# Chapter 13. Statistics

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## Answer 1PT5.

We have from definition of matrix that, a vertical set of numbers is called a column. Again, an entry in a matrix is called an element. A horizontal set of numbers is called a row, a constant multiplied by each element in the matrix is called a scalar. The number of rows and columns gives us the dimensions of the matrix.

Therefore, we can identify each item as below:

1. A vertical set of numbers $\boxed{\text{b. column}}$
2. An entry in a matrix $\boxed{\text{a. element}}$
3. A horizontal set of numbers $\boxed{\text{c. row}}$
4. A constant multiplied by each element in the matrix $\boxed{\text{e. scalar}}$
5. Number of rows and columns $\boxed{\text{d. dimensions}}$

## Answer 1STP.

Consider the line

$$y = 4x - 6$$

Here, slope $m = 4$, and $c = -6$

Now from the definition of perpendicular lines, the slope will be negative reciprocal.

Therefore, the line perpendicular to the line $y = 4x - 6$ is

$$y = \frac{1}{-m}x + c$$

$$y = -\frac{1}{4}x + c$$

Thus putting $c = 2$ we get the perpendicular line as

$$y = -\frac{1}{4}x + 2$$

Hence, the correct choice will be $\boxed{(\text{B})\, y = -\frac{1}{4}x + 2}$.

### Answer 1VC.

We have from definition that random sample is a sample that is as likely to be chosen as any other from the population.

Therefore, the statement can be filled up like below:

“A $\boxed{\text{simple random sample}}$ is a sample that is as likely to be chosen as any other from the population.”

### Answer 2STP.

Consider the ratio be:

$$\frac{3x}{5x}$$

The sum of the numbers will be: $3x+5x$

$$=8x$$

It is given that, if 8 is subtracted from the sum of the numbers, the result is 32.

Therefore, the equation will be:

$$8x-8=32$$

Solving the above equation we get

$$\begin{aligned} 8x-8&=32 \\ 8x-8+8&=32+8 && [\text{Add 8 in both sides}] \\ 8x&=40 \\ \frac{8x}{8}&=\frac{40}{8} && [\text{Divide both sides by 8}] \end{aligned}$$

$$x=5$$

Now, the greater number is

$$\begin{aligned} 5x&=5(5) \\ &=25 \end{aligned}$$

Hence, the correct choice will be: $\boxed{(\text{B})}$.

### Answer 2VC.

We have from definition that standard deviation is the measures that describe the spread of the values in a set of data.

Therefore, the statement can be filled up like below:

“Measures that describe the spread of the values in a set of data are called $\boxed{\text{standard deviation}}$.”

## Answer 3STP.

Consider the expression $(x-8)^2$

Using the formula $(a-b)^2 = a^2 - 2ab + b^2$ we can expand the above expression as below:

$$\begin{aligned}(x-8)^2 &= x^2 - 2\cdot x\cdot 8 + 8^2\\ &= x^2 - 16x + 64\end{aligned}$$

Hence, the correct choice will be: $\boxed{(\mathrm{B})}$.

## Answer 3VC.

We have from definition that quartile separates a data set into four sets with equal number of members.

Therefore, the statement can be filled up like below:

"Each $\boxed{\text{quartile}}$ separates a data set into four sets with equal number of members."

## Answer 4STP.

Consider the graph $y = x^2 - 4$.

Here, the graph is shifted 4 units downwards. Therefore, the least *y* value is $-4$.

Hence, the correct choice will be $\boxed{(\mathrm{D}) - 4}$.

## Answer 4VC.

We have from definition that in inter quartile range, the items are selected according to a specified time or item interval.

Therefore, the statement can be filled up like below:

"In $\boxed{\text{inter quartile range}}$, the items are selected according to a specified time or item interval."

## Answer 5STP.

Consider the expression $3\sqrt{72} - 3\sqrt{2}$

The above expression can be write in simplified form as below:

$$\begin{aligned}&3\sqrt{72} - 3\sqrt{2}\\ &= 3\sqrt{2\cdot 2\cdot 2\cdot 3\cdot 3} - 3\sqrt{2}\\ &= 3\sqrt{2^2\cdot 2\cdot 3^2} - 3\sqrt{2}\\ &= 3\cdot 2\cdot 3\sqrt{2} - 3\sqrt{2} && \left[\text{As } \sqrt{x^2} = x\right]\\ &= 18\sqrt{2} - 3\sqrt{2}\\ &= (18-3)\sqrt{2} && [\text{Take common}]\\ &= 15\sqrt{2}\end{aligned}$$

Hence, the correct choice will be: $\boxed{(\mathrm{C})}$.

## Answer 5VC.

We have from definition that a biased sample has a systematic error within it so that certain populations are favored.

Therefore, the statement can be filled up like below:

"A biased sample has a $\boxed{\text{systematic}}$ error within it so that certain populations are favored."

## Answer 6PT.

Consider that the veterinarian needs a sample of dogs in his kennel to be tested for fleas, he selects the first 5 dogs who run from the pen.

The population will be the first 5 dogs who run from the pen.

As here only 5 dogs are likely to be taken into consideration, therefore the sample is unbiased.

Here the unbiased sample is voluntary, because here the sample is independent to each other. The individual dog is tested and the result may vary.

## Answer 6STP.

Considering the statement we can write

| | | |
|---|---|---|
| Original height(in m) | 12 | *x* |
| Length of shadow(in m) | 9 | 27 |

Therefore, the proportion is

$$\frac{12}{9}=\frac{x}{27}$$

$$\frac{4}{3}=\frac{x}{27}$$

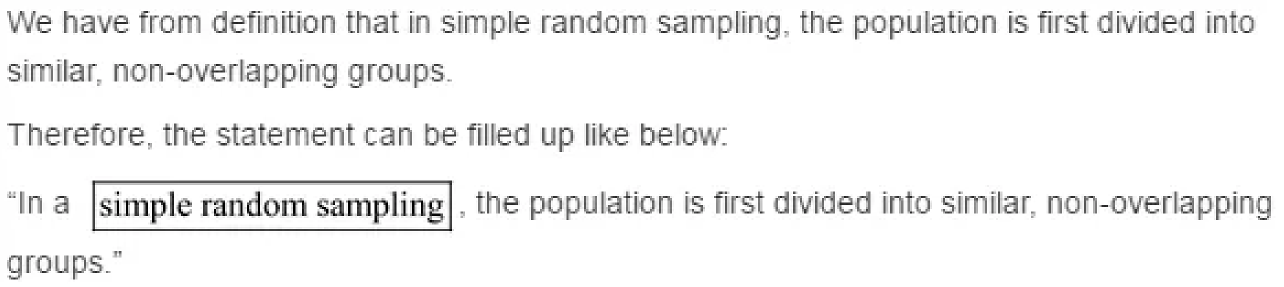
$$\frac{4}{3}\cdot 27=\frac{x}{27}\cdot 27 \qquad [\text{Multiply both sides by } 27]$$

$$4\cdot 9=x \qquad [\text{Simplify}]$$

$$x=36$$

Hence, the correct choice will be: $\boxed{(\text{B})\ 36\text{ m}}$.

## Answer 6VC.

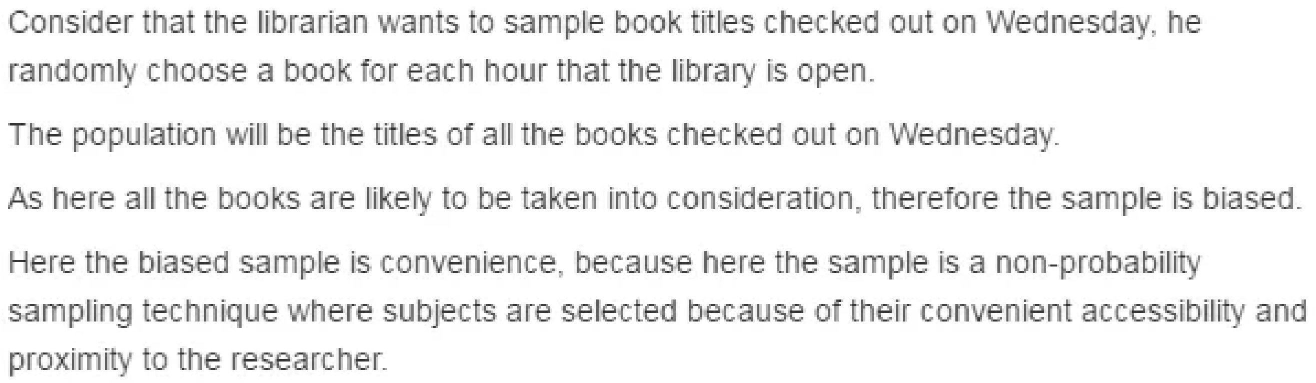


We have from definition that in simple random sampling, the population is first divided into similar, non-overlapping groups.

Therefore, the statement can be filled up like below:

“In a $\boxed{\text{simple random sampling}}$, the population is first divided into similar, non-overlapping groups.”

