect of medicine.**

**The given information will be shown as follows :**

| **Animals affected by Medicine** | **Animals not affected by Medicine** |
|---|---|
| Rabbits 7 | Rabbits 3 |
| Mice 5 | Mice 4 |
| **Total 12** | **Total 7** |

(1) Event that a rabbit shows effect of medicine $= A$

Event that a mouse shows effect of medicine $= B$

$\therefore$ Event that both the animals show the effect of medicine $= A \cap B$

The events $A$ and $B$ are independent. Whether the mice show the effect of medicine is not affected by the effect of medicine on rabbits. Hence,

$$P(A \cap B) = P(A) \times P(B)$$

From the total number of outcomes $n = 10$, $m = 7$ outcomes are favourable for event $A$.

$\therefore$ Probability of event $A$ $P(A) = \frac{m}{n}$

$= \frac{7}{10}$

From the total number of outcomes $n = 9$, $m = 5$ outcomes are favourable for event $B$.

$\therefore$ Probability of event $B$ $P(B) = \frac{m}{n}$

$= \frac{5}{9}$

$$\therefore P(A \cap B) = \frac{7}{10} \times \frac{5}{9}$$

$$= \frac{7}{18}$$

Required probability $= \frac{7}{18}$

(2) Let $C$ denote the event that one animal is affected by the medicine and the other animal is not affected by the medicine. The event $C$ can occur as follows :

Rabbit is affected by the medicine (event $A$) and mouse is not affected by the medicine (event $B'$)

**OR**

Rabbit is not affected by the medicine (event $A'$) and mouse is affected by the medicine (event $B$)

Thus, event $C = (A \cap B') \cup (A' \cap B)$

Since the events $A \cap B'$ and $A' \cap B$ are mutually exclusive,

$$P(C) = P(A \cap B') + P(A' \cap B)$$

$$= [P(A) \times P(B')] + [P(A') \times P(B)] \quad (\because A \text{ and } B \text{ are independent events})$$

Here, $P(A') = 1 - P(A)$

$= 1 - \frac{7}{10}$

$= \frac{3}{10}$

$P(B') = 1 - P(B)$

$= 1 - \frac{5}{9}$

$= \frac{4}{9}$

$$\therefore P(C) = [\frac{7}{10} \times \frac{4}{9}] + [\frac{3}{10} \times \frac{5}{9}]$$

$$= \frac{28}{90} + \frac{15}{90}$$

$$= \frac{43}{90}$$

Required probability $= \frac{43}{90}$

**Illustration 31 : A company produces a certain type of item in its two different factories $A_1$ and $A_2$ in the proportion 60% and 40% respectively. The proportions of defectives in the production of these factories are 2% and 3% respectively. One item is randomly selected after mixing the items produced in the two factories. Find the probability that this item is defective.**

Event that the selected item is produced in factory $A_1$ = $A_1$

$$\therefore \ P(A_1) = \frac{60}{100}$$

$$= \frac{3}{5}$$

Event that the selected item is produced in factory $A_2$ = $A_2$

$$\therefore \ P(A_2) = \frac{40}{100}$$

$$= \frac{2}{5}$$

Let $D$ denote the event that the item selected from the total production is defective.

Event that the selected item is defective when it is produced in factory $A_1$ = $D/A_1$

$$\therefore \ P(D/A_1) = \frac{2}{100}$$

$$= \frac{1}{50}$$

Event that the selected item is defective when it is produced in factory $A_2$ = $D/A_2$

$$\therefore \ P(D/A_2) = \frac{3}{100}$$

Event $D$ can occur as follows.

The selected item is produced in factory $A_1$ and it is defective.

**OR**

The selected item is produced in factory $A_2$ and it is defective.

Thus event $D = (A_1 \cap D) \cup (A_2 \cap D)$

Since the events $A_1 \cap D$ and $A_2 \cap D$ are mutually exclusive,

$$P(D) = P(A_1 \cap D) + P(A_2 \cap D)$$

$$= \left[P(A_1) \times P(D/A_1)\right] + \left[P(A_2) \times P(D/A_2)\right]$$

$$= \left[\frac{3}{5} \times \frac{1}{50}\right] + \left[\frac{2}{5} \times \frac{3}{100}\right]$$

$= \frac{3}{250} + \frac{6}{500}$

$= \frac{12}{500}$

$= \frac{3}{125}$

Required probability $= \frac{3}{125}$

**Illustration 32 : There are 12 screws in a box of which 4 screws are defective. Two screws are randomly selected one by one without replacement from this box. Find the probability that both the screws selected are defective.**

4 screws are defective in the box having 12 screws. Hence, the number of non-defective screws will be 8.

Total number of mutually exclusive, exhaustive and equiprobable outcomes for selecting the first screw are $n = {}^{12}C_1 = 12$.

If $A$ denotes the event that the first screw selected is defective then the number of favourable outcomes of $A$ is $m = {}^{4}C_1 = 4$.

Probability of event $A$ $P(A) = \frac{m}{n} = \frac{4}{12}$

The screws are selected without replacement which means that the first screw is not kept back into the box. Hence, the total number of mutually exclusive, exhaustive and equi-probable outcomes for selecting the second screw is $n = {}^{11}C_1 = 11$.

