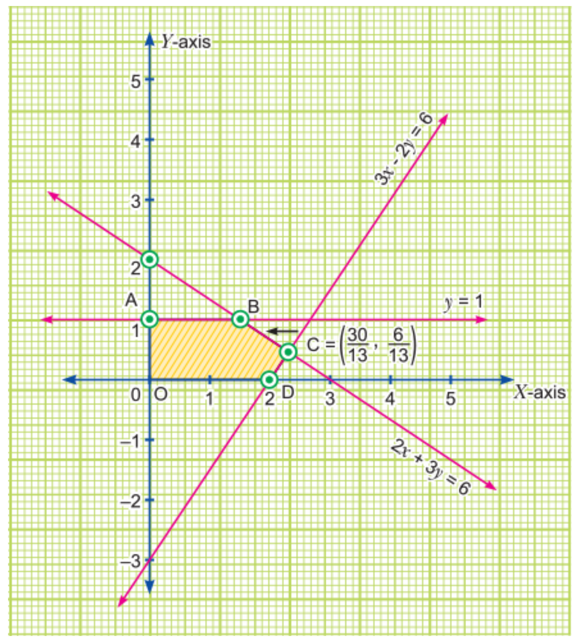
Long Answer Questions-I

[4 marks]

Q.1. Maximise Z = 8*x* + 9*y* subject to the constraints given below :

$2x + 3y \le 6$; $3x - 2y \le 6$; $y \le 1$; $x, y \ge 0$

Ans.



Given constraints are

$$2x + 3y \le 6$$

$$3x - 2y \le 6$$

$$y \le 1$$

$$x, y \ge 0$$

For graph of $2x + 3y \le 6$

We draw the graph of $2x + 3y = 6$

x	0	3
y	2	0

$2 \times 0 + 3 \times 0 \le 6 \Rightarrow (0,0)$ satisfy the constraints.

Hence, feasible region lie towards origin side of line.

For graph of $3x - 2y \le 6$

We draw the graph of line $3x - 2y = 6$.

x	0	2
y	–3	0

$3 \times 0 - 2 \times 0 \leq 6 \quad \Rightarrow \quad$ Origin (0, 0) satisfy $3x - 2y = 6$.

Hence, feasible region lie towards origin side of line.

For graph of $y \leq 1$

We draw the graph of line $y = 1$, which is parallel to x-axis and meet y-axis at 1.

$0 \leq 1 \Rightarrow$ feasible region lie towards origin side of $y = 1$.

Also, $x \geq 0$, $y \geq 0$ says feasible region is in Ist quadrant

Therefore, *OABCDO* is the required feasible region, having corner point $O(0, 0)$, $A(0, 1)$,

$B\left(\frac{3}{2}, 1\right)$, $C\left(\frac{30}{13}, \frac{6}{13}\right)$ $D(2, 0)$.

Here, feasible region is bounded. Now the value of objective function Z = $8x + 9y$ is obtained as.

Corner Point	Z = $8x + 9y$
O (0, 0)	0
A (0, 1)	9
$B(\frac{3}{2}, 1)$	21
$C(\frac{30}{13}, \frac{6}{13})$	22.6
$D(2, 0)$	16

Z is maximum when $x = \frac{30}{13}$ and $y = \frac{6}{13}$.

Q.2. Minimize and maximize $Z = 5x + 2y$ subject to the following constraints:

$x - 2y \leq 2$, $3x + 2y \leq 12$, $-3x + 2y \leq 3$, $x \geq 0, y \geq 0$

Ans.