## Answer 7PT.



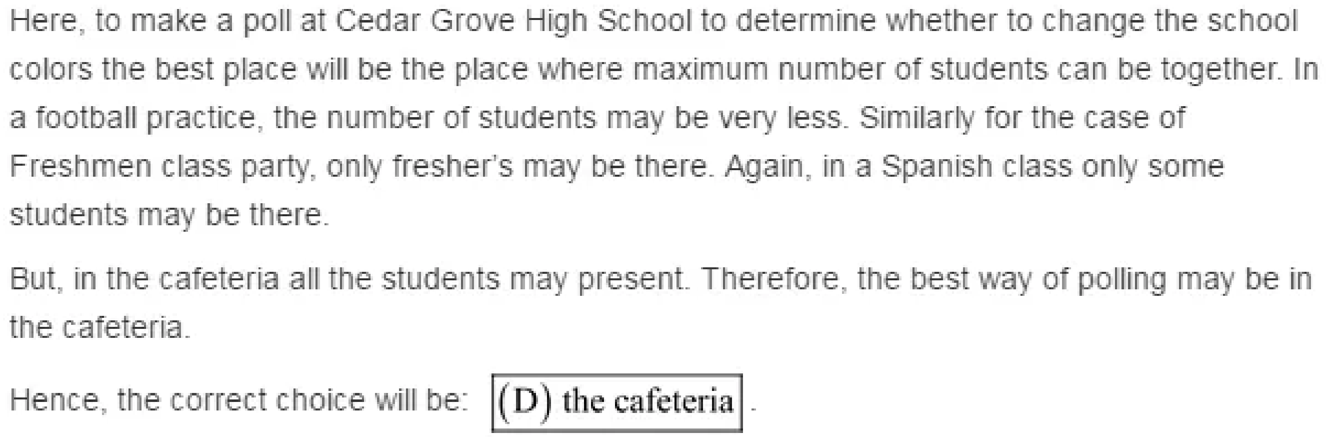
Consider that the librarian wants to sample book titles checked out on Wednesday, he randomly choose a book for each hour that the library is open.

The population will be the titles of all the books checked out on Wednesday.

As here all the books are likely to be taken into consideration, therefore the sample is biased.

Here the biased sample is convenience, because here the sample is a non-probability sampling technique where subjects are selected because of their convenient accessibility and proximity to the researcher.

## Answer 7STP.

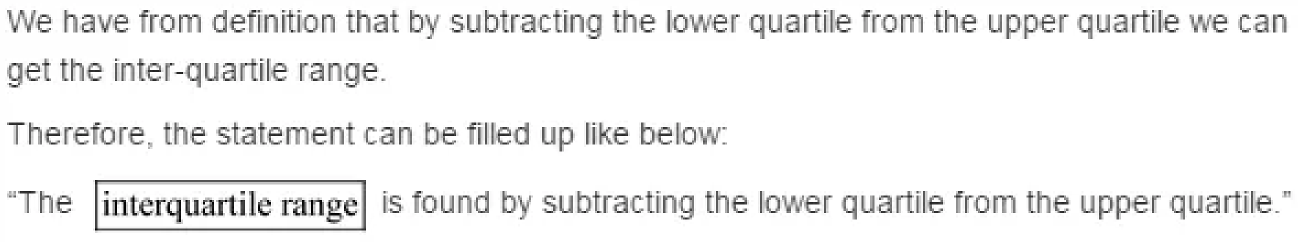


Here, to make a poll at Cedar Grove High School to determine whether to change the school colors the best place will be the place where maximum number of students can be together. In a football practice, the number of students may be very less. Similarly for the case of Freshmen class party, only fresher’s may be there. Again, in a Spanish class only some students may be there.

But, in the cafeteria all the students may present. Therefore, the best way of polling may be in the cafeteria.

Hence, the correct choice will be: $\boxed{\text{(D) the cafeteria}}$.

## Answer 7VC.



We have from definition that by subtracting the lower quartile from the upper quartile we can get the inter-quartile range.

Therefore, the statement can be filled up like below:

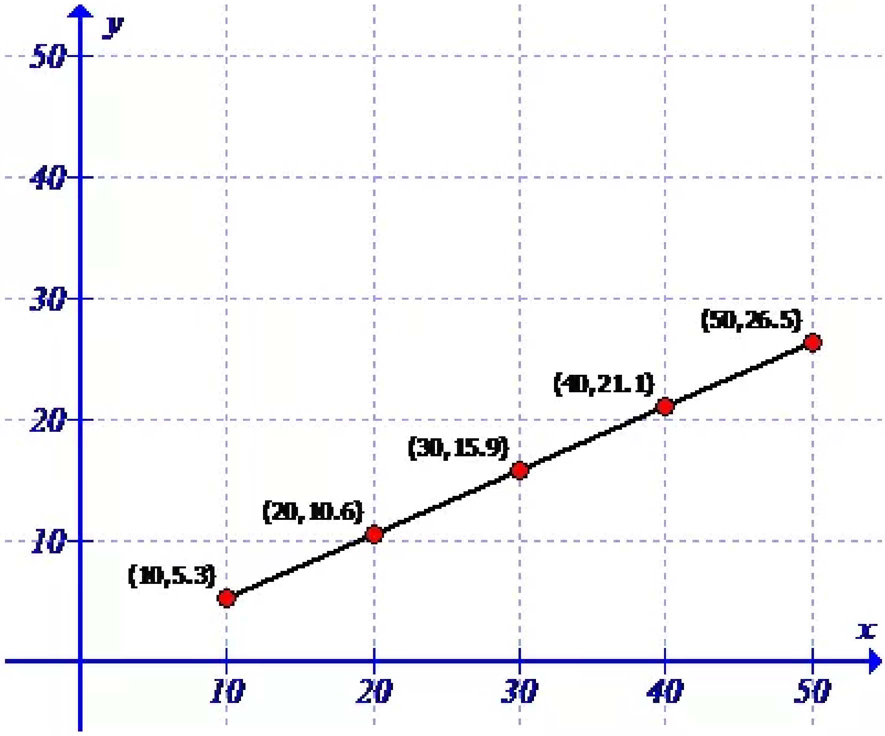
“The $\boxed{\text{interquartile range}}$ is found by subtracting the lower quartile from the upper quartile.”

## Answer 8STP.

Here, the table is

| | | | | | |
|---|---|---|---|---|---|
| Earth | 10 | 20 | 30 | 40 | 50 |
| Mars | 5.3 | 10.6 | 15.9 | 21.2 | 26.5 |

Plotting the values in a graph we get



From the graph we see that the graph is a linear function. Therefore, the correct choice will be:

$\boxed{\text{(A) Linear function}}$.

## Answer 8VC.

We have from definition that the sampling where only those who want to participate in the sampling is called the biased sample.

Therefore, the statement can be filled up like below:

"A $\boxed{\text{biased sample}}$ involves only those who want to participate in the sampling."

## Answer 9PT.

Here, the matrices are

$$Y = \begin{bmatrix} 3 & -2 & 1 \\ -1 & -2 & 4 \end{bmatrix}, \quad Z = \begin{bmatrix} 3 & 1 & 6 \\ 4 & -1 & -1 \end{bmatrix}$$

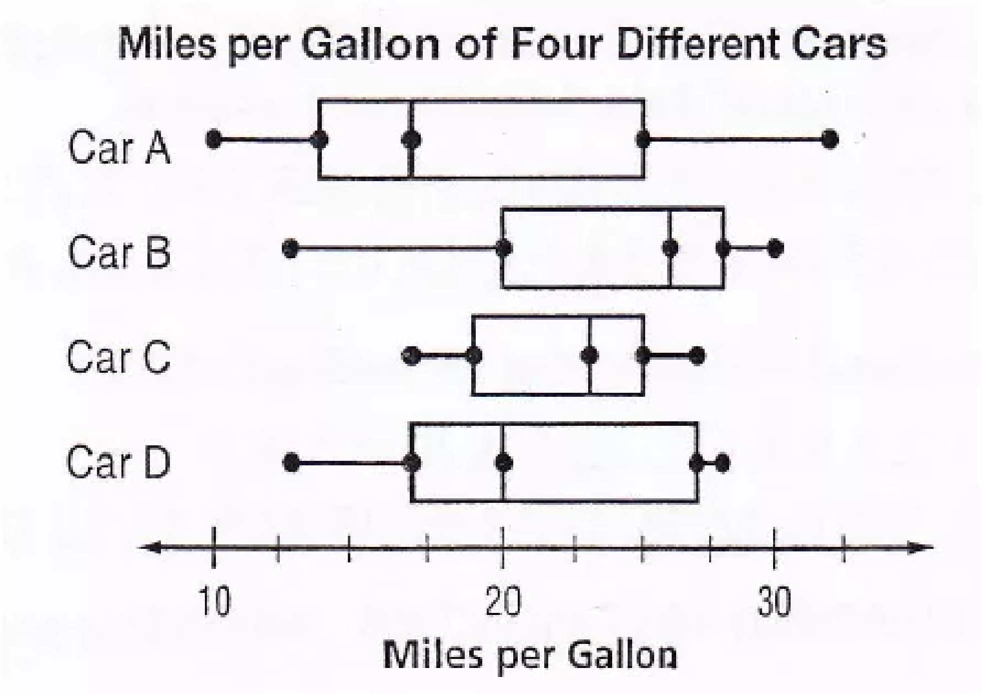
Therefore, $Y - Z$ is given as below:

$$\begin{aligned} Y - Z &= \begin{bmatrix} 3 & -2 & 1 \\ -1 & -2 & 4 \end{bmatrix} - \begin{bmatrix} 3 & 1 & 6 \\ 4 & -1 & -1 \end{bmatrix} \\ &= \begin{bmatrix} 3-3 & -2-1 & 1-6 \\ -1-4 & -2-(-1) & 4-(-1) \end{bmatrix} \\ &= \begin{bmatrix} 0 & -3 & -5 \\ -5 & -2+1 & 4+1 \end{bmatrix} \\ &= \begin{bmatrix} 0 & -3 & -5 \\ -5 & -1 & 5 \end{bmatrix} \end{aligned}$$

Hence, the resultant matrix is $\boxed{\begin{bmatrix} 0 & -3 & -5 \\ -5 & -1 & 5 \end{bmatrix}}$

## Answer 9STP.

Consider the Box-and-whisker plot below:



The variation for car A is almost:

$$25-13=12$$

The variation for car B is almost:

$$26-20=6$$

The variation for car C is almost:

$$25-18=7$$

The variation for car D is almost:

$$27-17=10$$

Therefore, the least variation in miles per gallon is of car $\boxed{\text{(C) C}}$.