Let $B$ denote the event that the second screw selected is defective.

The event $B$ occurs under the condition that the event $A$ has occurred. This is the occurrence of event $B/A$.

Since event $A$ has occurred earlier, there are 3 defective screws in the box.

Hence, the number of favourable outcomes for event $B/A$ is $m = {}^{3}C_1 = 3$.

Probability of $B/A$ $P(B/A) = \frac{m}{n}$

$= \frac{3}{11}$

Now, $A \cap B$ = Event that both the screws are defective

From the law of multiplication of probability,

$P(A \cap B) = P(A) \times P(B/A)$

$= \frac{4}{12} \times \frac{3}{11}$

$= \frac{1}{11}$

Required probability $= \frac{1}{11}$

**Illustration 33 : There are 3 boys and 2 girls in a friend-circle. Two persons are randomly selected from this friend-circle one by one with replacement to sing a song. Find the probability that the first person is a boy and the second person is a girl in the two persons selected to sing a song.**

The friend-circle consists of 3 boys and 2 girls that is total 5 persons. Two persons are selected one by one with replacement. This means that the person selected first is sent back to the group before selecting the second person. Hence, the events of selecting two persons one by one are independent events. The total number of mutually exclusive, exhaustive and equi-probable outcomes for selecting the first person is $n = {}^5C_1 = 5$.

Event that the first person selected to sing a song is a boy $= A$

The number of favourable outcomes for event $A$ is $m = {}^3C_1 = 3$

Probability of event $A$ $P(A) = \frac{m}{n}$

$= \frac{3}{5}$

The selection is with replacement here. This means that the total number of mutually exclusive, exhaustive and equi-probable outcomes for selecting the second person is $n = {}^5C_1$.

Event that the second person selected to sing a song is a girl $= B$

The number of favourable outcomes of $B$ is $m = {}^2C_1 = 2$

Probability of event $B$ $P(B) = \frac{m}{n}$

$= \frac{2}{5}$

Now, $A \cap B$ = Event that the first boy and the second girl are the two person selected to sing a song. Since the events $A$ and $B$ are independent,

$P(A \cap B) = P(A) \times P(B)$

$= \frac{3}{5} \times \frac{2}{5}$

$= \frac{6}{25}$

Required probability $= \frac{6}{25}$

**Illustration 34 : Two balanced dice are thrown simultaneously. Find the probability that at least one of the two dice shows the number 3.**

Event that the first die shows number 3 $= A$

Event that the second die shows number 3 $= B$

Event that at least one die shows number 3 $= A \cup B$

The number of favourable outcome for event $A$ is $m=1$

Probability of event $A$ $P(A)=\frac{m}{n}$

$=\frac{1}{6}$

The number of favourable outcomes for event $B$ is $m=1$

Probability of event $B$ $P(B)=\frac{m}{n}$

$=\frac{1}{6}$

Since the events $A$ and $B$ are independent, the events $A'$ and $B'$ are also independent. Moreover, $P(A')=1-P(A)=1-\frac{1}{6} = \frac{5}{6}$ and $P(B')=1-P(B)=1-\frac{1}{6} = \frac{5}{6}$.

Probability of the event that at least one die shows number 3 $=P(A\cup B)$

$=1-P(A'\cap B')$

$=1-\left[P(A')\times P(B')\right]$

$=1-\left[\frac{5}{6}\times\frac{5}{6}\right]$

$=1-\frac{25}{36}$

$=\frac{11}{36}$

Required probability $=\frac{11}{36}$

**Illustration 35 : Two cities $A$ and $B$ of different states have rains on 60% and 75% days respectively during the monsoon. For the cities $A$ and $B$, find the probability that on a certain monsoon day,**

**(1) both the cities have rains**

**(2) at least one city has rains**

**(3) only one city has rains.**

**Note : The events of rains on a day in these two cities are independent.**

Let event $A$ denote that it rains in city $A$ and event $B$ denote that it rains in city $B$. The given information can be stated as follows :

$P(A)=\frac{60}{100}=\frac{3}{5}$ $\quad\therefore P(A')=1-P(A)=1-\frac{3}{5}=\frac{2}{5}$

$P(B)=\frac{75}{100}=\frac{3}{4}$ $\quad\therefore P(B')=1-P(B)=1-\frac{3}{4}=\frac{1}{4}$

(1) Event that both the cities $A$ and $B$ have rains $A\cap B$

Since the events $A$ and $B$ are independent,

Probability of event $A\cap B$ $\quad P(A\cap B)=P(A)\times P(B)$

$=\frac{3}{5}\times\frac{3}{4}$

$=\frac{9}{20}$

Required probability $=\frac{9}{20}$

(2) Event that at least one of the cities $A$ and $B$ has rains $= A \cup B$

Probability of $A \cup B$ $\quad P(A \cup B) = 1 - P(A' \cap B')$

$$= 1 - [P(A') \times P(B')]$$

$$= 1 - \left[\frac{2}{5} \times \frac{1}{4}\right]$$

$$= 1 - \frac{1}{10}$$

$$= \frac{9}{10}$$

Required probability $= \frac{9}{10}$

(3) Event that only one of cities $A$ and $B$ has rains $= (A \cap B') \cup (A' \cap B)$

If the events $A$ and $B$ are independent then events $A$ and $B'$ as well as $A'$ and $B$ are also independent.