Here, objective function is

$$Z = 5x + 2y \qquad ...(i)$$

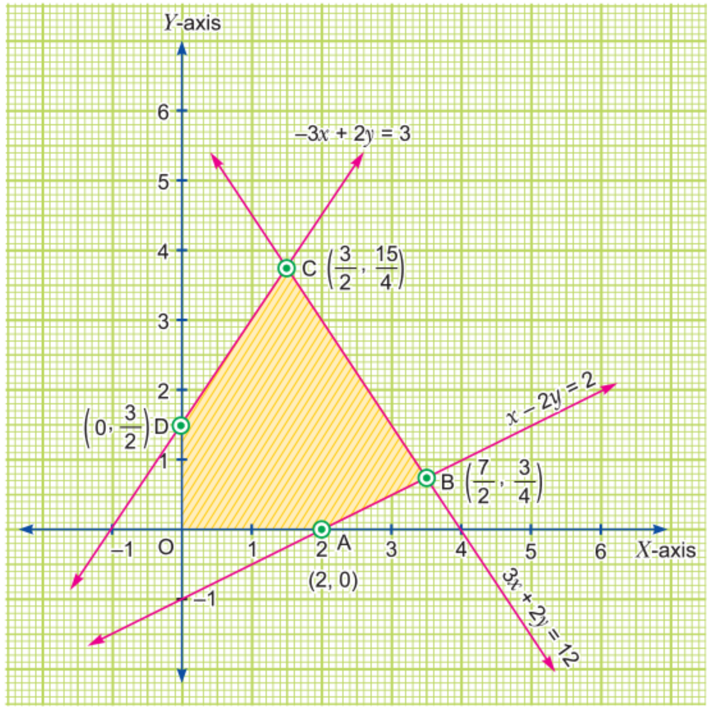
Subject to the constraints :

$$x - 2y \leq 2 \qquad ...(ii)$$

$$3x + 2y \leq 12 \qquad ...(iii)$$

$$-3x + 2y \leq 3 \qquad ...(iv)$$

$$x \geq 0,\ y \geq 0 \qquad ...(v)$$



Graph for $x - 2y \leq 2$

We draw graph of $x - 2y = 2$ as

x	0	2
y	–1	0

$0 - 2 \times 0 \leq 2$

[By putting $x = y = 0$ in the equation]

i.e., (0, 0) satisfy (*ii*) ⇒ feasible region lie origin side of line $x - 2y = 2$.

Graph for $3x + 2y \leq 12$

We draw the graph of $3x + 2y = 12$.

x	0	4
y	6	0

$3 \times 0 + 2 \times 0 \leq 12$ [By putting $x = y = 0$ in the given equation]

i.e., (0, 0) satisfy (*iii*) ⇒ feasible region lie origin side of line $3x + 2y = 12$.

Graph for $-3x + 2y \leq 3$

We draw the graph of $-3x + 2y \leq 3$

x	–1	0
y	0	1.5

$-3 \times 0 + 2 \times 0 \leq 3$[By putting $x = y = 0$]

i.e., (0, 0) satisfy (*iv*) ⇒ feasible region lies origin side of line $-3x + 2y = 3$.

$x \geq 0, y \geq 0$ ⇒ feasible region is in Its quadrant.

Now, we get shaded region having corner points *O, A, B, C* and *D* as feasible region. The co-ordinates of *O,*

A, B, C and *D* are $O(0, 0)$, $A(2, 0)$ $B\left(\frac{7}{2}, \frac{3}{4}\right)$, $C\left(\frac{3}{2}, \frac{15}{4}\right)$ and $D\left(0, \frac{3}{2}\right)$, respectively.

Now, we evaluate Z at the corner point as.

Corner Point	$Z = 5x + 2y$	
$O(0, 0)$	0	→ Minimum
$A(2, 0)$	10	
$B\left(\frac{7}{2}, \frac{3}{4}\right)$	19	→ Maximum
$C\left(\frac{3}{2}, \frac{15}{4}\right)$	15	
$D\left(0, \frac{3}{2}\right)$	3	

Hence, Z is minimum at $x = 0$, $y = 0$ and minimum value = 0

Z is maximum at $x = \frac{7}{2}$, $y = \frac{3}{4}$ and maximum value = 19

Q.3. Determine graphically the minimum value of the objective function

$Z = -50x + 20y$ ...(*i*)