## Answer 9VC.

We have from definition that extreme value that is much less or greater than the rest of the data is called an outlier.

Therefore, the statement can be filled up like below:

"The extreme value that is much less or greater than the rest of the data is an $\boxed{\text{outlier}}$."

## Answer 10PT.

Here, the matrix is

$$X = \begin{bmatrix} 4 & 2 & -1 \\ -2 & -2 & 0 \\ 0 & 1 & 2 \end{bmatrix}$$

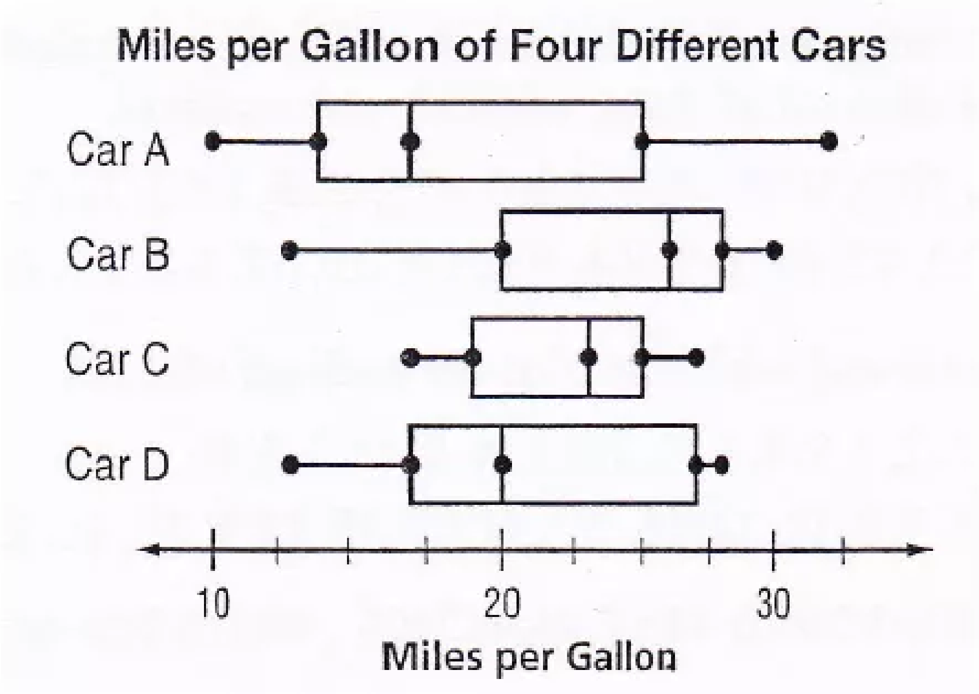
Therefore, $3X$ is given as below:

$$\begin{aligned} 3X &= 3\begin{bmatrix} 4 & 2 & -1 \\ -2 & -2 & 0 \\ 0 & 1 & 2 \end{bmatrix} \\ &= \begin{bmatrix} 3(4) & 3(2) & 3(-1) \\ 3(-2) & 3(-2) & 3(0) \\ 3(0) & 3(1) & 3(2) \end{bmatrix} \\ &= \begin{bmatrix} 12 & 6 & -3 \\ -6 & -6 & 0 \\ 0 & 3 & 6 \end{bmatrix} \end{aligned}$$

Hence, the resultant matrix is $\boxed{\begin{bmatrix} 12 & 6 & -3 \\ -6 & -6 & 0 \\ 0 & 3 & 6 \end{bmatrix}}$

## Answer 10STP.

Consider the Box-and-whisker plot below:



Now from the figure above,

The median of car A is almost: 17

The median of car B is almost: 25.5

The median of car C is almost: 23

The median of car D is: 20

Therefore, the highest median miles per gallon is for car model $\boxed{\text{(B) B}}$.

## Answer 10VC.

We have from definition that the difference between the greatest and the least values of a data set is called the range.

Therefore, the statement can be filled up like below:

"The $\boxed{\text{range}}$ is the difference between the greatest and least values of a data set."

## Answer 11E.

Consider that the laboratory technician needs a sample of results of chemical reactions, here 8 test tubes with results of chemical reactions is the sample.

The population will be the results of all chemical reactions performed.

As here all the 8 test tubes are likely to be taken into consideration, therefore the sample is biased.

Here the biased sample is convenience, because here the sample is a non-probability sampling technique where subjects are selected because of their convenient accessibility and proximity to the researcher.

11PT.

Here, the matrix is

$$Z = \begin{bmatrix} 3 & 1 & 6 \\ 4 & -1 & -1 \end{bmatrix}$$

Therefore, $-2Z$ is given as below:

$$\begin{aligned} -2Z &= (-2)\begin{bmatrix} 3 & 1 & 6 \\ 4 & -1 & -1 \end{bmatrix} \\ &= \begin{bmatrix} (-2)(3) & (-2)(1) & (-2)(6) \\ (-2)(4) & (-2)(-1) & (-2)(-1) \end{bmatrix} \\ &= \begin{bmatrix} -6 & -2 & -12 \\ -8 & 2 & 2 \end{bmatrix} \end{aligned}$$

Hence, the resultant matrix is $\boxed{\begin{bmatrix} -6 & -2 & -12 \\ -8 & 2 & 2 \end{bmatrix}}$

## Answer 11STP.

Here, the expression is

$$x^3 + 8x^2 + 16x$$

The above expression can be factorized as below:

$$\begin{aligned} & x^3 + 8x^2 + 16x \\ &= x\left(x^2 + 8x + 16\right) && \left[\text{Take } x \text{ common}\right] \\ &= x\left(x^2 + 2 \cdot x \cdot 4 + 4^2\right) \\ &= x(x+4)^2 && \left[\text{Use } (a+b)^2 = a^2 + 2ab + b^2\right] \end{aligned}$$

$$= \boxed{x(x+4)(x+4)}$$

## Answer 12E.

Consider that every 50th bar on the conveyor belt in the candy factory is removed and weighed; the sample is the every 50th bar on the conveyor.

The population will be the results of every 50th bar on the conveyor.

As here only every 50th bars are likely to be taken into consideration, therefore the sample is unbiased.

Here the unbiased sample is voluntary, because here the sample is independent to each other. The individual chocolate bar is tested and the result may vary.

## Answer 12PT.

Here, the matrices are

$$W=\begin{bmatrix}2 & 3 & 1\\ -1 & 0 & -1\\ 2 & -2 & 0\end{bmatrix}\quad Z=\begin{bmatrix}3 & 1 & 6\\ 4 & -1 & -1\end{bmatrix}$$

Since *W* is $3\times3$ matrix and *Z* is a $2\times3$ matrix, the matrices do not have the same dimensions. Therefore, the operation $2W-Z$ is not possible.

## Answer 12STP.

Here, the equation is

$$6x^2+x-2=0$$

The above equation can be solved by factoring as below:

$$\begin{aligned}6x^2+x-2&=0\\ 6x^2+(4-3)x-2&=0 && [\text{Split up the middle term}]\\ 6x^2+4x-3x-2&=0\\ 2x(3x+2)-1(3x+2)&=0 && [\text{Take common}]\\ (2x-1)(3x+2)&=0 && [\text{Factor out}]\end{aligned}$$

$$\begin{aligned}2x-1&=0 &\quad \text{or}\quad 3x+2&=0\\ 2x&=1 &\quad \text{or}\quad 3x&=-2\\ x&=\frac{1}{2} &\quad \text{or}\quad x&=-\frac{2}{3}\end{aligned}$$

Therefore, the solution is $\boxed{x=\frac{1}{2},-\frac{2}{3}}$.

## Answer 13E.

Here, the matrices are

$$A=\begin{bmatrix}1 & 3 & -1\\ 2 & 0 & 4\\ -1 & -1 & 3\end{bmatrix},\quad B=\begin{bmatrix}1 & 1 & -3\\ 2 & 3 & -1\\ -1 & -2 & 0\end{bmatrix}$$

Therefore, $A+B$ is given as below:

$$A+B=\begin{bmatrix}1 & 3 & -1\\ 2 & 0 & 4\\ -1 & -1 & 3\end{bmatrix}+\begin{bmatrix}1 & 1 & -3\\ 2 & 3 & -1\\ -1 & -2 & 0\end{bmatrix}$$

$$=\begin{bmatrix}1+1 & 3+1 & -1+(-3)\\ 2+2 & 0+3 & 4+(-1)\\ -1+(-1) & -1+(-2) & 3+0\end{bmatrix}$$

$$=\begin{bmatrix}2 & 4 & -1-3\\ 4 & 3 & 4-1\\ -1-1 & -1-2 & 3\end{bmatrix}$$

$$=\begin{bmatrix}2 & 4 & -4\\ 4 & 3 & 3\\ -2 & -3 & 3\end{bmatrix}$$

Hence, the resultant matrix is $\boxed{\begin{bmatrix}2 & 4 & -4\\ 4 & 3 & 3\\ -2 & -3 & 3\end{bmatrix}}$

## Answer 13PT.

Here, the matrices are

$$Y=\begin{bmatrix}3 & -2 & 1\\ -1 & -2 & 4\end{bmatrix},\quad Z=\begin{bmatrix}3 & 1 & 6\\ 4 & -1 & -1\end{bmatrix}$$

Therefore, $Y-2Z$ is given as below:

$$Y-2Z=\begin{bmatrix}3 & -2 & 1\\ -1 & -2 & 4\end{bmatrix}-2\begin{bmatrix}3 & 1 & 6\\ 4 & -1 & -1\end{bmatrix}$$

$$=\begin{bmatrix}3 & -2 & 1\\ -1 & -2 & 4\end{bmatrix}-\begin{bmatrix}2(3) & 2(1) & 2(6)\\ 2(4) & 2(-1) & 2(-1)\end{bmatrix}$$

$$=\begin{bmatrix}3 & -2 & 1\\ -1 & -2 & 4\end{bmatrix}-\begin{bmatrix}6 & 2 & 12\\ 8 & -2 & -2\end{bmatrix}$$

$$=\begin{bmatrix}3-6 & -2-2 & 1-12\\ -1-8 & -2-(-2) & 4-(-2)\end{bmatrix}$$

$$=\begin{bmatrix}-3 & -4 & -11\\ -9 & -2+2 & 4+2\end{bmatrix}$$

$$=\begin{bmatrix}-3 & -4 & -11\\ -9 & 0 & 6\end{bmatrix}$$

Hence, the resultant matrix is $\boxed{\begin{bmatrix}-3 & -4 & -11\\ -9 & 0 & 6\end{bmatrix}}$

## Answer 13STP.

Here, the expression is

$$\sqrt{4\sqrt{9}}$$

It can be simplified as below:

$$\begin{aligned}
&\sqrt{4\sqrt{9}}\\
&=\sqrt{4\sqrt{3^2}} && \left[\text{As } 3^2=9\right]\\
&=\sqrt{4\cdot 3} && \left[\text{Use } \sqrt{x^2}=x\right]\\
&=\sqrt{2^2\cdot 3} && \left[\text{As } 2^2=4\right]\\
&=2\sqrt{3} && \left[\text{Use } \sqrt{x^2}=x\right]
\end{aligned}$$

Therefore, the simplified expression is $\boxed{2\sqrt{3}}$.