Probability of $(A \cap B') \cup (A' \cap B)$ $\quad = P(A \cap B') + P(A' \cap B)$

$$= [P(A) \times P(B')] + [P(A') \times P(B)]$$

$$= \left[\frac{3}{5} \times \frac{1}{4}\right] + \left[\frac{2}{5} \times \frac{3}{4}\right]$$

$$= \frac{3}{20} + \frac{6}{20}$$

$$= \frac{9}{20}$$

Required probability $= \frac{9}{20}$

**Exercise 1.4**

**1.** There are two children in a family. If the first child is a girl then find the probability that both the children in the family are girls.

**2.** Two six-faced balanced dice are thrown simultaneously. If the sum of numbers on both the dice is more than 7 then find the probability that both the dice show same numbers.

**3.** Among the various vehicle-owners visiting a petrol pump, 80% vehicle-owners visit to fill petrol in their vehicle and 60% vehicle-owners visit to fill air in their vehicles. 50% vehicle-owners visit to fill air and petrol in their vehicle. Find the probability for the following events :

(1) If a vehicle-owner has come to fill petrol in his vehicle then that vehicle-owner will fill air in his vehicle.

(2) If a vehicle-owner has come to fill air in his vehicle then that vehicle-owner will fill petrol in his vehicle.

4. 80% customers hold saving account and 50% customers hold current account of a nationalised bank. 90% of the customers hold at least one of the saving account and the current account. If one of the account holders randomly selected from this bank holds a current account, find the probability that he holds a saving account.

5. If $P(A)=\frac{2}{3}$, $P(B)=\frac{3}{5}$ and $P(B/A) = \frac{3}{4}$ for two events in the sample space of a random experiment then find $P(A/B)$.

6. If $P(M)=P(F)=\frac{1}{2}$, $P(A/M)= \frac{1}{10}$ and $P(A/F)=\frac{1}{2}$ for events $A$, $M$ and $F$ then find $P(A\cap M)$ and $P(A\cap F)$.

7. There are 2 gold-coins and 4 silver-coins in a box. The other box contains 3 gold and 5 silver coins. One coin is selected from each box. Find the probability that one of the selected coins is a gold coin and the other is a silver coin.

8. One joint family has 3 sons and 2 daughters whereas the other joint family has 2 sons and 4 daughters. One joint family is selected from two joint families and a child is randomly selected from that family. Find the probability that the selected child is a girl.

9. There are 10 icecream cones in a box of which 3 cones weigh less than the specification and the rest of the 7 cones have the specified weight. Two cones are randomly selected one by one with replacement. Find the probability that both the cones selected weigh less than the specified weight.

10. There are 10 CDs in a CD rack in which 6 are action film CDs and 4 are drama film CDs. Two CDs are randomly selected one by one without replacement from this box. Find the probability that the first selected CD is of action film and the second CD is of drama film.

11. If two balanced dice are thrown then find the probability that

(1) at least one die shows number 5

(2) the first die shows the number 5 or 6 and the other die shows an even number.

12. A problem in Mathematics is given to Tania, Kathan and Kirti to solve. The probabilities of them solving the problem correctly are $\frac{2}{3}, \frac{3}{4}$ and $\frac{1}{2}$ repectively. Find the probability that the problem is solved correctly.

13. Person $A$ can hit the target in 3 out of 5 attempts whereas person $B$ can hit the target in 5 out of 6 attempts. If both of them attempt simultaneously, find the probability that the target is hit.

14. Person $A$ speaks truth in 90% cases whereas person $B$ speaks truth in 80% cases. Find the probability that persons $A$ and $B$ differ in stating the same fact.

15. If three events $A$, $B$ and $C$ of a random experiment are independent events and $P(A)=0.2, P(B)=0.3$ and $P(C)=0.5$ then find $P(A\cup B\cup C)$.

*

## 1.7 Statistical Definition of Probability

We have seen the mathematical definition of probability earlier. This definition can help to find the probability only in the cases where the outcomes of the sample space of a random experiment are equi-probable and their number is known. But we find several cases in practice where the outcomes of the sample space are infinite and unknown. For example, there are many fish of different types in a huge lake. We have to find the probability of catching a certain type of fish when a fisherman throws net in the lake to catch fish. The mathematical definition of probability can not be used here as the total number of fish in the lake is unknown. Moreover, we come across many cases in practice where the outcomes of the random experiment are not equi-probable. For example, a trader transports certain goods from his godown to his sales centre. The event that these goods safely reach the sales centre and event that it does not safely reach the sales centre are not equi-probable events. It is not possible to evaluate probability using the mathematical definition of probability in such cases. Let us consider another definition of probability, called the statistical definition of probability, which is generally more useful in such situations.