Subject to the constraints

$2x - y \geq -5$ ...(*ii*)

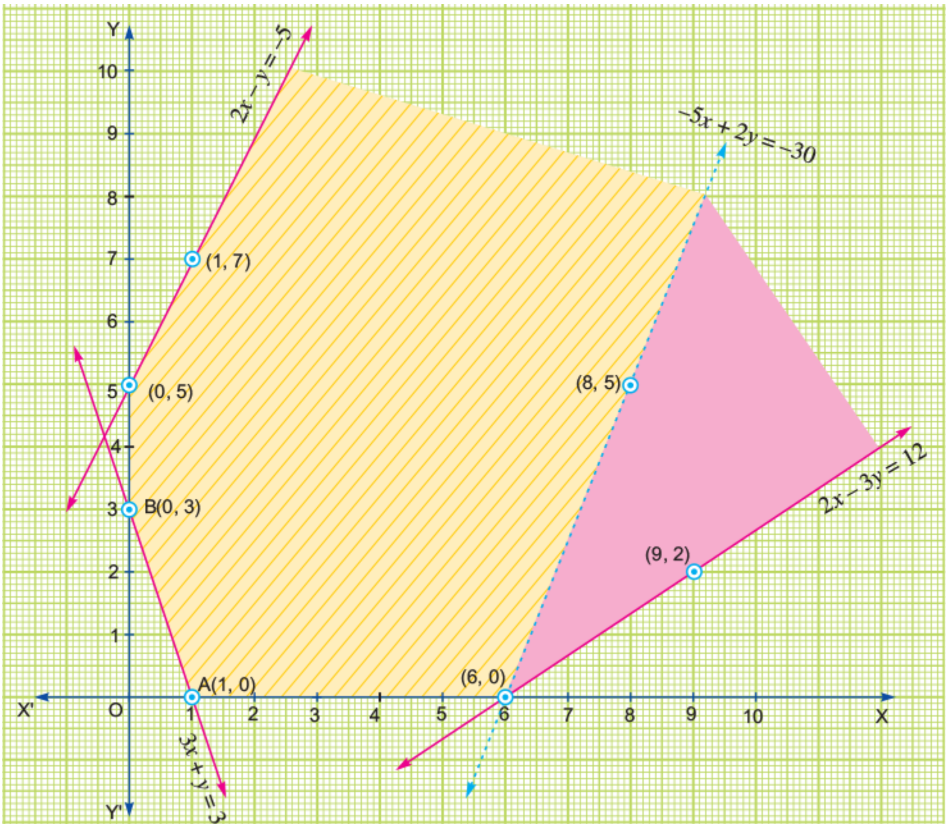
$3x + y \geq 3$ ...(*iii*)

$2x - 3y \le 12$ ...(*iv*)

$x \ge 0, y \ge 0$ ...(*v*)

Ans.

First of all, let us graph the feasible region of the system of inequalities (*ii*) to (*v*). The feasible region (Shaded) is shown in the figure. Observe that the feasible region is unbounded.



We now evaluate Z at the corner points.

Corner Point	$Z = -50x + 20y$
(0, 5)	100
(0, 3)	60
(1, 0)	–50
(6, 0)	–300 ← Smallest

From this table, we find that –300 is the smallest value of *Z* at the corner point (6, 0). Can we say that minimum value of *Z* is (–)300? Note that if the region would have been bounded, this smallest value of *Z*is the minimum value of *Z*. But here we see that the feasible region is unbounded. Therefore, –300 may or may not be the minimum value of *Z*. To decide this issue, we graph the inequality.

$$-50x + 20y < -300$$

i.e., $-5x + 2y < -30$

and check whether the resulting open half plane has points in common with feasible region or not. If it has common points, then –300 will not be the minimum value of Z otherwise, –300 will be the minimum value of Z.

As shown in the figure, it has common points. Therefore, $Z = -50x + 20y$ has no minimum value subject to the given constraints.