## Answer 14E.

Here, the matrix is

$$B=\begin{bmatrix}1 & 1 & -3\\ 2 & 3 & -1\\ -1 & -2 & 0\end{bmatrix}$$

Therefore, $3B$ is given as below:

$$\begin{aligned}
3B&=3\begin{bmatrix}1 & 1 & -3\\ 2 & 3 & -1\\ -1 & -2 & 0\end{bmatrix}\\
&=\begin{bmatrix}3(1) & 3(1) & 3(-3)\\ 3(2) & 3(3) & 3(-1)\\ 3(-1) & 3(-2) & 3(0)\end{bmatrix}\\
&=\begin{bmatrix}3 & 3 & -9\\ 6 & 9 & -3\\ -3 & -6 & 0\end{bmatrix}
\end{aligned}$$

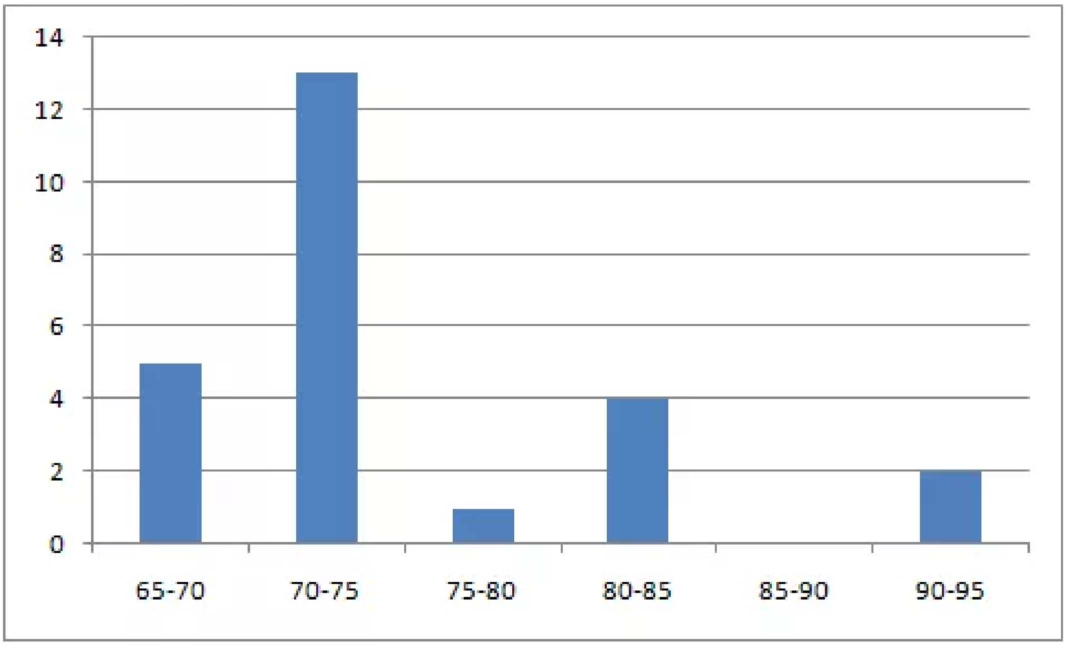
Hence, the resultant matrix is $\boxed{\begin{bmatrix}3 & 3 & -9\\ 6 & 9 & -3\\ -3 & -6 & 0\end{bmatrix}}$

## Answer 14PT.

Consider the given data to make the table below:

| Set of intervals | Tally | Frequency |
|---|---|---|
| $65 \le d < 70$ | 𝍸 | 5 |
| $70 \le d < 75$ | 𝍸 𝍸 \|\|\| | 13 |
| $75 \le d < 80$ | \| | 1 |
| $80 \le d < 85$ | \|\|\|\| | 4 |
| $85 \le d < 90$ |  | 0 |
| $90 \le d < 95$ | \|\| | 2 |

Now the histogram can be drawn as below:



## Answer 15E.

Here, the matrix is

$$D=\begin{bmatrix}2 & 1\\ -2 & 0\end{bmatrix}$$

Therefore, $-2D$ is given as below:

$$\begin{aligned}-2D&=(-2)\begin{bmatrix}2 & 1\\ -2 & 0\end{bmatrix}\\ &=\begin{bmatrix}(-2)2 & (-2)1\\ (-2)(-2) & (-2)0\end{bmatrix}\\ &=\begin{bmatrix}-4 & -2\\ 4 & 0\end{bmatrix}\end{aligned}$$

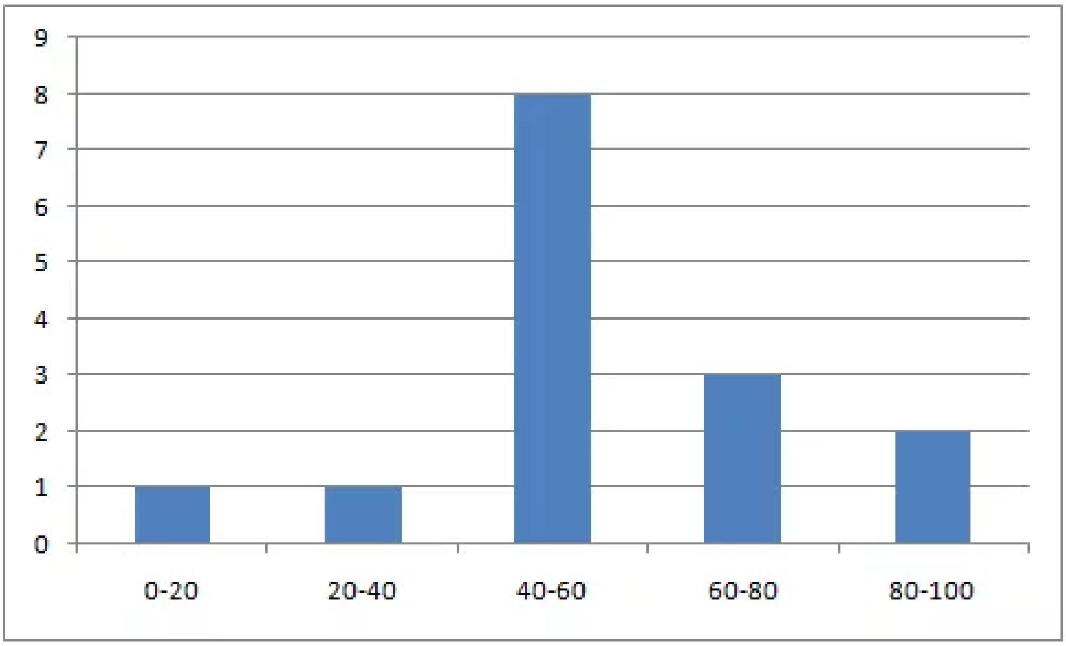
Hence, the resultant matrix is $\boxed{\begin{bmatrix}-4 & -2\\ 4 & 0\end{bmatrix}}$

## Answer 15PT.

Consider the given data to make the table below:

| Set of intervals | Tally | Frequency |
|---|---|---|
| $0 \le d < 20$ | \| | 1 |
| $20 \le d < 40$ | \| | 1 |
| $40 \le d < 60$ | 𝍸 \|\|\| | 8 |
| $60 \le d < 80$ | \|\|\| | 3 |
| $80 \le d < 100$ | \|\| | 2 |

Now the histogram can be drawn as below:



## Answer 15STP.

The difference between $\frac{3a-2}{2a+6}$ and $\frac{1+5a}{-6-2a}$ is calculated as below:

$$
\begin{aligned}
&\frac{3a-2}{2a+6}-\frac{1+5a}{-6-2a} \\
&\quad=\frac{3a-2}{2a+6}-\frac{1+5a}{-(2a+6)} && [\text{Take } -1 \text{ common}] \\
&\quad=\frac{3a-2}{2a+6}+\frac{1+5a}{2a+6} \\
&\quad=\frac{(3a-2)+(1+5a)}{2a+6} && [\text{Take LCM}] \\
&\quad=\frac{3a-2+1+5a}{2a+6} \\
&\quad=\frac{8a-1}{2a+6} && [\text{Simplify}]
\end{aligned}
$$

Therefore, the difference is $\boxed{\frac{8a-1}{2a+6}}$.