Let us start with an illustration. We have to find the probability that a customer will purchase while visiting a showroom selling ready made garments for a long time. To know this, we should obtain the data about the customers purchasing from this show-room. These data can be obtained by sample inquiry. As the size of the sample increases, we can say that the information from the sample inquiry is more close to the true (population) information. Suppose it is found that 79 customers purchase out of 100 customers in the sample inquiry. When the number of customers in the sample inquiry was 500 then it was found that 403 customers purchased. The data obtained by increasing the sample size ($n$) are as follows :

| **Size of the sample (No. of customers visiting the show-room)** | **No. of customers purchasing $r$ (Frequency)** | **Proportion of customers purchasing $\frac{r}{n}$ (Expected Frequency)** |
|---|---|---|
| 100 | 79 | 0.79 |
| 500 | 403 | 0.806 |
| 1000 | 799 | 0.799 |
| 5000 | 3991 | 0.7982 |
| 10,000 | 8014 | 0.8014 |

It can be seen from the above data that as the size of the sample $n$ increases, the proportion or expected frequency of customers purchasing the ready-made garments takes values close to 0.8. We accept this value as the probability of the event that the customer visiting the show-room will purchase. Thus, the probability is obtained in the form of relative frequency. The definition of porbability based on the relative frequency is called the statistical definition of probability. It is also called the empirical definition. The definition is as stated below :

Suppose a random experiment is repeated $n$ times under identical conditions. If an event $A$ occurs in $m$ trials out of $n$ trials then the relative frequency $\frac{m}{n}$ of event $A$ gives the estimate of the probability of event $A$, $P(A)$. When the larger and larger value of $n$ is taken, that is when $n$ tends to infinity, the limiting value of $\frac{m}{n}$ is called the probability of event $A$.

In notation,

$$P(A) = \lim_{n\to\infty} \frac{m}{n}$$

The limiting value of the ratio $\frac{m}{n}$ when $n$ tends to infinite value is denoted by $\lim_{n\to\infty} \frac{m}{n}$. In practice, the relative frequency $\frac{m}{n}$ itself is taken as the probability of event $A$. Now we shall consider the examples showing the use of the statistical definition of probability.

**Illustration 36 : The sample data obtained about marks scored by a large group of candidates appearing for a public examination of 100 marks are given in the following table.**

| Marks | 20 or less | 21–40 | 41–60 | 61–80 | 81–100 |
|---|---|---|---|---|---|
| No. of Candidates | 83 | 162 | 496 | 326 | 124 |

**One candidate is randomly selected from those appearing for the public examination. Find the probability that this candidate has scored :**

**(1) less than 41 marks**

**(2) More than 60 marks**

**(3) Marks from 21 to 80.**

The number of candidates selected in the sample is $n = 83 + 162 + 496 + 326 + 124 = 1191$.

(1) Event $A$ = The selected candidate scores less than 41 marks.

$P(A)$ = Relative frequency for the candidates scoring less than 41 marks.

$$= \frac{\text{No. of candidates scoring less than 41 marks}}{\text{Total number of candidates in the sample}} = \frac{m}{n}$$

$m$ = No. of candidates scoring less than 41 marks

$= 83 + 162$

$= 245$

Now, $P(A) = \frac{m}{n}$

$= \frac{245}{1191}$

Required probability $= \frac{245}{1191}$

(2) Event $B$ = The selected candidate scores more than 60 marks

$P(B)$ = relative frequency for candidates scoring more than 60 marks.

$$= \frac{\text{No. of candidates scoring more than 60 marks}}{\text{Total number of candidates in the sample}} = \frac{m}{n}$$

$m$ = No. of candidates scoring more than 60 marks

$= 326 + 124$

$= 450$

Now, $P(B) = \frac{m}{n}$

$= \frac{450}{1191}$

$= \frac{150}{397}$

Required probability $= \frac{150}{397}$

(3) Event $C$ = The selected candidate scores from 21 to 80 marks

$P(C)$ = relative frequency for candidates scoring from 21 to 80 marks.

$$= \frac{m}{n} = \frac{\text{No. of candidates scoring from 21 to 80 marks}}{\text{Total number of candidates in the sample}}$$

$m$ = No. of candidates scoring from 21 to 80 marks

$= 162 + 496 + 326$

$= 984$

Now, $P(C) = \frac{m}{n}$

$= \frac{984}{1191}$

$= \frac{328}{397}$

Required probability $= \frac{328}{397}$

**Illustration 37 : A factory runs in two shifts. The sample data about the quality of items produced in these shifts are shown in the following table :**

| Quality | Shift | | Total |
|---|---|---|---|
| | I | II | |
| **Defective items** | 24 | 46 | 70 |
| **Non-defective items** | 2176 | 2754 | 4930 |
| **Total** | **2200** | **2800** | **5000** |

**One item is randomly selected from the production of the factory.**

**(1) If the item is taken from the production of the first shift then find the probability that it is defective.**

**(2) If the item is defective then find the probability that it is taken from the production of the first shift.**

The total number of units in the sample = 5000

We shall define the events as follows :

Event $A$ = The selected item is from the production of first shift

$$P(A) = \frac{\text{No. of items produced in the first shift}}{\text{Total number of items in the sample}} = \frac{m}{n}$$