## Answer 16E.

Here, the matrices are

$$C = \begin{bmatrix} 3 & -2 \\ 1 & 4 \end{bmatrix},\quad D = \begin{bmatrix} 2 & 1 \\ -2 & 0 \end{bmatrix}$$

Therefore, $C - D$ is given as below:

$$\begin{aligned} C - D &= \begin{bmatrix} 3 & -2 \\ 1 & 4 \end{bmatrix} + \begin{bmatrix} 2 & 1 \\ -2 & 0 \end{bmatrix} \\ &= \begin{bmatrix} 3-2 & -2-1 \\ 1-(-2) & 4-0 \end{bmatrix} \\ &= \begin{bmatrix} 1 & -3 \\ 1+2 & 4 \end{bmatrix} \\ &= \begin{bmatrix} 1 & -3 \\ 3 & 4 \end{bmatrix} \end{aligned}$$

Hence, the resultant matrix is $\boxed{\begin{bmatrix} 1 & -3 \\ 3 & 4 \end{bmatrix}}$

## Answer 16PT.

Consider the set of data

1055, 1075, 1095, 1125, 1005, 975, 1125, 1100, 1145, 1025, 1075

Order the set of data from least to greatest:

$$975 \quad 1005 \quad \underset{\underset{Q_1}{\uparrow}}{1025} \quad 1055 \quad 1075 \quad \underset{\underset{Q_2}{\uparrow}}{1075} \quad 1095 \quad 1100 \quad \underset{\underset{Q_3}{\uparrow}}{1125} \quad 1125 \quad 1145$$

The range is: $1145 - 975$

$= \boxed{170}$

The median is: $\boxed{1075}$

The lower quartile is: $\boxed{1025}$

The upper quartile is: $\boxed{1125}$

The inter quartile range is: $1125 - 1025$

$= \boxed{100}$

The outliers are checked as below:

$$\begin{aligned} 1025 - 1.5(100) &= 1025 - 150 \\ &= 875 \end{aligned}$$

Again,

$$\begin{aligned} 1125 + 1.5(100) &= 1125 + 150 \\ &= 1275 \end{aligned}$$

Therefore, there is no any outlier.

## Answer 16STP.

Here the expression is $\dfrac{x-3}{\dfrac{x^2-7x+12}{x^2-16}}$.

It can be simplified as below:

$$
\begin{aligned}
&\frac{x-3}{\dfrac{x^2-7x+12}{x^2-16}} \\
&=\frac{x-3}{\dfrac{x^2-(4+3)x+12}{x^2-4^2}} && [\text{Split up the middle term}] \\
&=\frac{x-3}{\dfrac{x^2-4x-3x+12}{(x+4)(x-4)}} && [\text{Distributive law}] \\
&=\frac{x-3}{\dfrac{x(x-4)-3(x-4)}{(x+4)(x-4)}} && [\text{Take common}] \\
&=\frac{x-3}{\dfrac{(x-3)(x-4)}{(x+4)(x-4)}} \\
&=\frac{x-3}{\dfrac{x-3}{x+4}} \\
&=(x-3)\cdot\frac{(x+4)}{(x-3)} \\
&=x+4
\end{aligned}
$$

Therefore, the result is $\boxed{x+4}$.

## Answer 17E.

Here, the matrices are

$$C=\begin{bmatrix}3 & -2\\ 1 & 4\end{bmatrix},\quad D=\begin{bmatrix}2 & 1\\ -2 & 0\end{bmatrix}$$

Therefore, $C+D$ is given as below:

$$
\begin{aligned}
C+D&=\begin{bmatrix}3 & -2\\ 1 & 4\end{bmatrix}+\begin{bmatrix}2 & 1\\ -2 & 0\end{bmatrix} \\
&=\begin{bmatrix}3+2 & -2+1\\ 1+(-2) & 4+0\end{bmatrix} \\
&=\begin{bmatrix}5 & -1\\ 1-2 & 4\end{bmatrix} \\
&=\begin{bmatrix}5 & -1\\ -1 & 4\end{bmatrix}
\end{aligned}
$$

Hence, the resultant matrix is $\boxed{\begin{bmatrix}5 & -1\\ -1 & 4\end{bmatrix}}$

## Answer 17PT.

Consider the set of data

0.4, 0.2, 0.5, 0.9, 0.3, 0.4, 0.5, 1.9, 0.5, 0.7, 0.8, 0.6, 0.2, 0.1, 0.4

Order the set of data from least to greatest:

$$\begin{array}{ccccccccccccccccccc} 0.1 & 0.2 & 0.2 & 0.3 & & 0.4 & 0.4 & 0.4 & 0.5 & 0.5 & 0.5 & 0.5 & 0.6 & 0.7 & & 0.8 & 0.9 & 1.9 & 1.9 \\ & & & & \uparrow & & & & & \uparrow & & & & & \uparrow & & & & \\ & & & & Q_1 & & & & & Q_2 & & & & & Q_3 & & & & \end{array}$$

The range is: $1.9-0.1$

$=\boxed{1.8}$

The median is: $\boxed{0.5}$

The lower quartile is: $=\dfrac{0.3+0.4}{2}$

$=\dfrac{0.7}{2}$

$=\boxed{0.35}$

The upper quartile is: $=\dfrac{0.7+0.8}{2}$

$=\dfrac{1.5}{2}$

$=\boxed{0.75}$

The inter quartile range is: $0.75-0.35$

$=\boxed{0.4}$

The outliers are checked as below:

$$\begin{aligned} 0.35-1.5(0.4) &= 0.35-0.6 \\ &= -0.25 \end{aligned}$$

Again,

$$\begin{aligned} 0.75+1.5(0.4) &= 0.75+0.6 \\ &= 1.35 \end{aligned}$$

Therefore, the outlier is $\boxed{1.9}$.

## Answer 18E.

Here, the matrices are

$$B=\begin{bmatrix} 1 & 1 & -3 \\ 2 & 3 & -1 \\ -1 & -2 & 0 \end{bmatrix},\quad C=\begin{bmatrix} 3 & -2 \\ 1 & 4 \end{bmatrix}$$

Since *B* is $3\times 3$ matrix and *C* is a $2\times 2$ matrix, the matrices do not have the same dimensions. Therefore, it is impossible to add these matrices.

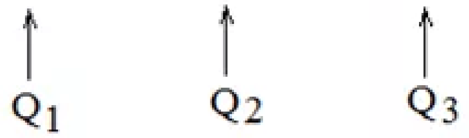
## Answer 18PT.

Here, the data are

1, 3, 2, 2, 1, 9, 4, 6, 1, 10, 1, 4, 5, 10, 1, 3, 6

Order the set of data from least to greatest:

1 1 1 1 1 2 2 3 3 4 4 5 6 6 9 10 10



Now the mean is: $3$

Lower quartile is $Q_1 = \frac{1+1}{2}$

$= \frac{2}{2}$

$= 1$

Again, the upper quartile is $Q_2 = \frac{6+6}{2}$

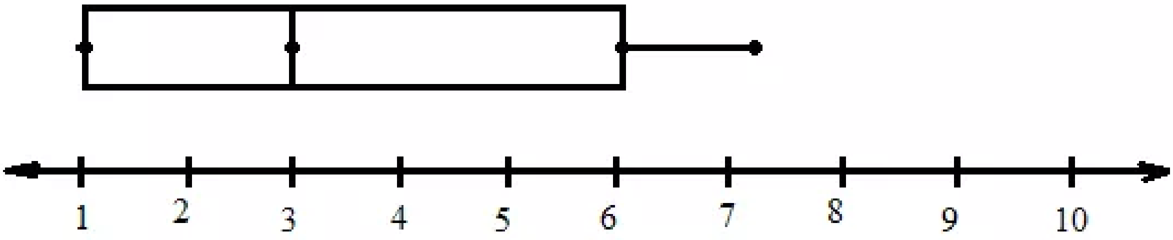
$= \frac{12}{2}$

$= 6$

The inter quartile range is $Q_2 - Q_1 = 6 - 1$

$= 5$

Hence, the box-and-whisker plot can be drawn as below:



## Answer 18STP.

Here the matrices are

$$A=\begin{bmatrix}-1 & 6 & 9\\ 5 & 0 & -3\\ 1 & -8 & -7\end{bmatrix},\quad B=\begin{bmatrix}2 & -4 & 5\\ -1 & 8 & 8\\ -5 & 3 & 1\end{bmatrix}$$

Now $A+B$ is calculated as below:

$$\begin{aligned}A+B&=\begin{bmatrix}-1 & 6 & 9\\ 5 & 0 & -3\\ 1 & -8 & -7\end{bmatrix}+\begin{bmatrix}2 & -4 & 5\\ -1 & 8 & 8\\ -5 & 3 & 1\end{bmatrix}\\
&=\begin{bmatrix}-1+2 & 6+(-4) & 9+5\\ 5+(-1) & 0+8 & -3+8\\ 1+(-5) & -8+3 & -7+1\end{bmatrix}\\
&=\begin{bmatrix}1 & 6-4 & 14\\ 5-1 & 8 & 5\\ 1-5 & -5 & -6\end{bmatrix}\\
&=\boxed{\begin{bmatrix}1 & 2 & 14\\ 4 & 8 & 5\\ -4 & -5 & -6\end{bmatrix}}\end{aligned}$$

Again, $A-B$ is calculated as below:

$$\begin{aligned}A-B&=\begin{bmatrix}-1 & 6 & 9\\ 5 & 0 & -3\\ 1 & -8 & -7\end{bmatrix}-\begin{bmatrix}2 & -4 & 5\\ -1 & 8 & 8\\ -5 & 3 & 1\end{bmatrix}\\
&=\begin{bmatrix}-1-2 & 6-(-4) & 9-5\\ 5-(-1) & 0-8 & -3-8\\ 1-(-5) & -8-3 & -7-1\end{bmatrix}\\
&=\begin{bmatrix}-3 & 6+4 & 4\\ 5+1 & -8 & -11\\ 1+5 & -11 & -8\end{bmatrix}\\
&=\boxed{\begin{bmatrix}-3 & 10 & 4\\ 6 & -8 & -11\\ 6 & -11 & -8\end{bmatrix}}\end{aligned}$$

Finally, $-3B$ is calculated as below:

$$-3B=(-3)\begin{bmatrix}2 & -4 & 5\\ -1 & 8 & 8\\ -5 & 3 & 1\end{bmatrix}$$

$$=\begin{bmatrix}-3(2) & -3(-4) & -3(5)\\ -3(-1) & -3(8) & -3(8)\\ -3(-5) & -3(3) & -3(1)\end{bmatrix}$$

$$=\boxed{\begin{bmatrix}-6 & 12 & -15\\ 3 & -24 & -24\\ 15 & -9 & -3\end{bmatrix}}$$

## Answer 19E.

Here, the matrix is

$$A=\begin{bmatrix}1 & 3 & -1\\ 2 & 0 & 4\\ -1 & -1 & 3\end{bmatrix}$$

Therefore, $5A$ is given as below:

$$5A=5\begin{bmatrix}1 & 3 & -1\\ 2 & 0 & 4\\ -1 & -1 & 3\end{bmatrix}$$

$$=\begin{bmatrix}5(1) & 5(3) & 5(-1)\\ 5(2) & 5(0) & 5(4)\\ 5(-1) & 5(-1) & 5(3)\end{bmatrix}$$

$$=\begin{bmatrix}5 & 15 & -5\\ 10 & 0 & 20\\ -5 & -5 & 15\end{bmatrix}$$

Hence, the resultant matrix is $\boxed{\begin{bmatrix}5 & 15 & -5\\ 10 & 0 & 20\\ -5 & -5 & 15\end{bmatrix}}$

## Answer 19PT.

Here, the data are

14, 18, 9, 9, 12, 22, 16, 12, 14, 16, 15, 13, 9, 10, 11, 12

Order the set of data from least to greatest:

$$9\quad 9\quad 9\quad 10\ \underset{\underset{Q_1}{\uparrow}}{}\ 11\quad 12\quad 12\quad 12\ \underset{\underset{Q_2}{\uparrow}}{}\ 13\quad 14\quad 14\quad 15\ \underset{\underset{Q_3}{\uparrow}}{}\ 16\quad 16\quad 18\quad 22$$

Now the mean is: $\dfrac{12+13}{2}$

$$=\frac{25}{2}$$
$$=12.5$$

Lower quartile is $Q_1=\dfrac{10+11}{2}$

$$=\frac{21}{2}$$
$$=10.5$$

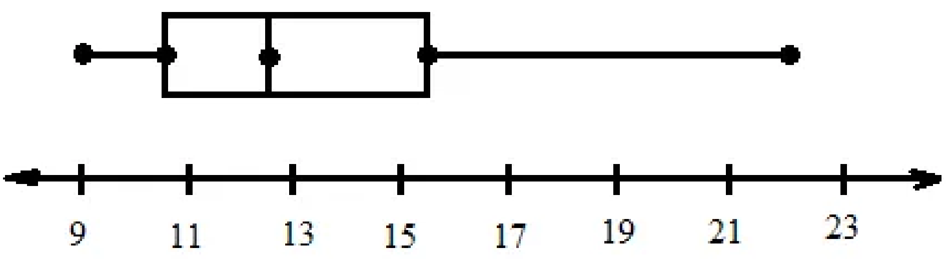
Again, the upper quartile is $Q_2=\dfrac{15+16}{2}$

$$=\frac{31}{2}$$
$$=15.5$$

The inter quartile range is $Q_2-Q_1=15.5-10.5$

$$=5$$

Hence, the box-and-whisker plot can be drawn as below:

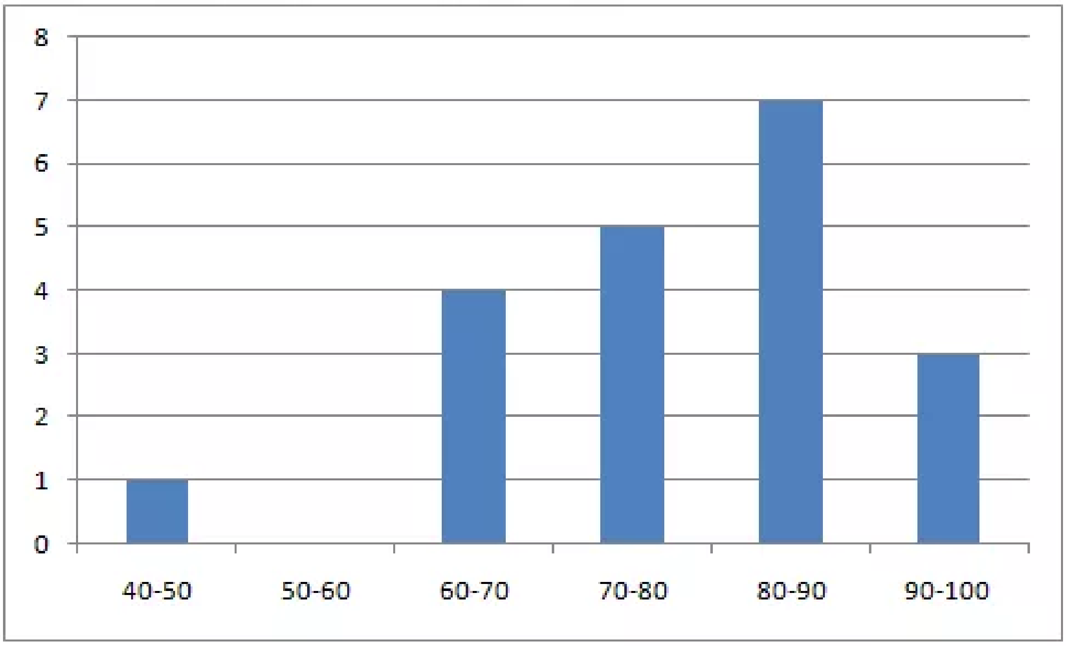


## Answer 19STP.

Consider the given data to make the table below:

| Set of intervals | Tally | Frequency |
|---|---|---|
| $40 \leq d < 50$ | $\vert$ | 1 |
| $50 \leq d < 60$ | | 0 |
| $60 \leq d < 70$ | $\vert\vert\vert\vert$ | 4 |
| $70 \leq d < 80$ | $\cancel{\vert\vert\vert\vert}$ | 5 |
| $80 \leq d < 90$ | $\cancel{\vert\vert\vert\vert}\ \vert\vert$ | 7 |
| $90 \leq d < 100$ | $\vert\vert\vert$ | 3 |

Now the histogram can be drawn as below:



The tallest bar contains 7 numbers of oranges. Therefore, the percentage of data in that bar is:

$$\frac{7}{20}\times 100\% = 7\times 5\%$$

$$= \boxed{35\%}$$

Arranging the data we get

45, 62, 63, 65, 68, 71, 73, 77, 78, 78, 80, 80, 83, 84, 85, 87, 87, 90, 91, 91

The mean is: $\frac{78+80}{2}$

$$=\frac{158}{2}$$
$$=79$$

Now 79 is in the measurement class $\boxed{70-80}$.

## Answer 20E.

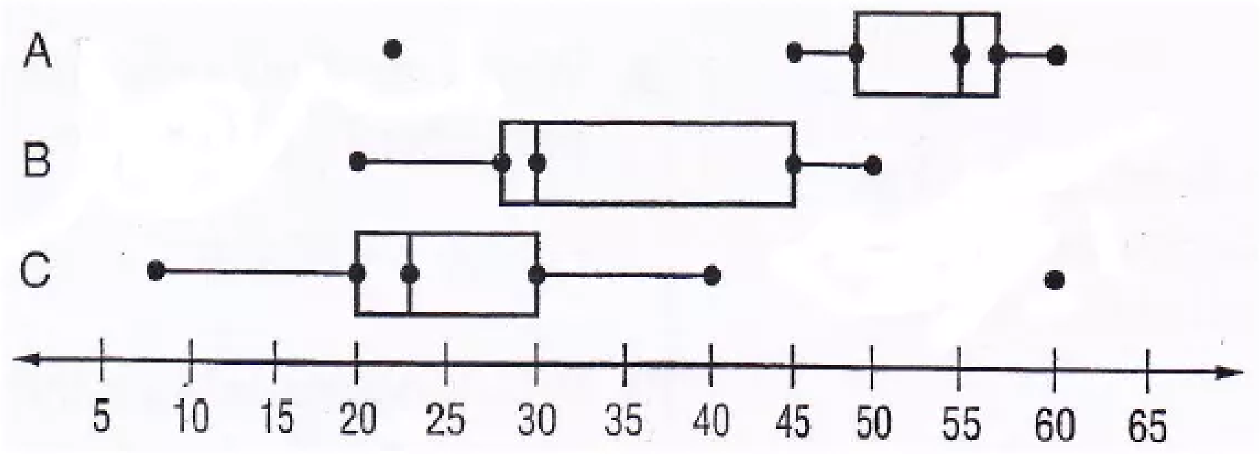
Here, the matrices are

$$A=\begin{bmatrix} 1 & 3 & -1 \\ 2 & 0 & 4 \\ -1 & -1 & 3 \end{bmatrix},\quad D=\begin{bmatrix} 2 & 1 \\ -2 & 0 \end{bmatrix}$$

Since *A* is $3\times 3$ matrix and *D* is a $2\times 2$ matrix, the matrices do not have the same dimensions. Therefore, it is impossible to subtract *D* matrix from the matrix *A*.