$m$ = No. of items produced in the first shift

= 2200

Now, $P(A) = \frac{m}{n}$

$= \frac{2200}{5000}$

Event $D$ = The selected item is defective

$P(D)$ = relative frequency for defective items

$$= \frac{\text{No. of defective items}}{\text{Total number of items in the sample}} = \frac{m}{n}$$

$m$ = No. of defective items

= 70

Now, $P(D) = \frac{m}{n}$

$= \frac{70}{5000}$

Event $A \cap D$ = The selected item is produced in the first shift and it is defective

$P(A \cap D)$ = relative frequency for event $A \cap D$

$$= \frac{\text{No. of items in event } A \cap D}{\text{Total number of items in the sample}} = \frac{m}{n}$$

$m$ = No. of items in event $A \cap D$

= 24

Now, $P(A \cap D) = \frac{m}{n} = \frac{24}{5000}$

(1) The event that the item is defective when it is taken from the first shift = $D/A$

Probability of $D/A$ using the formula of conditional event

$$P(D/A) = \frac{P(A \cap D)}{P(A)}$$

$$= \frac{\frac{24}{5000}}{\frac{2200}{5000}}$$

$$= \frac{24}{2200}$$

$$= \frac{3}{275}$$

Required probability $= \frac{3}{275}$

(This probability can be directly obtained as relative frequency $\frac{24}{2200}$ of the event $D/A$.)

(2) The event that the item is taken from the first shift when it is defective = $A/D$

Probability of $A/D$ using the formula of condition probability

$$P(A/D) = \frac{P(A \cap D)}{P(D)}$$

$$= \frac{\frac{24}{5000}}{\frac{70}{5000}}$$

$$= \frac{24}{70}$$

$$= \frac{12}{35}$$

Required probability $= \frac{12}{35}$

(This probability can be directly obtained as relative frequency $\frac{24}{70}$ of the event $A/D$.)

**Limitations :** The limitations of the statistical definition of probability are as follows :

(1) The value of probability can be obtained by the statistical definition of probability only if $n \to \infty$ that is if $n$ tends to infinity. But the infinite value of $n$ can not be taken in practice.

(2) The probability obtained by this definition is an estimated value. The exact value of probability cannot be known using this definition.

## Exercise 1.5

**1.** The sample data about monthly travel expense (in ₹) of a large group of travellers of local bus in a megacity are given in the following table :

| **Monthly travel expense (₹)** | 501–600 | 601–700 | 701–800 | 801–900 | 901 or more |
|---|---|---|---|---|---|
| **No. of travellers** | 318 | 432 | 639 | 579 | 174 |

One person from this megacity travelling by local bus is randomly selected. Find the probability that the monthly travel expense of this person will be (1) more than ₹ 900 (2) at the most ₹ 700 (3) ₹ 601 or more but ₹ 900 or less.

**2.** The details of a sample inquiry of 4979 voters of constituency are as follows :

| **Details** | **Males** | **Females** |
|---|---|---|
| **Supporters of party *A*** | 1319 | 1118 |
| **Supporters of party *B*** | 1217 | 1325 |

One voter is randomly selected from this constituency.

(1) If this voter is a male, find the probability that he is a supporter of Party $A$.

(2) If this voter is a supporter of Party $A$, find the probability that he is a male.

*

## Summary

- The events based on chance are called random events.
- The experiment which can be independently repeated under identical conditions and all its posible outcomes are known but which of the outcomes will appear can not be predicted with certainty before conducting the experiment is called a random experiment.
- The set of all possible outcomes of a random experiment is called a sample space of that experiment.
- A subset of the sample space of random experiment is called an event of that random experiment.
- $U$ is a finite sample space and $A$ and $B$ are two events in it. If events $A$ and $B$ can never occur together that is if $A \cap B = \phi$ then the events $A$ and $B$ are called mutually exclusive events.
- If the group of favourable outcomes of events of a random experiment is the sample space then the events are called exhaustive events.
- The elementary events are mutually exclusive and exhaustive.
- If there is no apparent reason to believe that out of one or more events of a random experiment, any one event is more or less likely to occur than the other events then the events are called equi-probable events.
- The number of mutually exclusive, exhaustive and equi-probable outcomes in the sample space $U$ of a random experiment is $n$. If $m$ outcomes among them are favourable for the event $A$ then probability of event $A$ is $\frac{m}{n}$.
- The range of values of $P(A)$, the probability of any event $A$ of a sample space $U$, is 0 to 1. That is $0 \le P(A) \le 1$.
- $A$ and $B$ are any two events in a finite sample space $U$. If the probability of occurrence of event $A$ does not change due to occurrence (or non-occurrence) of event $B$ then $A$ and $B$ are independent events.
- Suppose a random experiment is repeated $n$ times under identical conditions. If an event $A$ occurs in $m$ trials out of $n$ trials then the relative frequency $\frac{m}{n}$ of event $A$ gives the estimate of the probability of event $A$, $P(A)$. When the larger and larger value of $n$ is taken that is when $n$ tends to infinity, the limiting value of $\frac{m}{n}$ is called the probability of event $A$.

## List of Formulae

(1) Complementary event of $A$ $A' = U - A$

(2) Difference event of $A$ and $B$ $A - B = A \cap B' = A - (A \cap B)$ (only event $A$ occurrs.)

(3) Difference event of $B$ and $A$ $B - A = A' \cap B = B - (A \cap B)$ (only event $B$ occurrs.)

(4) The probability of an event $A$ of the sample space of a random experiment is $P(A) = \frac{m}{n}$.

(5) Law of addition of probability

For two events $A$ and $B$,

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

For any three events $A$, $B$ and $C$,

$$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(A \cap C) - P(B \cap C) + P(A \cap B \cap C)$$

If two events $A$ and $B$ are mutually exclusive,

$$P(A \cup B) = P(A) + P(B)$$

If three events $A$, $B$ and $C$ are mutually exclusive,

$$P(A \cup B \cup C) = P(A) + P(B) + P(C)$$

If two events $A$ and $B$ are mutually exclusive and exhaustive,

$$P(A \cup B) = P(A) + P(B) = 1$$

If three events $A$, $B$ and $C$ are mutually exclusive and exhaustive,

$$P(A \cup B \cup C) = P(A) + P(B) + P(C) = 1$$

(6) Conditional probability

Event $B$ occurs under the condition that event $A$ occurs

$$P(B/A) = \frac{P(A \cap B)}{P(A)}; \quad P(A) \neq 0$$

Event $A$ occurs under the condition that event $B$ occurs

$$P(A/B) = \frac{P(A \cap B)}{P(B)}; \quad P(B) \neq 0$$

(7) Law of multiplication of probability

For any two events $A$ and $B$,

$P(A \cap B) = P(A) \times P(B/A)$; $P(A) \neq 0$

$P(A \cap B) = P(B) \times P(A/B)$; $P(B) \neq 0$

- For independent events $A$ and $B$,

$P(A \cap B) = P(A) \times P(B)$

$P(A' \cap B') = P(A') \times P(B')$

$P(A' \cap B) = P(A') \times P(B)$

$P(A \cap B') = P(A) \times P(B')$

(8) According to statistical definition of probability,

$P(A) = \lim_{n \to \infty} \frac{m}{n}$

## Exercise 1

### Section A

**Find the correct option for the following multiple choice questions :**

**1.** Which event is given by a special subset $\phi$ of the sample space $U$ ?

(a) Certain event (b) Complementary event of $\phi$

(C) Union of events U and $\phi$ (d) Impossible event

**2.** What is the value of $P(A \cap A')$ for events $A$ and $A'$ ?

(a) 1 (b) 0 (c) 0.5 (d) between 0 and 1

**3.** Which of the following options is true for any event of the sample space ?

(a) $P(A) < 0$ (b) $0 \leq P(A) \geq 1$ (c) $0 \leq P(A) \leq 1$ (d) $P(A) > 1$

**4.** Which of the following options is not true for any two events $A$ and $B$ in the sample space $U$ where $A \subset B$ ?

(a) $P(A \cap B) = P(B)$ (b) $P(A \cap B) = P(A)$

(c) $P(A \cup B) \geq P(A)$ (d) $P(B - A) = P(B) - P(A)$

5. What is the other name of the classical definition of probability ?

(a) Mathematical definition (b) Axiomatic definition

(c) Statistical definition (d) Geometric definition

6. Which of the following statement for probability of elementary events $H$ and $T$ of random experiment of tossing a balanced coin is not true ?

(a) $P(T)=0.5$ (b) $P(H)+P(T)=1$ (c) $P(H\cap T)=0.5$ (d) $P(H)=0.5$

7. Which random experiment from the following random experiments has an infinite sample space ?

(a) Throwing two dice (b) Selecting two employees from an office

(c) To measure the life of electric bulb (d) Select a card from 52 cards

8. If $A\cup A'=U$ then what type of events are $A$ and $A'$ ?

(a) Independent events (b) Complementary events

(c) Certain events (d) Impossible events

9. If $P(A/B)=P(A)$ and $P(B/A)=P(B)$ then what type of events are $A$ and $B$ ?

(a) Independent events (b) Complementary events

(c) Certain events (d) Impossible events

10. Two events $A$ and $B$ of a sample space are mutually exclusive. Which of the following will be equal to $P(B-A)$ ?

(a) $P(A)$ (b) $P(B)$ (c) $P(A\cap B)$ (d) $P(A\cup B)$

11. What is the total number of sample points in the sample space formed by throwing three six-faced balanced dice simultaneously ?

(a) $6^2$ (b) $3^6$ (c) $6\times 3$ (d) $6^3$

12. If one number is randomly selected between 1 and 20, what is the probability that the number is a multiple of 5 ?

(a) $\frac{1}{2}$ (b) $\frac{1}{6}$ (c) $\frac{1}{5}$ (d) $\frac{1}{3}$

13. If events $A$ and $B$ are independent, which of the following options is true ?

(a) $P(A\cap B)=P(A)\times P(B)$ (b) $P(A\cup B)=P(A)+P(B)$

(c) $P(A\cup B)=P(A)\times P(B)$ (d) $P(A\cap B)=P(A)+P(B)$

14. What is the probability of having 5 Thursdays in the month of February in a year which is not a leap year ?

(a) 0 (b) $\frac{1}{7}$ (c) $\frac{2}{7}$ (d) $\frac{3}{7}$

**15.** If $P(A)=0.4$ and $P(B')=0.3$ for two independent events $A$ and $B$ of a sample space then state the value of $P(A\cap B)$.