## Answer 20PT.

Here, the box-and-whisker plot diagram is given as



For box-and-whisker plot A:

Lower quartile is $Q_1 = 50$

Again, the upper quartile is $Q_2 = 57$

The inter quartile range for A is $Q_2 - Q_1 = 57 - 50$

$= 7$

For box-and-whisker plot B:

Lower quartile is $Q_1 = 28$

Again, the upper quartile is $Q_2 = 45$

The inter quartile range for B is $Q_2 - Q_1 = 45 - 28$

$= 17$

For box-and-whisker plot C:

Lower quartile is $Q_1 = 20$

Again, the upper quartile is $Q_2 = 30$

The inter quartile range for C is $Q_2 - Q_1 = 30 - 20$

$= 10$

Hence, the greatest inter quartile range is for $\boxed{(\mathrm{B})\ \mathrm{B}}$.

## Answer 21E.

Here, the matrices are

$$C = \begin{bmatrix} 3 & -2 \\ 1 & 4 \end{bmatrix},\ D = \begin{bmatrix} 2 & 1 \\ -2 & 0 \end{bmatrix}$$

Therefore, $C + 3D$ is given as below:

$$\begin{aligned} C + 3D &= \begin{bmatrix} 3 & -2 \\ 1 & 4 \end{bmatrix} + 3\begin{bmatrix} 2 & 1 \\ -2 & 0 \end{bmatrix} \\ &= \begin{bmatrix} 3 & -2 \\ 1 & 4 \end{bmatrix} + \begin{bmatrix} 3(2) & 3(1) \\ 3(-2) & 3(0) \end{bmatrix} \\ &= \begin{bmatrix} 3 & -2 \\ 1 & 4 \end{bmatrix} + \begin{bmatrix} 6 & 3 \\ -6 & 0 \end{bmatrix} \\ &= \begin{bmatrix} 3+6 & -2+3 \\ 1+(-6) & 4+0 \end{bmatrix} \end{aligned}$$

$$= \begin{bmatrix} 9 & 1 \\ 1-6 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 9 & 1 \\ -5 & 4 \end{bmatrix}$$

Hence, the resultant matrix is $\boxed{\begin{bmatrix} 9 & 1 \\ -5 & 4 \end{bmatrix}}$

## Answer 22E.

Here, the matrices are

$$A=\begin{bmatrix}1&3&-1\\2&0&4\\-1&-1&3\end{bmatrix},\quad B=\begin{bmatrix}1&1&-3\\2&3&-1\\-1&-2&0\end{bmatrix}$$

Therefore, $2A-B$ is given as below:

$$2A-B=2\begin{bmatrix}1&3&-1\\2&0&4\\-1&-1&3\end{bmatrix}-\begin{bmatrix}1&1&-3\\2&3&-1\\-1&-2&0\end{bmatrix}$$

$$=\begin{bmatrix}2(1)&2(3)&2(-1)\\2(2)&2(0)&2(4)\\2(-1)&2(-1)&2(3)\end{bmatrix}-\begin{bmatrix}1&1&-3\\2&3&-1\\-1&-2&0\end{bmatrix}$$

$$=\begin{bmatrix}2&6&-2\\4&0&8\\-2&-2&6\end{bmatrix}-\begin{bmatrix}1&1&-3\\2&3&-1\\-1&-2&0\end{bmatrix}$$

$$=\begin{bmatrix}2-1&6-1&-2-(-3)\\4-2&0-3&8-(-1)\\-2-(-1)&-2-(-2)&6-0\end{bmatrix}$$

$$=\begin{bmatrix}1&5&-2+3\\2&-3&8+1\\-2+1&-2+2&6\end{bmatrix}$$

$$=\begin{bmatrix}1&5&1\\2&-3&9\\-1&0&6\end{bmatrix}$$

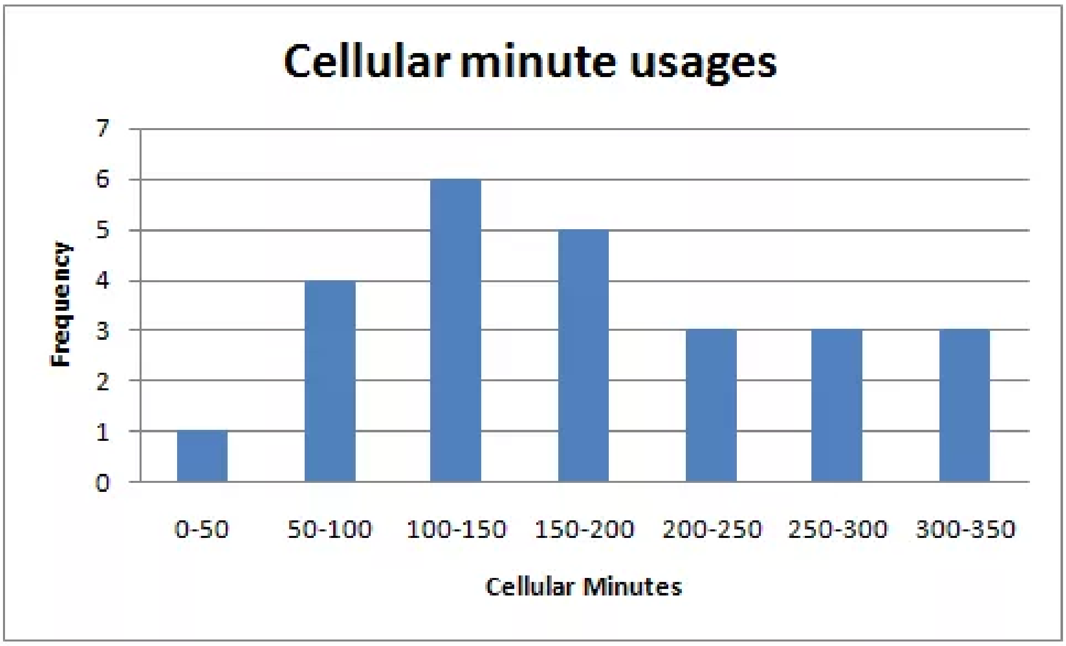
Hence, the resultant matrix is $\boxed{\begin{bmatrix}1&5&1\\2&-3&9\\-1&0&6\end{bmatrix}}$

**Answer 23E.**

Consider the given data to make the table below:

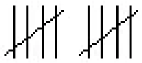
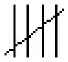
| Number of Cellular minutes intervals | Tally | Frequency |
|---|---|---|
| $0 \le d < 50$ | 𝍷 | 1 |
| $50 \le d < 100$ | 𝍷𝍷𝍷𝍷 | 4 |
| $100 \le d < 150$ | 𝍸 𝍷 | 6 |
| $150 \le d < 200$ | 𝍸 | 5 |
| $200 \le d < 250$ | 𝍷𝍷𝍷 | 3 |
| $250 \le d < 300$ | 𝍷𝍷𝍷 | 3 |
| $300 \le d < 350$ | 𝍷𝍷𝍷 | 3 |

Now the histogram can be drawn as below:

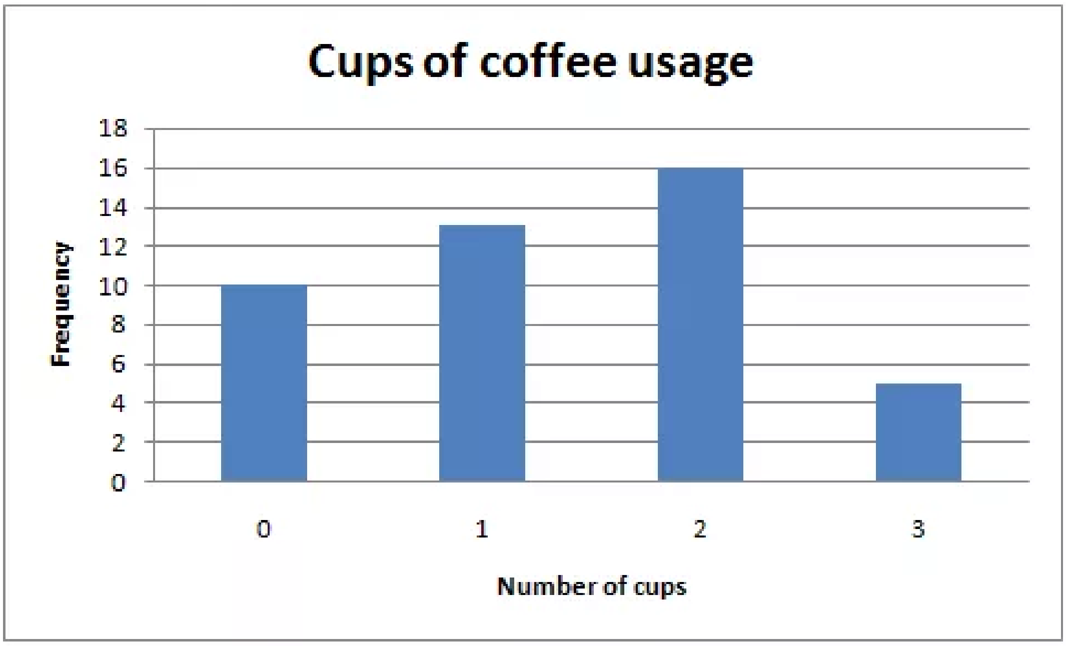


## Answer 24E.

Consider the given data to make the table below:

| Number of cups of coffee intervals | Tally | Frequency |
|---|---|---|
| 0 |  | 10 |
| 1 |  | 13 |
| 2 |  | 16 |
| 3 |  | 5 |

Now the histogram can be drawn as below:

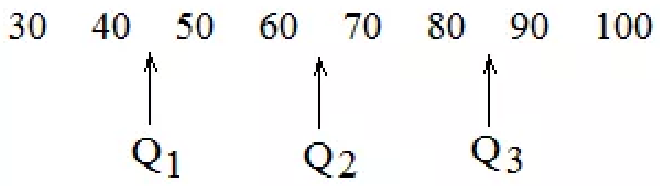


## Answer 25E.

Consider the set of data

30, 90, 40, 70, 50, 100, 80, 60

Order the set of data from least to greatest:



The range is: $100-30$

$=\boxed{70}$

The median is: $\frac{60+70}{2}$

$=\frac{130}{2}$

$=\boxed{65}$

The lower quartile is: $\frac{40+50}{2}$

$=\frac{90}{2}$

$=\boxed{45}$

The upper quartile is: $\frac{80+90}{2}$

$=\frac{170}{2}$

$=\boxed{85}$

The inter quartile range is: $85-45$

$=\boxed{40}$

The outliers are checked as below:

$$\begin{aligned}45-1.5(40)&=45-60\\&=15\end{aligned}$$

Again,

$$\begin{aligned}85+1.5(40)&=85+60\\&=145\end{aligned}$$

Therefore, there is $\boxed{\text{no}}$ any outlier.