(a) 0.12 (b) 0.42 (c) 0.28 (d) 0.18

**16.** For two events $A$ and $B$ of a samples space, state the event $(A\cap B)\cup(A\cap B')$.

(a) $\phi$ (b) $B$ (c) $A$ (d) $U$

**17.** According to the mathematical definition of probability, what is the probability of each outcome among the $n$ outcomes of a random experiment ?

(a) 0 (b) $\frac{1}{n}$ (c) 1 (d) can not say

## Section B

**Answer the following questions in one sentence :**

**1.** Give two examples of random experiment.

**2.** Draw the Venn diagram for $A$-$B$, the difference event of $A$ and $B$.

**3.** Define an event.

**4.** Write the sample space of a random experiment of throwing one balanced die and a balanced coin simultaneously.

**5.** Define conditional probability.

**6.** State the formula for the probability of occurrence of at least one event out of three events $A$, $B$ and $C$.

**7.** Define independent events.

**8.** Write the law of multiplication of probability for two independent events $A$ and $B$ in a sample space.

**9.** Interpret $P(A/B)$ and $P(B/A)$.

**10.** When can we say that three events $A$, $B$ and $C$ in a sample space are exhaustive ?

**11.** Arrange $P(A\cup B), P(A), P(A\cap B), 0, P(A)+P(B)$ in the ascending order.

**12.** Define :

(1) Random Experiment
(2) Sample Space
(3) Equi-probable Events
(4) Favourable Outcomes
(5) Probability (Mathematical definition)
(6) Probability (Statistical definition)
(7) Impossible Event
(8) Certain Event

**13.** For two events $A$ and $B$ in a sample space, $A \cap B = \phi$ and $A \cup B = U$. State the values of $P(A \cap B)$ and $P(A \cup B)$.

**14.** If two events $A$ and $B$ in a sample space are independent then state the formula for $P(A \cup B)$.

**15.** If $A = \{x \mid 0 < x < 1\}$ and $B = \{x \mid \frac{1}{4} \le x \le 3\}$ then find $A \cap B$.

**16.** For two independent events $A$ and $B$, $P(A) = 0.5$ and $P(B) = 0.7$. Find $P(A' \cap B')$.

**17.** If $P(A) = 0.8$ and $P(A \cap B) = 0.25$, find $P(A - B)$.

**18.** If $P(A) = 0.3$ and $P(A \cap B) = 0.03$, find $P(B/A)$.

**19.** If $P(A) = P(B) = K$ for two mutually exclusive events $A$ and $B$, find $P(A \cup B)$.

**20.** If $P(A' \cap B) = 0.45$ and $A \cap B = \phi$, find $P(B)$.

**21.** Two events $A$ and $B$ in a sample space are mutually exclusive and exhaustive. If $P(A) = \frac{1}{3}$, find $P(B)$.

**22.** 2% items in a lot are defective. What is the probability that an item randomly selected from this lot is non-defective ?

**23.** State the number of sample points in the random experiment of tossing five balanced coins.

**24.** State the number of sample points in the random experiment of tossing one balanced coin and two balanced dice simultaneously.

**25.** Is it possible that $P(A) = 0.7$ and $P(A \cup B) = 0.45$ for two events $A$ and $B$ in a sample space ?

**26.** Two cards are selected one by one with replacement from 52 cards. State the number of elements in the sample space of this random experiment.

**27.** For two independent events $A$ and $B$, $P(B/A) = \frac{1}{2}$ and $P(A \cap B) = \frac{1}{5}$. Find $P(A)$.

**28.** 1998 tickets out of 2000 tickets do not have a prize. If a person randomly selects one ticket from 2000 tickets then what is the probability that the ticket selected is eligible for prize ?

## Section C

**Answer the following questions :**

**1.** Define the following events and draw their venn diagram :

(1) Mutually exclusive events (2) Union of events

(3) Intersection of events (4) Difference event

(5) Exhaustive events (6) Complementary event

**2.** Give the illustrations of finite and infinite sample space.

**3.** Give the illustrations of impossible and certain event.

**4.** State the characteristics of random experiment.

**5.** State the assumptions of mathematical definition of probability.

**6.** State the limitations of mathematical definition of probability.

**7.** State the limitations of statistical definition of probability.

**8.** Explain the equiprobable events with illustration.

**9.** State the law of addition of probability for two events $A$ and $B$. Write the law of addition of probability if these two events are mutually exclusive.

**10.** State the law of multiplication of probability for two events $A$ and $B$. Write the law of multiplication of probability if these two events are independent.

**11.** State the following results for two independent events $A$ and $B$ :

(1) $P(A \cap B)$ (2) $P(A' \cap B')$

(3) $P(A \cap B')$ (4) $P(A' \cap B)$

**12.** If $P(A) = \frac{1}{3}$, $P(B) = \frac{2}{3}$ and $P(A \cap B) = \frac{1}{6}$ then find $P(A' \cap B')$.

**13.** If $P(B) = 2P(A/B) = 0.4$ then find $P(A \cap B)$.

**14.** If the events $A$ and $B$ are independent and $3P(A) = 2P(B) = 0.12$ then find $P(A \cap B)$.

**15.** If $5P(A) = 3P(B) = 2\,P(A \cup B) = \frac{3}{2}$ for two events $A$ and $B$ then find $P(A' \cup B')$.

**16.** If $P(A \cap B) = 0.12$ and $P(B) = 0.3$ for two independent events $A$ and $B$ then find $P(A \cup B)$.

**17.** If $A = \{x \mid 1 < x < 3\}$ and $B = \{x \mid \frac{1}{2} \le x < 2\}$ then find $A \cup B$ and $A \cap B$.

**18.** The probability of occurrence of at least one of the two events $A$ and $B$ is $\frac{1}{4}$. The probability that event $A$ occurs but event $B$ does not occur is $\frac{1}{5}$. Find the probability of event $B$.

**19.** If $P(B)=\frac{3}{5}$ and $P(A'\cap B)=\frac{1}{2}$, for two events $A$ and $B$, find $P(A/B)$.

**20.** 6 persons have a passport in a group of 10 persons. If 3 persons are randomly selected from this group, find the probability that

(1) all the three persons have a passport

(2) two persons among them do not have a passport.

**21.** The probability that the tax-limit for income of males increases in the budget of a year is 0.66 and the probability that the tax-limit increases for income of females is 0.72. The probability that the tax-limit increases for income of both the males and females is 0.47. Find the probability that

(1) the tax-limit increases for income of only one of the two, males and females.

(2) the tax-limit does not increase for income of males as well as females in the budget of that year.

**22.** The price of petrol rises in 80% of the cases and the price of diesel rises in 77% of the cases after the rise in price of crude oil. The price of petrol and diesel rises in 68% cases. Find the probability that the price of diesel rises under the condition that there is a rise in the price of petrol.

**23.** As per the prediction of weather bureau, the probabilities for rains on three days, Thursday, Friday and Saturday in the next week are 0.8, 0.7 and 0.6 respectively. Find the probability that it rains on at least one of the three days in the next week.

(**Note :** The events of rains on three days, Thursday, Friday and Saturday of a week are independent.)

## Section D

**Answer the following questions :**

**1.** 6 *LED* televisions and 4 *LCD* televisions are displayed in digital store $A$ whereas 5 *LED* televisions and 3 *LCD* televisions are displayed in digital store $B$. One of the two stores is randomly selected and one television is selected from that store. Find the probability that it is an *LCD* television.

**2.** One number is randomly selected from the natural number 1 to 100. Find the probability that the number selected is either a single digit number or a perfect square.

**3.** A balanced coin is tossed thrice. If the first two tosses have resulted in tail, find the probability that tail appears on the coin in all the three trials.

4. If events *A, B* and *C* are independent events and $P(A) = P(B) = P(C) = p$ then find the value of $P(A \cup B \cup C)$ in terms of *p*.

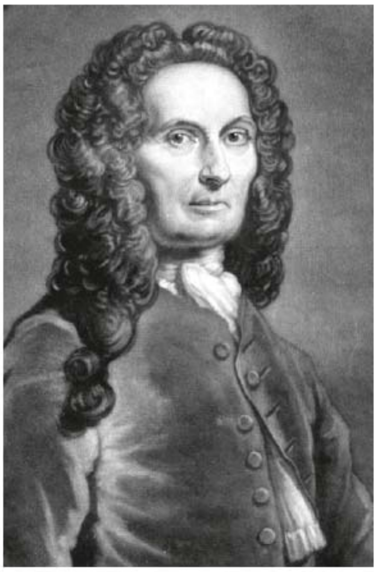
5. The genderwise data of a sample of 6000 employees selected from class 3 and class 4 employees in the government jobs of a state are shown in the following table :

| Class of Employees | Gender | | Total |
|---|---|---|---|
| | **Males** | **Females** | |
| **Class 3** | 3600 | 900 | 4500 |
| **Class 4** | 400 | 1100 | 1500 |
| **Total** | 4000 | 2000 | 6000 |

One employee is randomly selected from all the class 3 and class 4 employees in government jobs of this state.

(1) If the selected employee is a male, find the probability that he belongs to class 3.

(2) If it is given that the selected employee belongs to class 3, find the probability that he is a male.



**Abraham de Moivre**
**(1667 -1754)**

Abraham de Moivre was a French mathematician known for de Moivre's formula, one of those that link complex numbers and trigonometry, and for his work on the normal distribution and probability theory. De Moivre wrote a book on probability theory, The Doctrine of Chances. De Moivre first discovered Binet's formula, the closed-form expression for Fibonacci numbers linking the $n^{th}$ power of the golden ratio $\phi$ to the $n^{th}$ Fibonacci number. He also was the first to postulate the Central Limit Theorem, a cornerstone of probability theory. In the later editions of his book, de Moivre included his unpublished result of 1733, which is the first statement of an approximation to the binomial distribution in terms of what we now call the normal or Gaussian function.

De Moivre continued studying the fields of probability and mathematics until his death and several additional papers were published after his death.

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