## Answer 26E.

Consider the set of data

3, 3.2, 45, 7, 2, 1, 3.4, 4, 5.3, 5, 78, 8, 21, 5

Order the set of data from least to greatest:

$$\begin{array}{ccccccccccccccc}
1 & 2 & 3 & 3.2 & 3.4 & 4 & 5 & & 5 & 5.3 & 7 & 8 & 21 & 45 & 78 \\
 & & & \uparrow & & & & \uparrow & & & & \uparrow & & & \\
 & & & Q_1 & & & & Q_2 & & & & Q_3 & & &
\end{array}$$

The range is: $78-1$

$=\boxed{77}$

The median is: $\frac{5+5}{2}$

$=\frac{10}{2}$

$=\boxed{5}$

The lower quartile is: $\boxed{3.2}$

The upper quartile is: $\boxed{8}$

The inter quartile range is: $8-3.2$

$=\boxed{4.8}$

The outliers are checked as below:

$$\begin{aligned} 3.2-1.5(4.8) &= 3.2-7.2 \\ &= -4 \end{aligned}$$

Again,

$$\begin{aligned} 8+1.5(4.8) &= 38+7.2 \\ &= 45.2 \end{aligned}$$

Therefore, the outlier is $\boxed{78}$.

## Answer 27E.

Consider the set of data

85, 77, 58, 69, 62, 73, 55, 82, 67, 77, 59, 92, 75, 69, 76

Order the set of data from least to greatest:

$$55\quad 58\quad 59\quad \underset{\underset{Q_1}{\uparrow}}{62}\quad 67\quad 69\quad 69\quad \underset{\underset{Q_2}{\uparrow}}{73}\quad 75\quad 76\quad 77\quad \underset{\underset{Q_3}{\uparrow}}{77}\quad 82\quad 85\quad 92$$

The range is: $92-55$

$=\boxed{37}$

The median is: $\boxed{73}$

The lower quartile is: $\boxed{62}$

The upper quartile is: $\boxed{77}$

The inter quartile range is: $77-62$

$=\boxed{15}$

The outliers are checked as below:

$$\begin{aligned}62-1.5(15)&=62-22.5\\&=39.5\end{aligned}$$

Again,

$$\begin{aligned}77+1.5(15)&=77+22.5\\&=99.5\end{aligned}$$

Therefore, there is $\boxed{\text{no}}$ any outlier.

## Answer 28E.

Consider the set of data

111.5 70.7, 59.8, 68.6, 63.8, 254.8, 64.3, 82.3, 91.7, 88.9, 110.5, 77.1

Order the set of data from least to greatest:

$$\begin{array}{ccccccccccccccccccccccc} 59.8 & & 63.8 & & 64.3 & & 68.6 & & 70.7 & & 77.1 & & 82.3 & & 88.9 & & 91.7 & & 110.5 & & 111.5 & & 254.8 \\ & & & & & \uparrow & & & & & & \uparrow & & & & & & \uparrow & & & & & \\ & & & & & Q_1 & & & & & & Q_2 & & & & & & Q_3 & & & & & \end{array}$$

The range is: $254.8 - 59.8$

$= \boxed{195}$

The median is: $= \dfrac{77.1 + 82.3}{2}$

$= \dfrac{159.4}{2}$

$= \boxed{79.7}$

The lower quartile is: $= \dfrac{64.3 + 68.6}{2}$

$= \dfrac{132.9}{2}$

$= \boxed{66.45}$

The upper quartile is: $= \dfrac{91.7 + 110.5}{2}$

$= \dfrac{202.2}{2}$

$= \boxed{101.1}$

The inter quartile range is: $101.1 - 66.45$

$= \boxed{44.55}$

The outliers are checked as below:

$$\begin{aligned} 66.45 - 1.5(44.55) &= 66.45 - 66.825 \\ &= -0.375 \end{aligned}$$

Again,

$$\begin{aligned} 101.1 + 1.5(44.55) &= 101.1 + 66.825 \\ &= 167.925 \end{aligned}$$

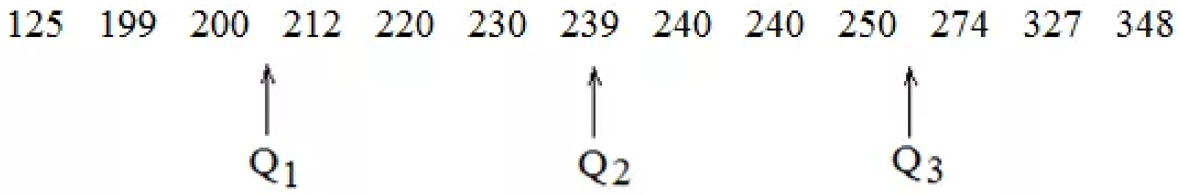
Therefore, the outlier is $\boxed{254.8}$.

## Answer 29E.

Here, the numbers of Calories are

250, 240, 220, 348, 199, 200, 125, 230, 274, 239, 212, 240, 327

Order the set of data from least to greatest:


125 199 200 212 220 230 239 240 240 250 274 327 348

Now Mean is: 239

Lower quartile is $Q_1 = \frac{200+212}{2}$

$= \frac{412}{2}$

$= 206$

Again, the upper quartile is $Q_2 = \frac{250+274}{2}$

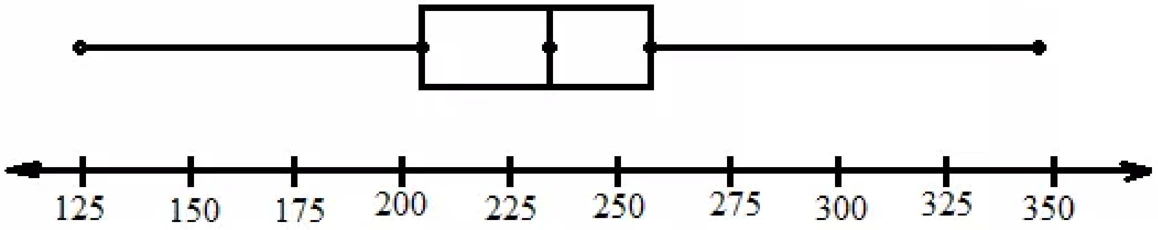
$= \frac{524}{2}$

$= 262$

The inter quartile range is $Q_2 - Q_1 = 262 - 206$

$= 56$

Hence, the box-and-whisker plot can be drawn as below:

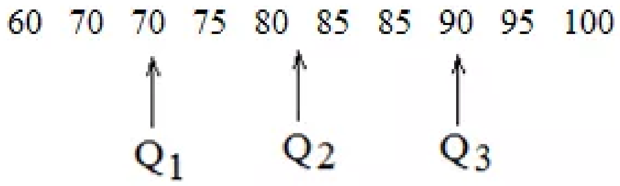


## Answer 30E.

Here, the scores are

60, 70, 70, 75, 80, 85, 85, 90, 95, 100

Order the set of data from least to greatest:



Now the mean is: $\frac{80+85}{2}$

$=\frac{165}{2}$

$=82.5$

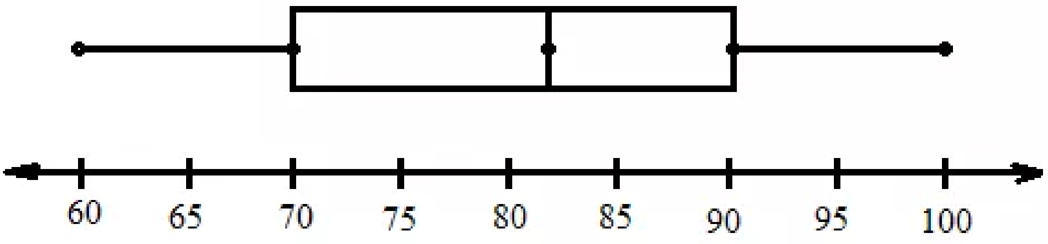
Lower quartile is $Q_1 = 70$

Again, the upper quartile is $Q_2 = 90$

The inter quartile range is $Q_2 - Q_1 = 90 - 70$

$= 20$

Hence, the box-and-whisker plot can be drawn as below:



## Answer 31E.

Here, the average temperatures are

52.4, 55.2, 61.1, 67.0, 73.4, 79.1, 81.6, 81.2, 78.1, 69.8, 61.9, 55.1

Order the set of data from least to greatest:

52.4 55.1 55.2 61.1 61.9 67.0 69.8 73.4 78.1 79.1 81.2 81.6

↑ ↑ ↑

$Q_1$ $Q_2$ $Q_3$

Now the mean is: $\dfrac{67.0+69.8}{2}$

$$=\frac{136.8}{2}$$
$$=68.4$$

Lower quartile is $Q_1=\dfrac{55.2+61.1}{2}$

$$=\frac{116.3}{2}$$
$$=58.25$$

Again, the upper quartile is $Q_2=\dfrac{78.1+79.1}{2}$

$$=\frac{157.2}{2}$$
$$=78.6$$

The inter quartile range is $Q_2-Q_1=78.6-58.25$

$$=20.35$$

Hence, the box-and-whisker plot can be drawn as below: